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Citation for the original published paper (version of record):
Holmqvist, C., Belzig, W., Fogelström, M. (2012). Critical charge and spin Josephson currents through a precessing spin. Journal of Physics: Conference Series, 400(2): 022027-.
http://dx.doi.org/10.1088/1742-6596/400/2/022027

Journal of Physics: Conference Series

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To cite this article: C Holmqvist *et al* 2012 *J. Phys.: Conf. Ser.* **400** 022027

26th International Conference on Low Temperature Physics (LT26) IOP Publishing
Journal of Physics: Conference Series **400** (2012) 022027 doi:10.1088/1742-6596/400/2/022027

Critical charge and spin Josephson currents through a precessing spin

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Abstract. We present a theoretical study of two superconductors coupled over a spin. The spin is treated classically and is assumed to precess with the Larmor frequency due to an external magnetic field. The precession results in spin-dependent Andreev scattering and a non-equilibrium population of the Andreev levels. Charge and spin currents at zero temperature were studied previously [1]. Here, we focus on the critical current as well as the corresponding spin currents at finite temperatures. At finite temperatures, the spin precession can enhance the supercurrent by a population redistribution. The enhancement leads to a modified current-phase relation and a non-monotonous critical current as function of temperature. This non-monotonous behavior is accompanied by a corresponding change in spin-transfer torques acting on the precessing spin and leads to the possibility of using temperature as a means to tune the back-action on the spin.

The electron's spin degree of freedom can be combined with conventional electronics in spintronics devices. The exchange interaction between the spins of conduction electrons and ferromagnets can for instance be used to read out and store information [2]. The field of spintronics has recently received increased attention with the appearance of magnetic molecules, whose molecular sizes and long magnetization relaxation times at low temperatures [3] make them suitable for high-density information storage [4]. Interesting spin-based phenomena can occur if a ferromagnet is combined with other states of matter. Ferromagnetism and superconductivity are usually two incompatible phases if one considers bulk properties. However, they can be combined in nanoscale junctions resulting in new spin-based effects [5].

Here, we study the interplay between the dynamics of the spin of a nanomagnet and the transport properties of a Josephson junction. We consider two superconducting leads coupled over a nanomagnet whose magnetization direction processes with the Larmor frequency, ω_L, and evaluate the critical current as function of temperature. In the case of $\omega_L = 0$, the critical charge current, $j_c^{ch}(T)$, follows the Ambegaokar-Baratoff temperature dependence [6], $j_c^{ch}(T) \propto \Delta(T)\tanh[\Delta(T)/2T]$. However, the critical current for finite values of ω_L shows an enhancement at high temperatures which we find is due to a redistribution of the quasiparticle population generated by the dynamics of the nanomagnet (see Fig. 1). Our starting point is the model used by Zhu *et al.* [7], but as in Ref. [1] we extended it to include arbitrary tunneling strengths. We consider two superconducting leads coupled over a nanomagnet (see the inset of Fig 1(a)), which may be either a single magnetic molecule or a ferromagnetic nanoparticle. The nanomagnet is subjected to an external effective magnetic field, $\boldsymbol{H}$, which couples to the spin of the nanomagnet, $\boldsymbol{S}$, through the Hamiltonian $\mathcal{H}_B = -\gamma \boldsymbol{H} \cdot \boldsymbol{S}$, where γ is the gyromagnetic ratio. In the classical limit, the external magnetic field results in a torque acting on the spin if the field is applied at an angle ϑ with respect to $\boldsymbol{S}$. This torque gives an equation of motion which can be written as $\dot{\boldsymbol{S}} = -\gamma \boldsymbol{H} \times \boldsymbol{S}$. The spin hence precesses with the Larmor frequency given by $\omega_L = \gamma|\boldsymbol{H}|$. The leads are described by BCS Hamiltonians and the tunneling Hamiltonian, $\mathcal{H}_T$, can be expressed as $\mathcal{H}_T = \sum_{k\sigma;k'\sigma'} c^\dagger_{L,k\sigma} V_{k\sigma;k'\sigma'} c_{R,k'\sigma'} + h.c.$ where $V_{k\sigma;k'\sigma'} = (V_0\delta_{\sigma\sigma'} + V_s(\boldsymbol{S}(t)\cdot\boldsymbol{\sigma})_{\sigma\sigma'})\delta(k-k')$ and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ consists of the Pauli spin matrices. The first term of the hopping amplitude describes spin-independent tunneling, while the spin-dependent tunneling is described by the second term. The second term depends on the instantaneous direction of the spin of the nanomagnet and results in different tunneling amplitudes for spin-up and -down quasiparticles as well as spin flips. Here, we focus on the spin-dependent tunneling and assume that $V_0 = 0$.

