THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING IN
SOLID AND STRUCTURAL MECHANICS

Calibration and Reduction of Large-Scale Dynamic Models

Application to Wind Turbine Blades

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ABSTRACT

This thesis investigates the validity of structural dynamics models of wind turbine blades. An outlook on methods for model calibration to make models valid for their intended use is presented in the thesis. The intention is to make the models valid for robust predictions. The model validity is here assessed to be of hierarchical dual level. On one hand, a detailed structural dynamics model needs to be substantiated by good correlation between experimental results of wind turbine testing and theoretical simulation results using that model. On the other hand, after that detailed model has been validated, a model of significantly low order based on the detailed model has been validated by a good model-to-model correlation. With the connection between models, this implies that also the low order model is implicitly validated by testing. The development of a highly detailed structural dynamics model provides real physical insights to observation made during testing. This model is often developed using finite element analysis. A model verification and validation activity is done to create a three dimensional finite element model that is capable to predict the dynamics of wind turbine blade with sufficient accuracy. Integration of such large-scale models of wind turbine blades in aeroelastic simulations places an untenable demand on computational resources and, hence, means of speed-up become necessary. The common practice is to develop, calibrate and validate an industry-standard beam model against the simulated data obtained from the validated highly detailed rotor blade model. However, the validated beam model cannot well capture the coupling features of the highly detailed model because of its inherent limitations. Our scientific hypothesis is that it is possible to create low-order rotor blade models which preserve the vibrational pattern of the baseline model at its eigenfrequecies and also closely mimic its input-output behavior. Toward this end, a quasi optimal modal truncation algorithm is developed to yield reduced models which have the eigenmodes with highest contribution to the input-output map of the large-scale model. The predictive capability of the created reduced model is compared with that of the validated beam model.

KEYWORDS: Verification and validation, finite element model calibration, logarithmic least square estimator, maximum likelihood estimator, Bayesian model calibration, model reduction, large-scale dynamics model, wind turbine blade

To my loving family:

Mohammad, Soudabeh, Mahsa, and Mahshid

PREFACE

The Swedish Wind Power Technology Centre (SWPTC) is a research centre for design of wind turbines. The purpose of the centre is to support Swedish industry with knowledge of design techniques as well as maintenance in the field of wind power. The research in the centre is carried out within six thematic groups that represent design and operation of wind turbines. These are the *Power and Control Systems*, *Turbine and Wind Loads*, *Mechanical Power Transmission and System Optimisation*, *Structure and Foundation*, *Maintenance and Reliability* and the *Cold Climate* theme groups. The work presented here is a part of the theme *Structure and Foundation*. SWPTC's work is funded by the Swedish Energy Agency, by three academic and thirteen industrial partners. Region Västra Götaland also contributes to the centre through several collaboration projects. The support of the centre to this work is gratefully acknowledged

The work presented in this thesis was carried out at the Department of Applied Mechanics at Chalmers University of Technology during the years 2011 through 2013. I would like to express my sincere gratitude to my advisor Professor Thomas Abrahamsson for his invaluable mentoring throughout these years. I would also like to thank him for giving me the warm support and confidence to step into the professional world of science. I would like to thank my co-supervisor Dr. Anders T. Johansson for his guidance and enthusiastic encouragement. I enjoyed very much our numerous discussions at the coffee table.

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Finally, I would like to express my deepest gratitude to my parents and my sisters for their unconditional love and support throughout my life.

Gothenburg, December 2013

Majid Khorsand Vakilzadeh

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A M. Khorsand Vakilzadeh, A. T. Johansson, T. J. S. Abrahamsson, An input-output based modal dominancy criterion in modal truncation, *submitted for international publication*

Paper B M. Khorsand Vakilzadeh, A. T. Johansson, C. J. Lindholm, J. Hedlund, T. J. S. Abrahamsson, Development of simplified models for wind turbine blades with application to NREL 5MW offshore research wind turbine, *To be presented at* IMAC XXXII, *A Conference on Structural Dynamics*, 2014

Paper C A. T. Johansson, C. J. Lindholm, M. Khorsand, T. J. S. Abrahamsson, Modeling and calibration of small-scale wind turbine blade, *Proceeding of* IMAC XXXI, *A Conference on Structural Dynamics*, **36**(2), 2013

Appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of the work for the first two papers, including the planning of the papers, developing the theory, developing and carrying out the numerical implementations and simulations. In the third paper, the author was involved in the planning of the paper and doing the experimental side of the work.

CONTENTS

EXTENDED SUMMARY

1 INTRODUCTION

The ultimate goal of model development in engineering is to predict the behavior of underlying real systems with a targeted level of confidence. Model Validation and Verification (V&V) provides an enabling methodology to yield such credible computational models [1]. Figure 1 shows the general picture of the V&V activity during model development and demonstrates how a model developer and an experimenter collaborate to develop a credible computer model. By taking into account all the important aspects of the underlying real system, computational objectives, and the required degree of agreement between model and experiment, the developer and the experimenter should ideally agree on a conceptual model. Once the development of the conceptual model is finished, the model developer starts to construct the corresponding computer model and the experimenter designs the test conditions to achieve the information needed to validate or falsify the model.

On the simulation (right) part of Figure 1, the model developer starts to build the mathematical model of the underlying physics of interest. In mechanics, the mathematical model consists of constitutive laws, conservation equations of mass, momentum, and energy and so forth. The mathematical models are often in the form of Partial Differential Equations (PDE) with a specified set of boundary and initial conditions.

