Gearshift Sequence Optimisation for Vehicles with Automated Non-Powershifting Transmissions

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Gearshift sequence optimisation for vehicles with automated non-powershifting transmissions

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Abstract: Vehicle powertrains are increasingly automated. The gearshift strategy that is built into an automated manual (non-powershifting) transmission (AMT) has to determine when to shift and to which gear. AMT has the potential to reduce environmental impact and improve vehicle performance. However, the gearshift strategy is far from obvious, especially for heavy-duty trucks with many gears.

This paper presents an analytic tool for finding the ultimate gearshift sequence for any given vehicle and driving situation. The paper includes some case studies that show credible results. These studies address:

- uphill acceleration
- engine torque down control

The optimisation criterion used is minimum acceleration time.

The tool is based on a trajectory optimisation method called dynamic programming. The cost function is formulated using simulations results from a dynamic system model, which can be implemented in Simulink. Comparisons with measurements secure a reasonably good practical relevance.

Keywords: automated manual transmission; computer aided gearshifting; optimisation; gearshift sequence; simulation; performance; dynamic programming.

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Biographical notes: Bengt Jacobson took his PhD on gearshifting dynamics in automatic vehicle transmissions at Chalmers University of Technology in Sweden 1993. He stayed as post-doc, and later Associate Professor, in Vehicle Dynamics at Chalmers. The research area was simulation and control of automotive powertrains and the teaching covered vehicle dynamics in a wider sense. Mid 2001 Jacobson moved to Vehicle Dynamics at Volvo Cars, to lead projects on complete vehicle control.

Michael Spickenreuther took his Diplom Ingenieur degree in Mechanical Engineering at TU München, Germany. He moved on to BMW to do industrial PhD research in the field of simulation based automotive development, connected to Insitute of Vehicle Engineering at TU München.

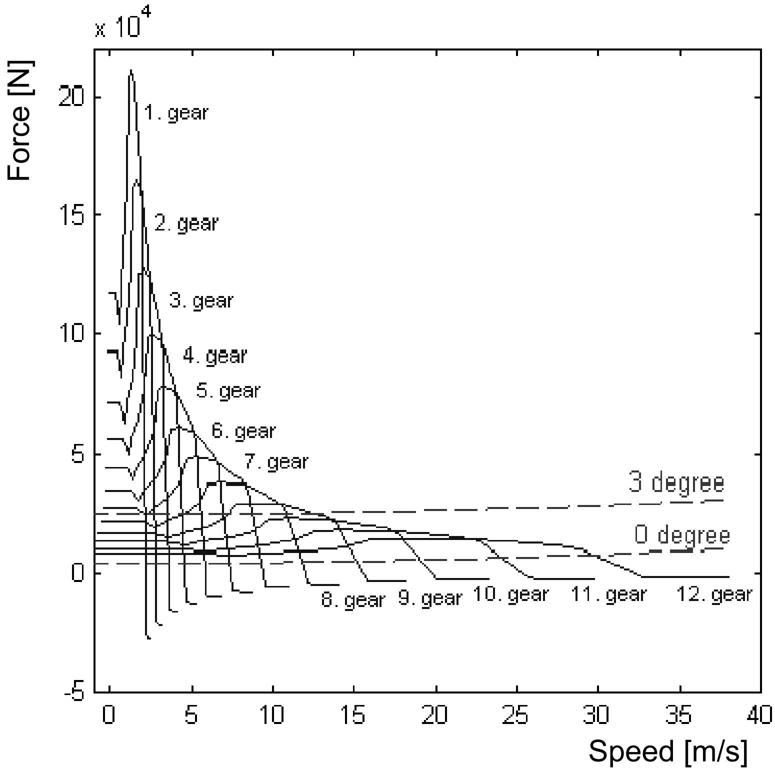
1 Introduction

1.1 Background and driving forces

In order to improve fuel efficiency and driveability there is a tendency to involve more gears in fixed-step transmissions for passenger cars. Heavy trucks have had a large number of gears, from 6 to 12, for many years. To make driving more comfortable, and to ensure that suitable gears are selected, these gearboxes are becoming automated. Such a transmission is referred to as an Automated Manual Transmission (AMT) or a transmission with Computer Aided Gearshifting (CAG). In that scenario, the control software plays as large a role as the hardware. The software has to decide when to shift and to which gear.

For given hardware and optimisation criteria, such as fuel economy or maximum acceleration, a particular gear is optimal for a certain instantaneous driving situation. For the example in Figure 1, gear 11 is an optimal choice for maximum acceleration when driving at 20 m/s. However, if we consider a time interval instead, and take the gearshift dynamics into account, it is not as easy to define optimal gears. For example, still referring to Figure 1, consider an acceleration from 15 m/s in 10th gear to 25 m/s in 12th gear. Then it is not obviously worth shifting sequentially from 10th to 12th gear via gear 11. Perhaps it would be preferable to jump gear 11.

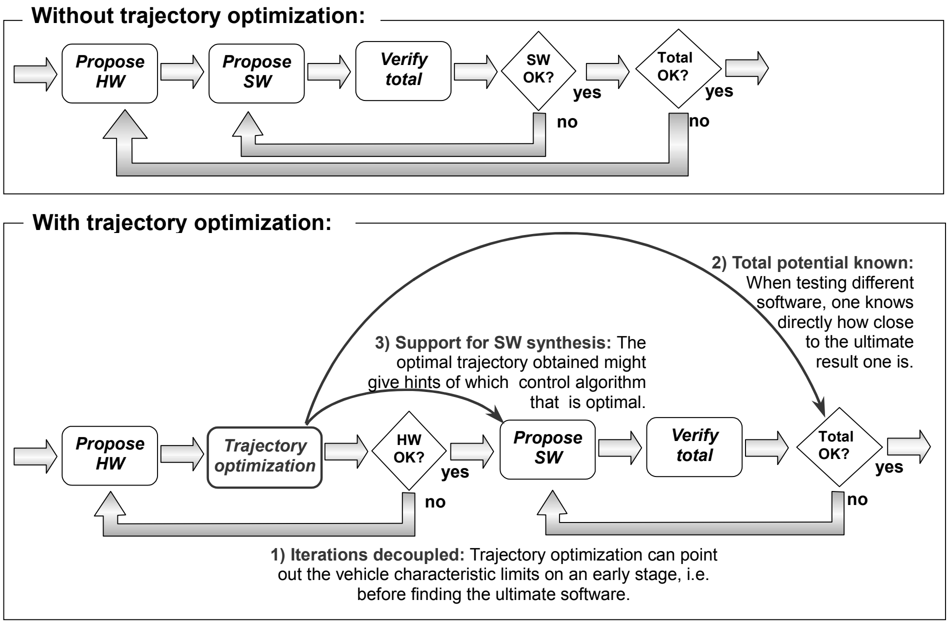
Figure 1 Traction force vs speed diagram for a heavy-duty truck with a 12-liter engine and a transmission with 12 forward gears. Dashed lines show driving resistance for different uphill slope



The issue of finding optimum operation for a system over a time interval is called *trajectory optimisation*, as opposed to *parameter optimisation*. Finding optimal gearshift sequence is one automotive problem to which it may be desirable to apply trajectory optimisation. Additional automotive examples where trajectory optimisation may be applicable are throttle and spark advance control for traction force response and comfort [1], and energy storage control in a hybrid vehicle for fuel economy over a complete driving cycle [2–4].