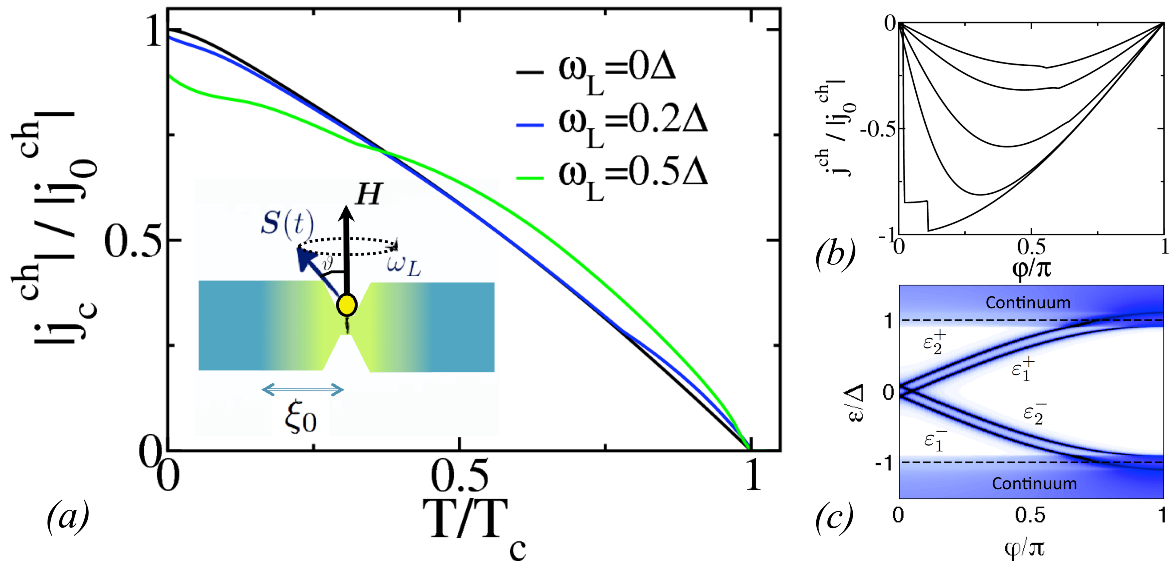

Figure 1. (a) Critical current as function of temperature. The inset shows a Josephson junction coupled to a nanomagnet whose magnetization direction is described by a classical spin, $\boldsymbol{S}$. An external magnetic field, $\boldsymbol{H}$, is applied at an angle ϑ with respect to $\boldsymbol{S}$. The resulting torque causes the spin to precess with the Larmor frequency, ω_L. ξ_0 is the superconducting coherence length. (b) Current-phase relations at temperatures (bottom to top) $T/T_c = 7.5 \cdot 10^{-4}$, 0.25, 0.50, 0.75 and 0.85. Note that the junction is in a π state and that a small peak develops at high temperatures. (c) Density of states, $\rho = -\frac{1}{\pi}\Im\{g^{i,R} + g^{o,R}\}$, as function of energy ε and superconducting phase difference φ. The sharp states inside the superconducting gap are Andreev levels, denoted by $\varepsilon^{+/-}_{1/2}$. The precessing spin produces scattering of the continuum states into the gap as well as the Andreev level splitting. Figures (b) and (c) are plotted with $\omega_L = 0.2\Delta$. All figures are plotted in the high transparency limit, $v_s = 1$, and with $\vartheta = \pi/4$.

We use non-equilibrium Green's functions in the quasiclassical approximation [8] to calculate the junction properties. The quasiclassical Green's functions are propagators describing quasiparticle trajectories and can be separated into incoming propagators, $\check{g}^i$, with trajectories leading into the junction interface, and outgoing propagators, $\check{g}^o$, defined by having trajectories leading out of the interface. The propagators $\check{g}^{i,o}$ are matrices in Keldysh space and consist of retarded ($\hat{g}^R$), advanced ($\hat{g}^A$) and Keldysh components ($\hat{g}^K$). The hat ("ˆ") denotes matrices in Nambu-spin space. The propagators on either side of the interface are connected by solving the T-matrix equation in the quasiclassical formulation [9]. The difference between the incoming and the outgoing propagators lead the expressions for the charge Josephson current and the spin Josephson current per conduction channel in lead $\alpha =$ (left, right) given by

$$j^{ch}_\alpha(t) = \frac{e}{2\hbar}\int \frac{d\varepsilon}{8\pi i}\mathrm{Tr}[\hat\tau_3(\hat g^{i,<}_\alpha(\varepsilon,t) - \hat g^{o,<}_\alpha(\varepsilon,t))] \quad \text{and} \quad \boldsymbol{j}^s_\alpha(t) = \frac{1}{4}\int \frac{d\varepsilon}{8\pi i}\mathrm{Tr}[\hat\tau_3\hat{\boldsymbol\sigma}(\hat g^{i,<}_\alpha(\varepsilon,t) - \hat g^{o,<}_\alpha(\varepsilon,t))], \tag{1}$$