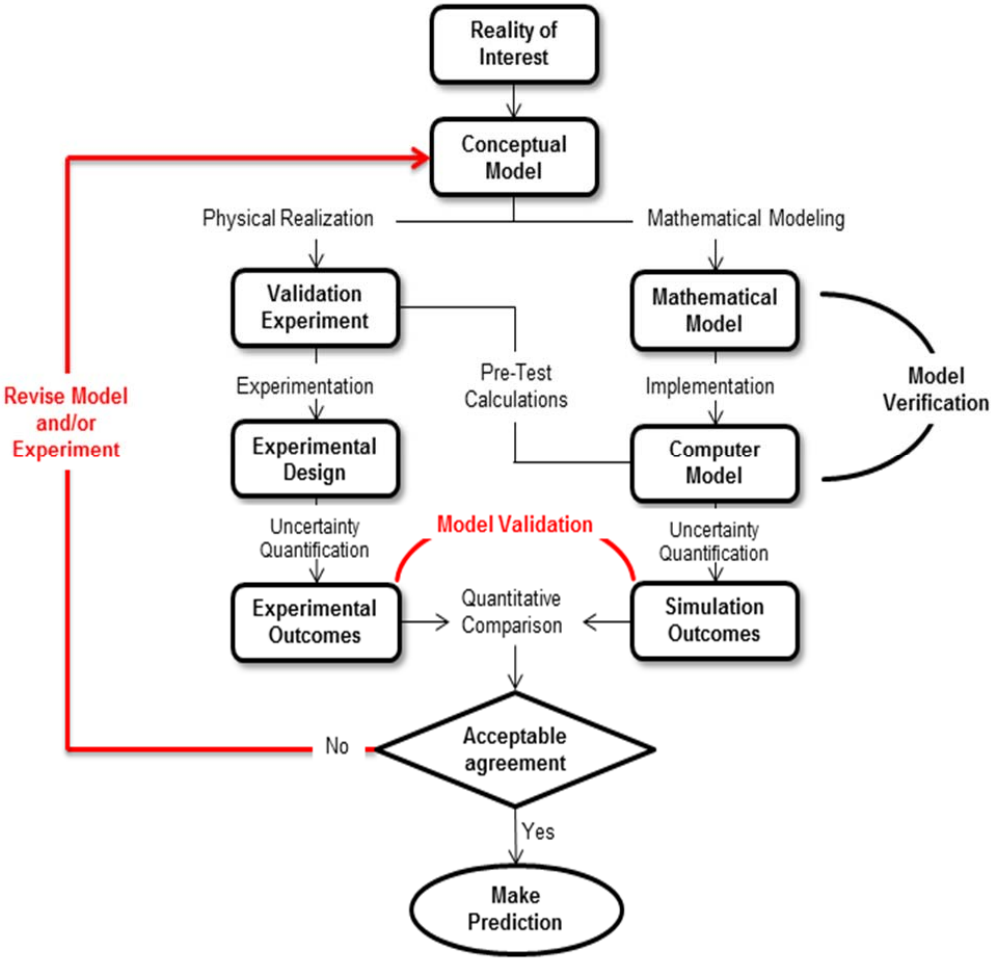

Figure 1. *Detailed procedure of model development, Verification and Validation and model calibration activity* [1]

The set of mathematical equations derived in the previous step are implemented in form of computer models using appropriate discretization algorithms both in time and space, such as finite difference or finite element methods. The approximated equations are solved using appropriate numerical algorithms. Here a verification activity plays a crucial role. In this thesis the verification definition presented by 1998 AIAA Guide [1] is adopted as "Verification is the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model". In fact, verification is the diagnostic process of error identification in the computer model by comparing its numerical outcomes with the analytical or already verified benchmark solutions. Verification involves two main activities, code and calculation verification. Code verification is the process of removing errors from the computer code and calculation verification attempts to quantify and minimize the errors introduced by discretization and numerical algorithm. Space and time convergence analyses are main examples of calculation verification activity. The last but not the least in the right side of the figure is to study the impact of uncertainties introduced by the employed numerical algorithm, modeling strategy and other assumptions on the simulation outcome. Uncertainties can be studied using different theories, such as probability, fuzzy theory or evidence theory, in which they mainly differ based on their model of uncertainty, for example in probability theory the variation in Young's modulus is represented using a proper probability distribution function.

On the experimental (left) side of Figure 1, the experimenter designs a physical experiment to provide information required to validate the computer model. Thus, all the assumptions in the conceptual model must be defined and controlled during the experiment campaign. Pre-test calculations can be done to assist the experiment side. For example, theoretical methods such as Effective Independence (EI) indices [2] can be employed to determine the most effective locations for the sensors. The test campaign need not be set up only to provide response data but its outcome can also quantify test-to-test variation for a single test specimen or for different test specimens of same type. Uncertainty quantification is then performed to assess the effect of the measurement error, fabrication errors and design tolerances.

Once the uncertainties in the simulation and experiment outcomes are quantified, the model validation process starts. In this study again we take up the definition given by 1998 AIAA Guide for the model validation process, "Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model". The decision about credibility of the computer model is much more complicated than the Boolean logic represented in the figure. Statistical methods such as hypothesis testing, cross validation and Bayesian validation are developed in the literature to help decision makers examine the predictive capability of the model in the validation step. If the validation outcome is not acceptable, the model or experiment will be revised. The process of model revision involves revision of the model boundary values or initial conditions, parameter estimates, underlying assumptions to improve the agreement of the simulation and experiment outcomes. The procedure of estimating model parameters is also called model calibration or model updating in structural dynamics. On the other hand, the experiment revision is to modify the test design or measurement system to improve the fidelity of experiment outcome to the simulation outcomes. The decision whether experiment or simulation should be revised is made by model developer or experimenter.

The remaining introduction part of this thesis will focus on the model calibration procedure.

2 ANATOMY OF MODEL CALIBRATION

If the validation decision is to revise the computer model, one possiblity is to update the model parameters to bring model into better agreement with the experiment. This practice is called model calibration or, more generally, the inverse problem. Thus, model calibration is the process of infering model parameters using the observed data and its ultimate goal is to improve the predictive capability of the model. Figure 2 demonstrates the anatomy of the inverse problem.

2.1 Forward problem

Suppose that the relationship between the model parameters $\boldsymbol{\theta} \in \mathbb{R}^{n_\mathrm{p}}$ and the system output $\boldsymbol{y} \in \mathbb{R}^{n_\mathrm{y}}$ can be denoted by $\boldsymbol{y}(\boldsymbol{\theta}, \boldsymbol{u})$, where $\boldsymbol{u} \in \mathbb{R}^{n_\mathrm{u}}$ is the system input. In other words, given the model parameters, the function $\boldsymbol{y}(\boldsymbol{\theta}, \boldsymbol{u})$ is the forward problem's system response when the system is excited by $\boldsymbol{u}$. In the following, input and output are discretized time histories of the excitation and the model response.