In addition to optimising vehicle characteristics, there are process arguments for using trajectory optimisation in product development (see Figure 2, items 1–3).

Figure 2 Potential influence by trajectory optimisation on a principal development process. HW means hardware, such as engine map, gearbox ratios, etc. SW means software, i.e., control algorithms



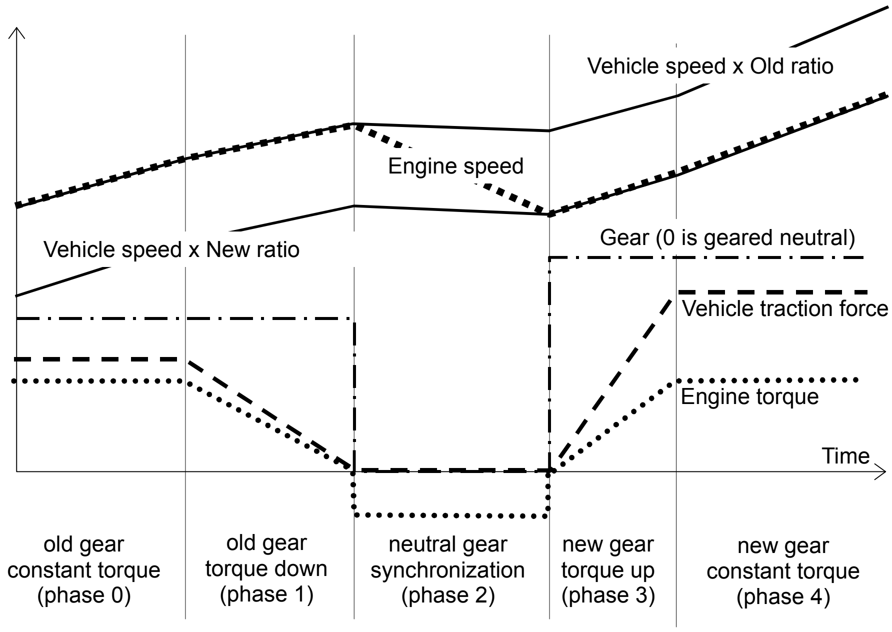
1.2 Aim and goal

The work presented aims at developing optimisation as an automotive system design tool. The concrete goals are to create a general computer code for trajectory optimisation and to verify that it can be successfully applied to the problem of finding the optimal gearshift sequence for a heavy-duty truck with automated manual gearbox.

2 Automated manual transmissions

An automated gearshift consists of five phases (see Figure 3). In phases 0 and 4, the gears are engaged and the engine and vehicle accelerate synchronously with the ratio. In phases 1 and 3, the torque is decreased and increased, respectively. In phase 2, no gear is engaged and the angular speed of the engine is reduced down to synchronisation speed for the next gear, using negative engine torque.

Figure 3 Phases of the gearshift process in an upshift



The process is very similar for a downshift, but in that case the engine speed is increased during the synchronisation. The system of gearshift processes is analysed in, e.g., Ref. [5].

Since the vehicle acceleration is interrupted during each gearshift, it might be preferable to jump some of the available gears. If aiming for maximum acceleration performance, it may appear that one should change gear when reaching the speed where the curves for each gear intersect (see Figure 1). However, this is not obvious, since the vehicle speed changes during the gearshift. In conclusion, the software controlling an automated gearshift has to consider both when to shift and to which gear.

3 Principal ideas for solution

There are many methods for finding an optimum trajectory for a dynamic system. There are *optimal control* methods, e.g., those used in Refs. [1,4], as well as methods based on *evolutionary algorithms*, e.g., those used in Ref. [6]. In the present work *dynamic programming* is used, as in Refs. [1,7–9], describing the method of dynamic programming.

Dynamic programming employs:

- a cost function, which defines the system characteristics to be optimised
- a dynamic system model, which defines the system to be optimised
- a state grid, which defines the variable space in which the optimal solution is sought

Dynamic programming works with the principle of this optimality [9]:

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

The concrete output of the present work is the computer code, which is written in Matlab [10]. The dynamic system model is implemented in Simulink [11].

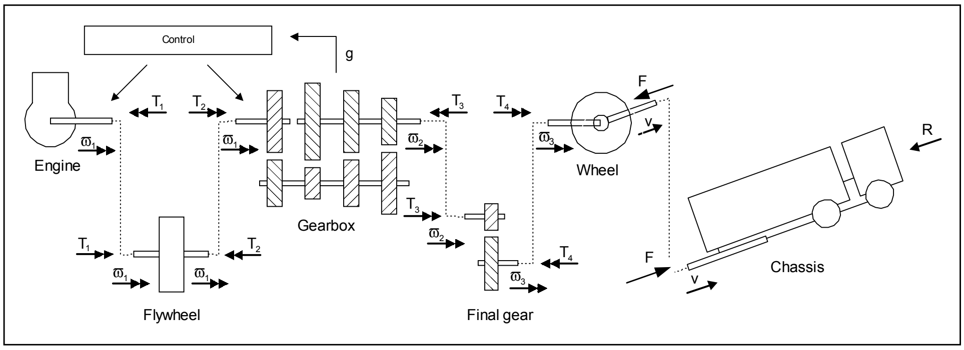
3.1 Cost function

The optimisation aims at minimising the time for taking the system from given start conditions to given end conditions.

3.2 Dynamic model

A 40-ton truck with a 6-cylinder, 12-l diesel engine and 12 forward gears as been modelled. The system is decomposed as in Figure 4.

