To appear in Proceedings *"Mechatronics 2000", 6-8 September 2000, Atlanta, USA, Elsevier*

MODELING OF CONTROLLED MOTION OF SEMI-PASSIVELY ACTUATED SCARA-LIKE ROBOT

Mathias R. Lidberg * , **Viktor E. Berbyuk** **

* *Department of Mechanics, Chalmers University of Technology*
SE-412 96, Göteborg, Sweden
E-mail: mlidb@mec.chalmers.se
Home page: www.mec.chalmers.se/~lidberg

** *Institute for Applied Problems of Mechanics and Mathematics*
3-B Naukova Street, 79601, Lviv, Ukraine
E-mail: berbyuk@iapmm.lviv.ua
Home page: www.mec.chalmers.se/~berbyuk

Abstract: The controlled motion of a new structure of manipulator robot is under study. In comparison with the well-known SCARA robot the proposed robotic system has the following new features: in addition to powered drives it comprises several unpowered (passive) spring-damper-like drives. An additional link has also been incorporated into the structure that gives the possibility to obtain a semi-passively actuated closed-loop chain robot. Special emphasis is put on a study of the interaction between the controlling stimuli of the powered drives and the torques exerted by the unpowered drives needed to provide the energy-optimal motion of the robot. Computer simulations have demonstrated the numerical efficiency of the developed algorithms and have proved several advantages of the considered semi-passively actuated closed-loop robot.

1. INTRODUCTION

The development of new robotic systems with closed-loop chains and different types of actuation is currently one of the main activities of many research institutions and industrial manufacturers (Tran and Kehl, 1998). Potential applications of closed-loop chain manipulators arise whenever there is a need for large structural stiffness or high performance dynamics and when it is desirable to bring the actuators as close as possible to the base (Nakamura and Ghodoussi, 1989); (Gosselin, 1996). If a closed-loop robot has more actuated joints than its degree-of-freedom the joint torques are no longer uniquely determined. Such kind of robot is called an overactuated robot in the sense that there are more actuators than necessary. To design an effective control law for overactuated closed-loop chain manipulators it seems reasonable to explore the inherent dynamics of the mechanical structure of the system and the optimal interaction between different kind of actuators (Zhang *et al.*, 1999). Previously (Berbyuk *et al.*, 1998) have proposed an optimization approach for the design of rotational spring-damper passive drives providing the programmed motion of a bipedal walking robot. The problem was formulated as an approximation procedure for the controlling torque acting at the joints of the robot during its optimal motion. The motion and the respective torques were determined by the solution of the optimal control problem for the dynamical system model of the robot (Berbyuk *et al.*, 1999).

In this paper the dynamics and control problems are studied for a new structure of manipulator robot. In comparison with the well-known SCARA robot the proposed structure is characterized by incorporation of an additional link that gives a closed-loop chain robot. Special emphasis is put on a study of optimal load distribution and interaction between controlling stimuli of powered drives and torques exerted by the unpowered actuators.

2. STATEMENT OF THE PROBLEM

Consider the manipulator robot depicted in Fig. 1. The robot comprises four links that are modeled by the rigid bodies OA, AB, OD and EC. There are one degree-of-freedom rotational joints at the points O and A, and translational joints at the point B.

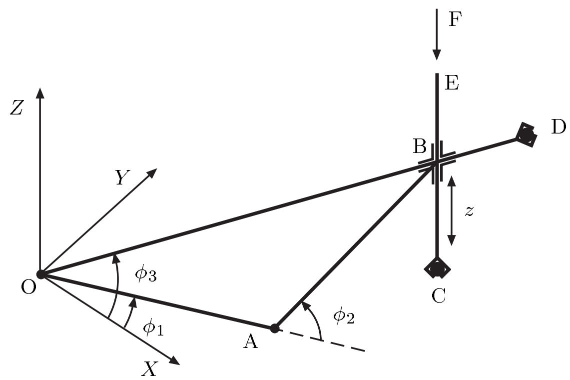

Fig. 1. The Sketch of the SCARA-Like Robot

Let OXYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the robot's links OA, AB and OD moves in the horizontal plane OXY under the action of the torques $u_1(t), u_2(t)$ and $u_3(t)$ applied to the links OA, AB, and OD, respectively. The link EC moves along the direction of the axes OZ under the action of the force $F(t)$. The controlling stimuli $u_i(t), i = 1, 2, 3$ and $F(t)$ are exerted by the powered drives of the robot. The robotic system also comprises spring-damper actuators at joints O and A. The torques exerted by these actuators p_1 , p_2 and p_3 act on the links OA, AB and OD, respectively. They will be treated as the controlling stimuli of unpowered (passive) drives of the robot.

