

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN SOLID AND
STRUCTURAL MECHANICS

Numerical simulation of crack growth and wear in rails

JIM BROUZOULIS

Department of Applied Mechanics
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2012

Numerical simulation of crack growth and wear in rails
JIM BROUZOULIS
ISBN 978-91-7385-747-5

© JIM BROUZOULIS, 2012

Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr. 3428
ISSN 0346-718X
Department of Applied Mechanics
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: +46 (0)31-772 1000

Chalmers Reproservice
Göteborg, Sweden 2012

Numerical simulation of crack growth and wear in rails
Thesis for the degree of Doctor of Philosophy in Solid and Structural Mechanics
JIM BROUZOULIS
Department of Applied Mechanics
Chalmers University of Technology

ABSTRACT

Wear and rolling contact fatigue (RCF) of rails are huge problems for the railway industry with increased world-wide occurrence during the last decades. It is, therefore, of high interest to develop tools for the prediction of wear and RCF which can be used to estimate rail life.

This thesis is concerned with the numerical simulation of rail deterioration due to wear and RCF. The main focus has been on the modeling of crack growth in rails, which has included development of a numerical tool in terms of a 2D Finite Element (FE) model of the rail that can be used to simulate the growth of (short and long) surface cracks in rails. Wear is taken into account through the process of crack truncation (partial removal of the crack). Furthermore, the change in geometry due to crack growth and wear is considered through a remeshing procedure. In the numerical framework, the concept of material forces is adopted from which a crack driving force can be derived. Based on this crack driving force, numerical procedures for simulation of crack growth under monotonic, cyclic and typical RCF loading have been developed. Different propagation laws have been proposed and used to study crack growth in rails under various loading conditions and crack geometry. The influence of the highly deformed (anisotropic) surface layer, often present in railway rails, on the crack growth direction has also been studied. Results from the numerical simulations indicate that anisotropy has a large influence on the crack growth direction and needs thus be accounted for in order to be able to simulate the growth of surface cracks in rails more accurately.

Furthermore, a numerical framework for simulation of rail degradation due to wear and plastic deformations has been developed and implemented. The procedure includes simulations of the wheel–rail dynamics using a Multi Body Simulation (MBS) software, together with detailed FE simulations of the plastic deformations. The procedure was applied for the calibration of a wear coefficient from experiments on a full scale wheel–rail test rig. Quantitatively good agreement was obtained between simulations and results from the full-scale tests in terms of worn-off area and shape of the worn profile.

Keywords: Rolling Contact Fatigue, wear, crack propagation, material forces, railway mechanics

Life is too serious to be taken seriously.

PREFACE

The work presented in this thesis was carried out at the department of Applied Mechanics at Chalmers University of Technology between the years 2007–2012. The work has been carried out within the project MU20 “Wear Impact on Rolling Contact Fatigue of Rails” and is part of the activities within the National Centre of Excellence in Railway Mechanics (CHARMEC) with voestalpine Bahnsysteme, Trafikverket and SL Technology as main supporting partners. Their financial and technical support is much appreciated.

ACKNOWLEDGEMENTS

First and foremost I would like to thank my humor, you are my recreational drug and without you I would never have been able to finalize this thesis, or amount to anything really, for that matter. This implicitly means acknowledgement to people like Seth MacFarlane, Matt Stone, Trey Parker and Matt Groening. In tight second place we have my main supervisor professor Magnus Ekh, who has been a tremendous support to me during the years. I have very much appreciated your open door policy which has lead to many rewarding discussions. I suppose I may have abused this privilege occasionally and therefore commend you on your large patience. Thanks to your mentoring I feel like a researcher ready to embark on new adventures. My only hope is that I one day will be half as good researcher as you are. Thank you.

Moving on to the rest of team MU20, I would like to acknowledge my co-supervisors Associate Professor Fredrik Larsson and Associate Professor Anders Ekberg. You have given me many valuable inputs on my work and for that I am most grateful. Continuing, all my co-authors are acknowledged, especially Mr. Peter T. Torstensson and Ms. NASIM LARIJANI, it has been a pleasure working with you. Also Björn Pålsson deserves my thanks for the many seedless discussions we have had.

At this point I would like to express some amount of appreciation for my colleagues who have made my time here less than painful. No, in all seriousness you have all contributed to making my years here a cosy and fuzzy experience, so thank you all.

Finally, I would like to thank my friends and family for all your love and kind support.

THESIS

This thesis consists of an extended summary and the following appended papers:

- Paper A** J. Brouzoulis, F. Larsson and K. Runesson (2010). Strategies for planar crack propagation based on the concept of material forces. *Computational Mechanics* **47.3**, 295-304
- Paper B** J. Brouzoulis, P.T. Torstensson, R. Stock and M. Ekh (2011). Prediction of wear and plastic flow in rails – Test rig results, model calibration and numerical prediction. *Wear* **271.1-2**, 92-99
- Paper C** J. Brouzoulis and M. Ekh (2012). Crack propagation in rails under Rolling Contact Fatigue loading conditions based on material forces. *International Journal of Fatigue* **45**, 98–105
- Paper D** J. Brouzoulis (2012a). Numerical prediction of the impact of wear on Rolling Contact Fatigue crack growth in rails. *To be submitted for international publication*
- Paper E** N. Larijani, J. Brouzoulis, M. Schilke and M. Ekh (2012). The effect of anisotropy on crack propagation in pearlitic rail steel. *To be submitted for international publication*

The appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of work in the papers **Paper A-D**, i.e. planning the papers, developing theory, developing the numerical implementations, and carrying out the numerical simulations. For **Paper E** the author of this thesis assisted in the development of the theory and the numerical implementation.

CONTENTS

Abstract	i
Preface	v
Acknowledgements	v
Thesis	vii
Contents	ix
I Extended Summary	1
1 Introduction	1
1.1 Background – motivation for research	1
1.2 Scope and limitations	2
2 Rail deterioration	3
2.1 RCF of rails	3
2.2 Wear of rails	9
2.3 Interaction between wear and RCF	10
3 Summary of appended papers	11
4 Practical use of the research	13
5 Concluding remark and future work	15
References	17
II Appended Papers A–E	21

Part I

Extended Summary

1 Introduction

1.1 Background – motivation for research

To understand and predict rail and wheel deterioration (including fatigue and wear) is a key issue in railway maintenance engineering. In particular, rolling contact fatigue (RCF) is a severe problem with increasing world-wide occurrence during the last decades (Fischer et al. 2006; Cannon et al. 2003; Ringsberg, Loo-Morrey, et al. 2000).

Considering rails specifically, RCF is currently one of the major factors affecting the maintainability and operational safety of the track, (Haidemenopoulos et al. 2006). Furthermore, maintenance costs associated with rail deterioration (repairs, grinding) are very high, (Olofsson and Lewis 2009). Considering the European Union, costs associated with rail defects (a large portion attributed to RCF) may reach €2 billion per year (Magel 2011). Therefore, having accurate and efficient tools for the prediction of wear and RCF in order to estimate the rail (and wheel) life is a pre-requisite for optimization of maintenance, planning of (re)investments, and also for the improvement of operational conditions in order to decrease deterioration and costs. Note that, under (principally) fixed traffic conditions statistics of the degradation history combined with empirical knowledge may be used for the purpose of optimization but when traffic conditions change (as they increasingly do) statistics are not enough.