Figure 4 Submodel decomposition of the truck with used variables. Notation: ϖ = rotational speed, T = torque, g = gear, F = force, v = speed, R = resistance force



Basically, the model can be described as follows:

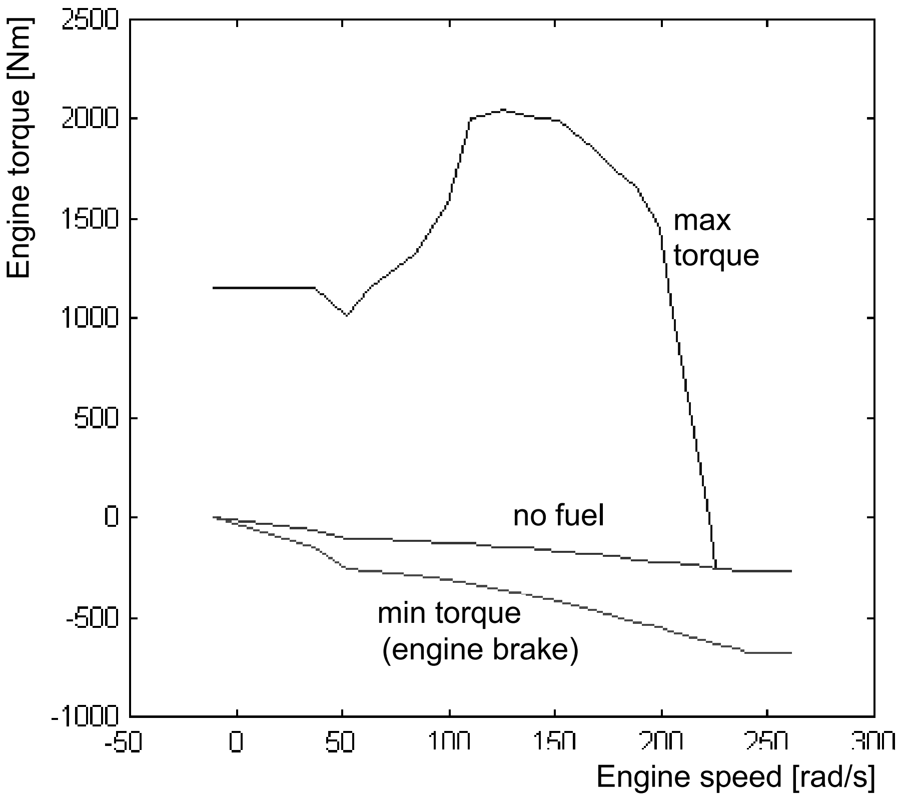
Control model: The engine is assumed to run at its maximum torque, except during the shift phases 1, 2 and 3 in Figure 3. In phases 1 and 3, the torque is ramped down and up, respectively. Two model parameters, t_{down} and t_{up}, are used to control the duration of phases 1 and 3. In phase 2, the engine is run at minimum torque.

Engine (including clutch) model: A steady state map for either maximum or minimum torque as function of engine speed models the engine combustion torque. The minimum torque, which is engine braking, is used only during the synchronisation phase. An engine alone cannot produce positive torque from zero engine speed, but slip in the clutch gives positive torque from zero speed at the clutch output shaft. This is included in the engine model, which finally gives us an engine characteristic as shown in Figure 5. The 'no fuel' curve is not used. It corresponds to minimum torque without adding the engine brake, which would make the synchronisation phase unnecessarily long. A flywheel replaces all rotational inertia in the engine, but this inertia is only considered in the synchronisation phase. This is the only engine dynamics considered in the present work. However, it is possible to include more engine dynamics in the optimisation, which is shown in Ref. [12].

Gearbox model, including final gear and driven wheels: The gearbox model normally works as an ideal transmission ratio without losses. When geared neutral, both input and output torque are zero.

Chassis model: The chassis is modelled as a translating mass, driven by the wheel traction force and braked by driving resistance, including the road slope (gradient), head wind and rolling resistance.

Figure 5 Output torque from engine and clutch as a unit



The whole model can finally be described as a dynamic system with the following state variables.

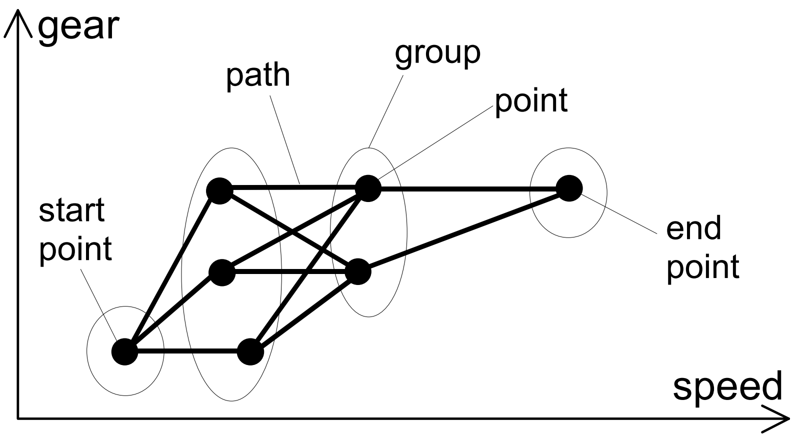
- vehicle speed, which is a (continuous) state variable during the entire simulation
- actual gear, which is a (discrete) state variable during the entire simulation
- engine speed, which is a (continuous) state variable during the gearshifting phase of synchronisation. Otherwise engine speed is constrained to vehicle speed

The dynamic model is further described in Appendix A.

3.3 State grid

In our case, the state grid is defined as the two-dimensional space over gear and speed (see Figure 6). All possible combinations of ‘paths’ between the ‘points’ in different ‘groups’ are tested in a special computationally efficient way. The totally optimal combination of paths is finally established. In addition to the points and groups in the state grid, how each path is passed also has to be specified. In our case, this is handled so that if a gearshift is involved, it is carried out in the beginning of the path and then the vehicle accelerates to the speed defined by the end point of the path. Evaluation of each path is done through a dynamic simulation, such as the one illustrated in Figure 17.

Figure 6 State grid



In the cases presented below, a much tighter grid is used than shown in Figure 6. Also, it is rectangular, $N \Delta M$, so that there is one single speed step size defined, which then defines the whole grid. A typical grid used below contains N = 200 groups (speed 0 to 20 m/s with speed step size 0.1 m/s) and M = 12 points in each group (12 gears). The total number of evaluated paths, i.e., evaluations of the cost function for a path, is approximately $M \Delta M \Delta N = 12 \Delta 12 \Delta 200 = 2.9 \Delta 10^4$. Without dynamic programming, each combination of paths from start to end point would have to be evaluated, which would mean that the number of path evaluations would be about $M^N = 12^{200} = 6.9 \Delta 10^{215}$. This shows the enormous computational savings achieved by using dynamic programming. Nevertheless, some engineering knowledge about the system has been used to further reduce the amount of computation, which also is suggested in Ref. [7]. For example, in the present work downshift paths are not considered in the acceleration optimisation cases. One could probably also gain even more efficiency by disregarding upshifts of more than 6–7 gear steps, depending on which driving situation and vehicle one is studying.

A more detailed description of dynamic programming is given in Appendix B.