In structural dynamics, the forward problems are derived using basic principles, such as Newton's Laws. Using the Finite Element (FE) method to discretize the boundary value problem, results in a set of ordinary differential equation of motion as

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{V}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{f}(t) \tag{1}$$

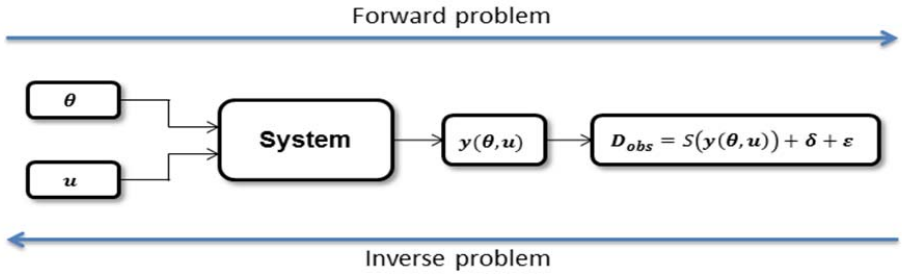

Figure 2. *Schematic of forward and inverse problem*

where $\boldsymbol{q}(t) \in \mathbb{R}^{n_q}$ is the displacement vector, $\boldsymbol{f}(t)$ is the external load vector which is governed by a Boolean transformation of input vector $\boldsymbol{f}(t) = \boldsymbol{P}_{\mathbf{u}}\boldsymbol{u}(t)$. Real positive-definite symmetric matrices $\boldsymbol{M}, \boldsymbol{V}, \boldsymbol{K} \in \mathbb{R}^{n_q \times n_q}$ are mass, damping and stiffness matrices, respectively. The finite element model can be parameterized using a set of identified parameter vector of $\boldsymbol{\theta} \in \mathbb{R}^{n_\mathrm{p}}$such that mass and stiffness matrices are denoted as $\boldsymbol{M}(\boldsymbol{\theta})$ and $\boldsymbol{K}(\boldsymbol{\theta})$. The state-space realization of the equation of motion in (1) can be written as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t), \quad \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

where $\boldsymbol{A} \in \mathbb{R}^{2n_q \times 2n_q}$, $\boldsymbol{B} \in \mathbb{R}^{2n_q \times n_\mathrm{u}}$, $\boldsymbol{C} \in \mathbb{R}^{n_\mathrm{y} \times 2n_q}$, and $\boldsymbol{D} \in \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{u}}$. $\boldsymbol{x}(t) = [\boldsymbol{q}(t)\ \dot{\boldsymbol{q}}(t)]^{\mathrm{T}} \in \mathbb{R}^{2n_q}$ is the state vector, and $\boldsymbol{y}(t) \in \mathbb{R}^{n_\mathrm{y}}$ is the model output. $\boldsymbol{A}$ and $\boldsymbol{B}$ are related to mass, damping and stiffness matrices as follows [3]

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{V} & -\boldsymbol{M}^{-1}\boldsymbol{K} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{P}_{\boldsymbol{u}} \end{bmatrix} \tag{3}$$

Whenever the input vector is exponentially bounded, the Laplace transform can be used to write the transfer function of the above system as $\boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$ for $s \in \mathbb{C}$. Fortunately, the forward problem is well-posed. In other words, given a set of parameters and an input vector, the solution to the forward problem is unique. Although time domain data can be used for inverse problems, some researchers in the field of structural dynamics prefer to use signal processing on the data to instead work with frequency domain data. For this reason, see Figure 2, the transformation $\boldsymbol{S}(.)$ is applied to computer model output to represent data in the general form.

2.2 Output model error

There are several sources of uncerainty that can influence the computer model and experiment outcomes [4-7]. One way to classify the unceratinty is as follows;

Model inadequacy - No model is a perfect representation of the underlying true physics. The modeling error depends on how accurately the computer model represents the corresponding real life system.

Residual variability - A computer model is supposed to predict the behavior of a system in response to a specified input condition. In practice, even if all the model inputs are fully specified, the experiment outcome varies over repeated experiments. This variation is called residual variability. The reason is that there is always a changing or unspecified test condition during the process of repitition.

Parametric variability - which is due to variation between individuals. For example the geometry of wind turbine blades varies due to imperfections of the manufacturing process and results in a variation of test outcome repeated for different blades.

Observation error - The model calibraion process is based on real observation data which are often contaminated with measurement noise. The measurement noise is typically modeled as being an independent (over time), zero-mean, and normally distributed random process.

Numerical errors - Complicated applications involved in structural dynamics are often solved using numerical approximations. For example, the finite element method is often used to discretize the partial differential equations. This discretization intriduces numerical errors.

Parameter uncertainties - Model parameters are given as inputs to the computational models. These parameters are normally to a certain extent uncertain. In the test situation, the test article represents a single realization of the parameter setting.

The relationship between the observed data, $\boldsymbol{D}^{obs} \in \mathbb{R}^{n_y}$, and the real system output, $\boldsymbol{\eta} \in \mathbb{R}^{n_y}$, can be represented using an uncertain output-error model as [5]

$$\boldsymbol{D}^{obs} = \boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{4}$$

where $\boldsymbol{\varepsilon} \in \mathbb{R}^{n_y}$ is the measurement error. In practice it is impossible to separate the observation error and residual variability. Therefore, the measurement error $\boldsymbol{\varepsilon}$ is a unified model for these two sources of uncertainty. Deviation of the computer model output $\boldsymbol{S}$ from the real system output $\boldsymbol{\eta}$ can be expressed as

$$\boldsymbol{\eta} = \boldsymbol{S}(\boldsymbol{y}(\boldsymbol{\theta}, \boldsymbol{u})) + \boldsymbol{\delta} \tag{5}$$

where $\boldsymbol{\delta} \in \mathbb{R}^{n_y}$ is a model inadequacy. In the statistical approaches to the inverse problems the statistical description of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\delta}$ are required. There are two ways to construct appropriate models for the mentioned error terms. In this thesis, the most common way in the structural dynamics will be addressed. Beck *et al.* [8, 9] established a unified probability model for both model inadequacy and measurement error. In their approach, the principle of maximum information entropy is used to construct a probability model such that yields the most uncertainty subjected to the imposed parameterized constraints. In other word, selection of any other type of probability model leads to a decreased amount of prediction uncertainty. The principle of maximum information entropy provides a structured way to build the probability models that cover the epistemic[1] uncertainty in the real system behavior. Under maximum information entropy assumption, $\boldsymbol{\varepsilon} + \boldsymbol{\delta}$ are independent and identically distributed as $\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma})$.

