Calculation of noise barrier performance in a turbulent atmosphere by using substitute sources with random amplitudes

Calculation of Noise Barrier Performance in a Turbulent Atmosphere by Using Substitute Sources with Random Amplitudes

Jens Forssén

Dept. of Applied Acoustics, Chalmers University of Technology, Göteborg, S-41296, Sweden. Tel +46 31 7722196. Fax +46 31 7722212. E-mail jf@ta.chalmers.se

Abstract

This paper studies the influence of turbulence on the sound reduction by a thin screen with varying height. In the model used, the field due to the acoustic source is substituted by a distribution of sources above the barrier (here called substitute sources). The amplitudes of the substitute sources are randomly perturbed to simulate the effect of a turbulent atmosphere. At the receiver the mean power is calculated from a number of realisations. The results are compared to those from using a mutual coherence function between all substitute sources. In this study only two-dimensional situations are considered. The Kirchhoff approximation is used to calculate the strengths of the substitute sources.

Introduction

Screens and buildings along the roadside are used as noise barriers for reducing the traffic noise in residential areas. For a good prediction of the performance of noise barriers, the non-homogeneous nature of the outdoor air is needed to be taken into account.

In terms of physical modelling, the problem situation with a noise barrier in an outdoor environment can be seen as consisting of two interacting processes: diffraction (due to the barrier) and sound propagation in an inhomogeneous medium. A direct numerical solution of the whole problem would in general be very expensive computationally (using e.g. a finite element method), and therefore a model is preferable where the two processes can be separated to some extent, without too large approximations.

The approach is that the field at a receiver, due to a source, can be described as a superposition of fields from a distribution of sources on a surface located between the source and the receiver. The surface will here be called the substitute surface, and the sources on it substitute sources. (See Figure 1.)

If the substitute surface is located between the barrier and the receiver, there will be a free path from all substitute sources to the receiver. In a previous work [1], a model using a mutual coherence function for a turbulent atmosphere is developed. In the work presented here, the effect of a turbulent atmosphere is simulated by realisations of random changes of the amplitudes and phases of the substitute sources. The mean power at the receiver is calculated from a large number of independent realisations. Both different models can be referred to as a *substitute-sources model*.

In this model the turbulent atmosphere is assumed to cause an increased noise level behind the barrier due to a decorrelation of the contributions from the substitute sources. This implies that, in the absence of turbulence, the contributions from the substitute sources must be interfering negatively.

The strengths of the substitute sources are, as an approximation, calculated as for a barrier in a homogeneous atmosphere. This approximation would be acceptable for weak inhomogeneity (a weak turbulence) and if the distance from the source to the barrier is short compared to the total source-receiver distance.

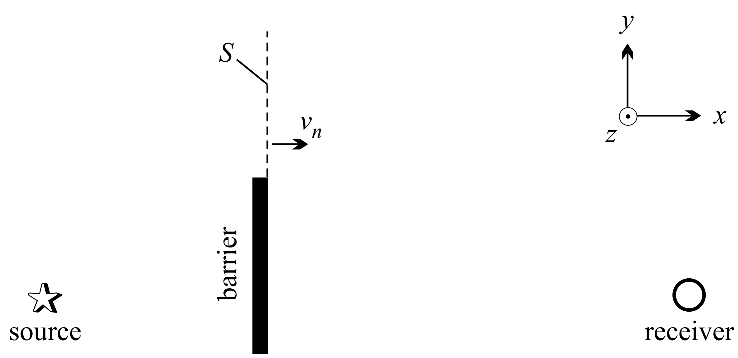

Figure 1. The substitute surface S and the normal velocity v_n there.

Theory and implementation

The strengths of the substitute sources need to be determined, i.e. the normal velocity of the sound field at the substitute surface is needed as the source distribution for the Rayleigh integral (see equation 1). In this study the normal velocity is approximated by the free field due to the source, i.e. the Kirchhoff approximation, and the normal velocity due to the introduction of the barrier is neglected.