3.4 Case studies

One of the most crucial driving situations for gearshift sequences is acceleration on a steep uphill slope. If the upshift is tried at too low a vehicle speed or to too high a gear, a synchronisation failure could occur, and the gearbox would have to be downshifted again. In a worst case scenario, so much vehicle speed is lost that the vehicle has to be stopped, after which the driver will find it difficult to move off again on the steep uphill gradient. The safest way is to shift up to the next higher gear at a high engine speed. However, such gearshift strategy can be far from optimal in many respects, such as vehicle performance, energy consumption and emissions. The influence of uphill slope is studied in the first case below.

Another issue of interest is the engine torque down control during the gearshifts, i.e., in phase 1 in Figure 3. If a long time is spent in the torque down phase, the shift comfort is improved. However, there is a trade-off with vehicle performance, since each shift takes longer time. Without gearshift sequence optimisation there would definitely be a problem in establishing how torque down time influences the acceleration performance, since the duration of each gearshift influence the best choice of gears to be jumped. The influence of 'torque down time' is studied in the second case study below.

4 Case study: influence of uphill slope

The study analyses three different uphill slopes: 0∀, 1.5∀and 3∀. The top speed and the highest possible gear will vary according to the degree of slope, which can be seen from Figure 1. The possible speed and gear ranges are shown in Table 1. A speed step size of 0.1 m/s is used in the state grid in the optimisations below.

Table 1 Top speeds and gears on different slopes

Slope [degrees]	*Speed* [m/s]	*Gear*
0	0–29.5	1–12
1.5	0–19.3	1–11
3	0–12.2	1–9

4.1 Results

Results from optimisations up to top speed for slopes of 0∀, 1.5∀and 3∀are shown in Figures 7, 8 and 9, respectively. Figure 7 is only plotted to show up to 25 m/s, since there are no gearshifts on the continuation up to 29.5 m/s.

Figures 10, 11 and 9 show optimisations up to 12.2 m/s for slopes of 0∀, 1.5∀and 3∀, respectively. A speed of 12.2 m/s can be reached on all slopes tested, so these latter diagrams are easier to compare.

The variable ShiftState in the plots shows the phases according to Figure 3.