2.3 Inverse problem

The inverse problem is opposite to the forward problem and is here meant to be the reconstruction of the physical parameters of the model using the observed data, see Figure 2. Herein, we use “inverse problem” to refer to general form of parameter estimation problem. Unfortunately, the solution to the inverse problem is ill-posed. In other words, there are many different sets of model parameters that are consistent with the observed data inasmuch as data are typically sparse and not informative about the full extent of the model. There are several ways to achieve solutions to the inverse problem. The next section reviews three different approaches to its solution.

3 INVERSE PROBLEM SOLUTION TECHNIQUES

3.1 Deterministic optimization

In the deterministic setting, one seeks to minimize an appropriate norm of discrepancy function, $\left\|\boldsymbol{D}^{obs} - \boldsymbol{S}\left(\boldsymbol{y}(\boldsymbol{\theta})\right)\right\|_R^2$, between the observed data and model predictions. One way to alleviate the ill-posedness of inverse problem is to add a regularization term, $\mathcal{R}(\boldsymbol{\theta})$ to the discrepancy function. The regularization term penalizes the distance from a given parameter setting.

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \left\{ \left\|\boldsymbol{D}^{obs} - \boldsymbol{S}\left(\boldsymbol{y}(\boldsymbol{\theta})\right)\right\|_R^2 + \mathcal{R}(\boldsymbol{\theta}) \right\} \tag{6}$$

The solution to this inverse problem is the best parameter setting which simultaneously make the discrepancy function and the regularization term small [10]. **Paper C** considers this optimization technique to minimize the linear distance between the eigenvalues of the FE model and the estimated eigenvalues from observed data.

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1. *Epistemic uncertainty* is a systematic uncertainty in the model of a real system. Basically, the sources of epistemic uncertainty could be made known but they are unknown in practice. These uncertainties are associated to lack of knowledge. In contrast, *aleatory uncertainty* is due to inherent randomness in the real system. This type of uncertainty can not be reduced other than by putting in unreasonable resources. The Brownian motion of molecules or momentary distribution of gust wind velocity at a wind turbine site are examples of processes subjected to aleatoric uncertainty. A quantity with aleatoric uncertainty will show up as being seemingly random in repeated experiments under the same overall conditions.

In the discretized frequency domain, the logarithmic least-square (LOG) optimization is another response to the challenging ill-posed inverse problems. In this framework, the optimization problem can be formulated as [3, 11]

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{p}}{\operatorname{argmin}}(\boldsymbol{\vartheta}^T\boldsymbol{\vartheta}) \tag{7}$$

where the deviation metric $\boldsymbol{\vartheta}(\boldsymbol{\theta})$ is

$$\boldsymbol{\vartheta}(\boldsymbol{\theta}) = \ln \operatorname{vect}(\boldsymbol{D}^{obs}) - \ln \operatorname{vect}\left(\boldsymbol{S}\left(\boldsymbol{y}(\boldsymbol{\theta})\right)\right) \tag{8}$$

in which $\operatorname{vect}(.)$ is a vectorizing operator that collects all the $n_{\mathrm{y}} \times n_{\mathrm{u}}$ elements of the system transfer function at all sampling frequencies into a column vector belonging to $\mathbb{C}^{n_{\mathrm{y}} n_{\mathrm{u}} n_{\mathrm{f}} \times 1}$, where n_{f} is the number of discrete sampling frequencies. Taking the logarithm of the system transfer function restricts the range of the frequency response function which increases the likelihood of obtaining a unique solution and, thus, makes the inverse problem better posed. However, the LOG estimator is theoretically inconsistent. Nevertheless, it can be shown [12] that if the system output has a signal-to-noise ratio larger than 10dB, the LOG estimator is "practically" consistent and the maximum magnitude of the bias term is smaller than 2×10^{-6}. This estimator is considered in the **Paper B**.

However, the solution to the deterministic optimization problems stated in (6) or (7) is the best parameter setting. Thus, the deterministic optimization is often called point estimator of the model parameters. For the point estimator, no statistical spread in the test data is carried over to the estimator. Making predictions based on the point estimator model is not best practice since it does not represent our confidence in the predictions. Instead, we are interested to extract the statistical description of the model parameters from the available information in the observed data. The next section sheds light on two common approaches to formulate a statistical inference problem; the Frequentist and the Bayesian inference approaches.

3.2 Frequentist approach

The Frequentist approach is one way to recovering the statistical discreption of the model parameters from the sample data. In this approach a probability value given to an outcome of any experiment is the relative frequency of that outcome in a long sequence of experiment repetitions yielding statistically independent results [13]. In such a manner, the Frequentist approach to statistical inference assigns a probability measure to an infered value of parameter setting through the notion of repetition. The Maximum Likelihood Estimator (MLE) is one often used estimator in the Frequentist literature. In general, MLE searches for the parameter setting which maximizes the likelihood function. Intuitively, MLE estimation of the model parameters is the parameter setting which maximizes the agreement between the computer model output and the observed data. To formulate an estimation problem in this framework, the joint probability density function of the error in (4) and (5) is required. To this end, let rewrite the output error model in the discrete form as

$$\boldsymbol{D}_i^{obs} = \boldsymbol{S}_i + \boldsymbol{\delta}_i + \boldsymbol{\varepsilon}_i, \quad i = 1, \dots, n_{obs} \tag{9}$$

where n_{obs} is the number of the observation points. Typically, $\boldsymbol{\delta}_i + \boldsymbol{\varepsilon}_i$ is supposed to be independent Gaussian distributed random variables, based on the principle of maximum information entropy, with zero mean and covariance matrix $\boldsymbol{\Gamma}$. Thus, the likelihood function $L(\boldsymbol{D}^{obs}|\boldsymbol{\theta})$ can be written as [14]

$$L(\boldsymbol{D}^{obs}|\boldsymbol{\theta}) \propto \prod_{\boldsymbol{i=1}}^{n_{obs}} \exp[-\frac{1}{2}\left\|\boldsymbol{D}_i^{obs} - \boldsymbol{S}_{\boldsymbol{i}}\right\|_{\boldsymbol{\Gamma^{-1}}}^2] \tag{10}$$

The MLE estimator $\widehat{\boldsymbol{\theta}}$ is the parameter setting that maximizes $L(\boldsymbol{D}^{obs}|\boldsymbol{\theta})$ or equivalently minimizes the discrepancy function $\|\boldsymbol{D}^{obs} - \boldsymbol{S}\|_{\boldsymbol{\Gamma^{-1}}}^2$. It can be shown that the MLE estimator $\widehat{\boldsymbol{\theta}}$ is asymptotically distributed as $\widehat{\boldsymbol{\theta}} \sim \mathcal{N}(\boldsymbol{\theta}, \boldsymbol{I^{-1}}(\boldsymbol{\theta}))$, where $\boldsymbol{I}(\boldsymbol{\theta})$ is the Fisher information matrix evaluated at the true value. Thus, it can be concluded that the MLE is an asymptotically unbiased and efficient estimator.

