Thesis for the Degree of Licentiate of Engineering

Accuracy and Delay: An Inherent Trade-off in Cooperative UWB Navigation

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To my Family
Abstract

Location-aware applications and wireless sensor networks are becoming essential in our daily lives from a commercial, and public perspectives. The need of localization information to drive the applications is a key requirement. New technologies have emerged to tackle the problem of the limitations of the Global Positioning System (GPS) solutions. Ultra-wideband (UWB) is one of those emerging RF-technologies. The thrive in search for better accuracy involves improved ranging algorithms, higher transmission powers, and the use of cooperation among nodes. The goal of this thesis is to investigate the trade-off between medium access control (MAC) delay and accuracy for UWB systems based on hands-on experience and practical implementation with state-of-the-art equipment, based on two-way-ranging and a spatial time division multiple access scheme (STDMA).

Paper A investigates the connection between accuracy and MAC delay for noncooperative scenarios. We quantify, by means of lower bounds how traditional methods to improve accuracy such as increased number of anchors, and increased communication range comes at a significant cost in terms of delay. Techniques such as selective ranging and eavesdropping help alleviate the trade-off and reduce the MAC delay in favor of mobile networks with tolerable accuracies. Paper B extends the work for cooperative scenarios, where nodes cooperate with each other by means of shared information. This sharing has an impact not only on the position accuracy but also on the MAC delay which we quantify by means of lower bounds, both for the accuracy and MAC delay. Once again, selective ranging is evaluated to reduce the MAC delay for finite cooperative networks. We show how indiscriminate cooperation leads to large MAC delays, which has a direct impact on the update rate for high mobility scenarios. Finally, Paper C unifies all findings by including derivations of the accuracy and MAC delay lower bounds for noncooperative and cooperative networks, evaluating selective ranging and eavesdropping to cope with the trade-off in different conditions. Numerical evaluations are included for several distinct operations. Furthermore, we characterize the trade-off behavior for dense-location aware networks for both noncooperative and cooperative cases by means of scaling laws. We conclude by introducing a delay/accuracy parameter which can uniquely quantify the trade off between accuracy and MAC delay as a function of the agent and anchor density. Noncooperative eavesdropping shows to outperform cooperative networks in terms of accuracy with reasonable delays. Finally, in terms of scaling, we found that, under certain conditions, standard cooperative positioning exhibits the worst possible trade-off among the considered strategies.

Keywords: Ultra-wideband, S-TDMA, MAC delay, navigation, positioning, localization.
List of Included Publications

This licentiate thesis is based on the following appended papers. They are:


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Part I.

Overview
1. Introduction

Location-aware technologies have revolutionized many aspects of our daily lives by means of applications within the commercial, public and military sectors. From location-based gaming [1], social networking [2], through inventory tracking [3], health-care [4] and habitat monitoring [5], precision agriculture [6], to emergency services [7, 8]. Furthermore, advancements in technology have influenced the utilization of low-cost wireless sensors to sense important information within the environment such as air quality, temperature, humidity, etc. [9]. Therefore, positioning information has become a key requirement to enable the use of location-aware and navigation applications as well as the deployment of wireless sensor networks (WSN) since collected information is only meaningful when attached jointly with position information [10].

The viability of these applications, the utilization of WSN, and the core of navigation rely extensively on the low device cost and the capability of self-organizing without significant human effort. Moreover, nodes within the networks must be localized in scenarios where they cannot be manually positioned randomly due to time constraints.

The traditional solution technology involves the use of Global Navigation Satellite Systems (GNSSs), such as the Global Positioning System (GPS). Even though GNSSs offer a solution to the localization problem with tolerable accuracy for numerous implementations, including a GPS receiver on each device/sensor is cost and energy prohibitive for most applications. Furthermore, its use is limited to outdoor environments where line-of-sight (LOS) conditions to the satellites apply. Consequently, the development of new techniques and radio frequency-based (RF) technologies has been a prolific research topic in recent years. Common RF-based technologies include the use of wireless local area networks (WLAN), radio frequency identification (RFID), cellular based, bluetooth, and ultra-wide bandwidth systems (UWB), just to name a few. Moreover, other non-RF solutions exist such as inertial navigation systems and cameras, although they are outside the scope of this thesis.

