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Six-dimensional $(2,0)$ theory on tori

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Abstract. The six-dimensional $(2,0)$ theories are a comparatively new and rather abstract type of quantum theory with important relations both to supersymmeric Yang-Mills theory in lower dimensions and to string- and M-theory in higher dimensions. After a short introduction to these theories, we focus on the case when they are considered on flat tori [1][2]. In particular, we give an example of how their ground state degeneracies can be computed, and also briefly discuss the spectrum of BPS-states. Finally, we comment on the automorphic transformation properties of the partition function of such a theory under the mapping class group of a six-torus.

The maximal dimension of a space(-time) which admits superconformal symmetry is $d = 1{+}5$. The symmetry algebra is then

$$osp(2,6|2n) = so(2,6) \oplus sp(2n) \oplus \text{ odd generators}$$

for some $n = 1, 2, \ldots$, where the first two terms are the conformal algebra in six-dimensions and the R-symmetry algebra [3].

Indications for the existence of a quantum theory with such symmetry (for $n = 2$) follows by considering type IIB string theory on a $(1+9)$-dimensional space-time with a codimension four singularity of some ADE-type. A self-consistent six-dimensional theory without dynamical gravity on the locus of the singularity then decouples from the bulk theory [4]. These so called $(2,0)$ theories are highly unique: Apart from their ADE-type, they have no other discrete or continuous parameters.

Some reasons to study the $(2,0)$ theories:

- They are quite different from other theories we know of, and still rather mysterious. Understanding them is likely to lead to much new mathematics and physics.
- They give a good opportunity to learn about important aspects of string theory without having to deal with quantum gravity.
- They are related to Yang-Mills theory in lower dimensions. In particular, they give a geometric understanding of S-duality of $N = 4$ super Yang-Mills theory in four dimensions.
- The study of $(2,0)$ theory might be our best way towards a rigorous definition of quantum theory with infinitely many degrees of freedom.

A way to introduce a parameter in a $(2,0)$ theory is to consider it on a space-time of the form $M^{1,5} = M^{1,4} \times S^1$ with a compact direction of radius R. At longer distances, the effective

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theory is then given by maximally supersymmetric Yang-Mills theory on $M^{1,4}$ with coupling constant $g = R^{1/2}$ and action

$$S = \frac{1}{R} \int_{M^{1,4}} \mathrm{Tr}\left(F \wedge *F + \ldots\right).$$

The gauge group is of the form G/C, where G is simply connected with center subgroup C:

type	G	C
A_{n-1}	$SU(n)$	$\mathbb{Z}_n$
D_{2k}	$Spin(4k)$	$\mathbb{Z}_2 \times \mathbb{Z}_2$
D_{2k+1}	$Spin(4k+2)$	$\mathbb{Z}_4$
E_6	E_6	$\mathbb{Z}_3$
E_7	E_7	$\mathbb{Z}_2$
E_8	E_8	1.

In this way, $(2,0)$ theory can be seen as providing an ultra-violet completion of the Yang-Mills theory. The negative power of R indicates that $(2,0)$ theory has no Lagrangian description [5].

Let $M^{1,4} = \mathbb{R} \times M^4$, where the first factor denotes time. Quantum states of the Yang-Mills theory on this space are characterized by their magnetic 't Hooft flux

$$m \in H^2(M^4, C)$$

which determines the topological class of the gauge bundle over M^4, and their electric 't Hooft flux

$$e \in \mathrm{Hom}(H^1(M^4, C), U(1)) \simeq H^3(M^4, C)$$

which determines the transformation properties under 'large' gauge transformations [6]. From the perspective of $(2,0)$ theory on $M^{1,5} = M^{1,4} \times S^1$, we instead have a self-dual 't Hooft flux

$$f = e + m \in H^3(M^4 \times S^1, C).$$

A particularly interesting case is to consider $M^{1,5} = \mathbb{R} \times M^4 \times S^1$ with $M^4 = T^4$ a flat four-torus, since this preserves 16 supersymmetries. We can think of this as Yang-Mills theory on $\mathbb{R} \times T^4$ or as $(2,0)$ theory on $\mathbb{R} \times T^5$ with

$$T^5 = T^4 \times S^1.$$

The quantum states are characterized by their

- self-dual 't Hooft flux $f \in H^3(T^5, C)$
- energy $E \in \mathbb{R}^+$
- spatial momentum $p \in H^1(T^5, \mathbb{R})$
  (with the fifth component given by the Yang-Mills instanton number over T^4)
- $sp(4)$ R-symmetry representation R.

Supersymmetry implies the energy bound

$$E \geq |p|.$$

There are three classes of states:

- Vacuum states have $E = p = 0$ and are annihilated by all supercharges.
- BPS states have $E = |p| > 0$ and are annihilated by half of the supercharges.

- non-BPS states have $E > |p|$.

The spectra of vacua and BPS states are invariant under smooth deformations of the geometry of T^5, and can thus be followed from weak to strong coupling in the Yang-Mills perspective. We will discuss the computation of the spectrum of vacua from the Yang-Mills perspective. (Similar reasoning may be applied to the BPS-states, although we do not yet have any non-trivial checks on the results.)

At weak coupling, vacuum states are localized at orbifold singularities of the moduli space of flat connections over T^4. The low energy theory is given by maximally supersymmetric matrix quantum mechanics based on the subgroup S of the gauge group G/C left unbroken by the configuration at the singularity. This quantum mechanical model has a number n_S, depending on S, of normalizable ground states. Summing over the orbifold singularities gives the complete spectrum of vacua, which may be decomposed according to their electric and magnetic 't Hooft fluxes e and m.