Then the expected power at the receiver of the sum of the waves propagated through the turbulent atmosphere from all the substitute sources needs to be estimated. The spatial correlation functions for the amplitude and phase of the substitute sources are used to construct the random realisations of the perturbations.

The theoretical description of the problem is held for three-dimensional situations. The numerical results presented here are, however, for two-dimensional situations, and the necessary modifications of the theory are shown.

If the substitute surface (denoted S) is a plane and the particle velocity v_n normal to the plain is known, then the monopole source strengths of the substitute sources are known, and the response p at the receiver position can be calculated as a Rayleigh integral:

$$p = \frac{j\omega\rho_0}{2\pi}\int_S v_n G \, dS \, , \tag{1}$$

where ω is the angular frequency of a time-oscillation $e^{j\omega t}$ with time t, ρ_0 is the medium density, v_n is the normal velocity, and G is a Green function. Any Green function suitable for the situation can be used, for instance for a mean wind profile, found either analytically, numerically or by measurements. Here, however, a Green function for a homogeneous atmosphere is used.

If it is assumed that the barrier has a hard plane surface toward the receiver, then the surface of integration S can be placed so that it coincides with the barrier's surface toward the receiver, as shown in Figure 1. This leads to a simplified problem since the particle velocity is zero on the hard barrier surface.

The normal velocity v_n on the surface S can be seen as consisting of two parts: The free field contribution v_{n0}, set to zero on the barrier, and the contribution due to the diffraction from the barrier v_{nd}:

$$v_n = v_{n0} + v_{nd}.$$

In general, the free field velocity contribution v_{n0} can be obtained straightforwardly, while the diffraction contribution v_{nd} is more complicated to obtain.

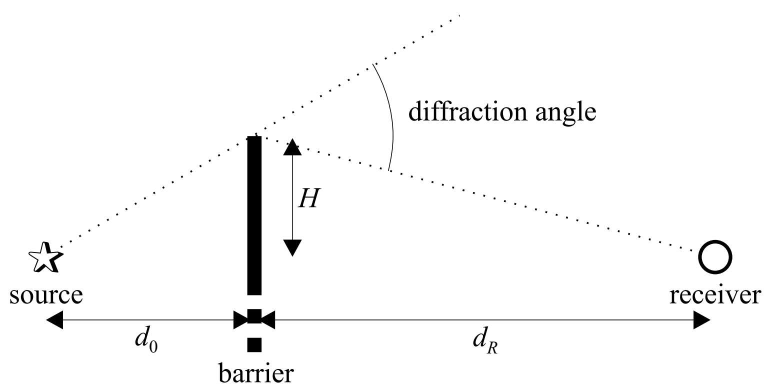

Figure 2. Geometry for the calculations.

The Kirchhoff approximation is $v_n \approx v_{n0}$, which often can be applied when the distances from source and receiver to the screen are large compared to the height of the screen, i.e. for small diffraction angles (see Figure 2).

It should be noted that, strictly, this only holds for a semi-infinite screen. In real cases the field diffracted at the screen edge might be reflected in a ground surface and diffracted again at the edge, and thereby influence the field at the receiver. These higher-order diffraction terms increase in strength when the screen height is reduced. Therefore, the error when using the Kirchhoff approximation (or other common diffraction theories) for a screen on ground can be substantial for very low screens in comparison to the acoustic wavelength.

For situations without ground, it is concluded that the error is smaller than about 1 dB for diffraction angles smaller than 12° if the frequency is sufficiently high (so that the distance from screen to source and receiver is larger than about 30 wavelengths) [1].

Furthermore, when the Kirchhoff approximation is valid, a change of the acoustic properties of the barrier surfaces will be without effect, since the free field contribution will be unchanged. Also, changing a thin screen into a wedge will have little effect. The reasoning above explains the applicability of one-way PE methods to situations with low (but not too low) barriers. In these implementations the PE method calculates wave propagation in one direction (outward from the source) and a barrier is modelled by setting the pressure field equal to zero at the location of the barrier [2]. The free field above the barrier is calculated correctly, and as long as the

Kirchhoff approximation is valid, the free field will produce the correct result at the receiver. Consequently, when the Kirchhoff approximation is not valid one would assume that the one-way PE method would give a substantial error, when including a barrier.

