Gauge theories of Partial Compositeness: Scenarios for Run-II of the LHC

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Gauge theories of partial compositeness: scenarios for Run-II of the LHC

Gabriele Ferretti

Department of Physics, Chalmers University of Technology,
Fysikgården 1, 41296 Göteborg, Sweden

E-mail: ferretti@chalmers.se

ABSTRACT: We continue our investigation of gauge theories in which the Higgs boson arises as a pseudo-Nambu-Goldstone boson (pNGB) and top-partners arise as bound states of three hyperfermions. All models have additional pNGBs in their spectrum that should be accessible at LHC. We analyze the patterns of symmetry breaking and present all relevant couplings of the pNGBs with the gauge fields. We discuss how vacuum misalignment and a mass for the pNGBs is generated by a loop-induced potential. Finally, we paint a very broad, qualitative, picture of the kind of experimental signatures these models give rise to, setting the stage for further analysis.

Contents

1 Introduction

1.1 Motivation

The Higgs mechanism [1–3] in the Standard Model [4] (SM) does an excellent job at parameterizing the mass spectrum of elementary particles in a consistent way, but leaves many questions unanswered. We would like to understand why the Higgs mass is so low and to explain the huge disparity among fermion masses.

One possible explanation of the lightness of the Higgs boson is to realize it as a (pseudo) Nambu-Goldstone Boson (pNGB) of a broken global symmetry. This approach was pioneered in [5] and goes under the name of "Composite Higgs". One way to deal with the disparity of fermionic masses and, in particular, to explain the origin of the top quark mass without reintroducing fine-tuning is to also have additional "partners" mixing with

SM fermions. This new ingredient was introduced in [6] and goes under the name of "Partial Compositeness".

Much work has been done in this area using the effective field theory description based on the CCWZ formalism [7, 8]. There was also a huge effort to realize these construction using extra-dimensions. There are by now exhaustive reviews such as [9, 10] providing all the necessary background to these subjects.

A much less studied approach is that of constructing UV completions for these models using a strongly coupled "hypercolor" gauge theory with purely fermionic matter ("hyper-quarks"). The philosophy behind this proposal is so old fashioned that it almost appears new! Fermionic models of BSM go all the way back to the old technicolor idea and were also tried in the context of composite Higgs and partial compositeness. The recent model building activities try to combine the two. Few explicit proposals have been made so far: [11, 12] and [13] and a partial classification of the available options was made in [14]. (For earlier attempts using supersymmetry, see [15, 16]. Alternative avenues being explored are found in [17–19].)

The LHC is now entering a phase where the potential for discovery is at its highest point, due to the increase in luminosity and energy. It is thus timely to chart the various scenarios implied by the above class models. In this work we are particularly interested in presenting the underlying theories in detail and in identifying the broad features that may allow one to discern one class of models from the others. We leave instead a detailed phenomenological analysis for future work. For recent phenomenological work in the area a surely incomplete list is [20–25].

1.2 Overview of the results

In a nutshell, the models we are considering are based on an asymptotically free gauge theory with simple hypercolor group G_{HC} and fermionic matter in *two* inequivalent irreducible representations (irreps).[1] The requirement of two different irreps arises from the need to construct top-partners carrying both color and EW quantum numbers. With the notable exception of a model by L. Vecchi [13], this requires at least two separate irreps; one, generically denoted by ψ, carrying EW quantum numbers in addition to hypercolor, the other, χ, carrying ordinary color as well as hypercolor.

At low energies, the theory is expected to confine after having spent a part of the RG evolution in or near the conformal window, somewhat in the spirit of [26, 27]. This is the main dynamical assumption needed for some of the operators in the theory to develop the large anomalous dimensions required to solve the hierarchy problem. However, contrary to the above-mentioned proposal, here we use fermionic operators [6] to generate the mass of the top quark, eluding the potential problems with fine-tuning pointed out in [28, 29].

Here we are only interested in the behavior of the theory below the dynamically generated scale Λ, (expected to be of the order of $10 \, \text{TeV}$, to fix the ideas). The conformal behavior occurs above this scale, up to the "flavor" scale $\Lambda_{\text{UV}} > 10^4 \, \text{TeV}$. In this range

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[1]We work with Weyl fermions and count a complex irrep and its conjugate as one.

the theory could have additional d.o.f./operators driving the conformal behavior and being ultimately responsible for its ending at the scale Λ.

Below Λ, the strong IR dynamics of one of the two types of hyperquarks (ψ) induces the symmetry breaking needed to realize the composite Higgs scenario. The three minimal cosets preserving custodial symmetry are SU(5)/SO(5), SU(4)/Sp(4), and SU(4) $\times$ SU(4)$'$/SU(4)$_D$. The SM EW group is embedded into the unbroken symmetry. The vacuum is misaligned, inducing a Higgs v.e.v., by the combined action of the one loop potential induced by the SM gauge bosons and the top quark as well as possible hyperquark bare masses of UV origin.

The second irrep (χ) is needed to realize the QCD color group. Its dynamics may or may not lead to additional pNGBs.[2] Top partners arise as G_{HC} invariant trilinear combinations of the two types of hyperquarks. The top quark acquires a mass via a linear coupling of these partners to the SM fields $Q_L^3 \equiv (t_L, b_L)$ and $u_R^3 \equiv t_R$. The remaining SM fields may instead be coupled bilinearly and acquire a mass via the more standard mechanism. This hybrid solution, proposed in [12, 31, 32], has the extra advantage of suppressing unwanted contributions to dipole moments or flavor violating operators and could be realized at low energies via the mechanism explained in [33].

With the exception of the Wess-Zumino-Witten (WZW) term, we consider only SM tree level couplings that preserve a parity symmetry, P_π, changing sign to all the pNGBs except for the Higgs itself. Heavier pNGBs thus decay into lighter ones plus a SM gauge boson or a pair of SM fermions if the decay into a gauge boson is not kinematically allowed.

This parity symmetry is however broken in some cases by the anomaly encoded in the WZW term, and this allows the lightest pGNBs to decay via di-bosons with a very narrow, but still prompt, decay width. It is interesting to notice that [34], for the coset SU(4) $\times$ SU(4)$'$/SU(4)$_D$, the decay of some of the pNGBs is forbidden by the existence of another symmetry, G_π, thus providing a possible Dark Matter candidate. For the scope of this paper we only assume that in the SU(4) $\times$ SU(4)$'$/SU(4)$_D$ scenario the lightest pNGB odd under this additional symmetry is collider stable, leading to the usual signatures-$\not{E}_T$ or highly ionizing tracks depending on the charge. (The requirement of this pNGB being neutral is necessary only in order to have a DM candidate, not simply a collider stable particle.)

The leading production mode for the pNGBs associated with the EW coset are Drell-Yan production and vector boson fusion.

If the dynamic in the color sector also leads to symmetry breaking, (as we assume through the paper for illustration purposes, since this case leads to additional interesting phenomena), there will be additional colored pNGB with a mass higher than the EW ones since it is due to gluon loops. All models have a neutral pNGB in the octet of color that can be singly produced and decay via an anomalous coupling. Some models also include additional charged and colored pNBGs in the triplet or sextet that, under the assumption of P_π-parity, decay to two jets and a lighter EW pNGB. Their charges are fixed by the structure of the top partners.

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[2]Note that the condensate $\langle \psi\chi \rangle$ would break the hypercolor group and cannot arise in vector-like theories such as these [30].

An universal feature of all of these models is the presence of two additional scalars arising from the two spontaneously broken U(1) axial symmetries associated to the two fermionic irreps. One of these bosons is associated to a G_{HC} anomalous current and it is thus expected to acquire a large mass just like the η' in QCD. The remaining one is instead naturally light in the absence of additional UV mechanisms such as bare hyperquark masses. Both couple to gluons via the anomaly and could provide an explanation of the current 750 GeV di-photon excess [35, 36]. Indeed, such an interpretation has already been put forward in [37] for the case of the light U(1) boson. (More details about the role of pNGBs in explaining the excess are given in [38].)

1.3 Organization of the paper

The paper is organized as follows: in section 2 we present the class of models of interest. We then turn to study their different sectors beginning in section 3 with the pNGBs associated to the EW coset. We study the generation of the potential, its symmetries, present a couple of prototypical spectra, work out all the couplings of relevance for LHC physics and briefly comment on the main phenomenological aspects. In section 4 we discuss the colored objects in the different theories, pNGBs and top partners, show how their quantum numbers are related and how this affects the phenomenology. In section 5 we comment on the remaining two pNGBs universally present in this class of models.

Technical details are collected in the appendix. Appendix A lists all the gauge theories having a composite higgs and a top partner under the requirements discussed in section 2 and 3 and discusses their IR properties. Appendix B contains the conventions for the explicit construction of the EW cosets. Appendix C lists additional couplings (anomalous and non) that did not find a place in the main text.

2 The models, streamlined classification

In this section we summarize the models of interest in this paper. We take the opportunity to slightly expand and streamline the classification presented in [14].

We want to realize the "composite Higgs" coset by condensation of a set of fermionic hyperquarks ψ transforming in some irrep of a simple hypercolor gauge group G_{HC}. Recall that the three basic cosets one can realize with fermionic matter depend on the type of irrep to which the fermions belong. One possibility is to mimic ordinary QCD. Working with left-handed (LH) fermions only, a set of n *pairs* of LH fermions $(\psi_i, \tilde{\psi}^i)$ in a $(R, \bar{R})$ irrep of G_{HC}, with R complex (C) and $\bar{R}$ its conjugate, breaks the global symmetry $\text{SU}(n) \times \text{SU}(n)' \to \text{SU}(n)_D$ after condensation $\langle \tilde{\psi}^i \psi_j \rangle \propto \delta_j^i$. (The U(1) factors will be studied separately because of possible ABJ anomalies. Here we concentrate on the non-abelian factors.)