Figure 7 Optimal gearshift sequence at a slope of 0∀

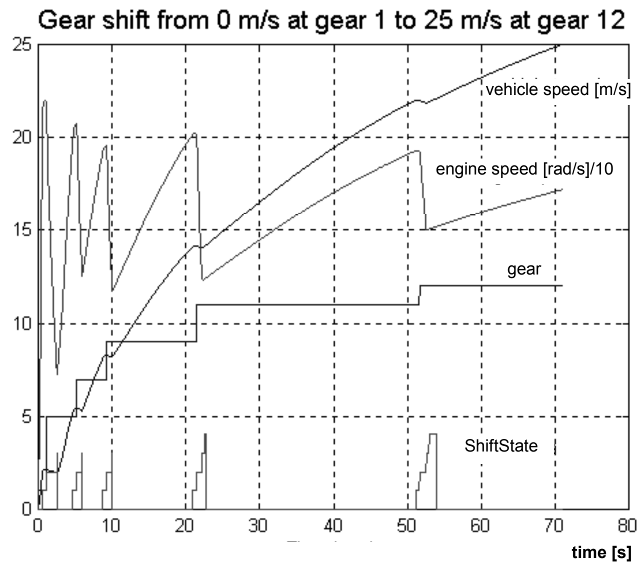


Figure 8 Optimal gearshift sequence at a slope of 1.5∀

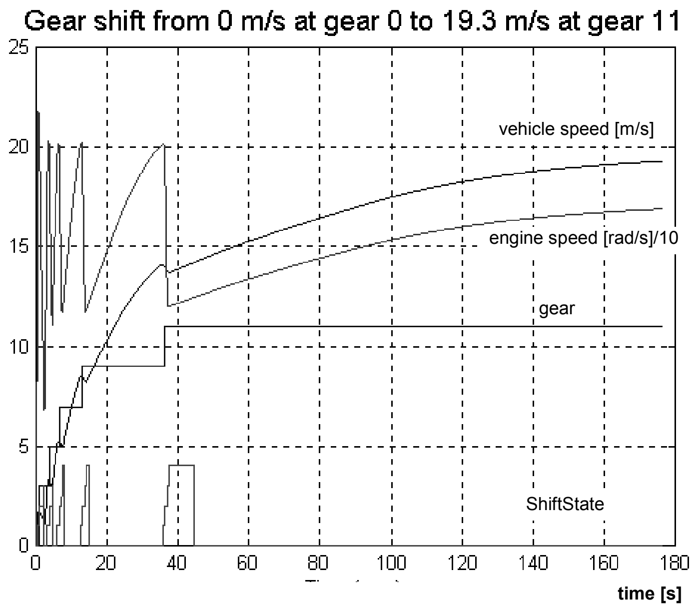


Figure 9 Optimal gearshift sequence at a slope of 3∀

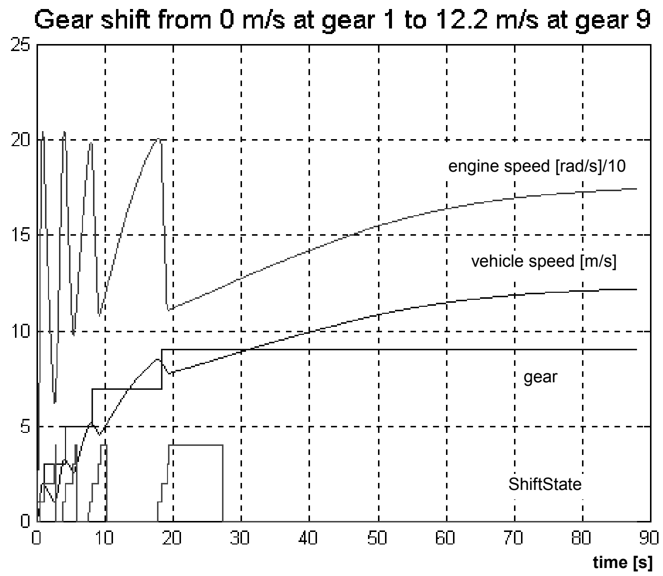


Figure 10 Optimal gearshift sequence at a slope of 0∀

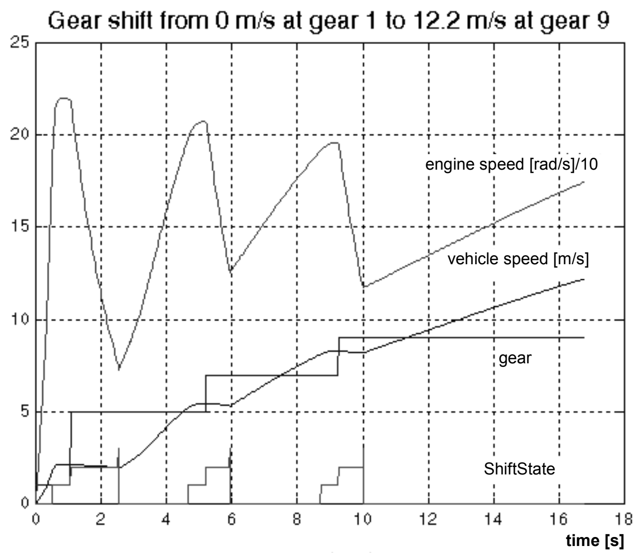
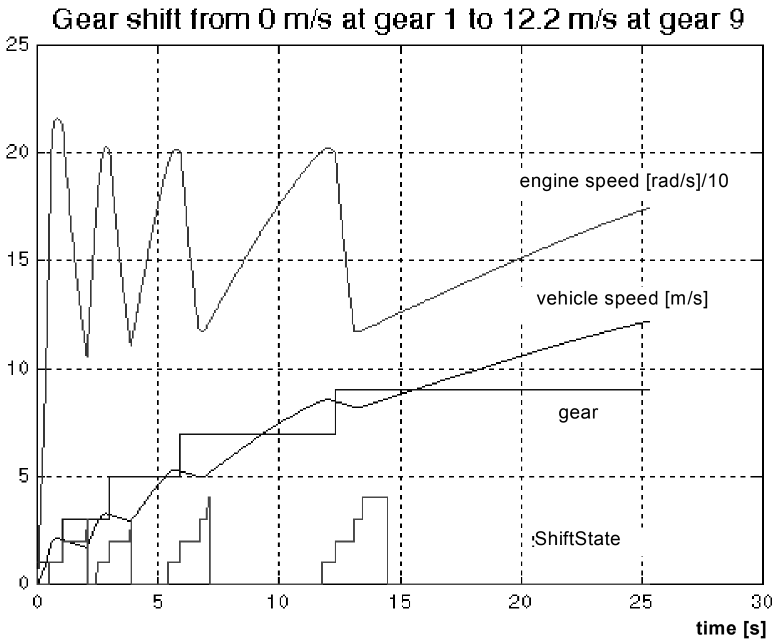


Figure 11 Optimal gearshift sequence at a slope of 1.5∀



4.2 Analysis of results

To emphasise the resulting sequence of the optimisation cases, some approximate values are collected in Table 2.

Table 2 Results of the optimisations in Section 4

Uphill slope [degrees]	*Gears* *(without start gear 1)*	*Engine speed* [rad/s] *(at start of ShiftState 1)*	*Duration* [s] *(of the ShiftState 1–3)*
0	5 7 9	220 205 195	2.0 1.7 1.7
1.5	3 5 7 9	210 200 200 200	1.8 1.8 1.8 1.8
3	3 5 7 9	200 200 200 200	2.1 2.1 2.1 2.1

The following general observations can be made:

∉# engine speed for gearshifts varies depending on the degree of slope

∉# only some gears are used

∉# the gears used vary depending on the degree of slope

It may be noted that increased slope leads to:

∉# decreased top speed

∉# longer time to reach a particular speed

∉# more gears being used

It was further noted, in some optimisations not presented in the diagrams, that it is not always possible to reach the top speed defined by intersection in the traction force diagram in Figure 1. As an example, consider a vehicle without 8th gear on an uphill slope of 3∀. In Figure 1, we can then observe a top speed of 12–13 m/s in 9th gear. However, the optimisation will illustrate that the vehicle is unable to shift from 7th to 9th gear as it loses too much speed during the shift. Consequently, the top speed cannot be reached through acceleration. The interesting thing is that we can find such limitations through the optimisation procedure. It can be noted that the vehicle can still drive at the real top speed, 12.2 m/s in 9th gear, providing it has reached this speed by other means, such as driving in 9th gear on a flat road, which gradually increases its slope to 3∀.

The results are qualitatively expected and logical, which is why they are also considered to be quantitatively reliable.

One might think that the engine speeds reached before shifting, typically 200 or 220 rad/s in Table 2, are surprisingly high. Figure 5 shows that engine power (speed multiplied by torque) is close to zero or even negative at those speeds. An alternative gearshift strategy could be an earlier shift to the same gear, which would bring the engine down to very low power levels, which would ruin the acceleration in a short time perspective. If shifting to a closer gear, the engine power would remain high, but more gearshifts would be required, which would also negatively affect acceleration, but over a longer time perspective. Consequently, the result which indicates that the engine should reach those high speeds before shifting seems reasonable from an overall point of view for the complete acceleration.

Further, one might reflect over the fact that many of the lower gears are jumped. It would be tempting to propose a redesigned gearbox with less tight gear steps among the lower gears. However, the road slopes tested were rather modest and larger slopes may reveal the importance of tight lower gears.

5 Case study: influence of torque down time

The study analyses three different torque down times: 0.5, 1.0 and 2.0 s. The torque up time is fixed to 0.3 s. The optimisations concern driving on a level road with acceleration from 0 m/s in 1st gear to 25 m/s in 12th gear. A speed step size of 0.1 m/s is used in the state grid in the optimisations.

5.1 Results

The optimisation results for torque down time 0.5, 1 and 2 s are plotted in Figures 12, 13 and 14, respectively.