where $\hat\tau_3 = \mathrm{diag}(1,-1)$ and $\hat{\boldsymbol\sigma} = \mathrm{diag}(\boldsymbol\sigma, -\sigma_y\boldsymbol\sigma\sigma_y)$. The lesser ("$<$") Green's functions can be obtained as $\hat g^< = \frac{1}{2}(\hat g^K - \hat g^R + \hat g^A)$. The details of this approach are described in Ref. [1]. Within the quasiclassical approximation, the tunneling strength V_s can be replaced with its Fermi-surface limit, $v_s = \pi N_F S V_s$, where N_F is the density of states at the Fermi surface.

We solve the time-dependent boundary conditions by applying a unitary transformation, $\hat{\mathcal U}(t) = \mathrm{diag}(\mathrm{e}^{-i\frac{\omega_L}{2}t\sigma_z}, \mathrm{e}^{i\frac{\omega_L}{2}t\sigma_z})$, which results in a transformation to a rotating frame of reference where the Fermi surface of the spin-up and -down bands are shifted with $\pm\omega_L/2$. In this rotating frame, the precessing spin, $\boldsymbol S(t)$, now appears static and points along the direction $\boldsymbol e_S = \cos\vartheta\boldsymbol e_z + \sin\vartheta\boldsymbol e_x$. The Andreev levels in the rotating frame are given by

$$\varepsilon^\pm_{1,2} = \pm\Delta\sqrt{1 + \Phi(v_s,\varphi,\vartheta,\omega_L/2\Delta) + \Xi_{1,2}(v_s,\varphi,\vartheta,\omega_L/2\Delta)}, \tag{2}$$

where $\Phi(v_s,\varphi,\vartheta,\omega_L/2\Delta) = -2v_s^2\left[\frac{v_s^2+(1+v_s^4)\cos 2\vartheta}{(1+v_s^4+2v_s^2\cos 2\vartheta)^2} + \frac{1}{(1+v_s^2)^2}\right]\cos^2\frac{\varphi}{2} + \left[\frac{\omega_L}{2\Delta}\right]^2$ and the upper Andreev levels, $\varepsilon^+_{1/2}$, carry charge current in the positive direction while the lower Andreev levels, $\varepsilon^-_{1/2}$, carry charge current in the opposite direction. In the limit of $\vartheta = 0$ and $\omega_L = 0$, the function Ξ reduces to 0 and the Andreev levels are given by $\varepsilon^\pm = \pm\Delta\sqrt{1-\mathcal D_s\cos^2\varphi/2}$, where $\mathcal D_s$ is a spin-transmission coefficient given by $\mathcal D_s = 4v_s^2/[1+v_s^2]^2$. For finite ω_L, the function Ξ is nonzero and gives a Zeemann splitting of the Andreev levels $\varepsilon^{+(-)}$ into $\varepsilon^{+(-)}_{1,2}$ as can be seen in Fig. 1(c) where the density of states, $\rho = -\frac{1}{\pi}\Im\{g^{i,R} + g^{o,R}\}$, is plotted as a function of phase difference φ and energy ε. The effective Zeemann splitting depends on the precession frequency ω_L and the angle ϑ. For $\omega_L \ll \Delta$, the splitting is $\sim \omega_L\cos\vartheta$. The precessing spin causes scattering between the Andreev levels as well as from the continuum states into the gap. This quasiparticle scattering also couples the continuum states to the Andreev levels if $|\Delta| - |\varepsilon^{1(2)}_{-(+)}| \leq \omega_L/2$ and the Andreev levels acquire a broadening.

The occupation of the Andreev levels determines the current-phase relation which is calculated using Eq. 1. As can be seen in Fig. 1(b), the junction is in a π state and displays abrupt jumps at temperatures $T/T_c \to 0$ [1]. As the temperature is increased, these jumps are smoothed out and instead a new peak develops at phase difference φ_p as $T \to T_c$. The peak is due to an increase of the charge current for phase differences $\varphi_p \leq \phi < \pi$ and at sufficiently high temperatures, the critical current is given by $j^{ch}(\varphi_p)$. This increase in charge current generates the enhancement of the critical current shown in Fig. 1(a).