If, on the other hand, we consider just a single set of n LH fermions ψ_i in a real (R) (respectively pseudo-real (PR)) irrep, the symmetry breaking is $\text{SU}(n) \to \text{SO}(n)$ (resp. $\text{SU}(n) \to \text{Sp}(n)$) since the condensate $\langle \psi_i \psi_j \rangle$ turns out to be symmetric (resp. anti-symmetric).

If we want to use such cosets to construct an EW sector for the composite Higgs, the possible minimal custodial cosets of this type are $\text{SU}(4) \times \text{SU}(4)'/\text{SU}(4)_D$, $\text{SU}(5)/\text{SO}(5)$

	$\psi \in \mathrm{R}$	$\psi \in \mathrm{PR}$	$\psi, \tilde{\psi} \in \mathrm{C}$
$\chi \in \mathrm{R}$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)_u$	$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)_u$	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(6)}{\mathrm{SO}(6)} \mathrm{U}(1)_u$
$\chi \in \mathrm{PR}$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)} \mathrm{U}(1)_u$	~~$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)} \mathrm{U}(1)_u$~~	~~$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(6)}{\mathrm{Sp}(6)} \mathrm{U}(1)_u$~~
$\chi, \tilde{\chi} \in \mathrm{C}$	$\frac{\mathrm{SU}(5)}{\mathrm{SO}(5)} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)'}{\mathrm{SU}(3)_D} \mathrm{U}(1)_u$	~~$\frac{\mathrm{SU}(4)}{\mathrm{Sp}(4)} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)'}{\mathrm{SU}(3)_D} \mathrm{U}(1)_u$~~	$\frac{\mathrm{SU}(4) \times \mathrm{SU}(4)'}{\mathrm{SU}(4)_D} \frac{\mathrm{SU}(3) \times \mathrm{SU}(3)'}{\mathrm{SU}(3)_D} \mathrm{U}(1)_u$

Table 1. The possible minimal cosets realized in this class of models. The hyperquarks ψ and χ transform under different irreps of G_{HC}. ψ also carries EW quantum numbers, while χ carries color. The three cases crossed out are those that do not give rise to top partners because the nature of their congruency classes prevents the formation of singlets.

and $\mathrm{SU}(4)/\mathrm{Sp}(4)$ for the three cases. For instance, $\mathrm{SU}(4)/\mathrm{SO}(4)$ is not acceptable since the pNGBs are only in the symmetric irrep $(\mathbf{3}, \mathbf{3})$ of $\mathrm{SO}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and thus we do not get the Higgs irrep $(\mathbf{2}, \mathbf{2})$.

Since we want to obtain the top partners as fermionic trilinears, we also need to embed the color group $\mathrm{SU}(3)_c$ into the global symmetry of the composite theory. For this purpose we introduce a second fermionic irrep χ coupling to color as well as hypercolor. The minimal field content allowing an anomaly-free embedding of unbroken $\mathrm{SU}(3)_c$ are $\mathrm{SU}(3) \times \mathrm{SU}(3)' \to \mathrm{SU}(3)_D \equiv \mathrm{SU}(3)_c$ for the complex case, $\mathrm{SU}(6) \to \mathrm{SO}(6) \supset \mathrm{SU}(3)_c$ for the real case and $\mathrm{SU}(6) \to \mathrm{Sp}(6) \supset \mathrm{SU}(3)_c$ for the pseudoreal case.

In all of these cases we need 6 LH fermions altogether, to be divided into three pairs $(\chi, \tilde{\chi})$ in the case of a complex irrep. Top-partners are constructed by G_{HC} invariant trilinears of type $\psi \chi \psi$ or $\chi \psi \chi$ depending on the model as shown in appendix A.

All combinations of R, PR and C irreps are in principle possible. The minimal cosets are shown in table 1. The three cases crossed out are those that do not give rise to top partners. This can be easily seen e.g. for the case in which both irreps are pseudo-real since the product of three pseudo-real irreps cannot contain a singlet. For each remaining case one can look for possible hypercolor gauge groups and irreps that satisfy the remaining constraint of asymptotic freedom. These are listed in appendix A for completeness. More details can be found in [14].

Table 1 also shows a "ubiquitous" non-anomalous $\mathrm{U}(1)_u$ factor arsing from the spontaneous breaking of the G_{HC}-anomaly-free abelian chiral symmetry. This symmetry is obtained by constructing the anomaly free linear combination of the two axial symmetries $\mathrm{U}(1)_A^\psi$ and $\mathrm{U}(1)_A^\chi$ rotating, respectively, all the ψ (or $\psi, \tilde{\psi}$) and χ (or $\chi, \tilde{\chi}$) by the same phase. For each pair of complex irreps there is also one vector-like $\mathrm{U}(1)_V^\psi$ or $\mathrm{U}(1)_V^\chi$ factor which is both anomaly free and unbroken.

To understand the type of pNGBs arising in the various cases, we look at the decomposition under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ of the irrep of H under which the pNGB's transform.[3] The decomposition is shown in table 2.

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[3]We denote specific irreps either by their dimensionality or by the symbols $\mathbf{F}$, $\mathbf{S}_n$, $\mathbf{A}_n$, $\mathbf{Ad}$ and $\mathbf{Spin}$ for the fundamental, $n-$symmetric, $n-$antisymmetric, adjoint and spin.

G/H	irrep of H	$H \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
$\mathrm{SU}(5)/\mathrm{SO}(5)$	$\mathbf{14} \equiv \mathbf{S}_2$	$\mathbf{3}_{\pm 1}(\phi_\pm) + \mathbf{3}_0(\phi_0) + \mathbf{2}_{\pm 1/2}(H) + \mathbf{1}_0(\eta)$
$\mathrm{SU}(4)/\mathrm{Sp}(4)$	$\mathbf{5} \equiv \mathbf{A}_2$	$\mathbf{2}_{\pm 1/2}(H) + \mathbf{1}_0(\eta)$
$\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$	$\mathbf{15} \equiv \mathbf{Ad}$	$\mathbf{3}_0(\phi_0) + \mathbf{2}_{\pm 1/2}(H) + \mathbf{2}'_{\pm 1/2}(H') + \mathbf{1}_{\pm 1}(N_\pm) + \mathbf{1}_0(N_0) + \mathbf{1}'_0(\eta)$

Table 2. The decomposition under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset H$ of the pNGBs associated to the coset G/H and transforming linearly under an irrep of H. Next to the charges we indicate the names used throughout the paper. For uniformity of notation we denote all $\mathrm{SU}(2)_L$ triplets by ϕ and doublets by H. The fields N in $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ form a triplet of $\mathrm{SU}(2)_R$.

The EW cosets above have been studied in many previous papers, see e.g. [39–43] for $\mathrm{SU}(4)/\mathrm{Sp}(4)$, [34, 44] for $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ and [12, 40, 45–48] for $\mathrm{SU}(5)/\mathrm{SO}(5)$. General non-minimal cosets are discussed in [49].

As for the color cosets, arising when the χ also condense, a generic prediction is the existence of an electrically neutral color octet pNGB. In addition, we have a pair of electrically charged pNGBs in the $(\mathbf{3}, \overline{\mathbf{3}})$ of $\mathrm{SU}(3)_c$ for the $\mathrm{SU}(6)/\mathrm{Sp}(6)$ case or in the $(\mathbf{6}, \overline{\mathbf{6}})$ for the $\mathrm{SU}(6)/\mathrm{SO}(6)$ case. The charges are discussed in section 4.

Top partners can be broadly divided into two separate groups: those of type $\psi\chi\psi$ and those of type $\chi\psi\chi$. (We are being schematic here, and only indicate the relative number of ψ or χ-type hyperquarks, without indicating the specific Lorentz and hypercolor contractions.) Top partners of the first type require coupling to top quark spurions in a two index irrep, while partners of the second type give rise to single index irreps.

There is a sense in which models of type $\psi\chi\psi$ are more promising than the others. Top-partners of type $\chi\psi\chi$ force one to chose the fundamental irrep for the spurions. For the $\mathrm{SU}(5)$ case this leads to the $\mathbf{5}$ that, although being compatible with the $Z \to b_L \bar{b}_L$ custodial symmetry [12, 50], gives rise to effective potentials that tend to break the usual custodial symmetry [51]. The case of $\mathrm{SU}(4) \times \mathrm{SU}(4)'$ leads to problems already at the $Z \to b_L \bar{b}_L$ level and we exclude these models from the list in appendix A. There are no $\chi\psi\chi$ cases for the $\mathrm{SU}(4)/\mathrm{Sp}(4)$ coset.