Much research has been conducted on the topic of RCF; however, studies concerning the effect of wear on RCF are more limited, see e.g. (Ringsberg and Bergkvist 2003; Donzella 2005; Franklin et al. 2001; Kapoor et al. 2003). Wear and RCF of rails are coupled problems but are often treated uncoupled. For example, wear will influence the propagation rate of surface cracks through crack truncation (partial removal of the crack). Typically this is considered by computing a net crack growth rate which is then extrapolated in one step. This is in many cases not sufficient, especially when the crack growth rate varies due to changes in loading etc. To gain a deeper understanding of the complex influence wear has on RCF, the project “Wear impact on rolling contact fatigue in rails” (MU20) was initiated within the National Centre of Excellence in Railway Mechanics CHARMEC (CHalmers Railway MEchanics), at Chalmers University of Technology. The work presented in this thesis has been conducted within that project. The project focus has been on the development and use of predictive models and numerical tools for the simulation of rail deterioration. In particular, the project included development of strategies for effective and accurate updating of deteriorating rail profiles which can take into account the coupling between wear and RCF.

Several previous CHARMEC projects have investigated RCF initiated crack growth in rails, cf. MU6, MU11, MU12 and MU17 (<http://www.charmec.chalmers.se/>). The approach towards prediction of crack growth in these projects was, in most cases, to adopt

the concept of material forces, see for example (Bergkvist 2005; Heintz 2006; Tillberg 2010a). In particular, the aim of project MU17 was the formulation of an appropriate crack driving force for use in general simulations of inelastic fracture processes. One of the strongest arguments for adopting the concept of material forces, in the formulation of a crack driving force, is that it is not restricted to (hyper)elasticity, like the classical \mathcal{J} -integral. Due to this fact it was deemed advantageous to adopt material forces as a framework for crack growth also in the current project.

1.2 Scope and limitations

Research on RCF of rail components is a tremendously vast topic. Hence the specific work carried out within one single project must be limited. Owing to this, the research conducted within the project MU20 has mainly focused on the area of numerical simulation of crack growth in rails. In summary, this thesis addresses the following topics:

- Propagation strategies for simple monotonic and cyclic as well as typical RCF loading.
- Wear impact on RCF crack growth through crack truncation.
- Effect of anisotropy on RCF crack growth direction.
- Simulation of 2D rail profile evolution due to wear and plastic deformations.

Below are four major simplifications that have been considered necessary in the conducted research, presented in this thesis, concerning the modeling of RCF crack growth in rails.

- Simulation of 3D crack propagation is associated with a high complexity and computational burden. By also considering that a study on RCF calls for the simulation of a large number of load passages, for a realistic operational situation, the crack growth simulations in this thesis have been restricted to 2D.
- The current formulation of the crack driving force, as presented in (Brouzoulis 2012b), for inelastic material behavior yields discretization dependent results for an RCF type of loading, see Section 2.1.2. Furthermore, for many rails a state of elastic shakedown is reached after initial hardening and wearing-in of the wheel–rail profiles. For these reasons, the simulations of crack growth presented in the appended papers have been limited to elastic material response.
- The wheel–rail contact interaction is generally highly complex with transient stick–slip mechanisms etc. In this thesis the contact conditions have in most parts been modeled using Hertzian theory which allows for the wheel load to be incorporated in a simplified manner.
- Surface cracks in rails are often observed in regular patterns; thus, single cracks are uncommon. Nevertheless, only the propagation of one surface crack has been

studied in this thesis. This limitation is mainly due to the large increase in numerical complexity associated with the propagation of multiple cracks. This approach will neglect phenomena such as crack tip shielding. On the other hand, it still facilitates an analysis of influencing factors on propagation of single cracks.

2 Rail deterioration

In the following, two main processes that cause rail deterioration will be described, namely RCF and wear. Research regarding RCF and wear of rails have been conducted extensively during recent years, (Magel 2011; Ekberg and Kabo 2005; Fletcher et al. 2008; Kapoor et al. 2003; Donzella 2005; Suresh 1998). In Figure 2.1 schematic illustrations of rail deterioration, caused by a combination of wear, crack propagation and plastic deformation, are presented. As the rail is loaded by passing wheels the crack tip extends into the rail while at the same time the crack mouth is gradually worn away (left figure). Large plastic deformations combined with rail head wear will result in a change in the profile shape over time (right figure).

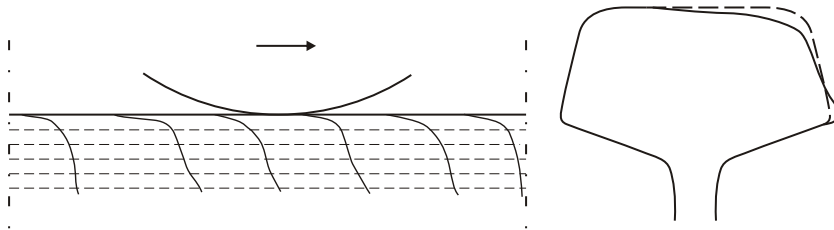


Figure 2.1: *Illustration of rail deterioration: (left) side view of the rail which shows crack truncation due to wear, and (right) typical profile change due to wear and plastic deformation.*

2.1 RCF of rails

RCF damage initiates in the rail head which is subjected to high compressive stresses combined with tangential stresses due traction. In the following, RCF is considered to include crack initiation and growth as well as related inelastic (plastic) deformations. Note that, wear is often considered to be a part of the RCF process; however, in the current discussion it is distinguished as a separate process. Also one may distinguish between crack face wear and railhead wear; only the latter is considered in the work carried out within the scope of this thesis. In the literature, several terms exist for the different (surface/sub-surface) cracks that may develop in the rail. For a detailed description of the different types of cracks, see (Magel 2011; Olofsson and Lewis 2009; D. Stone 2004; UIC 2002).

2.1.1 Plastic deformation

Due to the extremely high vertical and lateral forces that the rail is subjected to, stresses will often exceed the yield limit of the rail material and thereby cause plastic deformations. In particular at the rail surface, severe plastic deformations are often accumulated during the cyclic loading. This phenomena, where plastic deformations are accumulated and increased, is referred to as ratcheting and has been the subject of investigation in the literature, see e.g. (Kapoor 1997; Johansson, Ekh, and Runesson 2005; Johansson and Ekh 2006). These ratcheting deformations may eventually exhaust the strength of the rail material which will then cause local failure of the material and consequently crack initiation. This is one mechanism behind the formation of head checks (and probably other crack types as well). Note that, in many cases the accumulation of plastic deformation reduces over time and a state of elastic shakedown may be reached. Partly motivated by this, the simulations of crack growth in this thesis has been restricted to elastic material response.

It is noted, that large plastic deformations will influence the mechanical properties of the deformed surface material and as a consequence it has been shown that the fracture toughness of the rail surface becomes anisotropic, (Hohenwarter et al. 2011; Ekberg and Sotkovszki 2001). In **Paper E**, the effect of anisotropy on crack growth direction is investigated through the introduction of an anisotropic fatigue crack growth threshold, \mathcal{G}_{th} , in analogy to an anisotropic yield surface. Therefore in this paper, plastic deformations can be said to be accounted for in the simulation of crack growth, although in an implicit fashion. The adopted fatigue crack growth threshold curve, as a function of direction φ , is given by

$$\mathcal{G}_{th}(\varphi) = \mathcal{G}_{th,1} + (\mathcal{G}_{th,2} - \mathcal{G}_{th,1})(1 - e^{-A|\sin(\varphi)|}) \quad (2.1)$$

where $\mathcal{G}_{th,1}$, $\mathcal{G}_{th,2}$ are principal resistances in two perpendicular directions, and $A > 0$ is a regularization parameter. For an illustration of the fatigue crack growth threshold surface for isotropy and anisotropy, respectively, see Figure 2.2. Note that, the adopted surface captures the experimentally observed behavior with increased resistance towards crack growth in one direction and reduced resistance in the perpendicular direction. From the numerical simulations presented in **Paper E**, it is shown that the amount of anisotropy has a profound effect on the crack growth direction.