3.3 Bayesian approach

Bayesian inference provides a systematic framework to take into account various sources of uncertainty to characterize the uncertainty in the model parameters through updates of their joint probability density function after testing. Before reviewing

the theoretical formulation of Bayesian inference and its subsequent tools to make robust predictions, the interpretation of probability measure and the definition of stochastic system models in the Bayesian framework are briefly reviewed in the following sections.

3.3.1 Probability logic in the Bayesian framework

In contrast to the Frequentist interpretation of a probability measure, the Bayesian approach is based on the probability logic axioms [15]. In probability logic, a probability measure demonstrates the degree of plausibility of an event $\boldsymbol{n}$ given the information covered by a proposition $\boldsymbol{m}$, with probability of $\boldsymbol{n}$ given $\boldsymbol{m}$ to be $p(\boldsymbol{n}|\boldsymbol{m})$. If the information in $\boldsymbol{m}$ gives a complete knowledge about the event $\boldsymbol{n}$, and thus makes it deterministic, the probability logic collapses to the Boolean logic which means that information in $\boldsymbol{m}$ implies that $\boldsymbol{n}$ is either true or false. Presence of epistemic uncertainty, due to lack of knowledge, reduces information in the proposition $\boldsymbol{m}$ about $\boldsymbol{n}$, and then $p(\boldsymbol{n}|\boldsymbol{m})$ is not binary but instead $p(\boldsymbol{n}|\boldsymbol{m}) \in [0,1]$. Thus, epistemic uncertainty extends the Boolean logic to a multi-valued logic. The following sections will demonstrate that conditional probability logic axioms provide a rigorous framework to investigate the influence of uncertainty in order to make robust predictions [8, 9].

3.3.2 Stochastic model classes

In this section, the Bayesian model class $\mathcal{M}$ is introduced to investigate the uncertain input/output behavior of a system. Toward this end, the first step is to select a deterministic dynamical model such as a finite element model, a state-space model or an ARMAX model that gives a relation between the input $\boldsymbol{u}$ and the model output $\boldsymbol{S}(\boldsymbol{y}(\boldsymbol{\theta},\boldsymbol{u}))$ and involves the model parameters in parameter vector $\boldsymbol{\theta}$. The second step is to introduce the output error relation expressed in (4) and (5) to discrete values of the model output and observed data as

$$\boldsymbol{\zeta}_i = \boldsymbol{S}_i + \boldsymbol{\delta}_i + \boldsymbol{\varepsilon}_i, \quad i = 1, \dots, n_{obs} \tag{11}$$

From now on, we selected $\boldsymbol{\zeta}_i$ to denote the time history of the real system output since it can represent either the time history of the predicted output of the actual system, $\boldsymbol{\eta}_i$, or the time history of the output of the actual system measured from the sensors as observed data $\boldsymbol{D}_i^{obs}$. The predicted output of the actual system, $\boldsymbol{\eta}_i$, will be used in two distinct situations. The first situation is when measurement data are not available and a prior analysis is going to be executed for prediction of output of the actual system. More details on that will be considered in section 3.3.4. The second situation is when the measurement noise is negligible in comparison to the modeling error, $\boldsymbol{\delta}_i$. The observed data, $\boldsymbol{D}^{obs}$, will be used when the measurement noise and the modeling error are both important and a posterior analysis is made.

The next step is to construct the probability model of the output error $\boldsymbol{\delta}_i + \boldsymbol{\varepsilon}_i$ in the above formulation. Following the same line of reasoning as in section 2.2 the time history of the output error can be modeled as random variables that are independent and identically distributed as $\mathcal{N}(\mathbf{0}, \boldsymbol{\Gamma})$ [9]. Therefore, the probability model of the output error can be defined as

$$p(\boldsymbol{\zeta}_i \,|\boldsymbol{\theta}, \mathcal{M}) = \frac{1}{(2\pi)^{n_y/2}\boldsymbol{\Gamma}^{-1}} \exp[-\frac{1}{2}(\boldsymbol{\zeta}_i - \boldsymbol{S}_i)^T \boldsymbol{\Gamma}^{-1}(\boldsymbol{\zeta}_i - \boldsymbol{S}_i)] \tag{12}$$

where $\boldsymbol{S}_i \in \mathbb{R}^{n_y}$. Thus, the probability model for n_{obs} discrete observations of the real system output can be written

$$p(\boldsymbol{\zeta}|\boldsymbol{\theta}, \mathcal{M}) = \prod_{i=1}^{n_{obs}} p(\boldsymbol{\zeta}_i|\boldsymbol{\theta}, \mathcal{M}) \tag{13}$$

Here, a fundamental probability model involved in the Bayesian model class $\mathcal{M}$ can be defined which is a set of input/output probability models

$$\{p(\boldsymbol{\zeta}|\boldsymbol{u}, \boldsymbol{\theta}, \mathcal{M})|\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^{n_p}\} \tag{14}$$

in which $p(\boldsymbol{\zeta}|\boldsymbol{u}, \boldsymbol{\theta}, \mathcal{M})$ represents the degree of plausibility of the real system output $\boldsymbol{\zeta}$, given a particular time history of input, the Bayesian model class $\mathcal{M}$ and the value of parameter vector $\boldsymbol{\theta}$.

The last step in construction of the Bayesian model class $\mathcal{M}$ is the selection of the prior probability model, $p(\boldsymbol{\theta}|\mathcal{M})$, which states the intial relative degree of plausibility of the elements of each input/output probability model in the set (14). In other

words, the prior probability reflects our initial belief about the relative plausibility of the elements of the parameter vector $\boldsymbol{\theta}$. The prior statistical information about dynamics model parameters can often be obtained from engineering knowledge based on result of material testing, component testing, etc.