Figure 12 Optimal gearshift with a torque down duration of 0.5 s

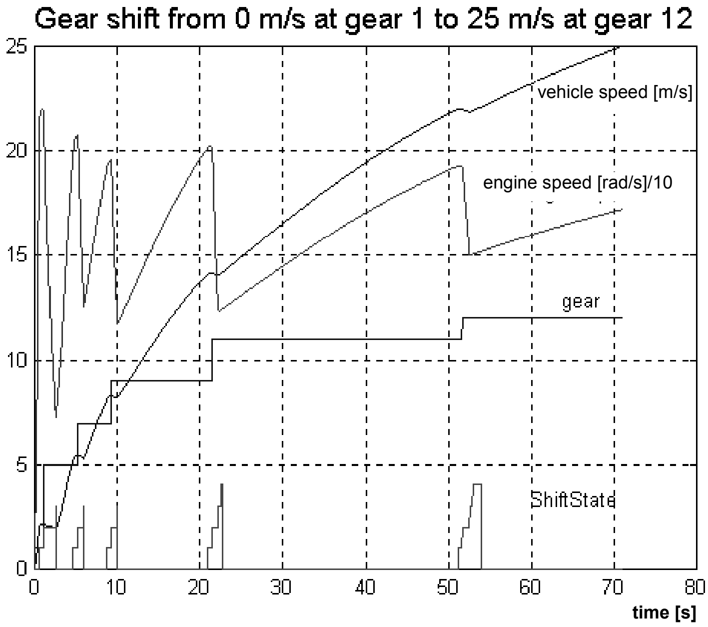


Figure 13 Optimal gearshift with a torque down duration of 1.0 s

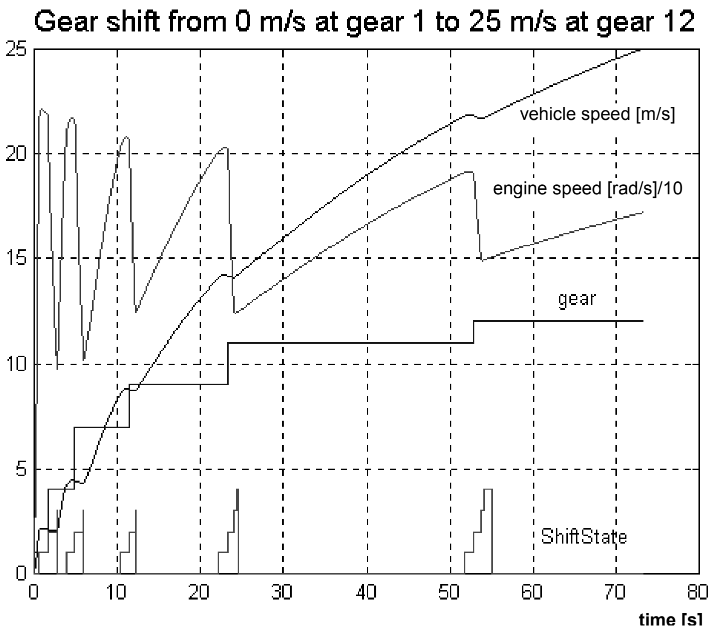
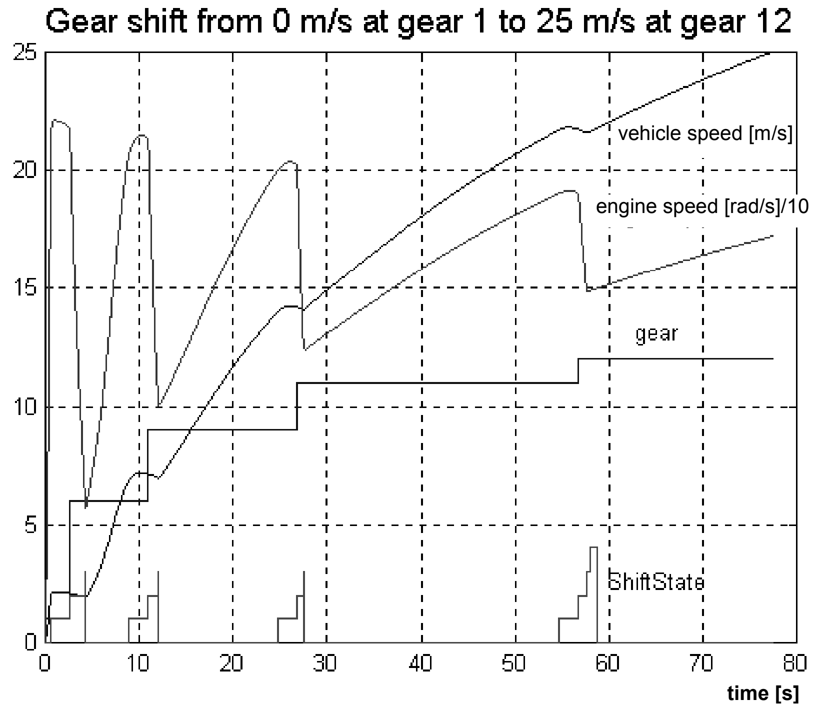


Figure 14 Optimal gearshift with a torque down duration of 2.0 s



5.2 Analysis of results

To emphasise the resulting sequence of the optimisation cases, some approximate values are collected in Table 3.

Table 3 Results of the optimisations in the Section 5

Torque down time [s]	*Gears (without start gear 1)*	*Engine speed* [rad/s] *(at start of ShiftState 1)*	*Duration* [s] *(of the ShiftState 1–3)*
0.5	5 7 9 11 12	220 205 195 200 190	1.6 1.3 1.3 1.5 1.5
1.0	4 7 9 11 12	215 210 205 200 190	2.0 2.0 1.8 1.8 2.1
2.9	6 9 11 12	220 215 205 190	3.3 3.0 2.5 2.7

The same general observations as for the previous case can be made.

It may be noted that increased torque down time leads to:

- longer time to reach a particular speed
- fewer gears used
- no influence on top speed

The results were qualitatively expected and logical, which is why they are also considered quantitatively reliable.

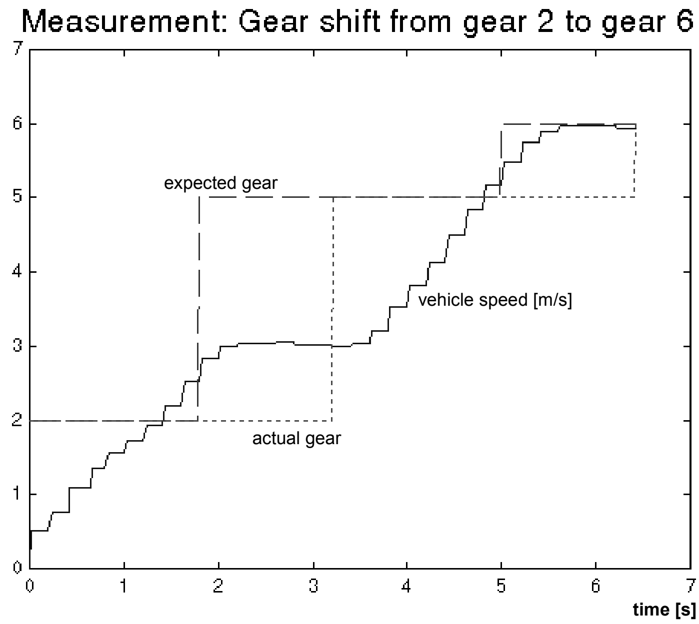
6 Comparison with measurement

The measured curves in Figure 15 derive from a full pedal acceleration with a truck, very similar to the truck modelled and optimised in the present work. Figure 16 shows the corresponding optimisation.