The following notations are employed: ϕ_1, ϕ_2 ,ϕ_3 and z are the angles and linear displacement that determine the position of the links OA, AB, OD and EC respectively (Fig. 1); $l_i, m_i, J_i, i = 1, 2, 3$, denote the length, the mass and the moment of inertia of the links OA, AB and OD relative to the vertical axis passing through their mass center, respectively; r_1, r_2, r_3 are the distances from the points O, A and O to the center of mass of the links OA, AB and OD, respectively; m_4, m_C, m_D denote the mass of the link EC and the mass of point-loads located at the end-effectors C and D of the robot, respectively.

Let ϕ_1, ϕ_2 and z be the Lagrangian generalized coordinates of the considered system. The equations of motion for the robot are given by the following expressions:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_{q_i}, \; i = 1, 2, 3 \qquad (1)$$

Here the functions Q_{q_i} constitute the generalized forces, L is refered to system Lagrangian $L = K - P$, where K and P are kinetic and potential energies of the robot, respectively; and $q_1 = \phi_1$, $q_2 = \phi_2$, and $q_3 = z$.

The kinetic and potential energies of the system, and the generalized forces Q_{q_i} can be written as follows:

$$K = K_{OA} + K_{AB} + K_{OD} + K_{EC} \qquad (2)$$
$$P = m_4 g z \qquad (3)$$
$$Q_{\phi_1} = u_1 + p_1 + u_3 + p_3 \qquad (4)$$
$$Q_{\phi_2} = u_2 + p_2 + b\left(\phi_1, \phi_2\right)\left(u_3 + p_3\right) \qquad (5)$$
$$Q_z = F \qquad (6)$$

In formula (2) we denote:

$$K_{OA} = \frac{1}{2}\left(m_1 r_1^2 + J_1\right)\dot{\phi}_1^2$$
$$K_{AB} = \frac{1}{2}\left(m_2 l_1^2 \dot{\phi}_1^2 + \left(J_2 + m_2 r_2^2\right)\left(\dot{\phi}_1 + \dot{\phi}_2\right)^2\right)$$
$$+ m_2 l_1 r_2 \dot{\phi}_1 \left(\dot{\phi}_1 + \dot{\phi}_2\right)\cos\phi_2$$
$$K_{OD} = \frac{1}{2}\frac{J_0}{l^2}\left(l_1^2\dot{\phi}_1^2 + l_2^2\left(\dot{\phi}_1 + \dot{\phi}_2\right)^2\right.$$
$$\left. + 2 l_1 l_2 \dot{\phi}_1\left(\dot{\phi}_1 + \dot{\phi}_2\right)\cos\phi_2 - \left(l_1 l_2 \dot{\phi}_2 \sin\phi_2\right)^2 / l^2\right)$$
$$K_{EC} = \frac{1}{2}\left(m_4 + m_C\right)\left(l_1^2\dot{\phi}_1^2 + l_2^2\left(\dot{\phi}_1 + \dot{\phi}_2\right)^2\right.$$
$$\left. + 2 l_1 l_2 \dot{\phi}_1\left(\dot{\phi}_1 + \dot{\phi}_2\right)\cos\phi_2 + \dot{z}^2\right)$$
$$l = \left(l_1^2 + l_2^2 + 2 l_1 l_2 \cos\phi_2\right)^{\frac{1}{2}}$$
$$b = \left(l^2 l_2 \cos(\phi_1 + \phi_2) + l_1 l_2 y_B \sin\phi_2\right) / \left(l^2 x_B\right)$$
$$x_B = l_1 \cos\phi_2 + l_2 \cos(\phi_1 + \phi_2)$$
$$y_B = l_1 \sin\phi_1 + l_2 \sin(\phi_1 + \phi_2)$$
$$J_0 = J_3 + m_3 r_3^2 + m_D l_3^2$$

The inherent dynamics of the passive drives of the robot can be modeled in different ways, e.g. by the differential constraints:

$$p_i + k_i \phi_i(t) + c_i \dot{\phi}_i(t) = 0 \qquad (7)$$

where k_i and c_i are the spring and the damper coefficients of the i-th passive drive, $i = 1, 2, 3$.