2.1.2 Material forces

During the last years much research has been conducted within the field of material (configurational) forces, (Maugin 1995; Gurtin and Guidugli 1998; Denzer et al. 2003; Menzel et al. 2004; Miehe 2007; Fagerström and Larsson 2008; Schütte 2009; Tillberg et al. 2010; Brouzoulis, Larsson, et al. 2010). By adopting the concept of material forces, a thermodynamically consistent framework for simulation of evolving configurations is provided (e.g. moving interfaces, growth etc.). In this context, crack propagation is included as a special case. This research field has gained much attention due to its versatile nature; for example, inelastic material behaviour, crack closure, crack face friction etc. can be introduced into the structural model (FE problem) in a straightforward way without the need to modify the propagation law.

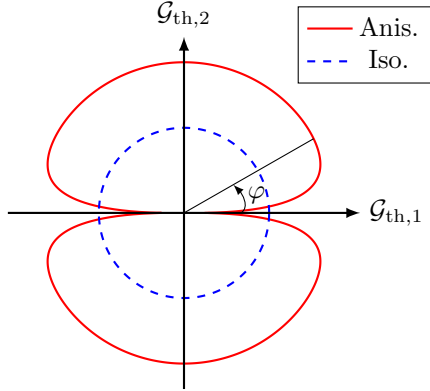


Figure 2.2: *Example of an isotropic and an anisotropic fatigue crack growth surface.*

Below is a summary of the governing equations for a generalized crack driving force based on material forces of an inhomogeneous dissipative material, (Tillberg et al. 2010; Menzel et al. 2004; Miehe 2007) for details. The material force \mathcal{G} may be decomposed into three parts (internal, volumetric and surface) in analogy with the principal of virtual work, i.e

$$\begin{aligned} \mathcal{G} &= \mathcal{G}_{\text{int}} + \mathcal{G}_{\text{vol}} + \mathcal{G}_{\text{sur}} \\ &= \int_{\mathcal{B}_X} -\Sigma \cdot (W \nabla_X) dV_X + \int_{\mathcal{B}_X} W \mathbf{B}_X dV_X + \int_{\partial \mathcal{B}_X} W \mathbf{T}_X dS_X \end{aligned} \quad (2.2)$$

where Σ is the Eshelby stress tensor, defined as

$$\Sigma = \psi_X \mathbf{I} - \mathbf{F}^T \mathbf{P} \quad (2.3)$$

with the strain energy ψ_X , identity tensor \mathbf{I} , deformation gradient \mathbf{F} and the first Piola-Kirchhoff stress tensor \mathbf{P} . Furthermore, \mathbf{B}_X is the configurational body force and includes contributions from dissipative processes, such as plastic deformations, \mathbf{T}_X is the configurational traction and \mathcal{B}_X is the domain of the body. Finally, W is a suitably chosen weight function with local support ($W \neq 0$ close to the crack tip).

The crack driving force describes the local energy dissipation due to a motion of the crack tip and is a vectorial quantity. It may be decomposed into one parallel component and one component perpendicular to the crack tip, cf. Figure 2.3,

$$\mathcal{G} = \mathcal{G}_{\parallel} \mathbf{e}_{\parallel} + \mathcal{G}_{\perp} \mathbf{e}_{\perp} \quad (2.4)$$

It thus contains more information than a scalar counterpart (such as the \mathcal{J} -integral). Therefore, it may give information regarding both rate and growth direction of the crack. In fact, this has been a strong argument as to why the concept of material forces should be adopted. However as shown in **Paper C**, the component of the crack driving force perpendicular to the crack tip (\mathcal{G}_{\perp}) is heavily path dependent in contrast to the parallel

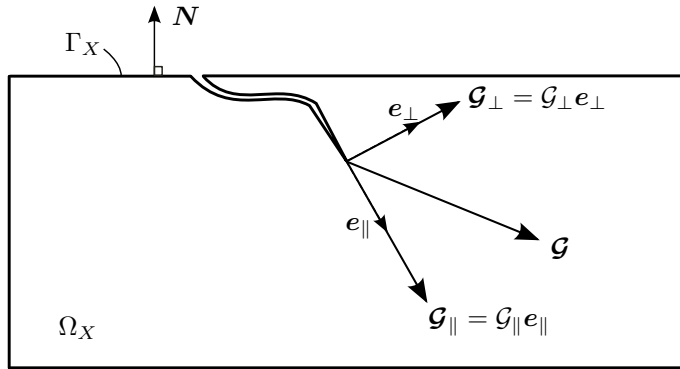


Figure 2.3: Unit base vectors, e_{\parallel} and e_{\perp} which are directed parallel and perpendicular to the crack tip respectively. In addition, the corresponding components \mathcal{G}_{\parallel} and \mathcal{G}_{\perp} of the crack driving force \mathcal{G} is shown.

component (\mathcal{G}_{\parallel}), cf. the \mathcal{J} -integral. This implies that the use of the perpendicular component is an unsuitable quantity to use in numerical simulations. Also the physical interpretation of the perpendicular component is somewhat unclear. It may represent a kinking or curving of the crack tip.

Furthermore, the expression for the crack driving force in Equation (2.2) may produce discretization dependent results for RCF loading. In Figure 2.4, the evolution of \mathcal{G}_{\parallel} can be seen over one load cycle for inelastic material behavior. Three different meshes have been studied and the pertinent degrees of freedom (dofs) are indicated. Figure 2.4(a) corresponds to tension–compression loading, whereas Figure 2.4(b) corresponds to RCF loading; clearly, no convergence is obtained for this more complex load case.

2.1.3 Crack growth

The topic of crack growth in rails under various loading conditions have been studied by several researchers during the years, see e.g. (Bogdański and Brown 2002; Canadinc et al. 2008; Donzella 2005; Ringsberg 2005; Fajdiga, Ren, et al. 2007; Fajdiga and Sraml 2009). In these references, the approach towards simulation of crack growth is fairly traditional. Typically an exponential (Paris’) type of propagation law is used expressed in terms of the stress intensity factors or the crack tip opening/sliding displacement.

In this thesis however, crack growth in rails is modelled based on the concept of material forces. From this concept, different propagation laws may be proposed, in terms of the generalized crack driving force \mathcal{G} , and is addressed in papers **Paper A**, **C** and **D**. In **Paper A**, a viscous propagation law was adopted and studied under monotonic and cyclic loading. Here the crack growth rate per unit time ($d\mathbf{a}/dt$) is given by

$$\begin{cases} \frac{d\mathbf{a}}{dt} = \frac{1}{\gamma} \langle |\mathcal{G}| - \mathcal{G}_{\text{th}} \rangle \mathbf{e}^* \\ \mathbf{e}^* = \frac{\mathcal{G}}{|\mathcal{G}|} \end{cases} \quad (2.5)$$