It is not meaningful to compare the two diagrams in detail, because the gear ratios in the measurement are somewhat different to the gear ratios of the model. Anyway, two types of comparison can be made:

- comparison to find how good the model is
- comparison between the control implemented by Volvo Truck Corporation and the optimised gearshift sequence

Figure 15 Measurement. Full pedal acceleration of a production truck, similar to the one studied in the present work

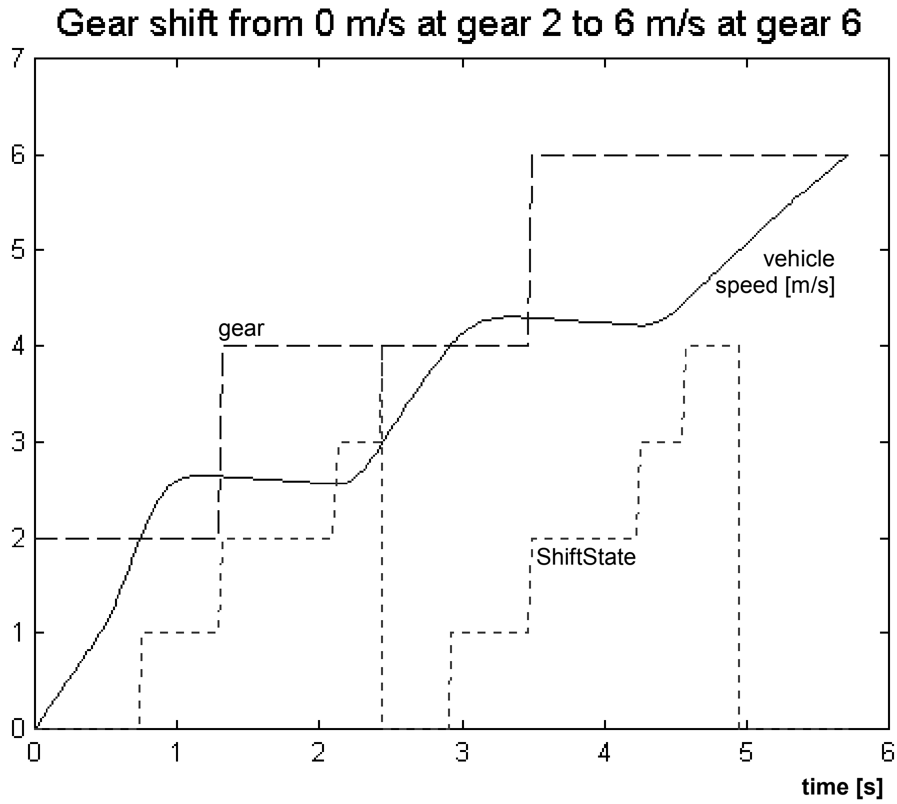


In the perspective of the first comparison, the acceleration in 2nd gear is a bit too high in the theory. This could be explained by the fact that theoretical curve assumes a very good control of the clutch when the vehicle speed is close to zero. Acceleration in 5th gear (measured) is slightly smaller than that in 4th gear (theoretical), which seems logical in view of the different gear ratios. In conclusion, the model is not perfect, but probably good enough to capture the essential problems inherent of a gearshift sequence optimisation.

The second comparison basically shows that the production truck chooses a higher gear, 5th instead of 4th. This could be considered to be within the acceptable range of inaccuracy of the theoretical study. It might also be due to the fact that the real truck is probably optimised also considering fuel economy.

Overall, the similarities of the measurements are considered to be good enough.

Figure 16 Theoretical optimisation



7 Conclusions

A general computer code for trajectory optimisation has been developed and successfully applied to gearshift sequence optimisation. The generality is proved by the fact that the code has also been used in another case: fuel consumption optimisation for a city bus with automatic (powershifting) transmission [12].

The analysis of the optimisation results seems reasonable in all aspects, which is why the optimisation method is regarded as credible.

A very positive aspect of the developed code is that the model implemented in Simulink can be used. Simulink is a very commonly used modelling and simulation software and it has an easy interface for defining dynamic models.

There is a limitation with the chosen optimisation method, dynamic programming. Although it is very computationally efficient, as compared to testing all possible combinations in the state grid, the required computational efforts increase rapidly if more dimensions are introduced in the state grid.

Issues which could be addressed in further research are:

- Implementation of engine dynamic characteristics, e.g., turbo dynamics. This is actually done in a basic form in Ref. [12].
- Optimisation of a gearbox ratio layout. This could be done by optimisation of a sequence with a very tightly stepped, virtual gearbox. The gear ratios chosen would then point out the optimal ratio layout for the real gearbox. Another attempt to approach optimisation of gearbox ratio layout is given in Ref. [13], based on pure simulation, a statistic test plan and regression.

Acknowledgement

Professor Göran Gerbert, Chalmers University of Technology, and Tekn. Lic. Anders Eriksson, Volvo Trucks, are gratefully acknowledged for comments on the present work and important know-how about automated gearshifting.

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Appendix A Model description

A heavy-duty truck from Volvo Truck Corporation is studied. The engine of the truck is a diesel engine, DC12C420, EM-ECPT5. The fully automated gearbox, Geartronic GSS-AGS, is based on the manual gearbox VT2514. For more detailed information, see Table 4 and Section 3.2.

The total model is implemented in Simulink and used as a subroutine within the optimisation code.

Table 4 Some details of the simulated truck. A = front area, c_d = drag coefficient, ρ = air density, v = speed, f = rolling resistance coefficient, m = mass and g = gravity

Engine	Rotational inertia	4 kg m^2
	Torque down time	0.5 s (except in case study 2, where this parameter is varied)
	Torque up time	0.3 s
Transmission to driven wheels	Gear ratios for the 12 forward gears	14.94, 11.72, 9.03, 7.09, 5.53, 4.34, 3.43, 2.69, 2.07, 1.63, 1.27, 1.00
	Final gear ratio	3.44
	Wheel radius	0.5 m
Chassis	Air resistance	$R_{air} = A \cdot c_d \cdot \frac{\rho}{2} \cdot v^2 = 10 \cdot 0.7 \cdot \frac{1.2}{2} \cdot v^2 = 4.2 \cdot v^2$
	Rolling resistance	$R_r = f \cdot m \cdot g = 0.01 \cdot m \cdot g$
	Grading resistance	$R_s = m \cdot g \cdot \sin(\text{slope})$
	Total resistance	$R = R_s + R_{air} + R_r$

