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ON THE LINEARIZED FIELD THEORY OF TRAVELING WAVE TUBES

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SVEN OLVING

GÖTEBORG 1960

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SVEN OLVING
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The present paper serves as an introduction and summary to a doctoral thesis which includes the following papers:

A. S. OLVING: Electromagnetic and Space Charge Waves in a Sheath Helix, Transactions of Chalmers University of Technology, No 225, 1960 (Reports from the Res. Lab. of Electronics No 49).

B. S. OLVING: A New Method for Space Charge Wave Interaction Studies, Part I, Transactions of Chalmers University of Technology, No 178, 1956 (Reports from the Res. Lab. of Electronics No 37).

C. S. OLVING: A New Method for Space Charge Wave Interaction Studies, Part II, Transactions of Chalmers University of Technology, No 228, 1960 (Reports from the Res. Lab. of Electronics No 51).

Introduction

The main lines of present day research in the field of microwave electronics concern the development of extremely low noise amplifiers and real high power generators. At the same time the extension of the practical usefulness of these devices into the millimeter wave range constitutes an acute and important technical problem. The traveling wave tube (TWT) [1], which is one of the most brilliant inventions in the microwave area, plays a more or less dominant role in connection with these trends.

The TWT in principle can be regarded as a kind of amplifying coaxial transmission line with an electron beam as inner conductor and with a slow wave stucture (e. g. a helix) as outer conductor. Thus the device is characterized by constructional simplicity. The absence of resonant circuit elements makes it exceptionally broad banded (bandwidth more than one octave).

The theory of the TWT is one of the most fascinating subjects in the whole field of microwave electronics and consequently has attracted a great number of authors in the course of time. However, in spite of the constructional simplicity of the device, its theory is markedly complex. It has been suggested that an absolutely complete theory of the TWT is almost out of the question [2]. It is therefore not surprising that many important TWT problems have not yet been dealt with theoretically. Some other problems have been treated, however, with a high degree of uncertainty concerning the correctness of the obtained results. The possibilities of further development of the TWT and related devices therefore depend to a great extent upon the evolution of a more refined basic theory. It is the object of the investigations reported in this thesis to advance the state of the theory.

The aim of paper A is to present a complete mode theory of the well-known sheath helix TWT. A number of new results are found concerning the three dominant waves of the device. Some of the results reveal important mistakes in previous theories. Contrary to the usual procedure, our theory includes the infinite set of higher order modes. Furthermore, the theory reveals some new interesting TWT problems associated with the behaviour of axially inhomogeneous slow wave structures.

Papers B and C are devoted to the development and analysis of a new idealized TWT model — the plane TWT. In this model the TWT phenomena can be described in terms of plane waves. This makes it possible to deal with many advanced physical TWT problems in the simplest conceivable manner.

* * *

Practically all of the extensive theoretical TWT work is concerned with so small signals that the partial differential equations which describe the high frequency behaviour of the device can be linearized. This is a serious limitation in the case of high power TWTs. Some research going back to NORDSIECK [3] has been carried out in the large signal case. We will, however, use the small signal assumption throughout our work since we are primarily interested in the low power tubes.

There are essentially two groups of small signal TWT theories. PIERCE [2] has worked out a method which simplifies the mathematics as far as possible and cuts down the number of parameters involved. The basic simplification is that all electrons are assumed to be acted upon by known fields. The main results obtained by Pierce agree qualitatively and often also quantitatively with the results of the more rigorous but also more complicated approach originally used by CHU and JACKSON [4] and RYDBECK [5]. The latter method is based directly on the solutions of Maxwell's equations and is the one used throughout this thesis.

Before going into the details of the thesis we should emphasize and explain the fact that the TWT, although now an almost classical device, is expected to retain an important position also in the light of the most modern developments in the field of high sensitivity microwave amplification. Remarkable progress has been made in this general area during the last five years. The maser [6], which has an equivalent noise temperature of a few °K, has been invented. It is, however, a technically complicated device. Although the maser is an invaluable instrument in some branches of research and technique, it is not suited for very wide applications. Another new type of low noise amplifier — the mavar [6, 7] — proves to be very promising.