On one hand, traditional positioning performance metrics when choosing an RF technology involves analysis in their complexity, robustness, cost, scalability, and accuracy. Complexity can be related to hardware, software, and operational aspects. A positioning system needs to be able to function normally even in non-ideal or harsh conditions, in other words, it needs to be robust. Cost can be related to money and power consumption. Finally, accuracy has been traditionally the most important requirement for positioning systems. Higher accuracy translates to a better system, although it is highly dependent on the application. Within the different related research tracks, algorithms and techniques have been developed over the years in
search for better accuracy [10–14]. One of this tracks involves cooperation among nodes to distribute and share information over the network [11].

On the other hand, practical implementations introduce communication related constraints due to the channel access required in the medium access control (MAC) layer and the transmission of information over the wireless network. Similarly to the positioning performance metrics, complexity, robustness, cost, scalability and MAC delay need to be considered within the design of WSNs and navigation systems.

In this thesis, we consider cooperative networks, where we study the trade-off between the position and the communication performance metrics. More specifically, we focus on the trade-off between accuracy, arguably the most important metric in terms of positioning, and the MAC delay due to the channel access requirement, that has a direct impact in the update rate when dealing with high mobility networks. This trade-off is investigated and quantified for one of the rapidly growing RF technologies for high precision positioning systems: UWB.

**Thesis Outline**

The positioning problem involves several factors and steps to be considered when designing WSN and navigation systems. In order to place our contributions in the proper context, the thesis is organized as follows: in Chapter 2, we formulate the positioning problem. Moreover, commonly used measurement models in literature are reviewed. Different RF technologies can be related to the types of measurements. A brief description of these technologies along with the benefits and drawbacks is presented. In Chapter 3 the communication constraint in positioning is briefly introduced. More specifically, the different types of MAC protocols suitable for UWB are reviewed. In Chapter 4, we introduce the performance metrics in order to quantify the accuracy and MAC delay within a positioning system. Finally, in Chapter 5, we summarize our contributions.
2. Positioning Basics

In this Chapter, we describe the basics behind the positioning problem and its different phases for RF-based technologies. Furthermore, we formulate the localization problem mathematically and describe the different types of technologies along with their related types of measurements.

The main goal is to localize the nodes, by estimating their two-dimensional geographical coordinates (it can be extended to three-dimensional position), in other words, to estimate their state. In this thesis, we will refer to the state as the two-dimensional location coordinates, although it can include more components such as velocity and orientation. Moreover, we introduce two different types of nodes: anchors, nodes with known states at all times; and agents, nodes with a priori unknown state information. The two-dimensional localization problem traditionally can be categorized into two different phases: the measurement phase and the localization phase [11].

2.1. Measurement Phase

In the first stage, packets are exchanged between neighboring nodes in the network. From the waveforms related to the corresponding packet exchanges, nodes can extract metrics based on the properties of the received signal related to the relative position between nodes. Measurements take advantage of metrics such as received signal strength (RSS), angle of arrival (AOA), and time-of-flight (TOF). In Section 2.4 we briefly describe different types of measurements for several RF-technologies. Measurements can be affected by different error sources such as interference, noise, multipath, non-line-of-sight (NLOS), clock drifts, and environmental conditions [10]. As an example, consider signal blockage for GPS, which prohibits GPS receivers to find a localization solution for the agent/user.

2.2. Localization Phase

The second stage, the localization phase, consists of the estimation of the position of the agents by means of a specific localization algorithm. An important concept within recent years is the use of cooperation. Cooperative algorithms rely on the notion of nodes helping each other by sharing information to determine their positions. Cooperation has received a lot of interest in different areas such as robotics and wireless networks [10,11,15–18]. A simple example of the concept of cooperation
for localization purposes is depicted in Figure 2.1, where agents cooperate with other agents in order to perform ranging measurements in addition to the measurements with anchors. Agents perform measurements with other agents as well, and are not only constrained to anchors. Sharing information is crucial among the agents.