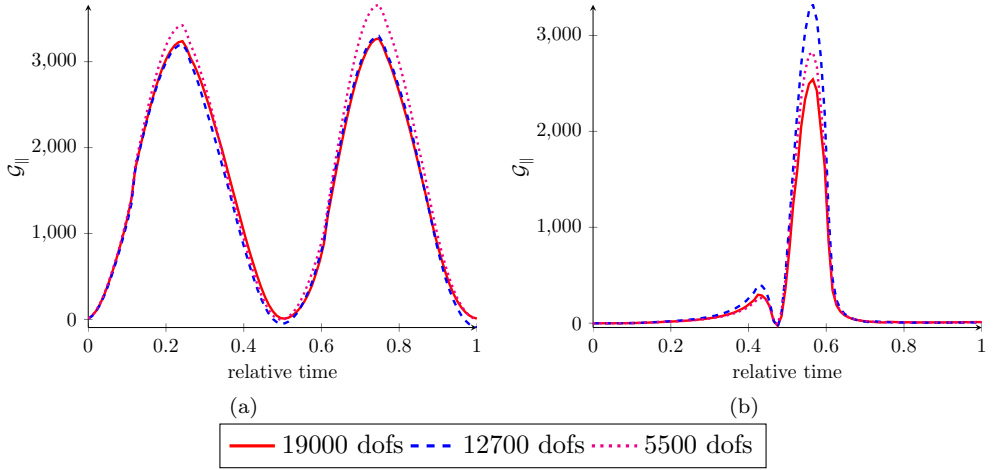


Figure 2.4: *Example of the evolution of the crack driving force (\mathcal{G}_{\parallel}) over one load cycle for three different mesh discretizations. (a) corresponds to simple tension–compression loading and (b) corresponds to RCF loading. The results correspond to inelastic material behavior, for details see (Brouzoulis 2012b).*

However, this type of propagation law is unsuitable for simulation of RCF crack growth as the predicted crack growth rate will depend on the loading rate; therefore, a rate independent propagation law was proposed in **Paper C**,

$$\begin{cases} \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \frac{1}{\gamma} \langle \dot{\Phi} \rangle \mathbf{e}^* \\ \Phi = \langle \mathcal{G} \cdot \mathbf{e}^* - \mathcal{G}_{\text{th}} \rangle \\ \mathbf{e}^* = \arg \max_{\mathbf{e}} \lim_{\epsilon \rightarrow 0} \mathcal{G}(\mathbf{a} + \epsilon \mathbf{e}) \cdot \mathbf{e} \end{cases} \quad (2.6)$$

As a continuation, a Paris' type of propagation law was adopted in **Paper D** which allowed for comparison with a calibrated propagation law (for rail grade R260). Here, the crack growth per load cycle ($d\mathbf{a}/dN$) is given as

$$\begin{cases} \frac{d\mathbf{a}}{dN} = \frac{1}{\gamma} \left(\Delta \mathcal{G}_{\parallel}^m - \Delta \mathcal{G}_{\text{th}}^m \right) \mathbf{e}^* \\ \mathbf{e}^* = \frac{\int_T \mathcal{G}^*(t) dt}{\left| \int_T \mathcal{G}^*(t) dt \right|} \\ \mathcal{G}^*(t) = \max_{\mathbf{e}} \lim_{\epsilon \rightarrow 0} \langle \mathcal{G}(\mathbf{a}(t) + \epsilon \mathbf{e}) \cdot \mathbf{e} - \mathcal{G}_{\text{th}} \rangle \mathbf{e} \end{cases} \quad (2.7)$$

In Equations (2.5)–(2.7), \mathbf{e}^* is the propagation direction; γ and m are material constants; ϵ is a small scalar value; and $\langle \bullet \rangle = \frac{1}{2}(\bullet + |\bullet|)$ denotes the MacCauley bracket. Equations (2.5) and (2.6) are formulated in the time domain and are integrated over one load cycle to obtain the growth rate per cycle (with time interval T), whereas Equation (2.7) is formulated in the cycle domain.

In order to compute \mathbf{e}^* numerically, a probing procedure is often utilized to determine the crack driving force in different directions. Here, the crack tip is extended, a small distance $\bar{\epsilon}$, in different predefined directions and the corresponding crack driving forces for each new tip are evaluated, cf. Figure 2.5. For details on the propagation laws and their algorithmic treatments, see the respective papers.

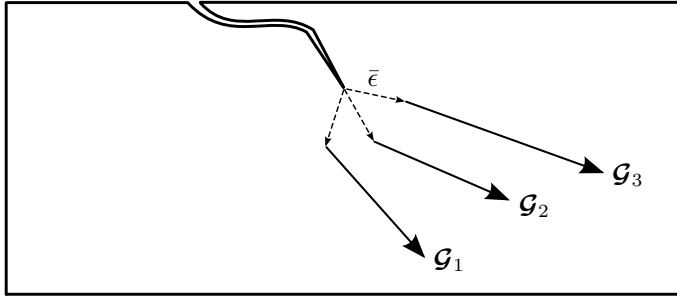


Figure 2.5: *Example of the probing procedure where the crack tip is extended in predefined directions (in this case three) and the corresponding crack driving forces, \mathcal{G} , are evaluated at each crack tip.*

The evolution of the crack is traced by updating the crack geometry based on an extrapolation of the propagation rate. To discretize the updated configuration, associated with the propagation of the crack, a remeshing technique has been adopted. This will reduce the bias associated with having a crack that is constrained to follow the element boundaries. Other, more sophisticated, approaches where discontinuities (such as cracks) can be reproduced within elements are available in literature. For example, the Extended Finite Element Method (XFEM) (Belytschko and Black 1999) or Nitsche’s method with overlapping elements as presented in (Hansbo and Hansbo 2004), are attractive alternatives to remeshing.

2.1.4 RCF damage criteria

In order to predict RCF initiation different models have been proposed in the literature, see (Ekberg and Kabo 2005) for an overview. Two categories of such models (both expressed in terms of the global quantities acting in the contact patch) are: one based on shakedown diagrams presented in e.g. (Ponter et al. 1985; Johnson 1985; Ekberg, Kabo, and Andersson 2002) and $T\gamma$ -models, based on the dissipated frictional energy in the contact, (Burstow 2004; Evans and Burstow 2006). To predict RCF, a damage parameter which expresses probability of RCF initiation is introduced in these models. For comparisons and extensions of these two types of models see (Dirks and Enblom 2011).

Another (qualitative) model based on the frictional energy is presented in (Pointner 2008) where a damage parameter is introduced. From this a damage map is constructed which indicates the risk of fatigue initiation in terms of wear and tangential stresses.

2.2 Wear of rails

Wear of rails is the root cause for a number of problems, often related to dynamic effects caused by profile degradation, such as: poor steering, wheel-climbing phenomena, corrugation etc, see e.g. (Olofsson and Lewis 2009) for an overview.

Concerning the modeling of wear in railway applications, Archard’s model for sliding wear is often adopted, (Archard 1956). However, other models such as the “frictional work”-model have been investigated in the literature, see e.g. (Enblom 2006; Johansson 2005). Archard’s model may be expressed as

$$\dot{w} = k_w \frac{p_N v_s}{H} \quad (2.8)$$

where H is the hardness of the material, p_N is the contact pressure and v_s is the slip velocity. Moreover, k_w is a wear coefficient usually considered to be dependent on v_s and p_N , (Lewis and Olofsson 2004). This coefficient is often represented in so-called wear maps. One example of a wear map is shown in Figure 2.6, which is reproduced from (Jendel 2002). Noteworthy are the large differences in k_w , as shown by the different regions in the map, which indicates a shift in dominating wear mechanism(s). Furthermore, there are also large variations within the regions, which was a main motivation for the work presented in **Paper B**.