When implementing the model, a finite substitute surface S is needed. If then the size of the surface S is varied (or if the source or the receiver is moved), the error due to the finite surface shows an oscillatory pattern, corresponding to the Fresnel-zones. The introduction of an artificial damping of the substitute sources leads to weaker oscillations and thereby a smaller surface is needed (see [1]).

There will be line-of-sight propagation from the substitute sources on the surface S to the receiver, that is, no barriers or other obstacles are shielding the sound propagation. The subject of line-of-sight propagation in a turbulent atmosphere has been studied extensively (e.g. [3-7]), and the theoretical results most useful here deal with the correlation between acoustic pressure signals that are received at different positions but are originating from a single point monopole source. These theoretical results can be applied to the reciprocal problem at hand: the correlation between signals coming from different sources to one receiver position. The correlation between signals from different sources is used to calculate the mean square pressure amplitude for single frequencies. The correlation between two source signals is usually described by the mutual coherence function Γ.

For the description of the turbulence, a homogeneous and isotropic turbulence is assumed, that is, the fluctuations are assumed to follow the same statistics in all points and in all directions. The turbulence is described by a fluctuating part of the index of refraction following a Gaussian correlation function with the standard deviation $\mu_0^2 = 3 \cdot 10^{-6}$ and the correlation length l=1.1 m. Other turbulence models than the Gaussian could be used, and this would then lead to different mutual coherence functions.

Temperature and velocity fluctuations affect the sound field in different ways. The mutual coherence function for velocity fluctuations is deduced in [8, 7] where also the older result for Gaussian temperature fluctuations is shown. Here, only temperature fluctuations are considered, but to include other medium fluctuations should not be a problem.

If we assume that the turbulence can be described by a Gaussian correlation function we can write the mutual coherence function as [9, eq. 11]

$$\Gamma(L,\rho) = \exp\left[-\sqrt{\pi}\,\mu_0^2 k^2 L l\left(1-\frac{\Phi(\rho/l)}{\rho/l}\right)\right], \tag{2}$$

where k is the wave number, ρ the transversal distance between two sources, L the longitudinal distance, and $\Phi(z) = \int_0^z \exp(-u^2)\,du$.

For N discrete source contributions p_i the mean square pressure amplitude can be formulated as [10]

$$\left\langle |p|^2 \right\rangle = \sum_{i=1}^{N} |p_i|^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} |p_i p_j| \cos\left[\arg\left(\frac{p_j}{p_i}\right)\right]\Gamma_{ij}. \tag{3}$$

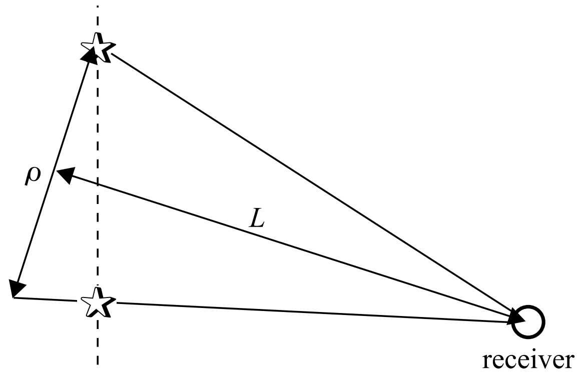

Figure 3. Longitudinal distance L and transversal distance ρ for a pair of sources and one receiver.

The mutual coherence function can be deduced via the parabolic equation and the Markov approximation [6]. Other methods than by using the parabolic equation can be applied [11], but, in any case, it is assumed that the transversal distance ρ is small compared to the longitudinal distance L. This approximation leads to an overestimation of the mutual coherence function.