The localization performance is dependent on the specific transmission technology and the implemented localization algorithm. The scope of this thesis is constrained to RF-based technologies, some of which are explained briefly in Section 2.5 and related to the different types of measurements available. Traditionally, the performance of a localization system is mainly given by its coverage and accuracy among some other inherent metrics, such as cost, complexity, scalability, and robustness. In this thesis, based on practical experimentation, we add a metric related to the MAC layer: the MAC delay. Our contributions then quantify the relation between the accuracy and the MAC delay to have a better understanding when designing navigation systems. Both performance metrics are introduced in Chapter 4.

![Figure 2.1: Cooperation involves sharing information and performing measurements among agents.](image)

### 2.3. Problem Formulation

We consider a wireless network consisting of $M$ anchors, nodes with known position, and $N$ agents, nodes with unknown time-varying positions. Mobility is introduced by letting the agents move in discrete time slots of duration $T$. The two-dimensional position of node $i$ at time slot $t$ is denoted by $\mathbf{x}_i^{(t)} = [x_i^{(t)} y_i^{(t)}]^T$. Furthermore, let $\mathcal{N}_i$ be the set containing all the neighboring nodes of node $i$, where nodes $i$ and $j$ can communicate directly with probability $P_{ij} = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2R^2}\right)$, where $R$ is the nominal communication range in meters, as in [19]. Cooperation can then be introduced into the problem when $\mathcal{N}_i$ contains other agents additionally to anchors. The measurements performed by agent $i$ at time slot $t$ with the neighboring nodes is denoted by $\mathbf{z}_i^{(t)}$. Then, the discrete time model for the agents is given by:
\[ x_i^{(t)} = f_{\text{state}}(x_i^{(t-1)}, \varepsilon_i^{(t)}) \]  \\
\[ z_i^{(t)} = f_{\text{meas}}(x_i^{(t)}, x_j^{(t)} \in \mathcal{N}_i, e_i^{(t)}) \]

where \( f_{\text{state}}(\cdot) \) describes the stochastic mobility of the \( i \)-th agent, \( f_{\text{meas}}(\cdot) \) is a deterministic function that defines the type of measurement between two nodes. The variables \( \varepsilon_i^{(t)} \) and \( e_i^{(t)} \) denote the process and measurement noise, and can take any probability function.

The goal is for every agent in the network to estimate its own state at time \( t \), based on the position of the \( M \) anchors nodes and the measurements collected in (2.2). The position of agent \( i \) is estimated by recursively predicting an a priori distribution, \( p(x_i^{(t)}|z_i^{(1:t-1)}) \), and then correcting to an a posteriori distribution \( p(x_i^{(t)}|z_i^{(1:t)}) \) using the available measurements \( z_i^{(t)} \). In this thesis, we focus on a single “snapshot” of the navigation problem, in other words, on a single time slot with fixed anchors and agents.

In the next Section we review different types of RF-based measurements and several RF technologies using these measurements.

## 2.4. Measurements

The localization algorithm depends greatly in the type of measurements. Generally, measurements follow the model introduced in (2.2). In this Section, we provide an overview of typical types of measurements related to RF technologies where LOS conditions exist. Note that this is not an exhaustive list of all existing types of measurements and in some cases nodes can use more than one type of measurement to solve the localization problem.

### 2.4.1. Received Signal Strength

Received signal strength (RSS) takes advantage of the power loss between the transmitter and receiver. Since the transmitted and the received powers are known by the sender and receiver, the attenuation can be calculated [7]. Main error sources for RSS measurements include shadowing and multipath. In general, RSS measurements are simple to implement and relatively inexpensive. However, they are sensitive to dynamic environments. WLAN and Bluetooth localization are mainly based on this type of measurements. The average received power at node \( j \) using the simple Okumura-Hata model based only on relative distance is given by [20]:

\[ z_{ij} = P_0 - 10\alpha \log_{10} \left( \frac{d_{ij}}{d_0} \right) + n_{ij} \]  

(2.3)
where \( d_{ij} = \|x_i - x_j\| \), \( P_0 \) is the power at distance \( d_0 \), and \( \alpha \) is the pathloss exponent, typically between 2 and 6; the error \( n_{ij} \) is modeled as a zero mean Gaussian random variable, i.e., \( n_{ij} \sim \mathcal{N}(0, \sigma^2_{ij}) \).