The mavar noise temperature is less than 100° K at 3 000 Mc/s. Contrary to the maser, which as yet must be cooled to the vicinity of absolute zero, the mavar is operated at room temperature. On the other hand both devices consume high frequency energy for amplification while the classical electronic amplifiers consume *dc* energy. The latter circumstance speaks very much in favour of the conventional electronic amplifiers.

About five years ago the lowest theoretical noise temperature of electronic space charge wave microwave amplifiers, such as the TWT, was believed to be of the order of 1 000° K. It has, however, recently been discovered [8, 9] that this limit can be removed by means of an artificial extension of the low potential region in front of the cathode. This is an extremely important discovery since it means, as far as we know at present, that there is no definite theoretical lowest noise limit for the amplifiers in question. As a practical limit 150° K at 3 000 Mc/s has been suggested which is about half of the noise temperatures already obtained experimentally in connection with backward wave TWTs. It is therefore very probable that the TWT will be widely used even for extremely low noise amplification after some additional years of research and technical development.

The operation of the TWT at lowest practically possible noise levels means to operate the device at its ultimate performance. It is thus necessary not only to study more deeply the noise minimizing mechanism at the cathode but also to investigate, in great detail, the modes of propagation of electromagnetic and space charge waves which carry signals and noise through the interaction section of the TWT. As has already been pointed out the present thesis is mainly concerned with the latter problems.

Summary of the Thesis

Paper A

Electromagnetic and Space Charge Waves in a Sheath Helix

This paper deals with the usual longitudinal TWT model — a well confined circular symmetric electron beam moving axially in a helically conducting circular cylinder sheath. Such an electronic transmission line will propagate an infinite number of circular symmetric wave modes in the direction of electron motion. Three of these modes — the dominant modes — have been the subject of numerous studies. One of the dominant waves increases exponentially with distance, provided the *dc* electron velocity lies within a certain region — the amplification region. This particular growing wave determines the amplification of a TWT. The remaining infinite number of wave modes — the higher order modes — have not been treated successfully in earlier work although they are always excited. However, the methods developed in paper A allow us to deal with all modes.

A theoretical prediction even of the most elementary properties of a TWT requires a rather detailed knowledge of at least the three dominant modes. Nevertheless, our analysis shows that previous theories fail both qualitatively and quantitatively concerning the dominant waves near and at the low electron velocity end of the amplification region. This region, for example, is found to be considerably wider (in certain cases by a factor of two) than earlier work indicates. Furthermore, it is shown that the continuation of the growing waves across the lower boundary of the amplification region is not made by the lowest pair of space charge wave modes as is the case according to the classical picture. We have also developed some new approximate methods to study the amplifying wave including the helix losses. One of the methods can be applied in the case of high *dc* current densities (large C), another is devoted to the in practice important case of radially inhomogeneous *dc* current density distributions.

The higher order modes are of interest, for example, in connection with the research aiming to improve the TWT noise figure, especially since it has been unknown whether these waves are amplifying or not. However, our analysis shows that all higher order modes are non-amplifying in a lossless helix. It has further been shown that they possess a very interesting mode transformation property. Approximate explicit expressions for the higher order wave propagation constants are also deduced.

The more specific contents of paper A are as follows.

1) Some discrepancies in previous TWT theories concerning C and Q are pointed out. C and Q are the well-known parameters of PIERCE [2] associated with the dominant modes.

2) A method is developed which makes it possible to plot the propagation constants of the non-growing waves (i. e. real propagation constants) versus the *dc* electron velocity without any approximations in the dispersion equation. This procedure leads to the important result that all higher order waves are non-growing. It also gives a clear indication about the location of the boundaries of the amplification region and, moreover, allows good guesses as to the approximations one should use in connection with the evaluation of the propagation constants of the growing waves (i. e. complex propagation constants). Furthermore, it immediately follows that the lower part of the amplification region is erroneously treated in previous theories.