This section showed that construction of the Bayesian model class is based on selection of two distinct types of models; the dynamics model, and the input/output and prior probability models. Thus, the conditioning on Bayesian model class $\mathcal{M}$ determines these choices. It should also be noted that the selection of probability models add some parameters to the parameter vector such as the elements of covariance matrix $\boldsymbol{\Gamma}$.

3.3.3 Bayesian Model updating

In the Bayesian framework [8, 9], inference about an unknown parameter is made using probability statements by computing the posterior Probability Density Function (PDF) $p(\boldsymbol{\theta}|\boldsymbol{D}^{obs})$. The prior knowledge about the relative plausability of the input/ouput models is updated using the observed data $\boldsymbol{D}^{obs}$ through the Bayes' rule

$$p(\boldsymbol{\theta}|\boldsymbol{D}^{obs}) = \frac{p(\boldsymbol{D}^{obs}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{D}^{obs})} \tag{15}$$

Since the Bayesain model updating is within a particular Bayesian model class $\mathcal{M}$, it is common to rewrite the Bayes' rule as

$$p(\boldsymbol{\theta}|\boldsymbol{D}^{obs}, \mathcal{M}) = \frac{p(\boldsymbol{D}^{obs}|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})}{p(\boldsymbol{D}^{obs}|\mathcal{M})} \tag{16}$$

Here, $p(\boldsymbol{D}^{obs}|\boldsymbol{\theta}, \mathcal{M})$ is the probability of the observed data given the parameters and the Bayesian model class $\mathcal{M}$. This probability function is also called the likelihood function. The probability $p(\boldsymbol{D}^{obs}|\mathcal{M}) = \int p(\boldsymbol{D}^{obs}|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})d\boldsymbol{\theta}$ is the probability of the data given the Bayesian model class $\mathcal{M}$ and $p(\boldsymbol{\theta}|\boldsymbol{D}^{obs}, \mathcal{M})$ is the probability of the unknown parameters given the data and the Bayesian model class. This conditioning on the observed data is what differs the Bayesian viewpoint from the classical Frequentist viewpoint.

3.3.4 Robust predictive analysis

In system's design based on Bayesian statistics, one seeks to analyze the performance of the real system based on the selected Bayesian model class. This is often assessed through a performance function $h(\boldsymbol{\zeta})$. A common example of a performance function used in reliability analysis is the indicator function, $h(\boldsymbol{\zeta}) = I(\boldsymbol{\zeta})$ in which $I(\boldsymbol{\zeta}) = 1$ if $\boldsymbol{\zeta}$ meets a given failure criterion and zero otherwise. The prior reliability analysis has a crucial role in the early product development phase. Due to model uncertainties, the performance measure is therefore formulated as the expected value of the performance function as [8, 9]

$$\mathrm{E}[h(\boldsymbol{\zeta})|\mathcal{M}] = \int h(\boldsymbol{\zeta})p(\boldsymbol{\zeta}|\mathcal{M})\, d\boldsymbol{\zeta} \tag{17}$$

where $p(\boldsymbol{\zeta}|\mathcal{M})$ is the prior predictive PDF and is computed using the total probability theoream as

$$p(\boldsymbol{\zeta}|\mathcal{M}) = \int p(\boldsymbol{\zeta}|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})\, d\boldsymbol{\theta} \tag{18}$$

This is often called the prior robust predictive PDF since the uncertainty of the parameters is integrated out. Hence, it contains all the probability information about the system output based on a model class $\mathcal{M}$ and system input $\boldsymbol{u}$. Substituting (18) in (17) results in the robust prediction of the system performance measure in regard to prior information as

$$\mathrm{E}[h(\boldsymbol{\zeta})|\mathcal{M}] = \int h(\boldsymbol{\zeta})p(\boldsymbol{\zeta}|\boldsymbol{\theta}, \mathcal{M})p(\boldsymbol{\theta}|\mathcal{M})\, d\boldsymbol{\theta}d\boldsymbol{\zeta} \tag{19}$$

A common approximation to this function is based on the Monte-Carlo method [13]. When the experiment outcome is available, the statistical description of the unknown model parameters of the selected model class $\mathcal{M}$ can be updated and the uncertainty in the model parameters can be reduced based on the experimental data. Then a similar analysis can be done

based on the posterior PDF to assess the predictive capabilities of the updated model. In this case, the posterior system performance measure can be written as

$$\mathrm{E}[h(\boldsymbol{\zeta})|\mathcal{M}] = \int h(\boldsymbol{\zeta}) p(\boldsymbol{\zeta}|\boldsymbol{\theta}, \boldsymbol{D}, \mathcal{M}) p(\boldsymbol{\theta}|\boldsymbol{D}, \mathcal{M})\, d\boldsymbol{\theta} d\boldsymbol{\zeta} \tag{20}$$

Again, this integral can be approximated using the Monte-Carlo simulation technique. Here, the difficulty is that the posterior PDF is often highly correlated and the high probability region is concentrated in a small region of the parameter space. Thus, generating independent samples from the posterior PDF is not always possible. This issue will be addressed in the next section.

Since all mathematical models are approximations of the real system, the robust predictive analyses stated in this section are thus sensitive to the selected model class. Therefore, a set of competing model classes can be constructed to describe the dynamics behavior of the real system. Then a posterior hyper-robust predictive PDF can be computed using the total probability theorem which gives the weighted average of the posterior input/output probability of each model class. This is called model averaging in the Bayesian framework. Since the predictions made by the averaged model are more robust to the uncertainties, this type of analysis is called the hyper-robust predictive analysis.

3.3.5 Sampling from the posterior distribution

The unknown *a priori* normalizing factor $p(\boldsymbol{D}^{obs}|\mathcal{M})$ in (16) makes the integrals with respect to the posterior PDF, which are common in the Bayesian inference, become analytically intractable. Laplace's method of asymptotic approximation has been used which approximates the posterior PDF using a Gaussian assumption. This method is valid when a large number of data are available. Another means of approximation is by the use of the Monte Carlo simulation technique. That requires simulation of samples from the posterior distribution $p(\boldsymbol{\theta}|\boldsymbol{D}, \mathcal{M})$. Drawing independent samples from the posterior PDF is inevitable in high dimensional parameter spaces since the evidence term $p(\boldsymbol{D}^{obs}|\mathcal{M})$ in Bayes rule is unknown and the high probability region is concentrated in a small region of the parameter space. The most commonly used simulation technique, the Markov Chain Monte-Carlo (MCMC) method, is developed to overcome these hurdles. The Metropolis-Hastings [16, 17], the Gibbs [18] and the Hybrid Monte Carlo algorithms [19] are MCMC algorithms that have been utilized for exploration of the posterior PDF in FE model updating problems.