3 The electro-weak sector

3.1 The potential

The pNGBs acquire a mass from a loop-induced [52] potential that breaks the shift symmetry explicitly. We consider three kinds of contribution to the potential. The first one is the contribution from the loop of gauge bosons, which is uniquely determined by the gauge structure up to an overall dimensionless positive constant B. It can be written as[4]

$$V_g = B\, f^4\, \mathrm{tr}(g^2 T_L^a U T_L^{a*} U^\dagger + g'^2 T_R^3 U T_R^{3*} U^\dagger) \tag{3.1}$$

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[4]We chose to use the pNGB decay constant f as the only dimensionfull parameter. This simplifies the notation but hides the scaling properties of the formulas. See appendix B for the conventions on the generators and the non-linear pNGB matrix U.

for the SU(4)/Sp(4) case and

$$V_g = -B\,f^4\,\mathrm{tr}(g^2 T_L^a U T_L^a U^\dagger + g'^2 T_R^3 U T_R^3 U^\dagger) \tag{3.2}$$

for $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$. For the SU(5)/SO(5) coset both expressions are equivalent in our conventions from appendix B. Actually, for all three cases the formula could be written in a uniform notation using the matrix Σ defined in appendix B instead of U, but we choose to work with U because of its easier transformation properties under the full symmetry group.

The constant B and related ones are the so-called low-energy-coefficients (LEC) (in units of f) that encode the information about the spectrum of the strongly interacting theory. Lacking direct experimental information, they could be estimated on the lattice. Some work in this direction has already been done in the context of a specific model [53, 54]. (For more general results on the lattice, see the review [55].) These models necessarily involve representations of the hypercolor group other than the fundamental and pose additional challenges. In the context of phenomenology they have also been used in e.g. [56–58]. For a clear discussion of how they are generated and can be computed in the context of effective theories of partial compositeness, we refer to [59] and references therein.

We also have the option of adding bare hyper-quark masses

$$V_m = -B'\,f^4\,\mathrm{tr}(\mu U + \mu^* U^*) \tag{3.3}$$

with μ a dimensionless matrix preserving the custodial symmetry and B' some other dimensionless constant. For definitiveness we take $\mu = \mathbf{1}$ if needed.

Lastly, we need to take into account the effect of the top quark, which leads to vacuum misalignment [60]. This can be done introducing spurionic fields transforming under a particular irrep of the unbroken flavor group. Here is one instance when having a candidate UV completion helps in picking the particular irreps to consider.

We restrict to the case where only the Higgs acquires a v.e.v. since we want to preserve the SM-like properties of the Higgs boson as well as the tree level mass relation $m_W^2 = \cos^2\theta_W m_Z^2$. Since we are only allowing the Higgs direction to be turned on, the matrix of v.e.v.s is easily exponentiated and we find it convenient to introduce a matrix $\Omega(\zeta)$ for all three cases denoting the vacuum misalignment and depending on $v = 246\,\mathrm{GeV}$ through $\sin\zeta = v/f$. In terms of the original Higgs field $\hat{h}$ gaining a v.e.v. we have $\zeta = \langle\hat{h}\rangle/f$. In other words $v = f\sin(\langle\hat{h}\rangle/f)$. The fields appearing into the effective lagrangian are always the canonically normalized fields with zero v.e.v.. The expression for Ω and U is found in appendix B.

It is then a straightforward matter to check which, among the irreps of $G = \mathrm{SU}(5)$, $\mathrm{SU}(4)$, $\mathrm{SU}(4) \times \mathrm{SU}(4)$ with up to two indices, contains spurions for the third generation quarks that preserve the custodial symmetry in the sense specified above. The decomposition of $G \to \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is useful at this point and it is reproduced in table 3 for convenience.

Spurions containing a $(\mathbf{2},\mathbf{2})$ are possible candidates for Q_L^3 and spurions containing $(\mathbf{1},\mathbf{1})$, $(\mathbf{1},\mathbf{2})$ or $(\mathbf{1},\mathbf{3})$ are candidates for t_R. Spurions in the $(\mathbf{2},\mathbf{1})$ should not be used because they violate the extra custodial requirement [50].

G	$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$		
SU(5)	$\mathbf{1}$	$\rightarrow$	$(\mathbf{1},\mathbf{1})$
	$\mathbf{5},\overline{\mathbf{5}}$	$\rightarrow$	$(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
	$\mathbf{10},\overline{\mathbf{10}}$	$\rightarrow$	$(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$
	$\mathbf{15},\overline{\mathbf{15}}$	$\rightarrow$	$2 \times (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{3})$
	$\mathbf{24}$	$\rightarrow$	$(\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{3},\mathbf{3})$
SU(4)	$\mathbf{1}$	$\rightarrow$	$(\mathbf{1},\mathbf{1})$
	$\mathbf{4},\overline{\mathbf{4}}$	$\rightarrow$	$(\mathbf{1},\mathbf{2}) + (\mathbf{2},\mathbf{1})$
	$\mathbf{6}$	$\rightarrow$	$2 \times (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
	$\mathbf{10},\overline{\mathbf{10}}$	$\rightarrow$	$(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$
	$\mathbf{15}$	$\rightarrow$	$(\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$
SU(4) $\times$ SU(4)$'$	$(\mathbf{1},\mathbf{1})$	$\rightarrow$	$(\mathbf{1},\mathbf{1})$
	$(\mathbf{1},\mathbf{4}),\ldots(\overline{\mathbf{4}},\mathbf{1})$	$\rightarrow$	$(\mathbf{1},\mathbf{2}) + (\mathbf{2},\mathbf{1})$
	$(\mathbf{1},\mathbf{6}),(\mathbf{6},\mathbf{1})$	$\rightarrow$	$2 \times (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
	$(\mathbf{1},\mathbf{10}),\ldots(\overline{\mathbf{10}},\mathbf{1})$	$\rightarrow$	$(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$
	$(\mathbf{1},\mathbf{15}),(\mathbf{15},\mathbf{1})$	$\rightarrow$	$(\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$
	$(\mathbf{4},\mathbf{4}),\ldots(\overline{\mathbf{4}},\overline{\mathbf{4}})$	$\rightarrow$	$2 \times (\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$

Table 3. Decompositions of the irreps of G to be used to identify candidate spurions.

The spurions irrep should be matched with the type of baryon arising in the UV completion. If, in a particular model, the composite top partners arise from bound states of type $\chi\psi\chi$, then the spurions to be used are those in the one index irrep (the fundamental). Vice-versa, if the top partners in a model are of type $\psi\chi\psi$, one should use two indices irreps, to be further restricted to symmetric, anti-symmetric, adjoint or bi-fundamental if required by the symmetries of the particular model. From table 5 in appendix A one can reconstruct the requirements case by case.

A spurion S in a two-index irrep of SU(n) transforms as $S \rightarrow gSg^T$ if in the $\mathbf{S}_2$ or $\mathbf{A}_2$ irrep and $S \rightarrow gSg^\dagger$ if in the $\mathbf{Ad}$. In the SU(4) $\times$ SU(4)$'$/SU(4)$_D$ case one should instead talk about $(\mathbf{F},\mathbf{F})$ or $(\mathbf{F},\overline{\mathbf{F}})$, whereby $S \rightarrow gSg'^T$ or $S \rightarrow gSg'^\dagger$. Similarly (see appendix B) the symmetry properties of the pNGB field U are $U \rightarrow gUg^T$ for the SU(5)/SO(5) and SU(4)/Sp(4) cosets and $U \rightarrow gUg'^\dagger$ for SU(4) $\times$ SU(4)$'$/SU(4)$_D$. Thus, we see that, to leading order, the potential for two-index representations is proportional to the expressions in table 4. Spurions like $(\mathbf{F},\overline{\mathbf{F}})$ must couple to top partners containing one ψ and one $\tilde{\psi}$. Spurions of the type $(R,\mathbf{1})$ or $(\mathbf{1},R')$ such as $(\mathbf{F},\mathbf{1})$, $(\mathbf{A}_2,\mathbf{1})$ etc., do not give rise to a non-trivial invariant since we need to multiply directly U and $U^\dagger$.

In the cases of SU(5)/SO(5) one could also consider spurions in the fundamental $\mathbf{F}$ of SU(5). In this case the leading contribution to the potential is of forth order and proportional to[5]

$$(S^\dagger U S^*)(S^T U^* S) \,. \tag{3.4}$$

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[5]We ignore possible non factorizable contributions and refer again to [59] for details.

	$S \in \mathbf{S}_2$	$S \in \mathbf{A}_2$	$S \in \mathbf{Ad}$
SU(5)/SO(5)	$\mathrm{tr}(SU^*)\,\mathrm{tr}(S^*U)$	0	$\mathrm{tr}(SUS^*U^*)$
SU(4)/Sp(4)	0	$\mathrm{tr}(SU^*)\,\mathrm{tr}(S^*U)$	$\mathrm{tr}(SUS^*U^*)$

	$S \in (\mathbf{F}, \mathbf{F})$	$S \in (\mathbf{F}, \overline{\mathbf{F}})$
SU(4) × SU(4)′/SU(4)$_D$	$\mathrm{tr}(US^T U^* S^\dagger)$	$\mathrm{tr}(SU^\dagger)\,\mathrm{tr}(S^\dagger U)$

Table 4. The spurion couplings at leading order for the two index irreps, to be associated to models where the top partners are of type $\psi\chi\psi$. The zeros arise in the case when U and S have opposite symmetry properties. Irreps of type $(R, \mathbf{1})$ or $(\mathbf{1}, R')$ such as $(\mathbf{F}, \mathbf{1})$, $(\mathbf{A}_2, \mathbf{1})$ etc. for $SU(4) \times SU(4)'/SU(4)_D$ do not give rise to a non-trivial invariant since we need to multiply directly U and $U^\dagger$.