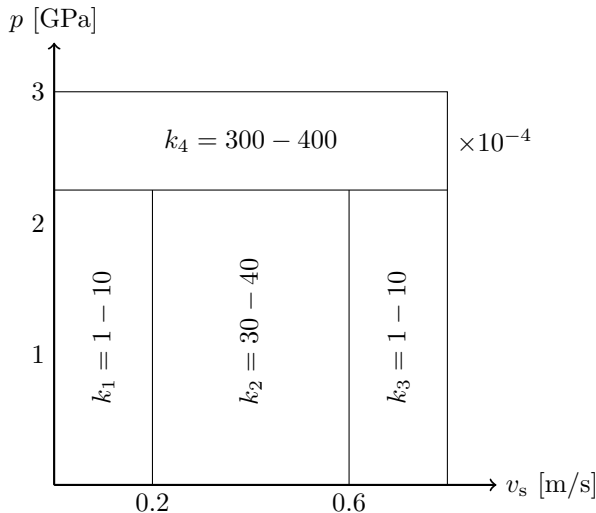


Figure 2.6: *Example of a wear map based on laboratory tests. Reproduced from (Jendel 2002).*

In **Paper B** a numerical procedure for the determination of Archard’s wear coefficient, based on experimental data from a full-scale wheel–rail test rig (Stock and Pippan 2011), is presented. The simulation model used in the procedure is able to account for both wear and plastic deformation of the rail profile.

2.3 Interaction between wear and RCF

Wear and RCF may be considered as parallel and competitive processes. At one hand, if wear rates are high incipient surface cracks in the rail are worn off and wear will in this sense have a beneficial effect. However, excessive wear may lead to a distortion of the rail profile which affects the wheel–rail contact. This in turn may lead to further aggravated wear and RCF. On the other hand, if wear rates are low, surface cracks may grow to a size where corrective grinding (artificial wear) of the rail is needed in order to reduce initiated cracks and prevent rail breaks. Finding a good balance between RCF crack growth and (artificial wear) is crucial. For example, optimization of grinding procedures such as preventive grinding strategies may lead to large savings in maintenance costs, see e.g. (INNOTRACK 2009). In addition, new rails may be ground in order to obtain more conformal contact and thus lower the contact pressure. Grinding may also be needed in order to remove mill scale that may have formed on the rail surface.

Several researchers have studied the interaction between wear and crack growth, see e.g. (Ringsberg and Bergkvist 2003; Donzella 2005; Stock and Pippan 2011; Franklin et al. 2001; Kapoor et al. 2003). In these references, models for the development of RCF damage in rails under specific loading conditions (e.g. given normal and tangential load, fluid pressurization of crack faces etc.) have been investigated; however, other models expressed in terms of system quantities have been proposed. One such model is the $T\gamma$ -model, mentioned in Section 2.1.4, which indirectly accounts for wear due to its dependence on the slip developed in the contact. In that model, a damage parameter is represented as a function of the frictional energy dissipated in the contact, i.e. tangential force T multiplied with the creepage γ . The shape of these curves are most often idealized such as in Figure 2.7. For calibration and application of the model see (Evans and Burstow 2006; Öberg and Andersson 2009; Lewis and Olofsson 2004).

One drawback of the $T\gamma$ -model, however, is the requirement of calibration against large number of experimental tests which are both time consuming and expensive. One alternative may be to construct such damage models based on numerical simulations rather than testing. This is the topic of **Paper D**, where numerically simulated $T\gamma$ -curves are presented for various loading conditions. The damage parameter chosen, in this study, is the net crack growth rate in the depth direction and is defined as

$$\left(\frac{d a_z}{d N}\right)_{\text{net}} = \frac{d a_z}{d N} - \frac{d w}{d N} \quad (2.9)$$

where the wear rate dw/dN has been subtracted from the crack propagation rate da_z/dN . In Figure 2.7, a typical simulated $T\gamma$ -curve is compared to an idealized $T\gamma$ -curve.

In the context of wear and RCF, it should also be noted that due to the large plastic deformation of the rail surface (as mentioned in Section 2.1) the rail surface experiences a large increase in hardness; as a consequence the wear resistance is increased (decreased wear rate). Note also that the plastic deformations of the rail surface will influence the crack growth resistance, (Wetscher et al. 2007). However, in this thesis the effect of increased hardness is disregarded. Again, this can partly be motivated if a state of elastic shakedown is considered, although residual stresses may need to be considered.

Moreover, some of the mechanisms that cause wear may themselves be considered as a form of RCF, for example wear by plastic ratcheting, see e.g. (Kapoor 1997; Franklin et al. 2001; Torrance 1996) for details.

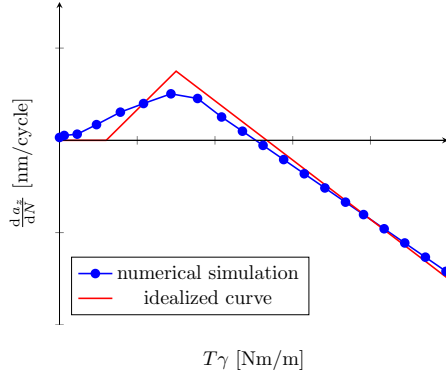


Figure 2.7: *Schematic example of net crack growth rate as a function of frictional work in the contact patch. The graph compares an idealized experimental curve to an idealized numerically simulated one. Negative values indicate that the crack will gradually be worn away.*

3 Summary of appended papers

Paper A: Strategies for planar crack propagation based on the concept of material forces

In this paper, a computational framework for the simulation of planar crack growth (which includes kinking) is presented. A viscous propagation law is adopted and expressed in terms of a crack driving force derived from the concept of material forces. The propagation law is shown to give rise to different propagation strategies depending on the choice of time discretization and assumptions regarding regularity of the crack driving force. In particular, three such strategies are presented: Explicit Proportional Extension (EPE), Implicit Proportional Extension (IPE) and Maximum Parallel Release Rate (MPRR). These strategies are evaluated under monotonic and cyclic loading and it is concluded that the proposed strategies produce almost identical results regarding propagation rate and direction. Furthermore, the results are shown to be robust with respect to time discretization.

Paper B: Prediction of wear and plastic flow in rails—Test rig results, model calibration and numerical prediction

This paper presents a numerical procedure for the simulation of rail profile evolution in conformal contacts accounting for both wear and plastic deformations. The procedure considers the effect of wear and plasticity in a staggered fashion. The conformal contact is treated by applying a multi-Hertzian approach. To account for

plastic deformations, a 2D elasto–plastic FE analysis is carried out in conjunction with a 3D local contact analysis in the commercial finite element (FE) software ABAQUS.

As a numerical example, a wear coefficient is calibrated from experimental data collected for a rail made from the pearlitic steel grade R260. The loading in the test rig resulted in a highly conformal wheel–rail contact. The contact conditions and the applied loading resulted in (numerically predicted) elastic shakedown (i.e. no further net accumulation of plastic deformation) after only a few load cycles. Therefore, plastic deformations were disregarded in the subsequent analysis. It could be noted that the validity of the contact model (multi-Hertzian) employed in the MBS simulations is somewhat questionable in the current case of conformal contact. However, quantitatively good results, in terms of worn-off area and the shape of the worn profile were obtained from the simulations, see Figure 3.1.

A similar procedure was adopted within the European project INNTRACK (Ekberg and Paulsson 2010) for the simulation of degradation of rail profiles in switches and crossings, see (Johansson, Pålsson, et al. 2011) for details.