The differential equations that are determined by the formulas (1)-(7) describe the controlled motion of the semi-passively actuated SCARA-like robot.

An analysis of the robot (Fig. 1) shows that the position, the velocity and the acceleration of the robot's end-effectors (points C and D) can be uniquely determined by specification of the functions $l(t), \phi_3(t)$ and $z(t)$. The set of cyclic pick-and-place operations of the robot can be given by the following conditions:

$$f(0) = f(T) = f_0,\ \dot{f}(0) = \dot{f}(T) = \dot{f}_0 \quad (8)$$

$$f(\tau) = f_\tau,\ \dot{f}(\tau) = \dot{f}_\tau \quad (9)$$

$$m_C = \begin{cases} m_C & ,0 \le t \le \tau \\ 0 & ,\tau < t \le T \end{cases} \quad (10)$$

$$m_D = \begin{cases} m_D & ,0 \le t \le \tau \\ 0 & ,\tau < t \le T \end{cases} \quad (11)$$

In formulas (8)-(11) f is a vector-function having as its components $l(t), \phi_3(t)$ and $z(t)$; $t = 0$ and $t = T$ are the times of the beginning and ending of the pick-and-place operation, respectively; τ is the duration of transferring of the loads; $f_0, \dot{f}_0, f_\tau, \dot{f}_\tau$ are given parameters that determine the initial (final) and intermediate phase states of the end-effectors.

As follows from formulas (8)-(11) the proposed set of pick-and-place operations of the robot is specified by the following parameters:

$$f_0, \dot{f}_0, f_\tau, \dot{f}_\tau, \tau, T, m_C, m_D \quad (12)$$

which are at disposal as input data.

The modeling task for the considered SCARA-like robot can be formulated as follows.

Problem 1. The values of all structural parameters $(l_i, r_i, m_i, J_i, k_i, c_i)$ of the robot and the input data (12) of the pick-and-place operation are given. Determine the motion:

$$\phi_1(t), \phi_2(t), z(t),\ t \in [0, T] \quad (13)$$

and the controlling stimuli of the powered drives of the robot

$$u_i(t),\ t \in [0, T], i = 1, 2, 3 \quad (14)$$

which provide the execution of the given pick-and-place operation subject to the differential constraints (1) and (7).

Based on the solution of Problem 1 different kind of cost functions can be evaluated. This will make it possible to study the sensitivity of dynamic and energetic characteristics of the robot with respect to the input data.

In the present paper the following functionals are utilized:

$$E_1 = \int_0^T \sum_{i=1}^{3} |u_i(t)|\,\mathrm{d}t \quad (15)$$

$$E_2 = \int_0^T \sum_{i=1}^{3} u_i^2(t)\mathrm{d}t \quad (16)$$

$$E_3 = \int_0^T \sum_{i=1}^{3} \left|u_i(t)\dot{\phi}_i\right|\mathrm{d}t \quad (17)$$

In many cases these functionals can be used to estimate the energy consumption for the controlled motion of mechanical systems (Athans *et al.*, 1963). The expressions for these cost functions involve only the controlling stimuli of the powered drives. This is one of the reasons why the controlling stimuli $p_i(t)$ is named passive.

3. METHODOLOGY

As follows from the description of the considered robot the number of degrees-of-freedom is less than the number of powered drives. It means that the robot in question is an overactuated mechanical system and the modeling task (Problem 1) leads to dynamic redundancy.

To solve Problem 1 an approach is proposed that is based on:

- solution of inverse kinematics for the initial, intermediate and final phase states of the robot end-effectors;
- path planning by means of polynomial and Fourier series approximation of the robots generalized coordinates;
- special regularization procedure of the dynamic redundancy of the system;
- and, finally, solution of the inverse dynamics problem.

An outline of the above approach is supported by the following details.