4 APPROXIMATION OF THE FORWARD MODELS

The Bayesian or Frequentist approaches to statistical inverse problems are often costly endeavors, especially when the forward models are large-scale. The main computation cost arises from the large number of forward simulations needed to evaluate an estimator, by the Frequentist approach, or exploration of the posterior distribution, by the Bayesian approach. Reduced order modelling and surrogate modelling are two common responses to this computational challenge. The former technique is to reduce the size of the forward model to decrease the simulation time. This technique has been investigated thoroughly in **paper A** and the next section will briefly review the general concept of projection based model reduction techniques. The surrogate model is sometimes called the emulator, the response surface model or the meta model. The surrogate model provides an approximation of the input-output map of the forward model as a black-box model which does not inherit the dynamical properties of the model and can be evaluated at a low computational cost [20]. A Gaussian process model is one of the available surrogate modeling techniques. Details of surrogate modeling is beyond the scope of this thesis.

4.1 Reduced-order modeling

The goal of model order reduction is to project the high order FE model onto a much lower dimensional system that closely replicates the input-output map of the original system. This model is subsequently utilized in the updating procedure of the physical parameters; see [21-25] for overviews.

The first step in the projection based model reduction techniques is to define the projection matrix $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_{n_\mathrm{r}}] \in \mathbb{R}^{n_\mathrm{x} \times n_\mathrm{r}}$ to approximate the state vector $\boldsymbol{x}(t) \in \mathbb{R}^{n_\mathrm{x}}$ using $\boldsymbol{x}(t) = \boldsymbol{\Phi}\boldsymbol{x}_\mathrm{r}(t)$, where $\boldsymbol{x}_\mathrm{r}(t) \in \mathbb{R}^{n_\mathrm{r}}$ with $n_\mathrm{r} \ll n_\mathrm{x}$. A corresponding left basis matrix $\boldsymbol{\Psi} \in \mathbb{R}^{n_\mathrm{x} \times n_\mathrm{r}}$ can be defined such that $\boldsymbol{\Psi}^T\boldsymbol{\Phi} = \mathbf{I}$. Given the linear system in (2), model reduction attempts to construct the reduced system of the form

$$\begin{aligned} \dot{\boldsymbol{x}}_\mathrm{r}(t) &= \boldsymbol{A}_\mathrm{r}\boldsymbol{x}_\mathrm{r}(t) + \boldsymbol{B}_\mathrm{r}\boldsymbol{u}(t) \\ \boldsymbol{y}_\mathrm{r}(t) &= \boldsymbol{C}_\mathrm{r}\boldsymbol{x}_\mathrm{r}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{aligned} \tag{21}$$

with the associated transfer function of $\boldsymbol{G}_{\mathrm{r}}(s) = \boldsymbol{C}_{\mathrm{r}}(s\boldsymbol{I} - \boldsymbol{A}_{\mathrm{r}})^{-1}\boldsymbol{B}_{\mathrm{r}} + \boldsymbol{D}_{\mathrm{r}}$. In this formulation $\mathbf{A} = \boldsymbol{\Psi}^T\boldsymbol{A}\boldsymbol{\Phi}$, $\boldsymbol{B}_{\mathrm{r}} = \boldsymbol{\Psi}^T\mathbf{B}$, and $\boldsymbol{C}_{\mathrm{r}} = \boldsymbol{C}\boldsymbol{\Phi}$. For the structural models, including the examples considered in this thesis, the reduction is determined such that the following properties are satisfied: (a) for all inputs, $\boldsymbol{u} \in \mathcal{L}_2(\mathbb{R}^+)$, the input-output mapping of system in (2) is approximated with small error, $\boldsymbol{y}_{\mathrm{r}}(t) \approx \boldsymbol{y}(t)$, (b) the reduction algorithm is computationally efficient, and (c) important features of the original system such as its eigensoultions and stability are preserved. There are various procedures to select the projection basis to reduce the size of the large-scale models. Static and dynamics condensations have been used extensively in structural engineering. In static condensation such as Guyan, the inertia effects are omitted in the construction of reduction basis. Generally this method is not suitable as reduction algorithm in dynamic problem since the natural frequencies of the original system can be altered substantially [25]. In the control community, techniques such as Krylov-based methods, optimal Hankel approach, balanced truncation [22], and proper orthogonal decomposition [26] have received considerable attention to produce the reduced-order models. These techniques are also inappropriate in updating the physical parameters since the eigenvalues of the high order FE model are not guaranteed to remain unchanged.

Modal truncation is probably among the oldest reduction techniques in structural dynamics and continues to receive attention since it can handle large-scale systems [27-31]. Compared to the other model reduction techniques, modal truncation shows the most promise because of the following properties; (1) it leaves the important eigenvalues of the original system unchanged [28, 32], (2) it has an $\mathcal{H}_\infty$ upper error bound [24, 33], and (3) it is computationally efficient. This method projects the original system onto a smaller subspace using projection bases consisting of the most dominant eigenmodes. It is of central concern to effectively select the dominant eigenmodes that can yield reduced-order models that are the most possible optimal ones in the sense of an appropriate input-output measure [27, 31]. This concern is addressed in the **paper A**.

5 SUMMARY OF APPENDED PAPERS

5.1 Paper A

In **Paper A**, *An input-output based modal dominancy criterion in modal truncation,* the problem of model truncation in structural dynamics is considered. Modal truncation shows the most promise to yield reduced order models in structural dynamics since this algorithm is structural preserving. A central problem in modal truncation is to select the projection bases, the dominant eigenmodes, such that the yielded reduced order model that closely mimics the behavior of the original model. A frequency-restricted $\mathcal{H}_2$ based modal dominancy index is suggested in this paper to alleviate the mode selection problem. Then, an efficient algorithm, called the quasi-optimal modal truncation, is proposed to yield minimal reduced models in case of multiple eigenvalues which employs the orthogonal bases of the modal input matrix as the projection basis. The proposed truncation algorithm is compared with competing truncation algorithms.