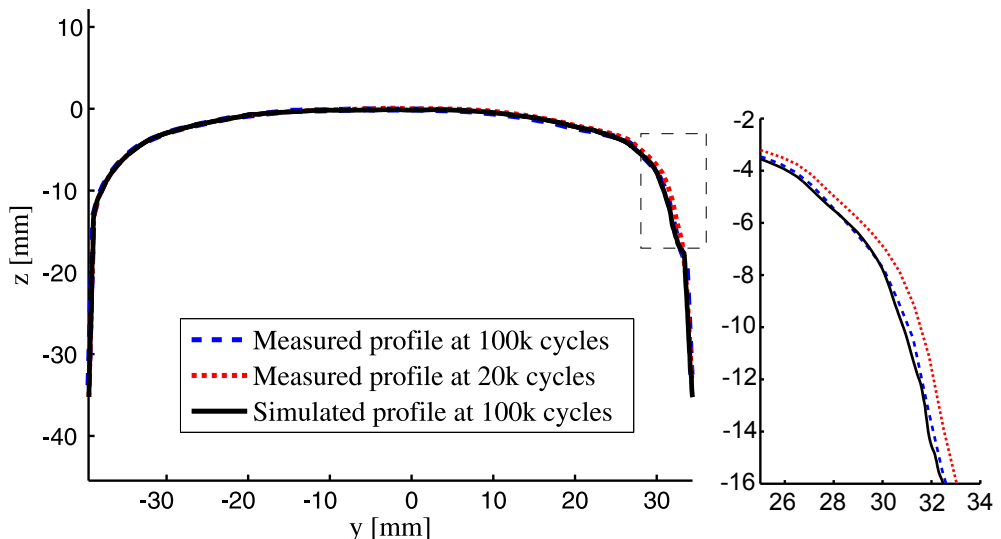


Figure 3.1: *Example of a measured and simulated rail profile corresponding to test in a linear full-scale test rig.*

Paper C: Crack propagation in rails under Rolling Contact Fatigue loading conditions based on material forces

In this paper, results from numerical simulations of RCF surface crack growth in an idealized rail are presented. A rate independent propagation law, expressed in the time domain, is proposed. The propagation law is integrated over a load cycle and then extrapolated a large number of cycles. This procedure is repeated until the predefined number of load cycles has been simulated. The propagation model has

been used for parametric studies of crack propagation to investigate the influence of key parameters such as loading and crack geometry. Results from these studies suggest that anisotropic effects due to the plastically deformed surface layer may need to be included in order to predict head check growth in rails accurately. In particular regarding the prediction of crack curving.

Moreover, it is shown in the paper that the perpendicular component of the crack driving force \mathcal{G}_\perp is highly dependent on a modeling parameter (size of the region where \mathcal{G} is evaluated) for a RCF load case. As a consequence, two of the propagation strategies adopted in **Paper A** must be discarded in simulations of RCF crack growth. Furthermore, it is shown that the contribution to the crack driving force from tractions along the crack faces is dependent on the computational mesh.

Paper D: Numerical prediction of the impact of wear on Rolling Contact Fatigue crack growth in rails

In this paper, the impact of wear on crack growth under various loading conditions is investigated by numerical simulations. Here, the effect of rail head wear is included through crack truncation. Predicted net crack growth rates are presented for different values of the dissipated contact energy $T\gamma$. Further, the influence of a number of parameters on the net crack growth rate is presented for different values of the dissipated contact energy $T\gamma$. Studied parameters include surface friction, contact pressure, fluid pressurization of the crack faces and the wear coefficient. From these studies it is concluded that the net growth rate is highly sensitive to all the studied parameters. This highlights the need for accurate estimates of contact forces and material parameters in order to quantitatively determine the impact wear has on crack growth in rails.

Paper E: The effect of anisotropy on crack propagation in pearlitic rail steel

At the rail surface, large plastic deformations accumulate under RCF loading. This will induce anisotropy in the mechanical properties of the material such as the resistance against crack growth. In this paper, the computational framework presented in **Paper C** is extended to include this observed anisotropy in the fatigue crack growth resistance. This is done through the introduction of an anisotropic fatigue crack growth threshold, comparable with an anisotropic yield surface, which is then used to study the effect of anisotropy on crack growth.

Results from the simulations show that the anisotropic layer, thickness as well as the amount of the anisotropy, has a prominent influence on the crack path. In effect, it determines what direction the crack will grow; up or down.

4 Practical use of the research

Much effort has been put into the development of a 2D FE model for simulation of crack growth in rails under a wide range of loading conditions. In summary, prominent features of the model include:

- Arbitrary curving/kinking of the crack (planar crack propagation) with automatic updating of the crack geometry.
- Crack truncation through rail head wear.
- An explicit model for crack closure (with possible extension to include crack face friction).
- Simplified modeling of fluid pressurization of the crack faces as the wheel traverses the rail.
- Inclusion of stresses due to rail bending induced by a passing boogie.
- Prescribed temperature loads and residual stresses may be accounted for.
- Anisotropic characteristics of the rail surface material are considered through the use of an anisotropic fatigue crack growth resistance.
- Allows for the contact patch to be divided into stick–slip regions for more realistic traction distribution.
- Different material models are supported, e.g. a plasticity model which incorporates non-linear isotropic and kinematic hardening.

The developed model may, for example, be used (independently) for qualitative studies of the following kind:

- The effect a change of rail material may have on crack propagation.
- The influence different degrees of anisotropy (from plastic deformations) have on crack propagation.

These types of studies may provide support in regard to the probability of spalling, crack arrest and transversal fracture. Furthermore, the developed model may be combined with an MBS software to form a more versatile numerical framework. This may open up for more quantitative studies such as:

- Simulation of the evolution of surface cracks in order to determine the number of cycles before some alarm limit is reached, e.g. a critical crack depth.
- Investigate how changes in running and surface conditions such as: axle load; velocity; surface roughness; etc. may affect the net crack growth rate.

Alternatively, the model may be used in combination with in–field traffic data for calibration of material parameters, e.g. the wear coefficient or the crack propagation law.

In summary, the developed model may have several applications with a common goal to optimize maintenance and asset management. Furthermore, the use of the model may lead to a better understanding of the physics behind the phenomenon RCF crack growth in rails.

5 Concluding remark and future work

This thesis is concerned with numerical simulation of the impact of wear on crack growth in rails. To this end a versatile 2D FE model has been developed. The model is capable of simulating crack growth and can account for crack truncation due to wear. The framework for numerical simulation of crack growth is based on the concept of material forces. In **Paper A**, three different strategies for planar crack propagation were developed for simple load cases and it was shown that they produce almost identical crack paths and are numerically robust.

In **Paper C**, growth of surface cracks in rails is studied. Here, a propagation law is proposed where the focus is on the propagation direction and not the growth rate. For the employed model to be applicable for quantitative simulation of crack growth, the propagation model must be modified and calibrated against experimental data.

Large plastic deformations of the rail surface lead to anisotropy in the mechanical properties of the rail material. This anisotropy may, in the current numerical framework, be accounted for in different ways. One possibility is to introduce an anisotropic material model which accounts for the development of plastic anisotropy with loading. This would then (automatically) be reflected in the crack driving force. Promising material models that take into account the development of plastic anisotropy of pearlitic steel are presented in (Johansson, Menzel, et al. 2005; Johansson and Ekh 2006; Larijani, Johansson, et al. 2012).

Another approach is to introduce an anisotropic fatigue crack growth threshold such that the resistance against crack growth is directional dependent. This approach is adopted in **Paper E** where an anisotropic fatigue crack growth threshold is included in order to study the effect of anisotropy on the crack propagation direction. It is shown that the effect of anisotropy may have a large influence on the crack path and it is, therefore, imperative to study this effect in more detail. Note also that this approach is more computationally efficient than to adopt a more sophisticated material model. Generally though, both approaches may need to be combined in order to accurately simulate the growth of surface cracks in rails.