The $\mathbf{F}$ for the coset SU(5)/SO(5) runs into trouble with the desire to have a vacuum that preserves custodial symmetry. In this case, coupling generically the pNGBS to spurions in the fundamental will induce a tadpole for the field $\phi_+^- - \phi_-^+$ which should be suppressed in order to avoid tree level corrections to the ρ-parameter. If we were to take this fact also as a strict guideline, we would be led to exclude *all* the cases in appendix A giving top partners of type $\chi\psi\chi$, although this may be a bit too drastic at this stage.

In the above formulas S could carry a SU(2)$_L$ index in the case it corresponds to Q_L^3. This index is then also summed over in the obvious way. Notice that terms proportional to $\mathrm{tr}(SU^*) + \mathrm{tr}(S^*U)$ or $(S^\dagger US^*) + (S^T U^* S)$ are not allowed due to the need to preserve the spurionic U(1).

3.2 The parity transformations P_π and G_π

We are now in the position of defining more concretely the parity symmetries of relevance for these models, starting with P_π. For the scope of this paper we will think of P_π as an accidental symmetry of the non-anomalous pNGB Lagrangian coupled to the SM. Its action changes sign to all the pNGB except the Higgs doublet(s) and can be realized in all three cases as $U \to \hat{P}_\pi U^\dagger \hat{P}_\pi$ with the matrix $\hat{P}_\pi$ defined as

$$\hat{P}_\pi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \hat{P}_\pi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \hat{P}_\pi = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{3.5}$$

for the three cosets SU(5)/SO(5), SU(4) × SU(4)′/SU(4)$_D$ and SU(4)/Sp(4) respectively.

To see that the transformation accomplishes its task note first that $\hat{P}_\pi \Omega^* = \Omega \hat{P}_\pi$ for SU(5)/SO(5) and SU(4)/Sp(4) and $\hat{P}_\pi \Omega^\dagger = \Omega \hat{P}_\pi$ for SU(4) × SU(4)′/SU(4)$_D$. This allows one to move the action of $\hat{P}_\pi$ pass the vacuum misalignment matrix directly onto the pNGB matrix Π (cf. appendix B) where its effect is to reverse the sign of the Higgs doublet(s). This, together with the hermitian conjugation on U that reverses the sign of all pNGBs,

has the desired combined effect. In all three cases P_π leaves the vacuum invariant and preserves the custodial symmetry group. In particular $D_\mu(\hat{P}_\pi U^\dagger \hat{P}_\pi) = \hat{P}_\pi(D_\mu U)^\dagger \hat{P}_\pi$.

Note that the hermitian conjugation is necessary in all three cases. But it is known that the WZW term breaks precisely this last transformation and thus P_π can never be an exact symmetry at the quantum level. Still, it is desirable for the Yukawa couplings to be left invariant by such transformation since this prevents the generation of custodial symmetry breaking v.e.v.s from the induced potential and greatly alleviates the constraints from flavor physics, e.d.m. etc. This condition can be realized by imposing the invariance of the spurion fields. In particular, for the two-index irreps in table 4 we require $S = \pm \hat{P}_\pi S^\dagger \hat{P}_\pi$ (either sign) for the $\mathbf{S}_2$, $\mathbf{A}_2$ or $(\mathbf{F}, \overline{\mathbf{F}})$ or $S = \pm \hat{P}_\pi S^T \hat{P}_\pi$ (either sign) for the $\mathbf{Ad}$ or $(\mathbf{F}, \mathbf{F})$. Some, but not all, spurions obey these requirements. The spurions used in the next section to generate an example of potential have been chosen to satisfy these invariance requirements.

The second transformation of interest, G_π, is realized as $U \to \hat{G}_\pi U^T \hat{G}_\pi^\dagger$ and gives non trivial results only for $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ since in the other two cases $U^T = \pm U$ (see appendix B). For the $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ case we choose, following [34]

$$\hat{G}_\pi = i \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{3.6}$$

This transformation is interesting because it is also a symmetry of the WZW term and it may be preserved at the quantum level in the UV theory. If so, the lightest neutral pNGBs odd under it (a linear combination of ϕ_0, N_0, h' and A') could be a Dark Matter candidate.

3.3 Mass spectrum

Now that we have seen what the main contributions to the potential are and how to compute them, we present a couple of examples of mass spectrum based on a particular choice of spurions. This is not in any way a prediction of the models, it is merely presented to make the previous discussion more concrete and to show qualitatively how a mass spectrum could look like. We consider potentials that depend on three of the dimensionless constants B_i, to be specified below. We trade one linear combination for the misalignment angle $\sin \zeta = v/f$, measuring the amount of fine-tuning in the model. A second combination is fixed by imposing the mass of the Higgs boson to be at its measured value [61, 62] of $125\,\mathrm{GeV}$. The third combination is left free and varying it gives possible examples for the mass spectrum.

As a first example, consider the $\mathrm{SU}(5)/\mathrm{SO}(5)$ model with a potential

$$\begin{aligned} V = &-B_1\, f^4 \operatorname{tr}(g^2 T_L^a U T_L^a U^\dagger + g'^2 T_R^3 U T_R^3 U^\dagger) + B_2\, f^4 \operatorname{tr}(S_{t_R} U S_{t_R}^* U^*) \\ &+ B_3\, f^4 \operatorname{tr}(S_{t_L} U S_{t_L}^* U^* + S_{b_L} U S_{b_L}^* U^*) \end{aligned} \tag{3.7}$$

where we have chosen the spurion for t_R to be in the $(\mathbf{1}, \mathbf{1})$ component of the decomposition of the $\mathbf{Ad}$ irrep and the spurion for (t_L, b_L) to be in one of the two $(\mathbf{2}, \mathbf{2})$ components with
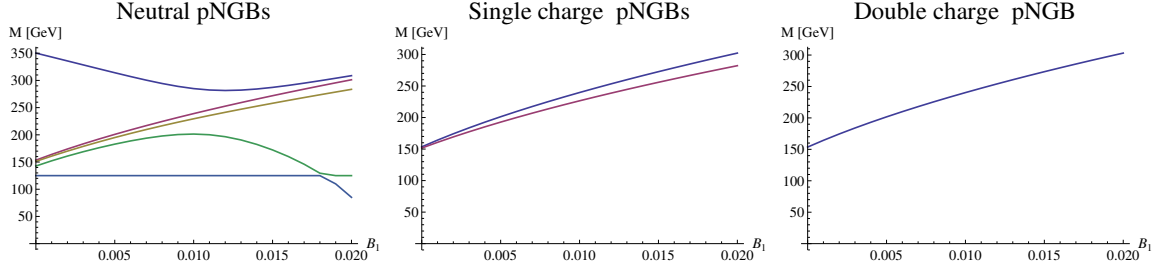
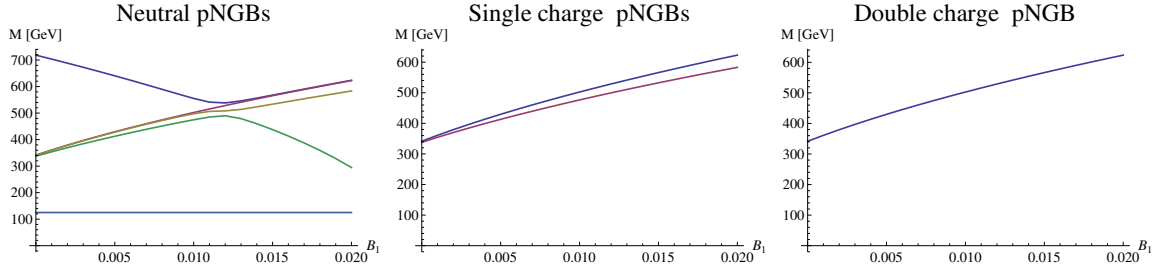**Figure 1**. Prototypical spectrum for the SU(5)/SO(5) model with $f = 800\,\text{GeV}$.

Figure 2. Prototypical spectrum for the SU(5)/SO(5) model with $f = 1600\,\text{GeV}$.

$T_R^3 = -1/2$ in order for b_L to obey the custodial relations $T_L(T_L + 1) = T_R(T_R + 1)$ and $T_L^3 = T_R^3$

$$
S_{t_R} = \frac{1}{2\sqrt{5}}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}, \qquad
S_{t_L} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1-i \\ 0 & 0 & 0 & 0 & 1-i \\ 0 & 0 & 1+i & -1+i & 0 \end{pmatrix},
$$

$$
S_{b_L} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 & 1-i \\ 0 & 0 & 0 & 0 & -1-i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1+i & 1+i & 0 & 0 & 0 \end{pmatrix}. \tag{3.8}
$$

Setting $f = 800\,\text{GeV}$ and $f = 1600\,\text{GeV}$, solving the constraints and varying B_1 we obtain the spectra in figure 1 and 2 respectively.