Covariance under the $SL_4(\mathbb{Z})$ mapping class group of T^4 is manifest in the Yang-Mills formulation, but $(2,0)$ theory indicates covariance under the $SL_5(\mathbb{Z})$ mapping class group of $T^5 = T^4 \times S^1$. This leads to predictions that appear quite non-trivial from the Yang-Mills point of view.

As an example, we consider the D_{2k+1} $(2,0)$ theories. There are 6 $SL_5(\mathbb{Z})$ orbits of self-dual 't Hooft flux $f \in H^3(T^5, \mathbb{Z}_4)$. But a single orbit may be realized in different ways in the corresponding $Spin(4k+2)/\mathbb{Z}_4$ Yang-Mills theory. In this way we get alternative expressions for the generating functions

$$\mathcal{N}_f(q) = \sum_{k=0}^{\infty} N_f(D_{2k+1}) q^{4k+2}$$

of the number $N_f(D_{2k+1})$ of vacua with 't Hooft flux f. (Here q is a formal parameter.) E.g. for a certain $SL_5(\mathbb{Z})$ orbit of f, we have three alternative expressions (modulo q^{4k}-terms) for $\mathcal{N}_f(q)$:

$$\begin{aligned}
\mathcal{N}_f(q) &= \frac{1}{8}\left(P^8_{\text{even}}(q) + P^8_{\text{odd}}(q)\right) \\
&= \frac{1}{4} P^4_{\text{even}}(q) P^4_{\text{odd}}(q) \\
&= Q^4(q)\left(P^3_{\text{odd}}(q^2) P^9_{\text{even}}(q^2) + 3P^5_{\text{odd}}(q^2) P^7_{\text{even}}(q^2)\right. \\
&\quad \left. + 3P^7_{\text{odd}}(q^2) p^5_{\text{even}}(q^2) + P^9_{\text{odd}}(q^2) P^3_{\text{odd}}(q^2)\right) \\
&= q^6 + 10q^{10} + 67q^{14} + 350q^{18} + \ldots .
\end{aligned}$$

Here

$$\begin{aligned}
P_{\text{even}}(q) &= \frac{1}{2}\prod_{k=1}^{\infty}(1+q^{2k-1}) + \frac{1}{2}\prod_{k=1}^{\infty}(1-q^{2k-1}) \\
P_{\text{odd}}(q) &= \frac{1}{2}\prod_{k=1}^{\infty}(1+q^{2k-1}) - \frac{1}{2}\prod_{k=1}^{\infty}(1-q^{2k-1}) \\
Q(q) &= \prod_{k=1}^{\infty}(1+q^{2k}).
\end{aligned}$$

A promising approach to understand the complete spectrum of states is to consider the partition functions

$$Z_f = \mathrm{Tr}_{\mathcal{H}_f} \exp(-tE + ix \cdot P + iAR),$$

where $\mathcal{H}_f$ is the Hilbert space of states with self-dual 't Hooft flux $f \in H^3(T^5, C)$, and t, x, and A are some formal parameters. After continuation to Euclidean time, these partition functions can be seen as pertaining to a particular decomposition of a flat six-torus $T^6 = S^1 \times T^5$ defined by the T^5 geometry together with t and x, where the first factor denotes the 'time' direction. Twisting by R-symmetry in the spatial directions determines, together with the parameters A, a flat $sp(4)$ connection over this T^6.

The set of partition functions Z_f for $f \in H^3(T^5, C)$ should have automorphic properties under the $SL_6(\mathbb{Z})$ mapping class group of T^6. Indeed, these partition function can be regarded as components of an element Z of a certain vector space V. The space V furnishes an irreducible representation of a discrete Heisenberg algebra generated by elements Φ_v for $v \in H^3(T^6, C)$ subject to the relations

$$\Phi_v \Phi_w = \exp\left(2\pi i \int_{T^6} v \wedge w\right) \Phi_w \Phi_v.$$

To construct a basis of V, we break the covariance and decompose

$$H^3(T^6, C) \ni v = f + g \in H^3(T^5, C) \oplus H^2(T^5, C).$$

We then have a basis E_f, $f \in H^3(T^5, C)$ of V. This is such that

$$\begin{aligned} E_f &= \Phi_f E_0 \\ \Phi_g E_0 &= E_0. \end{aligned}$$

The Z_f are the components of Z relative to this basis.

E.g. under a continuous shift of the 'time' cycle of T^6 by an integer linear combination β of the spatial cycles, Z_f is multiplied by an f-dependent phase factor:

$$Z_f \mapsto Z_v \exp\left(\pi i \int_{T^5} f \wedge f[\beta]\right).$$

The Hamiltonian interpretation is that the spatial momentum $p \in H^1(T^5, \mathbb{R})$ obeys the shifted quantization law

$$p - f \cdot f \in H^1(T^5, \mathbb{Z}),$$

where $f \cdot f \in H^1(T^5, \mathbb{R}/\mathbb{Z})$. The best known example of this phenomenon is the possible non-integrality of the fifth component of p (i.e. the instanton number over T^4) for a non-trivial G/C bundle (i.e. of non-trivial magnetic 't Hooft flux m). Another example is the possible non-integrality of the four spatial components of p in situations where both the electric and magnetic 't Hooft fluxes e and m are non-trivial.

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