### 2.4.2. Time of Flight

Round-trip time is the basis behind this type of measurement. In a completely synchronized network, the TOF of the signal can be computed. The estimation of the propagation time of the signals can be done in different ways and it is the basis of time of arrival (TOA), time difference of arrival (TDOA), and two-way time of arrival (TW-TOA). Time delay measurements such as the ones mentioned suffer greatly from NLOS conditions, noise, interference, multipath and clock drifts. This type of measurement is used in systems such as GPS [21], and UWB [22].

#### 2.4.2.1. Time of Arrival

The TOA is the measured time at which the signal arrives at the receiver. For a synchronized network the distance estimate derived from signal’s travel time transmitted from node \( i \) to node \( j \) at time \( t \) is given by [7]:

\[
z_{ij} = d_{ij} + n_{ij},
\]

where \( d_{ij} = \|x_i - x_j\| \) and \( n_{ij} \) is modeled as a zero mean Gaussian random variable, i.e., \( n_{ij} \sim \mathcal{N}(0, \sigma^2_{ij}) \). TOA requires a time reference among the nodes making it not practical in several scenarios.

#### 2.4.2.2. Time Difference or Arrival

In TDOA the measurement involves the distance difference between the agent and two synchronized anchors. It relies on the time difference between two TOA measurements. This measurement is the core of GPS since a receiver measures the TDOA of the received signals from two synchronized satellites. The latter helps to eliminate the clock bias nuisance parameter at the receiver. TDOA measurements are the basis of eavesdropping measurements introduced in the contributions. The distance estimation derived from the TDOA measurement is given by [7]:

\[
z_{ij}^k = d_{ij} - d_{ik} + n_{ij} - n_{ik},
\]

where \( d_{ij} = \|x_i - x_j\| \), \( d_{ik} = \|x_i - x_k\| \), \( n_{ij} \sim \mathcal{N}(0, \sigma^2_{ij}) \), and \( n_{ik} \sim \mathcal{N}(0, \sigma^2_{ik}) \).

#### 2.4.2.3. Two-way time of arrival

TW-TOA involves two TOA measurements between two nodes. Agent \( i \) sends a request to node \( j \), which responds back with an acknowledgement after a predefined time. Both nodes \( i \) and \( j \) estimate the TOA for the request and the acknowledgment,
respectively. Agent $i$ employs the round trip delay between itself and node $j$ to estimate their distance, the measurement is given by [14]:

$$z_{ij} = d_{ij} + \frac{cT_{\text{proc}}}{2} + \frac{n_{ij}}{2} + \frac{n_{ji}}{2},$$  

(2.6)

where $d_{ij} = \| \mathbf{x}_i - \mathbf{x}_j \|$ is the TOA error of the request from node $i$ to node $j$ and $n_{ij} \sim \mathcal{N}(0, \sigma_{ij}^2)$ is the TOA error from the acknowledgement from node $j$ to node $i$, $c$ is the speed of light, and $T_{\text{proc}}$ is a known processing time. TW-TOA overcomes the synchronization burden between nodes. As an example, TW-TOA is used in some UWB systems [22].

### 2.5. RF Technologies

In this Section, we briefly review common RF technologies employing the measurements described in Section 2.4. Note that this list is not exhaustive, but only for illustrative and introductory purposes.

#### 2.5.1. Global Positioning System

The Global Positioning System is one of the most widespread used technologies to solve the localization problem. GPS is a satellite-based radio navigation system. It consists of 3 different segments [21]: (i) the space segment, involving all the aspects concerning the GPS satellites, i.e., the anchors; (ii) the control segment, to check the status and functionality of the space segment; and (iii) the user segment, basically consisting of the users with GPS receivers and antennas: the agents. Based on TDOA measurements, GPS offers a solution for the localization problem, specially in outdoor environments [21].

Unfortunately, GPS-aided solutions are unsuitable in weak signal environments such as urban canyons or indoor environments due to weak signals, NLOS and multipath.

GPS receivers are widely used nowadays in smartphones, and vehicles with big players such as Garmin$^1$, Tom Tom$^2$, Magellan$^3$ and chip set producers such as Qualcomm$^4$, Broadcom$^5$, CSR$^6$, to name a few.