An underlying part of the mutual coherence function consists of the correlation functions for the log-amplitude and the phase fluctuations, which can be written, respectively, as

$$B_\chi(\rho) = \left\langle \chi(\vec{x})\chi(\vec{x}+\vec{\rho}) \right\rangle$$

$$B_S(\rho) = \left\langle S(\vec{x})S(\vec{x}+\vec{\rho}) \right\rangle, \tag{4}$$

where $\langle\ \rangle$ means expectation value, $\vec{x}$ describes any point and $\vec{\rho}$ is a transversal separation with magnitude ρ from that point. The amplitude and phase fluctuations are described by $\chi = \ln(A/A_0)$ and $S = \phi - \phi_0$, where A is the amplitude in presence of turbulence and A_0 in absence, and ϕ is the phase in presence of turbulence and ϕ_0 in absence. For the mutual coherence function (2) we have

$$B_\chi(\rho) = B_S(\rho) = \mu_0^2 k^2 L l \frac{\sqrt{\pi}}{2} \frac{\Phi(\rho/l)}{\rho/l}, \tag{5}$$

where L is assumed to be constant, at a value determined by the horizontal screen-receiver distance (here, 900 m).

Assuming that χ and S are Gaussian random variables, we can produce realisations of χ and S by filtering Gaussian white noise (independent samples) with a chosen filter function. Starting with a correlation function $B(\rho)$ we can calculate the impulse response of the wanted filter via a Fourier transform. First, $B(\rho)$ is sampled at points $\rho_m = \Delta\rho \cdot (0, 1, \ldots, M-1)$, where $\Delta\rho$ is simply chosen to be the discretisation needed to resolve the acoustic field (here, five points per wavelength is used). The sampling length, $M\Delta\rho$, is chosen large enough that $B(\rho)$ is small for $\rho \geq M\Delta\rho$, compared to $B(0)$. (Here, $M\Delta\rho$ is chosen to be 50 m, with l=1.1.) A window is applied to smoothly force the function to zero before $\rho = M\Delta\rho$. (Here, half a Hanning window starting at ρ=40 m is used.) Second, an autospectrum is calculated as the discrete Fourier transform of the sampled and windowed version of $B(\rho)$ is calculated. The inverse Fourier transform of the square root of this autospectrum is the impulse response $h(\rho_m)$ of the wanted (finite impulse response) filter.

The realisations of the random data are then produced by the convolution of $h(\rho_m)$ and Gaussian white noise with mean value 0 and variance 1. (Not all data are used; the tails with lengths one sample less than the filter length are cut.)

Here, 500 realisations of χ and S are used, and the mean value of the acoustic power is calculated therefrom. For each realisation of $\chi(y_i)$ and $S(y_i)$ at all discrete points y_i on the substitute surface, the amplitudes $p(y_i)$ of the sources are multiplied by $\exp[\chi(y_i) + jS(y_i)] \cdot \exp[-2B_\chi(0)]$.

The normalising factor $\exp[-2B_\chi(0)]$ is due to the mean power added when amplitude fluctuations are introduced. This can be seen from the relation

$$\left\langle e^{b\xi} \right\rangle = e^{b^2 \left\langle \xi^2 \right\rangle / 2} , \tag{6}$$

which is valid for a Gaussian variable ξ with zero mean and for any constant *b*. Since the power is proportional to the square of the absolute value of the amplitude, we have b=2 and $\left\langle \xi^2 \right\rangle = B_\chi(0)$ in the above.

When long data series are wanted, the above explained procedure for making random data with a wanted correlation function is advantageous. For shorter data series the commonly used procedures with Fourier modes [12, 13] or, similarly, by assigning a random phase to each sample of the square root of the autocorrelation function and then taking the inverse transform [14].

In the numerical study the geometry is two-dimensional, and the substitute surface is a line in the vertical direction *y*, with sources in discrete points y_i. Also the turbulence is assumed to be two-dimensional. For the mutual coherence function (2), the values of the input parameters are found from the projections on the vertical *xy*-plane. This is in accordance with Salomons' analysis [15] for a Gaussian turbulence that is rotationally symmetrical around the vertical axis through the source.

The acoustic pressure p_0 due to a coherent line source is calculated as the far-field approximation

$$p_0 = \frac{Q\sqrt{2}}{\sqrt{\pi k R}} e^{-j(kR - \pi/4)} \tag{7}$$

where *Q* is a source strength and *R* is the distance.