Moving on to $\text{SU}(4) \times \text{SU}(4)'/\text{SU}(4)_D$, we chose to present the mass spectrum induced by the following potential, consisting of the contributions from the gauge fields, some bare masses and a LH third family, assumed to give the dominant contribution.

$$
V = -B_1 f^4 \, \text{tr}(g^2 T_L^a U T_L^a U^\dagger + g'^2 T_R^3 U T_R^3 U^\dagger) - B_2 f^4 \, \text{tr}(U + U^*)
$$
$$
+ B_3 f^4 \left(\text{tr}(S_{t_L}^\dagger U) \, \text{tr}(S_{t_L} U^\dagger) + \text{tr}(S_{b_L}^\dagger U) \, \text{tr}(S_{b_L} U^\dagger) \right). \tag{3.9}
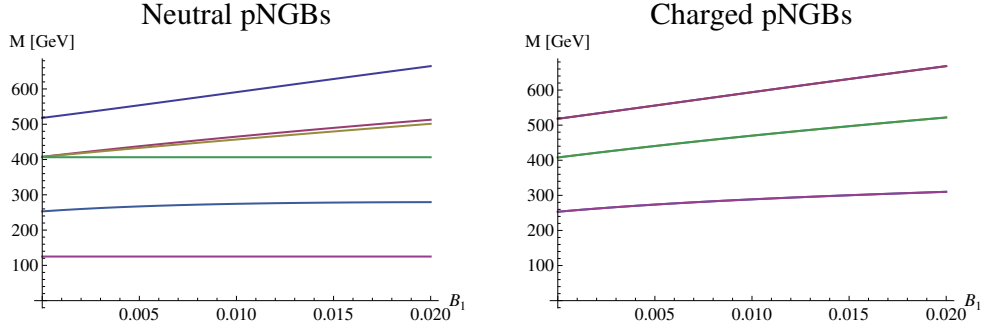$$

Figure 3. Prototypical spectrum for the $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ model with $f = 800\,\mathrm{GeV}$.
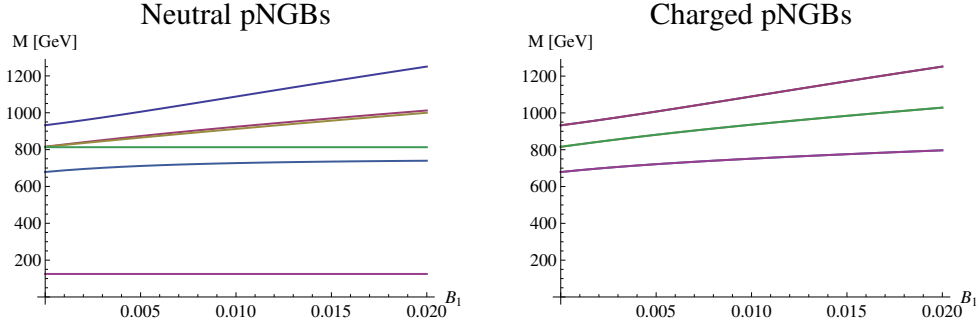
Figure 4. Prototypical spectrum for the $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ model with $f = 1600\,\mathrm{GeV}$.

The spurions for the LH quarks are chosen to belong to one of the $(\mathbf{2}, \overline{\mathbf{2}})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ found in the decomposition of $(\mathbf{4}, \overline{\mathbf{4}})$

$$S_{t_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \qquad S_{b_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{3.10}$$

The representative spectra for $f = 800\,\mathrm{GeV}$ and $f = 1600\,\mathrm{GeV}$ are given in in figure 3 and 4 respectively.

Not much needs to be done for the remaining $\mathrm{SU}(4)/\mathrm{Sp}(4)$. The η is the only pNGB particle other than the Higgs in our current approach its mass is essentially a free parameter. A full discussion of this case is given in [42].

3.4 Couplings involving pNGBs

The trilinear vertex $\pi\pi'V$ between two generic EW pNGBs and an EW vector boson is encoded in the structure of the currents. With the usual shorthand $\pi_1^* \overset{\leftrightarrow}{\partial}_\mu \pi_2 = \pi_1^* \partial_\mu \pi_2 - \partial_\mu \pi_1^* \pi_2$ we find, for $\mathrm{SU}(5)/\mathrm{SO}(5)$ the coupling to the Z-boson[6]

$$\mathcal{L} \supset \frac{ie}{2\mathrm{s_w c_w}} Z^\mu \big((\mathrm{c_{2w}} - \mathrm{c}_\zeta)\phi_-^0 \overset{\leftrightarrow}{\partial}_\mu \phi_+^0 + (\mathrm{c_{2w}} + \mathrm{c}_\zeta)\phi_0^- \overset{\leftrightarrow}{\partial}_\mu \phi_0^+ + 2\mathrm{c}_\zeta \phi_+^- \overset{\leftrightarrow}{\partial}_\mu \phi_-^+ + 2\mathrm{c_{2w}}\phi_-^- \overset{\leftrightarrow}{\partial}_\mu \phi_+^+ \big)$$

$$(3.11)$$

---

[6]See again appendix B for notation on the field content of the theory. We set $\mathrm{s}_\zeta = \sin\zeta$, $\mathrm{s_w} = \sin\theta_W$ etc.

and that to the $W^\pm$

$$\mathcal{L} \supset \frac{e}{2s_w} W^{-\mu} \big((1 + c_\zeta)(\phi_-^+ \overset{\leftrightarrow}{\partial}_\mu \phi_+^0 + \phi_0^0 \overset{\leftrightarrow}{\partial}_\mu \phi_0^+ - \phi_-^0 \overset{\leftrightarrow}{\partial}_\mu \phi_+^+)$$
$$- (1 - c_\zeta)(\phi_0^0 \overset{\leftrightarrow}{\partial}_\mu \phi_+^0 + \phi_+^- \overset{\leftrightarrow}{\partial}_\mu \phi_0^+ - \phi_0^- \overset{\leftrightarrow}{\partial}_\mu \phi_+^+)\big) + \text{h.c.} \tag{3.12}$$

For $\mathrm{SU}(4) \times \mathrm{SU}(4)'/\mathrm{SU}(4)_D$ we find instead, in agreement with the results of [34]

$$\mathcal{L} \supset \frac{ie}{2s_w c_w} Z^\mu \big((c_{2w} - c_\zeta) N_- \overset{\leftrightarrow}{\partial}_\mu N_+ + (c_{2w} + c_\zeta) \phi_- \overset{\leftrightarrow}{\partial}_\mu \phi_+ + c_{2w} H'_- \overset{\leftrightarrow}{\partial}_\mu H'_+ + i c_\zeta A' \overset{\leftrightarrow}{\partial}_\mu h'\big) \tag{3.13}$$

for the Z couplings, and

$$\mathcal{L} \supset -\frac{ie}{2s_w} W^{-\mu} \big((1 - c_\zeta) N_0 \overset{\leftrightarrow}{\partial}_\mu N_+ + (1 + c_\zeta) \phi_0 \overset{\leftrightarrow}{\partial}_\mu \phi_+ - c_\zeta h' \overset{\leftrightarrow}{\partial}_\mu H'_+ + i A' \overset{\leftrightarrow}{\partial}_\mu H'_+\big) + \text{h.c.} \tag{3.14}$$

for the $W^\pm$ couplings. The electromagnetic coupling is of course always given by $i e q_\pi A^\mu \pi^* \overset{\leftrightarrow}{\partial}_\mu \pi$ for any of the pNGBs π of charge q_π.

In all three cases the Higgs boson h does not mix with the other pNGBs and its couplings to the vector bosons at tree level are:[7]

$$\mathcal{L} = \frac{1}{4} g^2 f s_{2\zeta} h W^{+\mu} W_\mu^- + \frac{1}{8}(g^2 + g'^2) f s_{2\zeta} h Z^\mu Z_\mu + \frac{1}{4} g^2 c_{2\zeta} h^2 W^{+\mu} W_\mu^- + \frac{1}{8}(g^2 + g'^2) c_{2\zeta} h^2 Z^\mu Z_\mu. \tag{3.15}$$

The model $\mathrm{SU}(4)/\mathrm{Sp}(4)$ only contains the η as an additional pNGB. Its trilinear couplings vanish and at quartic level it can easily be written down:

$$\mathcal{L} = -\frac{1}{4} g^2 s_\zeta^2 \eta^2 W^{+\mu} W_\mu^- - \frac{1}{8}(g^2 + g'^2) s_\zeta^2 \eta^2 Z^\mu Z_\mu. \tag{3.16}$$

For the quartic couplings in the remaining models we refer to appendix C.

The P_π-parity odd pNGBs can decay to the transverse part of the vector bosons via the anomaly term yielding a vertex $\pi V V'$. This can be extracted from the WZW term [63, 64] by considering the piece containing one pNGB and two vector bosons. The relevant term is given in [65] in the elegant language of differential forms

$$S_{\mathrm{WZW}} \supset \frac{i \dim(\psi)}{48\pi^2} \int \mathrm{tr}(dA_L A_L dU U^\dagger + A_L dA_L dU U^\dagger + dA_R A_R U^\dagger dU + A_R dA_R U^\dagger dU$$
$$- dA_L dU A_R U^\dagger + dA_R dU^\dagger A_L U). \tag{3.17}$$

For $\mathrm{SU}(4)/\mathrm{Sp}(4)$ we set $A_L = A$, $A_R = -A^T = -\epsilon_0 A \epsilon_0$ and $U = \Omega \exp(2\sqrt{2} i \Pi/f) \epsilon_0 \Omega^T$. Expanding to first order in the pNGBs and integrating by parts yields

$$S_{\mathrm{WZW}} \supset \frac{\dim(\psi)}{16\pi^2 f} c_\zeta \int \eta \left(\frac{g^2 - g'^2}{2} Z_{\mu\nu} \tilde{Z}^{\mu\nu} + g g' F_{\mu\nu} \tilde{Z}^{\mu\nu} + g^2 W_{\mu\nu}^+ \tilde{W}^{-\mu\nu}\right) \mathrm{d}^4 x. \tag{3.18}$$

_____

[7] For uniformity we have chosen to normalize all three cases according to $m_W = \frac{1}{2} g f \sin \zeta$, implying $v = f \sin \zeta = 246 \,\mathrm{GeV}$. This is different from the normalization of f used in [12]. Our h is already shifted to have zero v.e.v. and is canonically normalized.
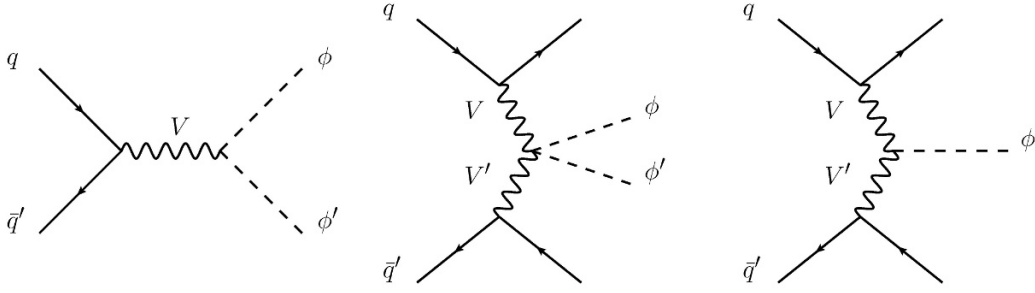
Figure 5. Production modes for the EW pNGBs. From left to right: Drell-Yan (DY), Vector Boson Fusion pair production via a renormalizable quartic interaction (VBFr) and Vector Boson Fusion via the anomaly (VBFa).