The method should be fruitful in connection with azimuthal higher order modes too although this would require considerably more work. It should also be possible to apply it successfully to several other electronic interaction systems.

3) The plot of the non-growing wave propagation constants versus electron velocity reveals a very interesting mode transformation property of these waves (for example a mode transformation of the lowest space charge wave pair to the next lowest space charge wave pair). The practical realization of the indicated transformation would require the use of helices with slowly varying pitch angles. Coupling between the different modes may, however, affect the transformation. Further studies are required for definite conclusions.

4) It is pointed out that the sum of the electromagnetic and kinetic power flows of a growing wave must vanish. This fact is used in order to evaluate C for homogeneous as well as inhomogeneous beams.

5) The dominant waves in a solid homogeneous beam near the region of maximum amplification are treated by RYDBECK's [5] early approximate method. It is shown that previous treatments [5, 10] based on this method have neglected some important terms. The inclusion of these terms increases certain quantities (Q) by 100 % and brings them to harmonize with the result of a different approach by OLVING [11] and a more recent theory by VAINSHTEIN [12]. The product QC is independent of the helix parameters.

6) An integral equation method is developed for the treatment of the dominant waves near the region of maximum amplification. This method makes it possible to work out explicit expressions for C and Q in the practically important case of arbitrary radial variation of the *dc* electron current density. The annular beam with finite thickness is dealt with in detail.

7) It is pointed out that the function $xJ_1(x)/J_0(x)$ is very accurately represented by the expression $\frac{1}{2}x^2/(1 - \frac{1}{8}x^2)$ provided that $|x| \lesssim 1.5$ but regardless of the phase of x. This expression is used to develop a method for treating the dominant waves near the upper boundary of the interaction region. The results are practically equivalent to the well-known work of FLETCHER [13], who uses an entirely different approach. It is concluded that Fletcher's results should not be used near the region of maximum amplification unless the beam is very thin.

8) The expansion $xI_1(x)/I_0(x) \simeq x - \frac{1}{2}$, subject to the condition $\text{Re}\ (x) \gtrsim 1$, forms the basis of a method used to describe the dominant waves near the lower end of the amplification region. The location of the lower boundary of this region is found to be practically independent of the *dc* current density contrary to the results of previous theories. When the beam diameter is large the validity of the expressions obtained by this method is extended to cover the region of maximum amplification. The results are then valid also for high *dc* current densities (large C) in which case the maximum gain becomes independent of the current density provided the beam does not fill the helix completely. It should be mentioned here that this method leads to a new cubic equation for the dominant wave propagation constants. The equation is not of the classical form although it contains the parameters C and Q.

9) The approximate domains of validity of the different methods are discussed. The Smith-type impedance diagram, well-known from

the theory of transmission lines, is shown to provide an easy and quick graphical method for finding the radial propagation index (Δ_0). The magnitude of this fundamental quantity must be checked in connection with the method described under 8). The results of the different methods compare favourably in regions where the corresponding domains of validity overlap.

10) Expressions and/or graphs, emanating from the different methods, are given for the maximum gain, the electron velocity corresponding to the maximum gain, the influence of the helix losses, the velocity boundaries of the amplification region, etc.

11) A comparatively simple approximate representation of the function $xJ_1(x)/J_0(x)$ for real x is deduced. It is then possible to obtain explicit expressions for the propagation constants of the infinite number of higher order waves.

Paper B

A New Method for Space Charge Wave Interaction Studies

Part I

In a TWT the signal propagates in the form of waves. The most elementary type of wave is the infinitely extended plane wave. A thorough understanding of the behaviour of plane waves under different conditions plays a very important role in the interpretation of more complicated wave phenomena.