Results

This paper mainly studies how the influence of turbulence varies with screen height. Here, the screen height is the vertical position of the screen edge compared to the line through the source and the receiver. Starting with representing the free field solution by substitute sources all the way from $-y_{\max}$ to $y_{\max}$ and then taking away source after source, starting at $-y_{\max}$, simulates an increasing screen height (under the Kirchhoff approximation). The maximum height $y_{\max}$ used for the substitute sources is about 200 m, found by numerical tests in the absence of turbulence.

Artificial damping is introduced in the strength of the substitute sources to decrease the small oscillations in the solution when the receiver distance is varied. The damping is chosen to start at 75 m, and the strengths of the substitute sources above that were multiplied by the factor, $\exp[-a*(y-75)]$ with a=0.05 m^{-1}. For the discretisation, a horizontal distance between the substitute sources of $\lambda/5$ was used.

The calculations are made for two frequencies, 500 Hz and 1000 Hz. The horizontal distance from the source to the screen is d_0=100 m and from screen to receiver d_r=900 m. No influence of a ground surface is modelled.

In Figure 4 the sound power for a varying screen height in a homogeneous atmosphere at the frequency 500 Hz is shown. For screen heights lower than 75 m, one can see the artificial decrease due to the damping of the strengths of the substitute sources.

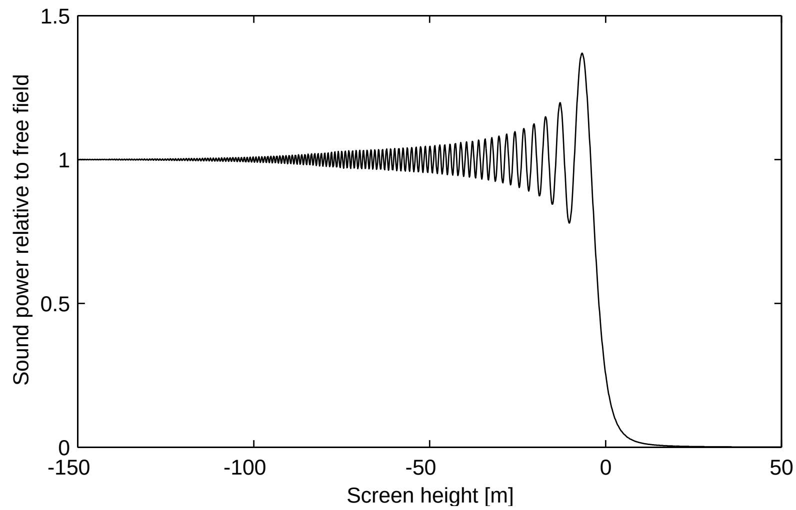

Figure 4. Sound power relative to free field without turbulence, at the frequency 500 Hz.

Figures 5 and 6 show the influence of turbulence for the frequency 500 Hz. The dashed line shows the result using the mutual coherence function (equations 2 and 3) and the solid, thinner line shows the result from random realisations. The result without turbulence (Figure 5) is plotted with a thick solid line.

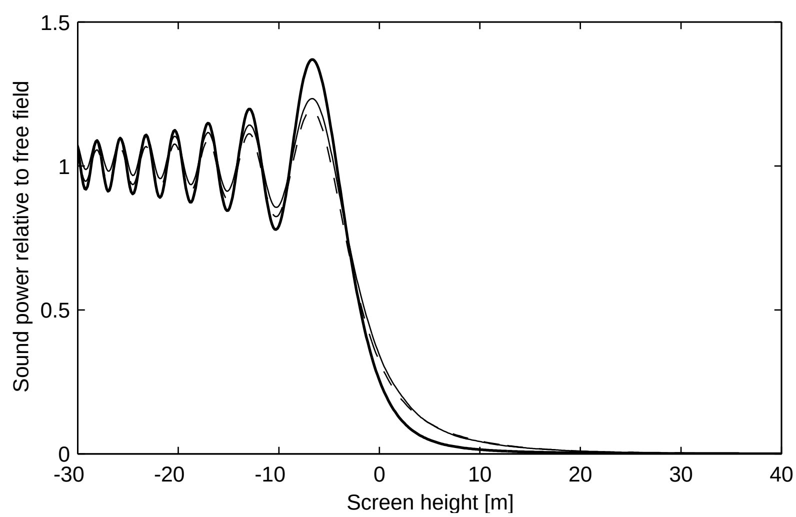

Figure 5. Sound power relative to free field with and without turbulence, at the frequency 500 Hz.