In equilibrium, $g^<_0$ is given by $g^<_0(\varepsilon,\varphi) = 2\pi\rho(\varepsilon,\varphi)\phi^<_0(\varepsilon)$ where $\phi^<_0(\varepsilon)$ is the Fermi distribution function. In non-equilibrium situations, one can define a similar function describing the population as $\phi^<(\varepsilon,\varphi) = g^<(\varepsilon,\varphi)/2\pi\rho(\varepsilon,\varphi)$. The population of the Andreev levels $\varepsilon^-_1(\varphi = 0.6\pi)$ and $\varepsilon^+_2(\varphi = 0.6\pi)$ at temperatures $T/T_c = 7.5\cdot 10^{-4}$ (solid black lines) and $T/T_c = 0.85$ (dashed black lines) is plotted as function of precession frequency in Fig. 2(a). At $T/T_c = 7.5\cdot 10^{-4}$, the lower Andreev level ε^-_1 is fully occupied and $\phi^<(\varepsilon^-_1)$ is independent of ω_L. The upper Andreev level ε^+_2 is correspondingly unoccupied. At temperature $T/T_c = 0.85$, the population of the two Andreev levels are given by $\phi^<_0(\varepsilon^{-(+)}_{1(2)})$ at $\omega_L = 0$. As ω_L increases, the population is constant until $\omega_L \geq |\Delta| - |\varepsilon^{1(2)}_{-(+)}|$ and the Andreev levels are coupled to the continuum states. The coupling between the Andreev levels and the continuum states leads to a redistribution of quasiparticles and an increased population of the lower Andreev level, ε^-_1, and a decreased population of the

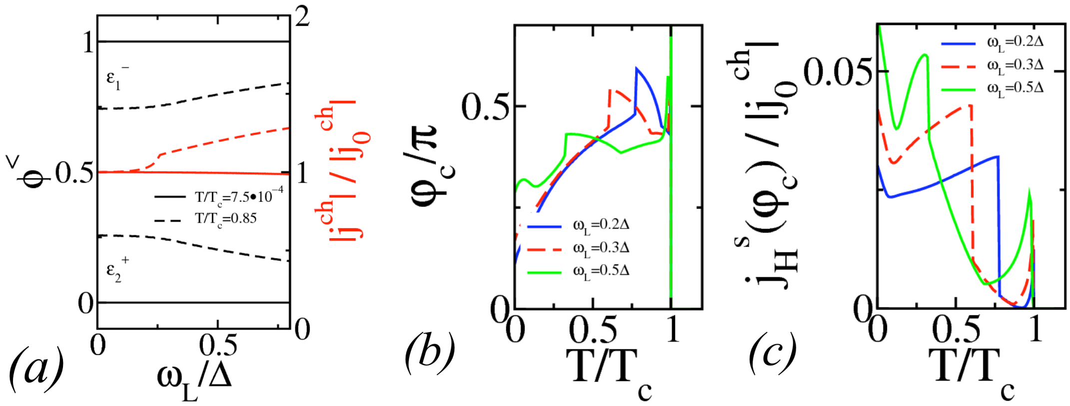

Figure 2. (a) Population, $\phi^<$, of the Andreev levels $\varepsilon_{1(2)}^{-(+)}(\varphi = 0.6\pi)$ at temperatures $T/T_c = 7.5 \cdot 10^{-4}$ (solid black line) and $T/T_c = 0.85$ (dashed black line). The charge current $|j^{ch}(\varphi = 0.6\pi)|$ is plotted for the same temperatures (red lines). (b) The superconducting phase difference corresponding to the critical current, φ_c, plotted as function of temperature. (c) The spin current component j_H^s as function of temperature. The values $v_s = 1$ and $\vartheta = \pi/4$ were used for all plots.

upper Andreev level, ε_2^+. Fig. 2(a) shows the charge current $j^{ch}(\varphi = 0.6\pi)$ at temperatures $T/T_c = 7.5 \cdot 10^{-4}$ (solid red line) and $T/T_c = 0.85$ (dashed red line). For $\omega_L \geq |\Delta| - |\varepsilon_{-(+)}^{1(2)}|$, the supercurrent is enhanced due to the population redistribution. The processes behind the enhanced critical current in Fig. 1(a) are similar to the ones in Ref. [10] in which a supercurrent in a Josephson junction is enhanced by microwave radiation.

The enhancement of the critical current in Fig. 1(a) is accompanied by a sudden change of the superconducting phase difference that corresponds to the critical current, φ_c. This phase difference is plotted as function of temperature in Fig. 2(b). As was shown in Ref. [1], a spin current was found to exist in the vicinity of the junction interface. The Josephson spin current, $\boldsymbol{j}^s$, was shown to have a term $j_H^s(\gamma \boldsymbol{H}) \times \boldsymbol{S}(t)$ which may be measured in a ferromagnetic resonance experiment [11]. The abrupt change in φ_c is correspondingly followed by a jump in j_H^s as plotted in Fig. 2(c). This abrupt step in j_H^s is another signature of the Andreev levels and their non-equilibrium population generated by the precessing spin.

Acknowledgments
The authors wish to thank the DFG and the Swedish Research Council (VR) for support.

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