The basic theories of many electron devices deal with idealized models which are thought to be infinitely extended in the direction transverse to the electron flow. The well-known plane diode theory of Llewellyn [14] may be mentioned as a classical example. Furthermore, one-dimensional plane wave theories have been given for most of the principal space charge wave devices such as the klystron amplifier, the double-stream amplifier, the velocity step amplifier, the resistive medium amplifier, etc., with the exception of the TWT. The aim of paper B therefore is to work out a true plane wave model of the TWT.

The TWT involves a slow wave structure carrying a transverse magnetic (TM) wave with its phase velocity much less than the velocity of light. In a true one-dimensional model this structure must be replaced by an infinite medium supporting a slow TM-wave. Clearly the medium must be anisotropic, otherwise the waves would be of TEM type.

A simple slow TM wave medium is developed in paper B. This medium is penetrated by an infinitely wide electron beam to form the desired one-dimensional TWT model. An elementary analysis of this model is carried out.

The essential features of paper B are as follows:

1) The slow TM wave medium is characterized by anisotropic conductivity — the conductivity is zero except in one direction. The angle between this direction and the direction of motion of the electrons corresponds to the angle between the helix wire and the axis in a helical type TWT.

2) The characteristic small signal TWT-waves are found directly as a consequence of the application of MAXWELL's equations and the equation of motion.

3) The plane TWT model bears a close analogy to the sheath helix TWT model, especially when the electron beam in the latter model fills up the helix completely.

4) The dispersion equation of the plane system contains also the resistive-medium amplifier waves.

5) The plane TWT has considerably higher gain per wavelength than other one-dimensional distributed interaction amplifiers provided the *dc* electron current density is low. At very high *dc* current densities the resistive medium and double stream amplifiers will have higher gain than the plane TWT.

Paper C

A New Method for Space Charge Wave Interaction Studies

Part II

In paper C the theory of the plane TWT is carried further. The following conclusions have been reached.

1) The plane system is modified in such a way as to make it possible to study the input and output properties analogous to those of a practical TWT. The signal input reflection coefficient in particular is surprisingly small under a variety of conditions.

2) The plane geometry we are dealing with makes it possible to give an exact treatment of some interesting power flow problems in connection with the growing waves.

3) A simultaneous application of a combination of several amplification mechanisms (TWT, resistive medium, double stream) will not increase the gain over the single mechanism level.

4) A new pair of growing waves will develop in the plane TWT when the *dc* current density exceeds a certain value. This is explained by interaction between the fast space charge wave and the backward electromagnetic wave. Both waves are of evanescent type.

5) Similar waves as outlined under 4) are searched for in a sheath helix TWT with negative result. However, it is shown in this connection that the maximum gain (in *dB*) of a high current helical TWT (beam filling helix completely) is proportional to $(i_0)^{1/6}$, i_0 being the *dc* current density. The corresponding proportionality factor is $(i_0)^{1/4}$ for the high current plane TWT and $(i_0)^0 =$ constant for the high current helical TWT (beam not filling helix completely). At low current densities the maximum gain obeys the well-known $(i_0)^{1/3}$-law for the helical as well as the plane TWT.

6) The fundamental fourth order wave equation for the waves in an axially inhomogeneous plane TWT is worked out. The case of slow axial variation of the conductivity direction of the slow TM wave medium is treated in detail. The most elementary form of the WKB-type solutions can be shown to correspond to power conservation in the individual modes. The important problem of the transition

from a non-growing wave pair to a growing wave pair and vice versa is solved by a more advanced analytical method. This is a fundamental problem in the theory of growing wave propagation in inhomogeneous media.

7) The relativistic plane TWT dispersion equation, including an arbitrary *dc* magnetic focusing field and collisional friction, is given. The nature of the eight different waves, contained in this equation, are analysed. Of these the so called transverse space charge waves have led to some controversial interpretations [15]. The physical meaning of these waves is therefore discussed in detail. The importance of the *ac* drift Lorentz force in connection with the transverse waves is pointed out.

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