Studying the sound power in Figure 5, the results for the turbulent atmosphere follows fairly closely the result for a homogeneous atmosphere. For a screen well below line-of-sight (i.e. well below zero height) all results approximate the free field solution. When increasing the screen height from minus infinity, the effect of turbulence can be seen as a decrease in the oscillation amplitude. When the screen height approaches zero and increases, the sound power for the turbulent atmosphere falls off more slowly than for the homogeneous atmosphere. For positive screen heights we then get a non-oscillating increase due to turbulence.

In figure 6 the increase due to turbulence is plotted in dB and we can see a large influence of turbulence with a peak around the screen height 12-13 m, and then a rapid decrease. The difference between the two curves in Figure 6 is small and mainly looks like random errors. The small offset at negative screen heights is also believed to be a random error, and not a bias error; each time the calculation is repeated, the offset differs (see also Figure 7, where the offset is larger, and has opposite sign).

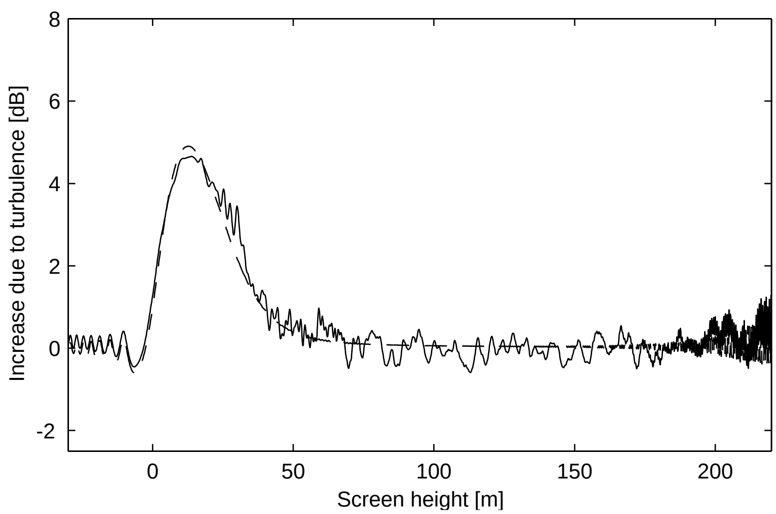

Figure 6. Increased sound level due turbulence, at the frequency 500 Hz.

Increasing the frequency to 1000 Hz (Figures 7 and 8) we can mainly notice an increased influence of turbulence. Also the region where we have a large influence (see Figure 8) becomes smaller: the peak is at around 8 m and dies down to less than 1 dB at about 30 m, compared to about 40 m at 500 Hz.

For very large screen heights, around 200 m and more, the numerical resolution is getting low and rounding errors dominate the solution (see Figures 6 and 8).