For $SU(4) \times SU(4)'/SU(4)_D$ we set $A_L = A_R = A$ and $U = \Omega \exp(2\sqrt{2}i\Pi/f)\Omega$. Expanding to first order in the pNGBs and integrating by parts we find exactly the same expression as (3.18). This was found in [34] and it is due to the extra symmetry G_π, defined in section 3.2, present in this case. In particular, no terms involving the pNGB ϕ and N arise in this model.

On the contrary, for the coset $SU(5)/SO(5)$, we need to set $A_L = A$, $A_R = -A^T = A$ and $U = \Omega \exp(2i\Pi/f)\Omega^T$. Here, no additional symmetry is present and all the pNGBs other than the Higgs boson appear in the WZW action. In this case, the trilinear anomalous couplings are presented in appendix C.

There are three possible production modes to be considered for these EW pNGBs, see figure 5. Two of them are pair production modes, one by an off-shell vector boson in the s-channel — Drell-Yan production (DY) — and the other by vector boson fusion via a renormalizable four boson interaction (VBFr). The third one is a single production mode by vector boson fusion via the anomaly (VBFa).

Perhaps surprisingly, VBFr tends to give a larger contribution than DY. Consider the interesting case of the doubly charged pNGB ϕ_+^+ present in $SU(5)/SO(5)$. (A model in which such a particle is present as an elementary object is the Georgi-Machacek model [66].) The tree level production can be easily estimated with `MadGraph` and `FeynRules` [67, 68] yielding, at $13\,\mathrm{TeV}$ for a mass of $500\,\mathrm{GeV}$ and $f = 800\,\mathrm{GeV}$: $\sigma_{\mathrm{DY}}(\phi_+^+\phi_-^-) = 1.3\,\mathrm{fb}$ and $\sigma_{\mathrm{VBFr}}(\phi_+^+\phi_-^-) = 3.0\,\mathrm{fb}$.

The single production of the doubly charged pNGBs via VBFa is totally negligible in this case: $\sigma_{\mathrm{VBFa}}(\phi_+^+) = 2.3 \times 10^{-5}\,\mathrm{fb}$. This last statement is no longer true for other pNGBs. For instance, in the case of the η of $SU(4)/Sp(4)$, (and a particle with exactly the same couplings is present in $SU(4) \times SU(4)'/SU(4)_D$ as well), with the same parameters as before, the double production is now negligible: $\sigma_{\mathrm{DY}}(\eta\eta) = 0$ (impossible) and $\sigma_{\mathrm{VBFr}}(\eta\eta) = 2.0 \times 10^{-2}\,\mathrm{fb}$, while $\sigma_{\mathrm{VBFa}}(\eta)$ is of the order of a few fb depending on the specific value of the anomaly.

The reason for this different behavior is due to the fact that the VBF diagrams that contribute the most are those where a photon is allowed to be present. For this same reason, the single charge pNGBs have non negligible cross section for all processes and the
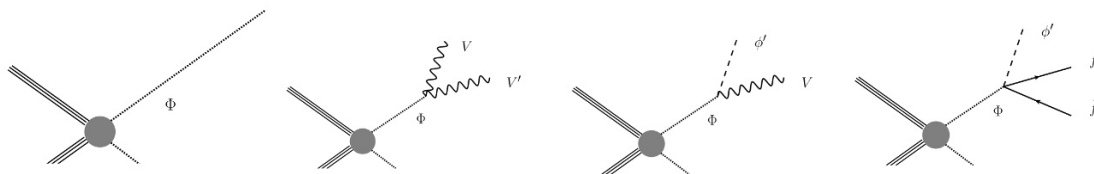
Figure 6. After production, an EW pNGB Φ can be collider stable, decay to two vector bosons V, V' via the anomaly, decay to a lighter pNGB ϕ' plus a vector boson V or to a pair of fermions $f, \bar{f}'$ plus a lighter pNGB ϕ'.

single production mode becomes relevant at higher masses. We have not tried to pin down the exact range of masses where one production mode is expected to be dominant with respect to the others because this depends on the details of the models such as mixing, which is not an issue for the η of SU(4)/Sp(4) or the $\phi_+^\pm$ of SU(5)/SO(5). However, given that $\sigma_{\text{VBFr}}(\phi_+^+ \phi_-^-)$ and $\sigma_{\text{VBFa}}(\eta)$ are roughly comparable for masses of $500\,\text{GeV}$, we expect the cross-over region to be within the energy range of the LHC.

The phenomenology of the cosets $SU(4) \times SU(4)'/SU(4)_D$ and $SU(5)/SO(5)$ is potentially very rich (some would say too rich...). Once produced, the EW pNGBs chain decay to lighter ones plus a SM vector boson, if kinematically allowed, or a pair of SM fermions. In the $SU(5)/SO(5)$ case, the lightest EW pNGB decays to two SM vector bosons via the anomaly. (This may actually become the dominant decay mode for heavier pNGBs as well if the spectrum is squeezed, $\Delta m \lesssim 10\,\text{GeV}$.) In the $SU(4) \times SU(4)'/SU(4)_D$ case, the lightest pNGB odd under G_π is collider stable under our assumptions and thus leads to missing energy or charged heavy tracks depending on its charge. If its decay into SM fermions is totally forbidden, it could even be a dark matter candidate [34]. This is in the spirit of [69] although their candidate for dark matter (the η of SU(4)/Sp(4)) is not viable for our UV completions because it decays through the anomalous couplings. (For pNGB dark matter see also [70]. Additional dark matter candidates have been conjectured to arise from the topological structure of similar cosets [71].) A pictorial description of the various possibilities is given in figure 6.

4 Top partners and colored mesons

We now turn to the discussion of objects carrying color, that is, bound states containing some of the constituents χ.

As we mentioned in the introduction, top-partners are realized via fermionic tri-linears in the hyperquarks. These can be of type $\psi\chi\psi$ or $\chi\psi\chi$ depending on the type of model under consideration, as shown in appendix A. So far we have been somewhat sloppy in indicating the structure of these objects, now it is time to be more specific.

We need at least six new fermions "χ" in order to embed the color group into the associated global symmetry group in an anomaly-free way. In the case of a complex irrep, leading to $SU(3) \times SU(3)'/SU(3)_D$, the χs are already naturally divided into 3 χ and 3 $\tilde{\chi}$

transforming in conjugate irreps R_χ and $\overline{R}_\chi$ of G_{HC} as well as the $\mathbf{3}$ and $\overline{\mathbf{3}}$ of color $SU(3)_c$. (As always, we are using Weyl LH notation.)

Even in the other two cases (real or pseudo-real irreps), it is still convenient to split the 6 fermions into a $\mathbf{3} + \overline{\mathbf{3}}$ of $SU(3)_c$. In these cases we allow ourselves the following notational ambiguity

$$\chi \in \mathbf{6} \equiv \begin{pmatrix} \chi \\ \tilde{\chi} \end{pmatrix} \in \begin{pmatrix} \mathbf{3}_X \\ \overline{\mathbf{3}}_{-X} \end{pmatrix} \tag{4.1}$$

for ease of notation. Note that these fermions must carry not only the color quantum numbers but also the additional $U(1)_X$ charge needed to obtain the proper weak hypercharge $Y = X + T_R^3$ for the top partners. The allowed values of X can be found looking at the construction of the top-partners as follows.

Consider the case where the top-partners are of type $\chi\psi\chi$. Using the notation (4.1), we can generally construct at most three types of LH objects transforming in the $\mathbf{3}$. They are contained in the products $\tilde{\chi}\psi\tilde{\chi}$, $\tilde{\chi}\psi^\dagger\chi^\dagger$, $\chi^\dagger\psi\chi^\dagger$, where we used the fact that $\overline{\mathbf{3}} \times \overline{\mathbf{3}} = \overline{\mathbf{6}} + \mathbf{3}$. Identifying the $T_L = T_R = 0$ component with the partner of t_R we see that we must chose $X(\chi) = -1/3$ and $B(\chi) = -1/6$ (baryon number) for the constituents χ.