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$^1$http://www.garmin.com/

$^2$http://www.tomtom.com/

$^3$http://www.magellangps.com/

$^4$http://www.qualcomm.com/

$^5$http://www.broadcom.com/

$^6$http://www.csr.com/
2.5.2. Wireless Local Area Network

Wireless Local Area Network (IEEE 802.11), operating in the 2.4 GHz industrial, scientific and medical (ISM) band has become a popular solution for localization. Based on RSS measurements, with typical accuracies of 3-30 meters, WLAN is an appealing RF technology due to the already globally existing infrastructure. Typical error sources for this RF technology include the non-adaptability to fast changing environments, due to interference and blockage situations, and the cost in time related to fingerprinting solutions, where databases with received signal strengths and relative positions need to be constructed [23].

Several commercial and research systems have been tested and deployed, e.g., by companies such as Senion Lab\(^7\), Navizon\(^8\), Ekahau\(^9\) and Skyhook Wireless\(^10\) that offer localization solutions based on WLAN.

2.5.3. Ultra-wide bandwidth

In recent years, ultra-wide bandwidth (IEEE 802.15.4a), transmitting a signal over multiple bands of frequencies from 3.1 to 10.6 GHz, has been shown to be a promising technology to deal with the positioning problem in GPS-challenged scenarios. In the time domain, UWB relies on the use of ultra-short pulses (less than 1 ns), which translate in the use of absolute bandwidths of more than 500 MHz in the frequency domain. It offers a wide range of advantages from the localization point of view. UWB transceivers can coexist with other RF signals without causing and suffering from interference because of the radio spectrum employed and the differences in signals. Moreover, based on TW-TOA ranging procedure, UWB allows for accurate and reliable ranging [22], where multipath signals can be identified and filtered. Employed signals can propagate through walls, clothing and equipment materials [23].

Disadvantages include interference due to metallic and liquid materials, and cost, given that it is a recent technology within the localization context.

Nowadays, commercial, development systems, and chipsets from companies such as Ubisense\(^11\) (based on TDOA), Time Domain\(^12\), and Decawave\(^13\) employ this RF technology.

Given the advantages of UWB within the localization context, this thesis is focused on the localization problem using UWB. Therefore, the next Section covers the communication constraints with a focus on UWB localization systems.

\(^7\)http://www.senionlab.com/
\(^8\)http://www.navizon.com/
\(^9\)http://www.ekahau.com/
\(^10\)http://www.skyhookwireless.com/
\(^11\)http://www.ubisense.net/
\(^12\)http://www.timedomain.com/
\(^13\)http://www.decawave.com/
3. The Multiple Access Channel

3.1. Introduction

In this Section, we review the communication constraints in positioning systems and then focus on MAC protocols suitable for current UWB systems with TW-TOA measurements. The time slot of duration $T$ introduced in Section 2.3 can be broken down into: the measurement time $T_{\text{meas}}$, the computation time $T_{\text{comp}}$, and the communication time $T_{\text{comm}}$.

The measurement time $T_{\text{meas}}$ is the time required by the agents in the network to collect all necessary measurements, the ranging information in the UWB case. The computation time $T_{\text{comp}}$ accounts for the time it takes to make the correction update to calculate the posteriori distribution $p(x_i^{(t)}|z_i^{(1:t)})$ with the available measurements. Finally, the communication time $T_{\text{comm}}$ relates to the delay caused by agents transmitting information to other nodes in the network (distributed approach) or to a fusion center (centralized approach) after all ranging information has been collected. The computation time is dependant on the localization algorithm in hand. Therefore, considering the analysis of fundamental bounds within the contributions, this delay is not included in the analysis. Furthermore, the information transmission associated to the communication time can be implemented using an alternative technology, thus, it is not included in this thesis analysis. Therefore, it is important to note that UWB is used exclusively for the measurement phase with a focus in the measurement time.