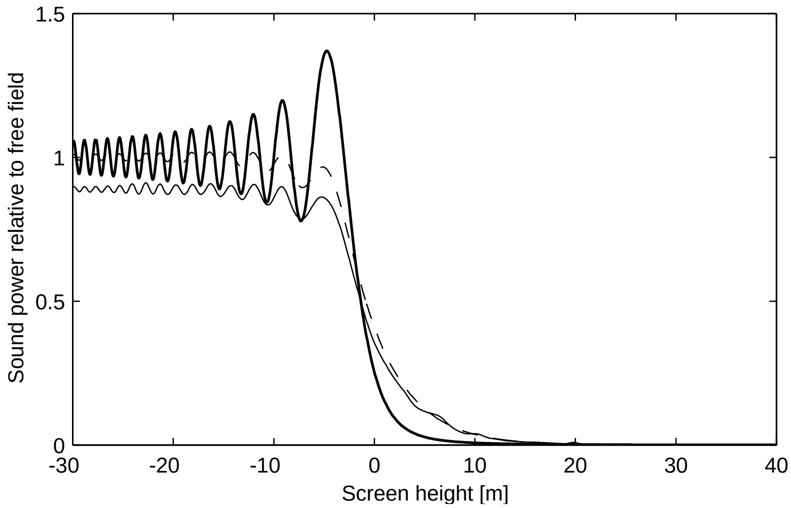

Figure 7. Sound power relative to free field with and without turbulence for the frequency 1000 Hz.

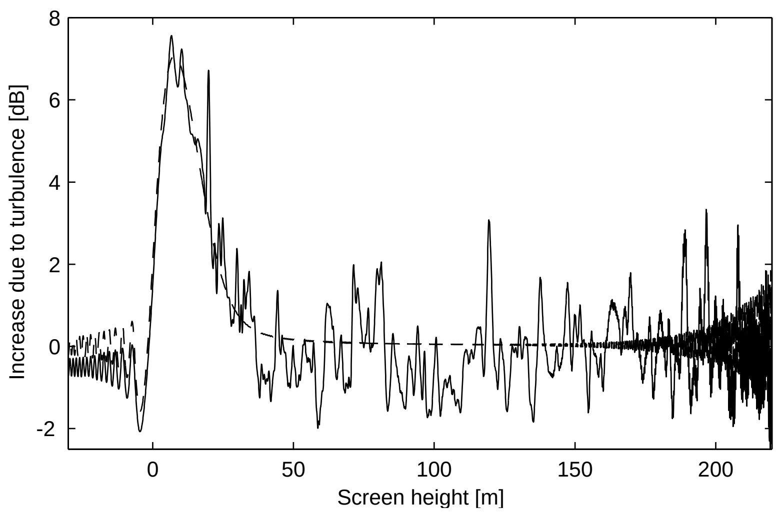

Figure 8. Increased sound level due turbulence, at the frequency 1000 Hz.

Discussion and conclusions

The results from using random realisations compares well with the ones from using the mutual coherence function. The results show a large influence of turbulence on the sound reduction by a screen.

An interesting result is that the turbulence shows influence on the acoustic power in the screen shadow only for a small range of screen heights. For higher screens, the turbulence does not alter the acoustic power. This result is, however, for a peculiar type of screen (or barrier), where the part of the sound wave that goes toward the screen is totally absorbed, without influencing the (free) field above the screen, i.e. the Kirchhoff approximation. Moreover, the Gaussian correlation function is a rather poor approximation of the turbulent atmosphere, and a more realistic description might lead to different results.

The two substitute-sources methods are not very expensive, computationally, for the situations studied here, and need about the same computation time. For three-dimensional situations, however, both methods will be much more expensive. The computation time for the method with random realisations would roughly increase linearly with the number of sources, whereas, roughly, a quadratic increase would be expected for the method based on the mutual coherence function since the number of source pairs would increase quadratically. Therefore using random realisations might be an attractive alternative.

In this paper the turbulence is assumed to be introduced into a homogeneous free space, and it is for this situation the used correlation functions and the mutual coherence function has been deduced. It is, however, a reasonable approximation that these functions also can be applied to a weakly modified medium, for instance for a moderate sound speed profile, with the corresponding Green function. For a more strongly modified medium, the correlation and coherence functions, as well as the Green function, could be estimated numerically. In this respect the substitute-sources model is applicable to a large variety of geometrical and atmospherical situations.

For future work it would be of interest to extend the model to three dimensions, with a point source and a three-dimensional turbulence. Also to take into account the correct diffraction above the barrier would be of interest to try, i.e. to not use the Kirchhoff approximation.

Moreover, it could be possible to include in the model a thick barrier of finite length, a finite impedance ground, a sound speed profile, and an anisotropic and inhomogeneous turbulence.

Acknowledgement

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