In **Paper D**, wear impact on crack growth in rails is studied through numerical simulation of crack growth and wear and the evaluation of a net crack growth rate. From parametric studies it is seen that the predicted net crack growth rate is highly sensitive to the value of the wear coefficient (in Archards' wear law). It is, therefore, important to have accurate values for the wear coefficient or alternatively more sophisticated wear models.

In relation to the modeling of wear, the determination of Archard's wear coefficient was in **Paper B** carried out for a case of conformal contact using multi-body dynamic simulations featuring a multi-Hertzian contact model. It is known that the contact pressure may deviate from a Hertzian distribution under certain contact conditions, e.g. conformal contact, significant plastic deformation, presence of surface asperities etc., see e.g. (Ayasse and Chollet 2005; Hannes and Alfredsson 2012). Thus, the modeling errors associated with the multi-Hertzian contact model, for this kind of loading, are unknown and must be investigated further.

Furthermore, the hardness of the rail surface increases due to plastic deformations;

however, this hardened surface will eventually be worn away by natural or artificial wear (grinding). How the process of wear influences the residual stress distribution and consequently crack growth needs to be studied in more detail; although, the numerical tool for these kind of studies are available within the developed FE model.

All cracks encountered in rails are 3D but they have in this thesis been studied through idealized 2D models. It is still unclear how well the adopted 2D models correlate to the real behaviour of the rails. To evaluate this, the 2D models need to be verified against 3D models and experimental results. Some work has been conducted in these lines where the magnitude of the 2D loads have been scaled in order to obtain comparable stress intensity factors between 2D and 3D models, see (Tillberg 2010b). Furthermore, single cracks are not observed in practice but rather a network of cracks which interact and may give rise to shielding effects, (Tillberg et al. 2009). Therefore, the effect of multiple cracks should be investigated further.

RCF and wear are relatively slow processes which may develop in the span of up to several million load cycles. Due to the extensive computational efforts related to simulations of such long-term processes, the development of efficient strategies for rail profile updating under many load cycles is a further important research task. This is particularly challenging if plasticity and ratcheting behavior are included in the material model since the response is then highly nonlinear.

Finally, all simulations of crack propagation in this thesis have been restricted to elastic material behavior. This is in many cases not fully realistic as there may often be plastic deformations at the rail surface (although elastic shakedown may be reached). Inelastic material behavior has not been included due to numerical issues with mesh dependent results in the evaluation of the crack driving force. This appears to be a consequence of the chosen formulation of the crack driving force. It is still unclear how an appropriate crack driving force should be formulated which is numerically robust regardless of the material model, physically sound and possible to calibrate against (available) test data. The concept of material forces has taken us far in this regard and it may very well provide (part of) the solution; however, further exploration is still needed.

References

- Archard, J. (1956). The wear of metals under unlubricated conditions. *Proceedings of the Royal Society A* **236**.1206, 397–410.
- Ayasse, J. and Chollet, H. (2005). Determination of the wheel rail contact patch in semi-Hertzian conditions. *Vehicle System Dynamics* **43**.3, 161–172.
- Belytschko, T. and Black, T. (1999). Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering* **62**0. July 1998, 601–620.
- Bergkvist, A. (2005). *On the crack driving force in elastic–plastic fracture mechanics with application to rolling contact fatigue in rails*. Tech. rep. Chalmers University of Technology.
- Bogdański, S. and Brown, M. (2002). Modelling the three-dimensional behaviour of shallow rolling contact fatigue cracks in rails. *Wear* **253**.1-2, 17–25.
- Brouzoulis, J. (2012a). Numerical prediction of the impact of wear on Rolling Contact Fatigue crack growth in rails. *To be submitted for international publication*.
- (2012b). RCF crack growth in rails under plastic material behaviour. *Technical report*.
- Brouzoulis, J. and Ekh, M. (2012). Crack propagation in rails under Rolling Contact Fatigue loading conditions based on material forces. *International Journal of Fatigue* **45**, 98–105.
- Brouzoulis, J., Larsson, F., and Runesson, K. (2010). Strategies for planar crack propagation based on the concept of material forces. *Computational Mechanics* **47**.3, 295–304.
- Brouzoulis, J., Torstensson, P. T., Stock, R., and Ekh, M. (2011). Prediction of wear and plastic flow in rails—Test rig results, model calibration and numerical prediction. *Wear* **271**.1-2, 92–99.
- Burstow, M. (2004). *Whole Life Rail Model application and development: Development of a rolling contact fatigue damage parameter*. Tech. rep. London: Rail Standards and Safety Board.
- Canadinc, D., Sehitoglu, H., and Verzal, K. (2008). Analysis of surface crack growth under rolling contact fatigue. *International Journal of Fatigue* **30**.9, 1678–1689.
- Cannon, D. F., Edel, K. O., Grassie, S. L., and Sawley, K. (2003). Rail defects: an overview. *Fatigue Fracture of Engineering Materials and Structures* **26**.10, 865–886.
- CHARMEC. <http://www.charmec.chalmers.se/>.
- Denzer, R., Barth, F., and Steinmann, P. (2003). Studies in elastic fracture mechanics based on the material force method. *International Journal for Numerical Methods in Engineering* **58**.12, 1817–1835.
- Dirks, B. and Enblom, R. (2011). Prediction model for wheel profile wear and rolling contact fatigue. *Wear* **271**.1-2, 210–217.
- Donzella, G. (2005). The competitive role of wear and RCF in a rail steel. *Engineering Fracture Mechanics* **72**.2, 287–308.
- D. Stone (2004). *Atlas of Wheel and Rail Defects*. Tech. rep. Paris, France: UIC.
- Ekberg, A. and Kabo, E. (2005). Fatigue of railway wheels and rails under rolling contact and thermal loading—an overview. *Wear* **258**.7-8, 1288–1300.