For the input parameters $f_0, \dot{f}_0, f_\tau, \dot{f}_\tau$ of the pick-and-place operation (8)-(11) the inverse kinematics problem is solved for the moments of time $t = 0$, $t = \tau$ and $t = T$. As a consequence the following parameters are calculated:

$$\begin{aligned} &\phi_{i0} = \phi_i(0), \dot{\phi}_{i0} = \dot{\phi}_i(0),\ i = 1, 2 \quad (18) \\ &\phi_{i\tau} = \phi_i(\tau), \dot{\phi}_{i\tau} = \dot{\phi}_i(\tau),\ i = 1, 2 \\ &z_0 = z(0), \dot{z}_0 = \dot{z}(0), z_\tau = z(\tau), \dot{z}_\tau = \dot{z}(\tau) \end{aligned}$$

The path planning problem is solved using the approximation of the generalized coordinates of the robot by a sum of a fifth order polynomial and a finite Fourier series given by the following formula:

$$q(t) = \sum_{j=1}^{5} C_{q\nu j}(t - t_0)^j \tag{19}$$

$$+ \sum_{k=1}^{N_{q\nu}} [a_{q\nu k}\cos(\omega_\nu(t - t_0)) + b_{q\nu k}\sin(\omega_\nu(t - t_0))]$$

for both intervals of time $t \in [0, \tau]$ and $t \in [\tau, T]$.

Here $q = (\phi_1, \phi_2, z)$, $\nu = 1$, $t_0 = 0$, and $\omega_\nu = \frac{2\pi}{\tau}$ for $t \in [0, \tau]$, and $\nu = 2$, $t_0 = \tau$, and $\omega_\nu = \frac{2\pi}{(T-\tau)}$ for $t \in [\tau, T]$; $N_{q\nu}$ are given positive integers.

Taking into account the conditions (8) and (18), from formula (19) follows that the parameters

$$C_{q\nu 4}, C_{q\nu 5}, a_{q\nu k}, b_{q\nu k},\ k = 1, 2, ..., N_{q\nu} \tag{20}$$

can serve as independent variables.

To avoid the dynamic redundancy of the considered robot an additional constraint

$$G[u_1(t), u_2(t), u_3(t)] = 0,\ t \in [0, T] \tag{21}$$

is imposed on the controlling stimuli of the powered drives. The scalar function G can be used for advanced optimization.

If the function G is chosen, the inverse dynamics problem can be solved by using the equations of motion (1), (7) and the constraint (21). This is the final step of solving the modeling task for the SCARA-like robot.

The proposed approach gives no unique solution of Problem 1. The solution will depend on the value of the parameters (20) and the function G.

4. ENERGY-OPTIMAL CONTROL FOR THE GIVEN MOTION OF THE ROBOT

The considered robot is an overactuated mechanical system. This makes it possible to optimize the controlling stimuli $u_i(t)$ of drives for an arbitrary given motion of the robot.

Below the motion of the robot is studied in the horizontal plane OXY. The equations of the plane motion of the robot can be written as follows:

$$f_1(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_1 + p_1 + u_3 + p_3 \tag{22}$$

$$f_2(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_2 + p_2 + b(\phi_i)(u_3 + p_3) \tag{23}$$

Here the functions f_1 and f_2 are determined by means of the formulas (1)-(6).

Problem 2. Assume that an arbitrary motion of the robot is given, i.e. the functions (13) are specified. Find the control stimuli (14) which minimize the functional (16) subject to the differential constraints (22) and (23).

It can be shown that the solution of Problem 2 is:

$$u_3^*(t) = (g_1 + bg_2) / (2 + b^2) \tag{24}$$

$$u_1^* = g_1 - u_3^*(t),\ u_2^* = g_2 - b(\phi_i)\, u_3^*(t) \tag{25}$$

Here the functions g_1 and g_2 have the expressions:

$$g_1 = f_1 - p_1 - p_3 \tag{26}$$
$$g_2 = f_2 - p_2 - b(\phi_i)\, p_3$$

The obtained controlling stimuli (24) and (25) provide the execution of an arbitrary given motion of the overactuated robot with minimal energy consumption E_2^*.