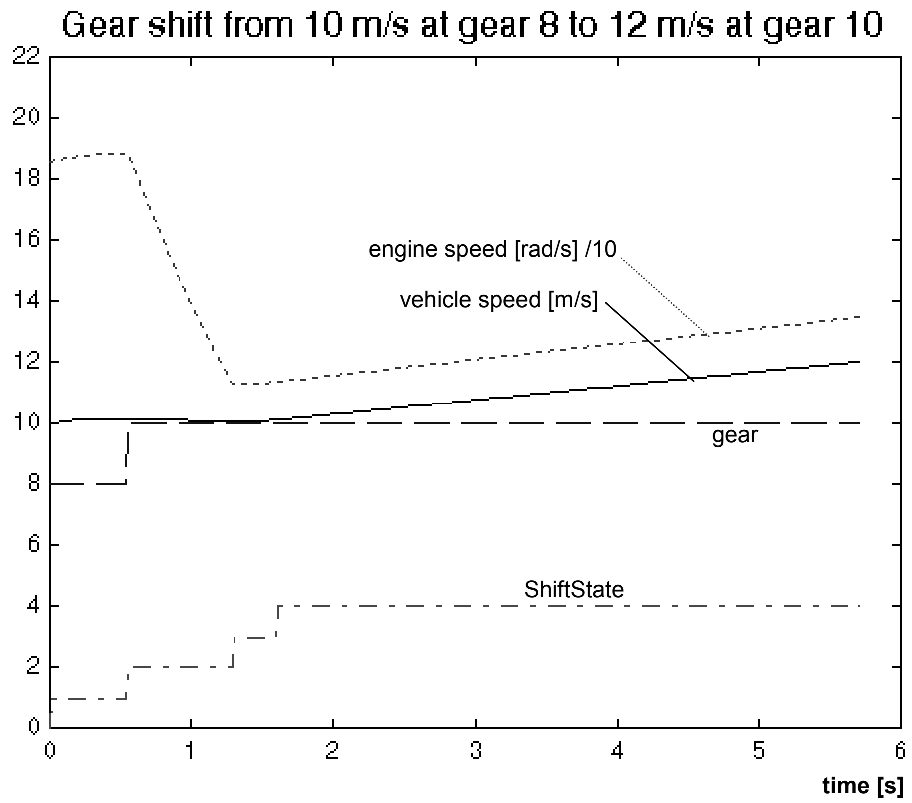
A.1 Example of a single gearshift

In order to demonstrate the model as a dynamic model, a single gearshift is simulated without any optimisation aspects. The result is shown in Figure 17. The ShiftState curve has five different levels, corresponding to the phases in Figure 3.

Appendix B Dynamic programming

Ref. [9] describes dynamic programming as a general method, but here an example is given in the context of the application of this work. The optimisation problem is to take a vehicle from 1 m/s in 1st gear to 3 m/s in 2nd gear.

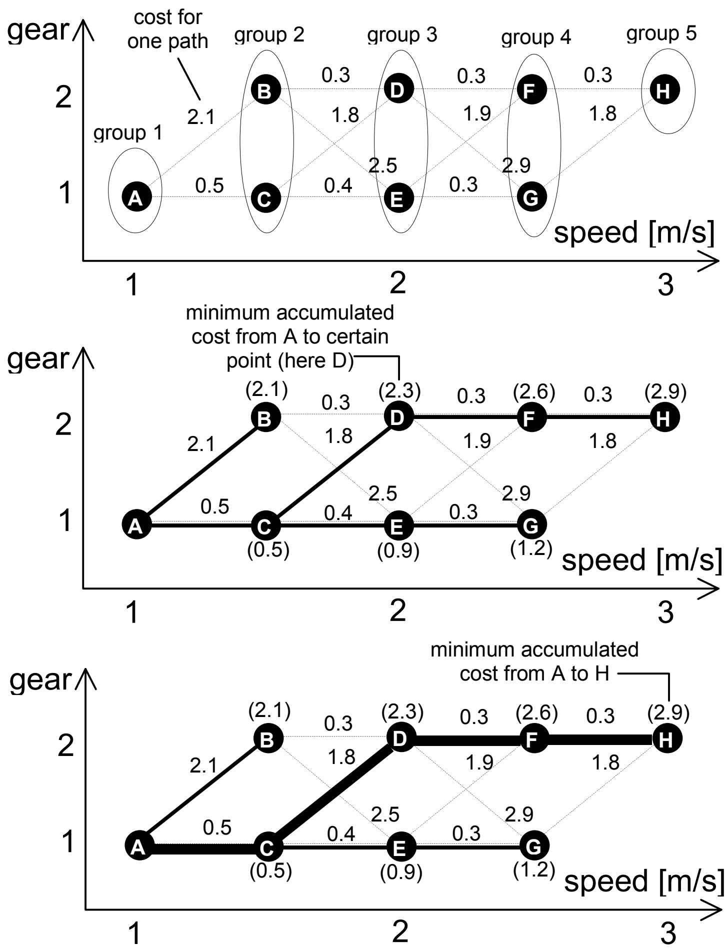
Figure 17 Single gearshift with engine brake



A state grid with speed step size of 0.5 m/s is used. Then the state grid is defined (see dots A–H in the upper diagram of Figure 18). The dynamic model defines the time for performing each path in the grid, which is visualised by writing the numerical values for each part in the upper diagram of Figure 18.

Now look at the middle diagram in Figure 18. There is a procedure for scanning over the grid, from A to H. First, find out which is the best way to come to each of the points in group 2 from any point in group 1. The first point in group 2 is point B. As there is only one starting point in group 1, the start has to be made from this point (A) and the cost is 2.1 s. Then 2.1 is given within parentheses at point B and the solid line is drawn to mark the path. The same procedure is done for the second point (C) in group 2. Then, start with group 3. How to arrive at point D? Now, we can either come from point B or point C. The accumulated cost for coming from point B to D is 2.1 + 0.3 = 2.4 s. From point C this is 0.5 + 1.8 = 2.3 s. Thus, the latter is best and 2.3 is given within parentheses at point D and the solid line between C and D is drawn. The same procedure for point E gives 0.9 s and a solid line from C to E. This can be continued until we reach the end point H. Then the accumulated cost is 2.9 s, which is then the cost for the ultimate trajectory.

Figure 18 State grid and optimisation process using dynamic programming. The solid lines in the middle diagram are candidates for belonging to the optimum solution. The thick lines in the lower diagram constitute the final optimum solution



The optimal trajectory itself is found in the lower diagram of Figure 18. By following the solid lines from point H back to point A and marking them extra thick, there will be only one path identified and this is the optimal trajectory.

It might be noted that the procedure can be defined in the opposite direction, scanning from point H to A first and then finding the ultimate trajectory by following thin dashed lines back to H. The cost for each part must then be simulated as a *final value problem*, instead of an *initial value problem*, which makes it less simple to use in combination with ordinary simulation models and simulation software.

B.1 Speed step size limitation

The choice of speed step size is important, since it influences the precision and computational time of the optimisation. A large step means a fast but less accurate optimisation and vice versa. A time step of 0.1–0.2 m/s has been found reasonable for the problem studied in the present work.

There is, however, another problem with small speed steps. The whole concept of defining the state grid in terms of speed and gear is only reasonable if the end speed is reached after the gearshift is completed. With speed steps that are too small, the end speed is reached earlier, already during phase 1 in Figure 3. This limits the speed step size to an approximate minimum of 0.1 m/s in the problem studied in the present work. Actually this has been the limiting cause for choosing step size in the present work. However, in the present work, the accuracy seems reasonable enough. With other system parameters, this could have been an obstacle forcing us to choose either another state grid or another optimisation method.