The measurement time is dependant on the MAC protocol used to access the channel. Thus, in this thesis we will refer to the measurement time related to a MAC protocol as the MAC delay. The next Section includes the review of several MAC protocols that can be implemented in UWB systems, their advantages and disadvantages based on our experimental campaigns and off-the-shelf UWB radios. The list is not exhaustive, a more detailed list with current research can be found within the contributions.

3.2. MAC Protocols

Practical implementations introduce communication related constraints due to the channel access required in the MAC layer and the transmission of information over the wireless network. A MAC protocol needs to be implemented in order to avoid primary and secondary interference. Primary interference refers to when a node transmits and receives at the same time. Secondary interference occurs when a
node receives multiple transmissions at the same time [24]. Traditional protocols include ALOHA, slotted ALOHA, time division multiple access (TDMA) and the use of different frequency bands, or distinct codes such as perfectly balanced ternary sequences (PBTS) for the preambles, time hopping codes, and scrambling codes for the data are found within UWB system implementations [22]. Given our experience with Time Domain off-the-shelf UWB radios [25], we briefly describe ALOHA, slotted ALOHA, time hopping, and spatial time division multiple access.

### 3.2.1. ALOHA and Slotted ALOHA

The IEEE 802.15.4a standard uses different schemes for multiple access. Traditional protocols include ALOHA or slotted ALOHA. The main principle in the ALOHA protocol is as follows: if the node has information to send, it sends it; if interference was detected by means of an acknowledgment packet, the node resends the information after a random “backoff” time. The slotted version reduces the probability of collision by dividing time into slots and allowing nodes to transmit only at the beginning of each slot. The latter requires synchronization within the nodes to have common knowledge on when a slot begins [26].

Even though traditional protocols, such as ALOHA or slotted ALOHA can be implemented within UWB systems, they offer poor efficiency in terms of the successful number of transactions and are therefore not considered within the scope of the thesis.

### 3.2.2. Time Hopping

Given the fact that UWB is a pulse based system with speed spectrum characteristics, with benefits such as antijamming and antiinterference, time hopping is an available scheme for modulation and multiple access purposes. By the use of pseudorandom time-hopping, each node (transceiver) is assigned a distinct time hopping code (pulse shifting patterns) to eliminate collisions in multiple accessing. In this way, two UWB links may share the same spectrum by using orthogonal time-hopping codes [27]. In this thesis time hopping is not considered due to the fact that 802.15.4a radios use a common preamble, similar to our off-the-shelf radios, to which any node can lock on.

### 3.2.3. Spatial Time Division Multiple Access

TDMA is a scheme for MAC where the transmission resource is divided into time-slots, and each link\(^1\) receives a dedicated slot. Spatial TDMA incorporates the use of the spatial reuse concept that takes into account that the transceivers that are spread geographically far apart can reuse the same time slot, provided they do not cause primary and secondary interference. The main disadvantages include the

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\(^1\)For positioning purposes each TW-TOA transaction including request and acknowledgement.
complexity from the algorithmic perspective, due to high mobility and distributed implementations. On the other hand, from the computational point of view it can be shown that finding an optimal scheduling solution is NP hard [29].

We consider an STDMA approach in which one TDMA slot is needed for a TW-TOA transaction, but two transactions can occur simultaneously if they do not interfere. The total STDMA delay within one time slot depends only on the network topology in the current time slot. In the next section we will describe the performance metrics, both for the accuracy, considering TW-TOA, and for the MAC delay, based on an STDMA approach.
4. Performance Metrics and their Trade-off

In this Section, we introduce the performance metrics to be used for the accuracy and the MAC delay. In order to appropriately quantify the trade-off between the accuracy and the MAC delay independently of the localization algorithm, we resort to lower bounds for the accuracy and MAC delay. Then, both bounds are related through the trade-off parameter to quantify the trade-off for arbitrary networks under different operation conditions.

4.1. Position Error Bound

The position error bound (PEB) relates to the accuracy in the UWB network by means of the Crámer-Rao Bound. It gives a lower bound on the variance achievable by any unbiased location estimator given the network topology and the measurements [10,12]. We give an overview of the PEB computation, details are included in the contributions for different operation conditions including noncooperative, and cooperative scenarios with selective ranging and eavesdropping.