The simplest way to reduce the overactuation of the considered robot is to exclude one of the powered drives. For instance, assuming that

$$u_3(t) \equiv 0,\ t \in [0, T] \tag{27}$$

the unique solution for the functions $u_1(t)$ and $u_2(t)$ can be obtained from the equations (22) and (23). In this case the functional (16) is

$$E_2^0 = \int_0^T \left(g_1^2 + g_2^2\right) \mathrm{d}t \tag{28}$$

where the functions g_1 and g_2 are given by the formulas (26). Comparing the value E_2^0 with the value of the functional (16) for the obtained optimal controlling stimuli $u_i^*(t)$, $(i = 1, 2, 3)$, it is easy to show the validity of the following expression

$$E_2^0 - E_2^* = \int_0^T \frac{(g_1 + bg_2)^2}{2 + b^2} \mathrm{d}t \tag{29}$$

The formula (29) shows that the energy consumption needed to execute an arbitrary given motion by the considered overactuated robot with obtained optimal controlling stimuli (24) and (25) is less than the energy consumption of the same robot but without powered drive acting on the link OD.

Problem 3. Assume that an arbitrary motion of the robot is given. Determine the torques $u_1(t)$, $u_2(t)$ and $p_3(t)$ which minimize the functional (16) subject to the equations (22), (23), the constraint (27) and the restrictions $p_1(t) = p_2(t) \equiv 0,\ t \in [0, T]$.

Solution of Problem 3 can be found explicitly and it is determined by the formulas

$$p_3^*(t) = (f_1 + bf_2)/(1 + b^2) \tag{30}$$

$$u_{1p_3}^*(t) = f_1 - p_3^*,\ u_{2p_3}^*(t) = f_2 - bp_3^* \tag{31}$$

The value of the functional (16) for the torques (27) and (31) is equal to

$$E_{2p_3}^* = E_2^0 - \int_0^T \frac{(f_1 + bf_2)^2}{1 + b^2} \mathrm{d}t \tag{32}$$

where E_2^0 is determined by formula (28) and $g_1 = f_1$, $g_2 = f_2$. Using the solutions of Problems 2 and 3 the following equality can be written

$$E_2^* - E_{2p_3}^* = \int_0^T \frac{(f_1 + bf_2)^2}{(1 + b^2)(2 + b^2)} \mathrm{d}t \tag{33}$$

Analysis of formulas (32) and (33) demonstrates that the energy consumption for an arbitrary motion of the semi-passively actuated robot having optimal unpowered (passive) drive (30) is less than the energy consumption of the fully actuated robot. The same statement is also valid for the energy consumption of the optimal overactuation (24), (25).

Problem 4. Consider Problem 3 with an additional constraint

$$p_3(t) = -k_3\phi_3(t), \ t \in [0,T] \tag{34}$$

where k_3 is the constant spring coefficient of a spring-like passive drive acting on the link OD of the robot.

Taking into account this constraint it can be shown that the solution of Problem 4 is determined by formula (31) and

$$p_3^*(t) = p_{3S}^*(t) = -\frac{D_2}{D_1}\phi_3(t) \tag{35}$$

$$D_1 = \int_0^T \left(1+b^2\right)\phi_3^2(t)\mathrm{d}t \tag{36}$$

$$D_2 = \int_0^T \left(f_1 + bf_2\right)\phi_3(t)\mathrm{d}t$$

The energy consumption E_{2S}^* of the robot having optimal spring-like passive drive (35) is equal to

$$E_{2S}^* = E_2^0 - D_2^2/D_1 \tag{37}$$

where E_2^0 is determined by the expression (28) and $g_1 = f_1$, $g_2 = f_2$.

From formulas (36) and (37) follow that the robot with the optimal spring-like passive drive has less energy consumption than the fully actuated robot executing the same given motion.

5. COMPUTER SIMULATIONS

Several numerical results are now presented that illustrate the effectiveness of the proposed methodology. In all presented variants the following input data for the pick-and-place operation are used: $T = 6$ s, $\tau = 4$ s, $l(0) = l(T) = 2$ m, $l(\tau) = 1.5$ m, $\dot{l}(0) = \dot{l}(\tau) = \dot{l}(T) = 0$, $\phi_3(0) = \phi_3(T) = -\frac{\pi}{4}$, $\phi(\tau) = \frac{\pi}{3}$, $\dot{\phi}_3(0) = \dot{\phi}_3(\tau) = \dot{\phi}_3(T) = 0$, $m_C = 5$ kg, $m_D = 0$ kg. It is assumed that the links OA, AB and OD of the robot in question are homogeneous bars and that their centre of mass is located at the midpoint of the links. The length and mass of links are as follows: $l_1 = 1$ m, $l_2 = 1.5$ m, $l_3 = 3$ m, $m_1 = 5$ kg, $m_2 = m_3 = 10$ kg.