Now, still within the $\chi\psi\chi$ case, if the G_{HC} irrep for the χ in question is real, giving rise to the coset $SU(6)/SO(6)$, this leads to colored pNGBs $\chi\chi \in \mathbf{6}_{-2/3}$ of baryon number $1/3$, as well as $\tilde{\chi}\tilde{\chi} \in \overline{\mathbf{6}}_{+2/3}$ of baryon number $-1/3$ and the ever-present $\tilde{\chi}\chi \in \mathbf{8}_0$ of baryon number 0. If the G_{HC} irrep is pseudo-real, giving rise to the coset $SU(6)/Sp(6)$, then the pNGB mesons are $\chi\chi \in \overline{\mathbf{3}}_{-2/3}$ etc. with the same baryon number assignments as before.

If instead the top partners are of type $\psi\chi\psi$, then the χ and $\tilde{\chi}$ in (4.1) must be in the $\mathbf{3}_{+2/3} + \overline{\mathbf{3}}_{-2/3}$ of $SU(3)_c \times U(1)_Y$ with baryon number $\pm 1/3$, leading, for a real irrep, to mesons $\chi\chi \in \mathbf{6}_{4/3}$ of baryon number $2/3$ and its complex conjugate plus the usual $\tilde{\chi}\chi \in \mathbf{8}_0$. From appendix A we see that no pseudo-real cases exist when the top-partners are of type $\psi\chi\psi$. The case in which the χ are in a complex irrep only leads to the neutral meson $\tilde{\chi}\chi \in \mathbf{8}_0$ without baryon number.

The masses for these colored objects should be in the multi TeV range getting contributions from gluon loops and possibly bare masses for χ but they could still be in the discovery range of LHC. The octets decay mostly to two gluons via the anomaly term

$$\mathcal{L}_{WZW} \supset \frac{g_s^2 \dim \chi}{16\pi^2 f_c} d^{ABC} \Pi^A G_{\mu\nu}^B \tilde{G}^{C\mu\nu} \tag{4.2}$$

but there is no such term available for the triplet or the sextet. Preserving P_π-parity, we can let them cascade to the lighter EW pNGBs via interactions of type $\pi q q' \phi$ where q and q' are SM quarks and ϕ is an appropriate EW pNGB with the right quantum numbers. If we allow for interactions violating P_π-parity, we do not need this additional pNGB. Summarizing, we have therefore the following three possibilities, in addition to the octet:

- *Case a)* χ in a real irrep and top-partners of type $\chi\psi\chi$. This gives rise to mesons π in the $\mathbf{6}_{-2/3}$ of $SU(3)_c \times U(1)_Y$ of baryon number $-1/3$. They can decay via $\Delta B = 1$ couplings

$$\pi^{*ab} Q_{La} Q_{Lb} \phi, \qquad \pi^{*ab} u_{Ra} u_{Rb} \phi, \qquad \pi^{*ab} d_{Ra} u_{Rb} \phi, \qquad \pi^{*ab} d_{Ra} d_{Rb} \phi \tag{4.3}$$
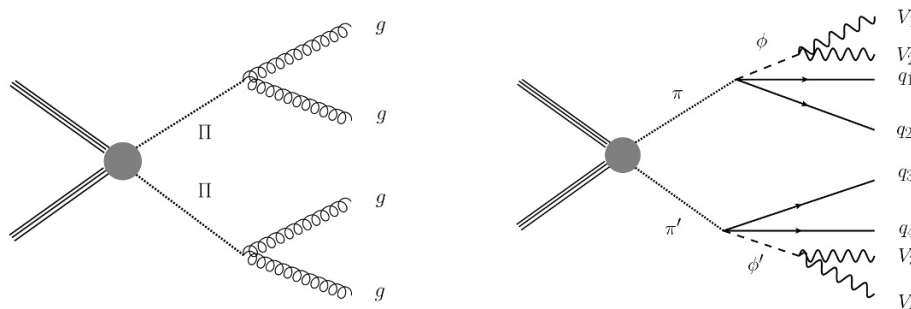
Figure 7. Pair production and decay of color octets (left) and triplet/sextet (right). The octet can also be singly produced by the anomalous coupling via gluon fusion.

where we denoted explicitly only the color index. The various EW pNGBs ϕ appearing in the vertex must be such that the particular vertex is invariant under the full SM gauge group. In the case of $Q_L Q_L$ coupling, we have the option of coupling to a $SU(2)_L$ triplet or a singlet, making the quark flavor indices symmetric or antisymmetric respectively. In all gory details for the triplet: $\pi_{ab}^* Q_{La}^{\alpha f i} Q_{Lb\alpha}^{f'j} \phi_{ij}$, symmetric in the exchange of ff'. In the absence of P_π-parity we could also consider the term $\pi^{*ab} d_{Ra} d_{Rb}$, symmetric in the flavor indices.

- *Case b)* χ in a real irrep and top-partners of type $\psi\chi\psi$. This gives rise to mesons π in the $\mathbf{6}_{4/3}$ of $SU(3)_c \times U(1)_Y$ of baryon number 2/3. They can decay via same couplings as case a) but now these couplings are baryon number preserving. Without P_π-parity one can only make the vertex $\pi^{*ab} u_{Ra} u_{Rb}$, symmetric in flavor.

- *Case c)* χ in a pseudo-real irrep and top-partners of type $\chi\psi\chi$. The extra mesons are now in the $\mathbf{3}_{2/3}$ of baryon number 1/3 and decay via the $\Delta B = 1$ interactions

$$\epsilon^{abc} \pi_a Q_{Lb} Q_{Lc} \phi \,, \qquad \epsilon^{abc} \pi_a u_{Rb} u_{Rc} \phi \,, \qquad \epsilon^{abc} \pi_a d_{Rb} u_{Rc} \phi \,, \qquad \epsilon^{abc} \pi_a d_{Rb} d_{Rc} \phi \quad (4.4)$$

with the appropriate EW pNGB. Without P_π-parity one can construct $\epsilon^{abc} \pi_a d_{Rb} d_{Rc}$ asymmetric in the flavor indices.

For all EW cosets there are some pNGBs that can be used to construct some of the couplings, so all the colored sextets and triplets can decay into two jets and an EW pNGB. Note that proton stability is assured since we preserve lepton number. However, the presence of $\Delta B = 1$ interactions raises the interesting possibility of neutron-anti-neutron oscillations. (See [72] for a recent discussion in the context of RPV-SUSY. Similar scalars objects have been discussed in e.g. [73, 74].) The situation is summarized in figure 7.

As far as fermionic colored objects go these models predict a slew of additional resonances but all of them, with the possible exception of the top partners, should be out of reach at LHC.

Exotic fermions of higher electric charge also need be taken into consideration. For the almost ubiquitous charge 5/3 state X, the main decay mode targeted by experiments so far is $X \to W\, t$ [75], but the existence of possible additional charged pNGBs opens alternative

channels such as $X \to t\,\phi_0^+$. The presence of doubly charged pNGBs in some constructions might even allow for $X \to b\,\phi_+^+$.

The operator creating the fermionic resonance should acquire a large negative anomalous dimension in the running from Λ_{UV} to Λ. This has been investigated at the perturbative level in [76] for the class of models in [12]. More recently [77] summarized the results for the QCD case, also within perturbation theory.

5 Two more pNGBs/ALPs

A universal feature of all of these models, simply due to the fact that they are constructed out of two different types of fermions, is the existence of two additional neutral pNGBs associated to the abelian axial currents from the axial $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$. One linear combination of these currents can be taken to be free of G_{HC} anomalies. The associated pNGB, to be denoted by a, will be naturally light and, in absence of further interactions would essentially be a composite axion [78] coupling to both gluons and EW gauge bosons via the anomaly

$$\mathcal{L} = \frac{g_s^2 N_s}{16\pi^2 f_a} a G_{\mu\nu}^A \tilde{G}^{A\mu\nu} + \frac{g'^2 N_B}{16\pi^2 f_a} a B_{\mu\nu} \tilde{B}^{\mu\nu} + \frac{g^2 N_W}{16\pi^2 f_a} a W_{\mu\nu}^i \tilde{W}^{i\mu\nu}. \tag{5.1}$$

Since the associated decay constant f_a is much smaller than the possible window of values allowed by the "invisible-axion" solution, we must give this particle a mass to avoid the usual constraints. As in technicolor models [79], a mass can be obtained from e.g. the four-fermi terms arising at the Λ_{UV} scale of the type ($c_i = \mathcal{O}(1)$)

$$H' = -\mathcal{L}_{4\mathrm{f}} = \frac{1}{\Lambda_{\mathrm{UV}}^2} (c_1 \chi^2 \tilde{\chi}^2 + c_2 \psi^4 + \dots). \tag{5.2}$$

For typical values of the parameters, using Dashen's formula [80] we estimate

$$m_a^2 = \frac{1}{f^2} \langle [Q, [Q, H']] \rangle \approx \frac{\Lambda^6}{f^2 \Lambda_{\mathrm{UV}}^2} \approx \frac{(5 \times 10^3 \, \mathrm{GeV})^6}{(800 \, \mathrm{GeV})^2 (10^8 \, \mathrm{GeV})^2} \approx (1.6 \, \mathrm{GeV})^2 \tag{5.3}$$

but a fairly large range of masses is possible. For instance, Naive Dimensional Analysis would lead to a lower estimate $m_a^2 \approx \Lambda^2 f^2 / \Lambda_{\mathrm{UV}}^2 \approx (40.\,\mathrm{MeV})^2$. This value needs to be raised at least by roughly a factor ≈ 3 in order not to conflict with the bounds on the visible axion, coming from beam dump experiments (discussed in [81]) or $K \to \pi a$ searches [82]. (See also [83] for cosmological bounds for ALPS at much higher scale f.) This however is easily achieved. In fact, in [37] the exciting possibility has been raised that this object is responsible for the $750\,\mathrm{GeV}$ bump in the di-photon signal recently reported by ATLAS and CMS [35, 36]. Such a large mass could be obtained by e.g. adding bare masses for the colored hyperquarks.