5.2 Paper B

In **Paper B**, *Development of simplified models for wind turbine blades with application to NREL 5MW offshore research wind turbine,* the common industrial practice for blade modeling is challenged. A 61.5m long wind turbine blade, previously reported by the National Renewable Energy Laboratory, is selected as a case study and a verified industry-standard three dimensional shell model is developed based on its actual geometry. Given the reported spanwise cross sectional properties of the blade, a simplified model is developed based on the Euler-Bernoulli beam element formulation. Then, the beam model is calibrated against the simulation data obtained from the high order model using a developed FE model updating framework, named FEMcali. While, the calibrated beam model exhibits an excellent agreement to the low frequency dynamics of the baseline model in terms of mode shapes and resonance frequencies, the simulation study provides evidence that a beam model cannot capture all the important features found in a large-scale 3D blade. This motivates a departure from conventional beam element formulation and suggests addressing the problem of producing simplified models in the framework of model reduction techniques. The quasi-optimal modal truncation algorithm developed in **Paper A** is applied to the high fidelity shell model to produce a low order model. This paper suggests that the yielded low order model suites well to wind turbine design codes in which efficiency is of major concern.

5.3 Paper C

In **Paper C**, *Modeling and calibration of small-scale wind turbine blade,* the modeling, calibration and validation of an Ampair A600W wind turbine is presented. A series of experimental test results have been incorporated in the modeling and calibration. To support the modeling, a number of material tests have been performed, ranging from thickness measurements of the blade's skin through chemical analysis for determination of the polymer materials in the blade and destructive mechanical testing for estimation of material properties. Furthermore, vibration data has been collected over a large frequency range through stepped-sine testing. From frequency response data, the frequency-domain state-space subspace

(N4SID) system identification method is used to build a test model to which the nominal FE model is calibrated. A highly detailed FE model is built using laminate theory in the skin and linear isotropic material modeling in the core. To ensure asymptotic convergence of the prediction of the first seven eigenfrequencies, a comprehensive mesh refinement study was performed. In this study, the linear error of eigenfrequencies obtained from experimental data and the FE model has been used as the cost function. Since the model was calibrated using only the first five eigenfrequencies as parameters, seven more eigenfrequencies that were available from experimental data could be used for validation purposes.

6 CONCLUSION AND FUTURE WORKS

In this thesis, the validity of structural dynamics models of wind turbine is studied. The model validity is of hierarchical dual level. On one hand, a detailed structural dynamics model needs to be substantiated by good correlation between experimental results of wind turbine testing and theoretical simulation results using that model. On the other hand, after that detailed model has been validated, a model of significantly low order based on the detailed model has been validated by a good model-to-model correlation. With the connection between models, this implies that also the low order model is implicitly validated by testing.

The use of two model levels is strongly motivated by their different focal points. The detailed structural dynamics model is motivated by its strong connection to the observed fundamental physical laws. The purpose of this model is to give physical insight to observations made during testing. **Paper C** discusses the development of a highly detailed FE model of Ampair A600W wind turbine blade. An ambitious measurement campaign including both non-destructive testing for dynamic properties and dedicated destructive tests for deduction of material properties are executed in this study to support the modelling. Destructive testing exhibited a large variation of the Young's modulus of the core and skin material which represents the extent of uncertainty available in calibration parameters. From vibration testing, it is concluded that the parametric variability of the A600W blades is fairly large. However, this paper was concerned only with calibration to a specific blade. The Active Set algorithm is used to minimize the linear distance of the first five eigenfrequencies obtained from the experimental and analytical model. This study concludes that the employed V&V activity substantiates the ability of the calibrated FE model to predict the vibrational pattern of the Ampair A600W wind turbine blade.

The low-order model is motivated by its fast evaluation. The main purpose of this model is to give a correct representation of the stimuli-to-response characteristics of the system in an efficient simulation environment. Such low-order model is highly suitable for integration with a fluid dynamics code for simulation of the entire wind turbine. **Paper B** discusses the development of a simplified model for the NREL 5MW wind turbine blade. A particular effort has been made to calibrate an industry-standard beam model against the simulated data obtained from a high fidelity three dimensional shell model by the use of Levenberg-Marquardt and multi point start procedure. Incorporation of a damping equalization technique and a logarithmic least-square optimization technique made the problem better posed for solution of the involved inverse problem. A model reduction was considered as another alternative to develop a simplified blade model. Toward this end, the quasi-optimal modal truncation algorithm proposed in **Paper A** is applied to the high fidelity shell model to produce a reduced model which simultaneously preserves the vibrational pattern of the retained eigenfrequencies and also mimics the input-output behavior of the shell model. The retained eigensolutions were selected based on their contribution to the $\mathcal{H}_2$-norm of the error transfer function. Two different load conditions were applied to the obtained simplified models; a random phase multi-sine excitation and a step excitation. Based on the obtained results, it is suggested to integrate the simplified blade model obtained by model reduction algorithm into aeroelastic simulations in order to keep the input-output error of the system at a low level.

Robustness to uncertainty is a crucial characteristic of the computer models developed to predict the dynamics of real systems such as wind turbine blades. Bayesian model calibration provides a systematic framework to incorporate different sources of uncertainty into the joint posterior probability of the model parameters. Its outcome enables us to make robust predictions. This fact motivates us to depart from the classical FE model calibration methodology presented in **Paper B** and **C**, and try to formulate the model calibration problem of the Ampair A600W wind turbine blade in the Bayesian framework. The Ampair A600W blades can be selected as case study since the vibration test data exhibites the parametric variability and the destructive tests provide the prior distribution of the uncertain parameters. However, the Bayesian framework is a computationally expensive endeavor in wind turbine application. Its reason is twofold. First, a tremendous number of large-scale forward simulations are required to explore the posterior PDF. An extension to the **Paper A** would be to modify the proposed quasi optimal modal truncation technique to be incorporated into the Bayesian framework. This technique helps us to evaluate the forward simulation at much lower cost. Since each new parameter setting demands the reconstruction of a new low-order model, the direct use of this method is not computationally efficient. Linearization of parametric dependence is one approach which can be used in parallel to the developed modal truncation technique to reduce the costly simulations involved in the Bayesain framework [20]. Second, the effectiveness of MCMC approaches is affected by the dimension of the model

parameter space and poses a challenge to generate independent samples from the posterior PDF. To this end, an appropriate MCMC technique to tackle this problem is under development in our research group [34].

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