Using the above input data the inverse kinematics problem was solved for the moments of time $t = 0$, $t = \tau$ and $t = T$. Then, the formula (19) was used to approximate the generalized coordinates $\phi_1(t)$ and $\phi_2(t)$ for both intervals of time: $t \in [0,\tau]$ and $t \in [\tau, T]$. This gives the set of functions $\phi_1(t)$ and $\phi_2(t)$ which depends on the variable parameters (20). The results are presented for one of the above mentioned pick-and-place operations, namely for the operation with the following values of the parameters (20): $a_{q\nu k} = b_{q\nu k} = 0$, $q = (\phi_1, \phi_2)$; $\nu = 1, 2$; $k = 1, 2, ..., N_{q\nu}$. The parameters $C_{q\nu 4}$ and $C_{q\nu 5}$ were determined by using formula (19) and the boundary conditions: $\ddot{l}(0) = \ddot{l}(\tau) = \ddot{l}(T) = 0$, $\ddot{\phi}_3(0) = \ddot{\phi}_3(\tau) = \ddot{\phi}_3(T) = 0$. The pick-and-place operation of the robot is then uniquely determined by the formula (19).

The path of the end-effector C which correspond to the path of the point B is depicted in Fig. 2 for the given pick-and-place operation. The time-history of the torques acting on the links OA, AB and OD of the robot are presented in Figure 3-5, respectively. The thin solid, heavy solid, thin dashed and heavy dashed curves correspond to the solution of Problem 1 with restriction (27) and Problems 2-4, respectively. Analysis of Figures 3-4 shows that the time-history of the control torques $u_1(t)$ and $u_2(t)$ have the same character for the fully actuated robot (thin solid curves) and for the robot with optimal overactuation (heavy solid curves).

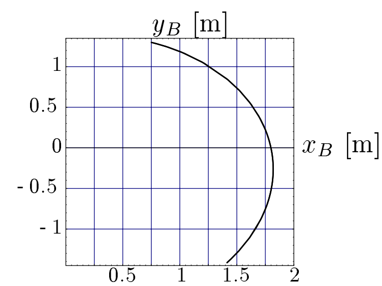

Fig. 2. The path of the end-effector C of the robot.

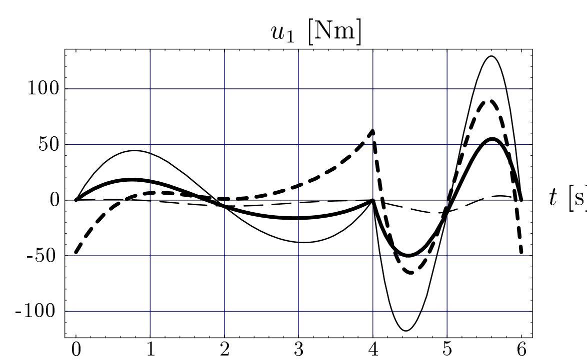

Fig. 3. The torque acting on the link OA.

The obtained values of the functionals (15)-(17) for the given motion of the robot are presented in Table 1. The data of columns 1-4 correspond to the solutions of Problem 1-4, respectively. Analysis of the data of Table 1 gives us the possibility to estimate quantitatively the gain in energy consumption of the considered robot due to optimal overactuation (formula (29)), incorporation of the optimal passive drive (formulas (32) and (33)),

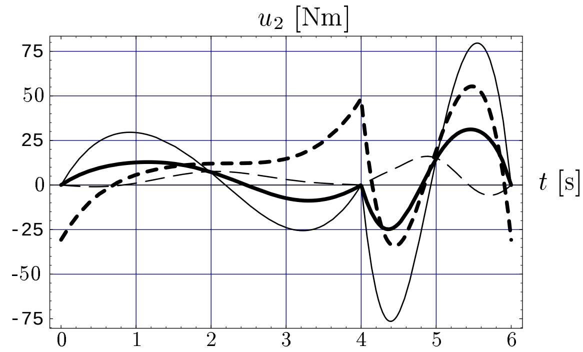

Fig. 4. The torque acting on the link AB.