Let $\mathbf{x} = [\mathbf{x}_1^T \mathbf{x}_2^T \cdots \mathbf{x}_N^T]^T$ be the vector containing the positions of all agents and its estimate, based on the observation $\mathbf{z}$. The Fisher information matrix (FIM) is defined as [28]

$$J = -E_{\mathbf{x}, \mathbf{z}} \left\{ \nabla_\mathbf{x} \nabla_\mathbf{z}^T \log p(\mathbf{z}, \mathbf{x}) \right\}$$

(4.1)

for random $\mathbf{x}$, and as

$$J(\mathbf{x}) = -E_{\mathbf{z}} \left\{ \nabla_\mathbf{x} \nabla_\mathbf{x}^T \log p(\mathbf{z}|\mathbf{x}) \right\}$$

(4.2)

for nonrandom $\mathbf{x}$.

The generic expression for the PEB of a network is given by:

$$\mathcal{P} = \sqrt{\text{tr} \{J^{-1}\}} / N,$$

(4.3)

where $N$ is the number of agents with a priori unknown positions and $\mathbf{J}$ is the FIM calculated in (4.1) or (4.2). The FIM structure can be described as

$$\mathbf{J} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{bmatrix},$$

(4.4)
where $A \in \mathbb{R}^{2 \times 2}$, $B \in \mathbb{R}^{2 \times (N-1)}$, and $C \in \mathbb{R}^{2(N-1) \times 2(N-1)}$, then the equivalent Fisher information matrix (EFIM) for agent 1 is given by $J^E_1 = A - BC^{-1}B^T$ [12]. Once can easily verify by means of the Schur complement that $[J^E_1]^{-1}$ is the top-left $2 \times 2$ block diagonal element of $J^{-1}$ and the EFIM for any agent can be computed by simple agent reordering.

The PEB of agent $i$ is defined as

$$P_i = \sqrt{\text{tr}\{[J^E_i]^{-1}\}}.$$  \hspace{1cm} (4.5)

The PEB is expressed in meters and that $P$ and $P_i$ are related through $P = \sqrt{\sum_i P_i^2 / N}$.

An example of the PEB is depicted in Figure 4.1. Three anchors at coordinates (2, 2), (5, 8), and (8, 2) are positioned in a $10 \times 10$ m two-dimensional area. The PEB for an agent located anywhere on the area is depicted in the colormap, considering distance measurements with unit variance ranging errors.

![Figure 4.1: Example of the PEB for a 10 x 10 m two dimensional area with anchors located at positions (2, 2), (5, 8), and (8, 2).](image)

### 4.2. Minimum MAC Delay

The minimum MAC delay bound relates to the minimum time required within the network to collect all measurements assuming an STDMA MAC approach given a connectivity model and a network topology. More specifically, it computes the minimum number of time slots needed within the network to perform all ranging transactions among nodes. Unfortunately, computing the exact number of time slots is an NP-hard problem [29]. Therefore, we resort to upper and lower bounds. The
Chapter 4 Performance Metrics and their Trade-off

![Graph showing log(Accuracy) vs log(Delay) with trade-off parameter δ(ρ).]

**Figure 4.2.** Interpretation of the trade-off parameter. Larger δ(ρ) leads to a faster reduction in PEB as the delay increases than a smaller δ(ρ).

minimum MAC Delay is expressed in number of time slots which can be translated into seconds after defining the time slot duration. Details are explained within the contributions. Here we present a brief overview.

### 4.2.1. Upper Bound

The upper bound is computed as \( \Omega = \min(\Omega_1, \Omega_2) \), where \( \Omega_1 \) is the total number of links to be scheduled in the network, and \( \Omega_2 \) is the number of time slots described by the Erdős-Nešetřil [30] conjecture, given by \( \Omega_2 = \frac{5}{4}\Delta^2 \) for even \( \Delta \) and \( \Omega_2 = \frac{1}{4}(5\Delta^2 - 2\Delta + 1) \) for odd \( \Delta \), where \( \Delta \) is the maximum node degree of the network related to the links to be scheduled for ranging transactions.