- Ekberg, A., Kabo, E., and Andersson, H. (2002). An engineering model for prediction of rolling contact fatigue of railway wheels. *Fatigue & Fracture of Engineering Materials and Structures* **25.10**, 899–909.
- Ekberg, A. and Paulsson, B., eds. (2010). *Concluding Technical Report INNOTRACK, Innovative Track Systems*. Solna: Intellecta Infolog.
- Ekberg, A. and Sotkovszki, P. (2001). Anisotropy and rolling contact fatigue of railway wheels. **23**, 29–43.
- Enblom, R. (2006). “On simulation of uniform wear and profile evolution in the wheel-rail contact”. PhD thesis. Royal Institute of Technology (KTH), Sweden.
- Evans, J. and Burstow, M. (2006). Vehicle/track interaction and rolling contact fatigue in rails in the UK. *Vehicle System Dynamics* **44**.sup1, 708–717.
- Fagerström, M. and Larsson, R. (2008). Approaches to dynamic fracture modelling at finite deformations. *Journal of the Mechanics and Physics of Solids* **56.2**, 613–639.
- Fajdiga, G., Ren, Z., and Kramar, J. (2007). Comparison of virtual crack extension and strain energy density methods applied to contact surface crack growth. *Engineering Fracture Mechanics* **74.17**, 2721–2734.
- Fajdiga, G. and Sraml, M. (2009). Fatigue crack initiation and propagation under cyclic contact loading. *Engineering Fracture Mechanics* **76.9**, 1320–1335.
- Fischer, F. D., Daves, W., Pippin, R., and Pointner, P. (2006). Some comments on surface cracks in rails. *Fatigue & Fracture of Engineering Materials & Structures* **29.11**, 938–948.
- Fletcher, D., Hyde, P., and Kapoor, A. (2008). Modelling and full-scale trials to investigate fluid pressurisation of rolling contact fatigue cracks. *Wear* **265.9-10**, 1317–1324.
- Franklin, F., Widiyarta, I., and Kapoor, a. (2001). Computer simulation of wear and rolling contact fatigue. *Wear* **251.1-12**, 949–955.
- Gurtin, M. E. and Guidugli, P. P.-G. (1998). Configurational forces and a constitutive theory for crack propagation that allows for kinking and curving. *Journal of the Mechanics and Physics of Solids* **46.8**, 13431378.
- Haidemenopoulos, G. N., Zervaki, A. D., Terezakis, P., Tzani, J., Giannakopoulos, A. E., and Kotouzas, M. K. (2006). Investigation of rolling contact fatigue cracks in a grade 900A rail steel of a metro track. *Fatigue & Fracture of Engineering Materials and Structures* **29.11**, 887–900.
- Hannes, D. and Alfredsson, B. (2012). A fracture mechanical life prediction method for rolling contact fatigue based on the asperity point load mechanism. *Engineering Fracture Mechanics* **83**, 62–74.
- Hansbo, A. and Hansbo, P. (2004). A finite element method for the simulation of strong and weak discontinuities in solid mechanics. *Computer Methods in Applied Mechanics and Engineering* **193.33-35**, 3523–3540.
- Heintz, P. (2006). “Finite Element Procedures for Numerical Simulation of Crack Propagation and Bilateral Contact”. PhD thesis. Chalmers University of Technology.
- Hohenwarter, A., Taylor, A., and Stock, R. (2011). Effect of Large Shear Deformations on the Fracture Behavior of a Fully Pearlitic Steel. *Metallurgical and Materials Transactions A* **42**.June, 1609–1618.
- INNOTRACK (2009). *Dekiverable D4.5.3 – Fields of improvement in grinding practicies as input for LCC evaluations*. Tech. rep., p. 17.

- Jendel, T. (2002). Prediction of wheel profile wear—comparisons with field measurements. *Wear* **253**, 89–99.
- Johansson, A. H. (2005). “Out-of-round railway wheels – causes and consequences”. PhD thesis. Chalmers University of Technology.
- Johansson, A., Pålsson, B., Ekh, M., Nielsen, J. C. O., Ander, M. K. A., Brouzoulis, J., and Kassa, E. (2011). Simulation of wheel–rail contact and damage in switches and crossings. *Wear* **271.1-2**, 472–481.
- Johansson, G. and Ekh, M. (2006). On the modeling of evolving anisotropy and large strains in pearlitic steel. *European Journal of Mechanics A Solids* **25.6**, 1041–1060.
- Johansson, G., Ekh, M., and Runesson, K. (2005). Computational modeling of inelastic large ratcheting strains. *International Journal of Plasticity* **21.5**, 955–980.
- Johansson, G., Menzel, A., and Runesson, K. (2005). Modeling of anisotropic inelasticity in pearlitic steel at large strains due to deformation induced substructure evolution. *Euro. J. Mech. A/Solids* **24.6**, 899–918.
- Johnson, K. (1985). *Contact Mechanics*. Cambridge, UK: Cambridge University Press.
- Kapoor, A. (1997). Wear by plastic ratchetting. *Wear* **212.1**, 119–130.
- Kapoor, A., Fletcher, D., and Franklin, F. (2003). The Role of Wear in Enhancing Rail Life. *Tribological Research and Design for Engineering Systems*, 331–340.
- Larijani, N., Johansson, G., and Ekh, M. (2012). “Hybrid micro-macromechanical modelling of anisotropy evolution in pearlitic steel”.
- Larijani, N., Brouzoulis, J., Schilke, M., and Ekh, M. (2012). The effect of anisotropy on crack propagation in pearlitic rail steel. *To be submitted for international publication*.
- Lewis, R. and Olofsson, U. (2004). Mapping rail wear regimes and transitions. *Wear* **257.7-8**, 721–729.
- Magel, E. E. (2011). *Rolling Contact Fatigue: A Comprehensive Review*. Tech. rep. Washington, DC: Federal Railroad Administration.
- Maugin, G. A. (1995). Material forces : Concepts and applications. *American Society of Mechanical Engineers* **48.5**.
- Menzel, A., Denzer, R., and Steinmann, P. (2004). On the comparison of two approaches to compute material forces for inelastic materials. Application to single-slip crystal-plasticity. *Computer Methods in Applied Mechanics and Engineering* **193.48-51**, 5411–5428.
- Miehe, C. (2007). A robust algorithm for configurational force driven brittle crack propagation with R-adaptive mesh alignment. *International Journal for Numerical Methods in Engineering* **72.2**, 127–155.
- Öberg, J. and Andersson, E. (2009). Determining the deterioration cost for railway tracks. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* **223.2**, 121–129.
- Olofsson, U. and Lewis, R., eds. (2009). *Wheel–rail interface handbook*. Woodhead Publishing Limited and CRC Press LLC.
- Pointner, P. (2008). The Impact of Wear and Rolling Contact Fatigue on Rails – A Pragmatic Approach. *ZEVrail* **132.132**, 2–10.
- Ponter, A., Hearle, A., and Johnson, K. (1985). Application of the kinematical shakedown theorem to rolling and sliding point contacts. *Journal of the Mechanics and Physics of Solids* **33.4**, 339–362.

- Ringsberg, J. (2005). Shear mode growth of short surface-breaking RCF cracks. *Wear* **258**.7-8, 955–963.
- Ringsberg, J. and Bergkvist, A. (2003). On propagation of short rolling contact fatigue cracks. *Fatigue & Fracture of Engineering Materials & Structures* **26**, 969–983.
- Ringsberg, J., Loo-Morrey, M., Josefson, B., Kapoor, A., and Beynon, J. (2000). Prediction of fatigue crack initiation for rolling contact fatigue. *International Journal of Fatigue* **22**.3, 205–215.
- Schütte, H. (2009). Curved crack-propagation based on configurational forces. *Computational Materials Science* **46**.642-646.
- Stock, R. and Pippin, R. (2011). RCF and wear in theory and practice – The influence of rail grade on wear and RCF. *Wear* **271**.1-2, 125–133.
- Suresh, S. (1998). *Fatigue of Materials*. 2nd ed. Cambridge, United Kingdom: Cambridge University Press.
- Tillberg, J. (2010a). “Elastic-Plastic Fracture Mechanics–Application to Rolling Contact Fatigue in Rails”. PhD thesis. Chalmers University of Technology.
- Tillberg, J., Larsson, F., and Runesson, K. (2009). A study of multiple crack interaction at rolling contact fatigue loading of rails. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* **223**.4, 319–330.
- (2010). On the role of material dissipation for the crack-driving force. *International Journal of Plasticity* **26**.7, 992–1012.
- Tillberg, J. (2010b). “Elastic-Plastic Fracture Mechanics – Application to Rolling Contact Fatigue in Rails”. PhD thesis. Chalmers University of Technology.
- Torrance, A. A. (1996). The influence of surface deformation on mechanical wear. *Wear* **200**.1-2, 45–54.
- UIC (2002). *Rail defects*. Tech. rep. Paris, France: UIC.
- Wetscher, F., Stock, R., and Pippin, R. (2007). Changes in the mechanical properties of a pearlitic steel due to large shear deformation. *Materials Science and Engineering: A* **445-446**, 237–243.