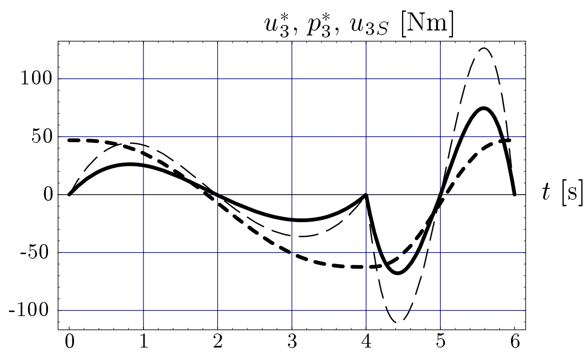

Fig. 5. The torque acting on the link OD.

Table 1. Energy Consumption

Problem#	1	2	3	4
E_1	442	337	48	268
E_2	27122	11065	362	10988
E_3	181	183	24	101

and due to utilization of the optimal spring-like passive drive (formula (37)).

6. CONCLUSION

The controlled motion of a SCARA-like closed-loop manipulator robot has been analytically and numerically studied with special emphasis on the interaction between controlling stimuli of powered drives and torques exerted by unpowered drives of the robot. Dynamic redundancy is utilized to perform energy optimization. Different elements and strategies in terms of arrangements of the powered and unpowered drives is investigated.

The effectiveness of the proposed methodology has been validated through computer simulations of a pick-and-place operation. The analytical study and the numerical results of the simulations show that the energy consumption of the given arbitrary motion in case of optimal overactuation is less than in case of full actuation. More importantly the energy consumption is even lower in the case of semi-passively actuation using a spring-like passive drive. This indicates that the semi-passively actuation might be a preferred alternative to overaction and full actuation not only for the manipulator robot in question but also for more general mechanical systems. The energy consumption with optimal passive drive also suggests that there is a potential to gain even more in terms of energy saving using advanced passive actuators, e.g. passive drive with switching of stiffness parameters.

Future work will involve a more detailed study of the effect of semi-passively actuation of the robot. In particular it is of interest to study the set of motion where semi-passively actuation is superior to overactuation. It is also of interest to study the effect of more advanced passive actuators.

7. ACKNOWLEDGEMENTS

The Volvo Research Foundation, Sweden, and the TFR (Swedish Research Council for Engineering Sciences) supported this work.

8. REFERENCES

Athans, M., P.L. Falb and R.T. Lacoss (1963). Time-fuel-and energy-optimal control of nonlinear norminvariant systems. *IEEE, Trans. Automat. Contr.* **8**, 196–202.

Berbyuk, V., A. Boström, B. Lytwyn and B. Peterson (1999). Optimization of control laws of bipedal locomotion systems. In: *Advances in Computational Multibody Dynamics* (Jorge A.C. Ambrsio and Werner O. Schiehlen, Eds.). pp. 713–728.

Berbyuk, V., B. Peterson and N. Nishchenko (1998). Linear viscoelastic actuator-based control system of a bipedal walking robot. In: *Mechatronics 98* (J. Adolfson and J. Karlsn, Eds.). Elsevier. pp. 379–384.

Gosselin, C. (1996). Parallel computational algorithms for the kinematics and dynamics of planar and spatial parallel manipulators. *ASME, J. Dynamics Systems, Measur. and Contr.* **118**, 22–28.

Nakamura, Y. and M. Ghodoussi (1989). Dynamic computation of closed-link robot mechanism with nonredundant and redundant actuators. *IEEE, Trans. on Robotics and Automation* **5**, 294–302.

Tran, T. and G. Kehl (1998). Drive dimension and path preparation for parallel manipulators. In: *Mechatronics 98* (J. Adolfsson and J. Karlsen, Eds.). Elsevier. pp. 293–298.

Zhang, Y., W. Gruver and F. Gao (1999). Dynamic simplification of three degree of freedom manipulators with closed chains. *Robotics and Autonomous Systems* **28**, 261–269.