### 4.2.2. Lower Bound

The lower bound \( \Upsilon_1 \) considers a constructive 1-hop neighborhood for each nodes \( i \). While this bound takes into account primary and secondary interference for a single agent, it results in every neighbor of node \( i \) to be scheduled in a distinct TDMA slot. We further introduce the lower bound \( \Upsilon_2 \), which considers a constructive 2-hop neighborhood for each node \( i \). It breaks the network in smaller subnetworks depending on the scheduling requirements and determines the number of TDMA links required to schedule each subnetwork avoiding primary and secondary interference. The maximum of the lower bounds of these subnetworks gives a lower bound for the entire network.
4.3. Trade-off parameter

Considering dense networks [13], where the node density increases by adding more nodes into the network while the area remains fixed, the relation between the PEB and the minimum MAC delay in the asymptotic regime (large number of nodes) can be quantified by means of the trade-off parameter (with similar basis as the parameter introduced in [31]). We introduce the trade-off parameter \( \delta(\rho) \in \mathbb{R} \), where \( \rho \) is called the agent growth rate and \( N = \kappa M^\rho \) for \( \kappa > 0 \). This parameter is dependent on the rate at which the number of agents \( N \) increases with respect to the number of anchors \( M \) and the asymptotic behavior of the PEB and MAC delay derived from the scaling laws presented in the contributions. It is easy to see that \( \rho < 1 \) translates into anchors being added faster than agents, while for \( \rho > 1 \), agents are added faster than anchors. Mathematically the trade-off parameter is defined as follows.

**Definition 1.**

\[
\delta(\rho) = -\lim_{M \to +\infty} \frac{\log f_P(M, \rho)}{\log f_M(M, \rho)}. \tag{4.6}
\]

where \( f_P(M, \rho) \) and \( f_M(M, \rho) \) relate to the PEB and the minimum MAC delay, respectively, as explained further in the contributions. The trade-off parameter \( \delta(\rho) \) can be interpreted as the slope of the accuracy versus delay line in a log-log scale, see Figure 4.2. A larger \( \delta(\rho) \) leads to a faster reduction in PEB as the delay increases than a smaller \( \delta(\rho) \).
5. Contributions

This thesis aims at investigating the accuracy and delay trade-off for cooperative UWB navigation systems. In Section 5.1, we list the papers that are appended to the thesis and summarize their contributions. Additional publications by the author, which are not included in this thesis, are listed in Section 5.2.

5.1. Included Papers

1. **Paper A: “On the trade-off between accuracy and delay in UWB navigation”**
   We investigate the relation between medium access control (MAC) delay and ultra-wide bandwidth (UWB) tracking accuracy. We quantify this relation by deriving fundamental lower bounds on tracking accuracy and MAC delay for arbitrary finite networks. Our main finding is that the traditional ways to increase accuracy (e.g., increasing the number of anchors or the transmission power) may lead to large MAC delays. We evaluate two methods to mitigate these delays.

2. **Paper B: “On the trade-off between accuracy and delay in cooperative UWB navigation”**
   In ultra-wide bandwidth (UWB) cooperative navigation, nodes estimate their position by means of shared information. Such sharing has a direct impact on the position accuracy and medium access control (MAC) delay, which needs to be considered when designing UWB navigation systems. We investigate the interplay between UWB position accuracy and MAC delay for cooperative scenarios. We quantify this relation through fundamental lower bounds on position accuracy and MAC delay for arbitrary finite networks. Results show that the traditional ways to increase accuracy (e.g., increasing the number of anchors or the transmission power) as well as inter-node cooperation may lead to large MAC delays. We evaluate one method to mitigate these delays.

   Ultra-wide bandwidth (UWB) systems allow for accurate positioning in environments where global navigation satellite systems may fail, especially when complemented with cooperative processing. While cooperative UWB has led to centimeter-level accuracies, the communication overhead is often neglected. We quantify how accuracy and delay trade off in a wide variety of operation conditions. We also derive the asymptotic scaling of accuracy and delay, indicating
that in some conditions standard cooperation offers the worst possible trade-off. Both avenues lead to the same conclusion: indiscriminately targeting increased accuracy incurs a significant delay penalty. Simple countermeasures can be taken to reduce this penalty and obtain a meaningful accuracy/delay trade-off.

5.2. Related Publications

Bibliography