The remaining linear combination, to be denoted by η', corresponds to the G_{HC} anomalous current and its associated "would-be" Goldstone boson acquires a mass via the 't Hooft mechanism [84]. The η' mass is given the Veneziano-Witten formula [85, 86] ($N \approx 10$, Ξ the topological susceptibility)

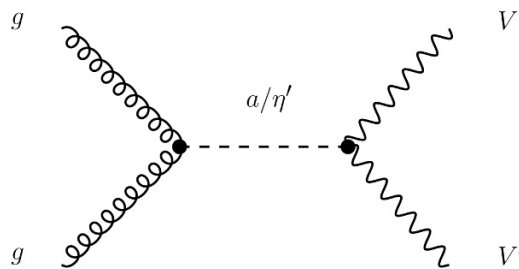$$m_{\eta'}^2 = \frac{2N}{f^2} \Xi \tag{5.4}$$

Figure 8. The main production mode of the two ALPs is via anomalous gluon fusion. Barring large mixing with other pNGBs, they subsequently decay into a pair of vector bosons with computable Branching Ratios.

that can be naively estimated to be of the same order of a typical resonance. However, subtleties may arise that lower the mass of this object and also make it within reach of the LHC.

Regardless of their mass, these objects are singly produced mostly by gluons via the anomaly and decay to di-bosons also via the anomaly (figure 8) with calculable branching ratios. This makes them a good window into UV physics since the branching ratios are related to the type of UV d.o.f. of the underlying theory. It would also be interesting to investigate in detail the mixing of these scalars with the other fields in the EW coset, as done recently in [87] in the context of the model [66]. This could lead to an enhancement in the cross-section for the EW pNGBs.

A All models of partial compositeness satisfying the requirements in the text

In this appendix we list all models of partial compositeness satisfying the requirements in the text. The main requirements are a simple hypercolor gauge group G_{HC} and two irreps ψ and χ giving rise to a custodial EW coset and top partners. In addition, we require the theory to be asymptotically free and of course free of gauge anomalies.

Comparing with [14] we have removed a few models that do not seem promising. Some are based on spinorial irreps of the orthogonal group for which, as discussed in [14], the MAC hypothesis leads to the wrong symmetry breaking pattern. Others are those having baryons of type $\chi\psi\chi$ with ψ in a complex irrep. This leads to top partners in the $(\mathbf{2}, \mathbf{1})$ violating the custodial symmetry [50].

G_{HC}	ψ	χ	Restrictions	Top-partners
	Real	Real		
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{S}_2$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} \geq 55$	$\chi\psi\chi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} \geq 15$	$\chi\psi\chi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\mathrm{HC}} = 7, 9$	$\chi\psi\chi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 7, 9$	$\psi\chi\psi$
	Real	Pseudo-Real		
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$2N_{\mathrm{HC}} \geq 12$	$\chi\psi\chi$
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\mathrm{HC}} \geq 4$	$\chi\psi\chi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\mathrm{HC}} = 11, 13$	$\chi\psi\chi$
	Real	Complex		
$\mathrm{SU}(N_{\mathrm{HC}})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\mathrm{HC}} = 4$	$\chi\psi\chi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$N_{\mathrm{HC}} = 10, 14$	$\chi\psi\chi$
	Pseudo-Real	Real		
$\mathrm{Sp}(2N_{\mathrm{HC}})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\mathrm{HC}} \leq 36$	$\psi\chi\psi$
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 11, 13$	$\psi\chi\psi$
	Complex	Real		
$\mathrm{SO}(N_{\mathrm{HC}})$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\mathrm{HC}} = 10$	$\psi\chi\psi$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\mathrm{HC}} = 4$	$\psi\chi\psi$
	Complex	Complex		
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$N_{\mathrm{HC}} \geq 5$	$\psi\chi\psi$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{S}_2, \overline{\mathbf{S}}_2)$	$N_{\mathrm{HC}} \geq 5$	$\psi\chi\psi$
$\mathrm{SU}(N_{\mathrm{HC}})$	$4 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\mathrm{HC}} = 5$	$\psi\chi\psi$

Table 5. All models obeying the consistency requirements discussed in appendix A. This list contains both conformal and confining theories. See text for a discussion of their IR properties.

If the di-photon excess [35, 36] will be confirmed with properties roughly in agreement with the 2015 data, only a fraction of models [37] will be able to fit the data. Further restrictions [88] could arise from imposing 't Hooft anomaly matching [89].

The list of models presented in table 5 contains both conformal and confining theories.

It is unfortunately not yet possible to exactly identify the conformal region in non-supersymmetric gauge theories. However, one can use some heuristic arguments to get indications on their behavior and it turns out that most of the models are rather clear-cut cases. Consider for instance the two-loop beta-function $\beta(\alpha) = \beta_1 \alpha^2 + \beta_2 \alpha^3$. ($\beta_1 < 0$ always.) A formal solution α^* to $\beta(\alpha^*) = 0$ exists for $\beta_2 > 0$ and, if not to large, it can be trusted and the theory can be assumed to be in the conformal regime. If $\beta_2 < 0$ or α^* is out of the perturbative regime, the model is likely to be confining. In between there is a region, difficult to characterize precisely, where the theory is conformal but strongly coupled.

$G_{\rm HC}$	ψ	χ	Restrictions	G/H
SO($N_{\rm HC}$)	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\rm HC} = 7, 9$	$\frac{\rm SU(5)}{\rm SO(5)} \frac{\rm SU(6)}{\rm SO(6)} {\rm U}(1)$
SO($N_{\rm HC}$)	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\rm HC} = 7, 9$	
Sp($2N_{\rm HC}$)	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\rm HC} = 4$	$\frac{\rm SU(5)}{\rm SO(5)} \frac{\rm SU(6)}{\rm Sp(6)} {\rm U}(1)$
SU($N_{\rm HC}$)	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\rm HC} = 4$	$\frac{\rm SU(5)}{\rm SO(5)} \frac{\rm SU(3) \times SU(3)'}{\rm SU(3)_D} {\rm U}(1)$
SO($N_{\rm HC}$)	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$N_{\rm HC} = 10$	
Sp($2N_{\rm HC}$)	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\rm HC} = 4$	$\frac{\rm SU(4)}{\rm Sp(4)} \frac{\rm SU(6)}{\rm SO(6)} {\rm U}(1)$
SO($N_{\rm HC}$)	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\rm HC} = 11$	
SO($N_{\rm HC}$)	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\rm HC} = 10$	$\frac{\rm SU(4) \times SU(4)'}{\rm SU(4)_D} \frac{\rm SU(6)}{\rm SO(6)} {\rm U}(1)$
SU($N_{\rm HC}$)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\rm HC} = 4$	
SU($N_{\rm HC}$)	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$N_{\rm HC} = 5, 6$	$\frac{\rm SU(4) \times SU(4)'}{\rm SU(4)_D} \frac{\rm SU(3) \times SU(3)'}{\rm SU(3)_D} {\rm U}(1)$

Table 6. Subclass of models that is likely to be outside of the conformal window, together with the coset they give rise to after spontaneous symmetry breaking.

In table 6 we list the subset of models that are likely to be *outside* of the conformal window. These models also obey the heuristic bound $11 C(G) > 4 (N_\psi T(\psi) + N_\chi T(\chi))$ proposed in [90, 91] as well as the rigorous bounds from the a-theorem [92–94] $a_{\rm UV} > a_{\rm IR}$.

The use of these models for BSM physics depends on their IR behavior. The simplest application would be to restrict oneself to the models in table 6. These models can be easily brought into the conformal window from the strong coupling side by adding additional matter. The most straightforward way of achieving this is to have additional fermions, possibly in the same irreps, with masses at the scale Λ. In this case one has a concrete way to put the theory in the "strongest possible" conformal point where the anomalous dimensions of the top-partners may be large enough. The theory then exits the conformal point at the scale Λ, where the additional d.o.f. decouple. However, models outside of this class might still be amenable to other applications and we decided to keep them in the full classification of table 5.

B Group theory conventions for the three cosets

In this appendix we collect the conventions for the explicit constructions of the three EW cosets studied in the text.

B.1 Notation for the SU(5)/SO(5) coset

In this case we realize the Lie algebra of the unbroken group SO(5) as the subset of antisymmetric imaginary generators of SU(5). This is just a particular choice of basis; a more general way of doing the decomposition is to introduce a symmetric matrix δ_0 and define the broken/unbroken generators as $T\delta_0 \mp \delta_0 T^T = 0$ respectively. We chose not to do this, and set $\delta_0 = \mathbf{1}$ from the onset but comment below on the general form of the pNGB

matrix in the general case. The generators of the custodial $SU(2)_L \times SU(2)_R$ are chosen to be

$$
T_L^1 = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad
T_L^2 = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad
T_L^3 = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},
$$

$$
T_R^1 = \begin{pmatrix} 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad
T_R^2 = \begin{pmatrix} 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad
T_R^3 = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 \\ 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{(B.1)}
$$

The broken generators are the real symmetric traceless generators of SU(5). We write the pNGBs as

$$
H = \begin{pmatrix} 0 & 0 & 0 & 0 & -iH_+/\sqrt{2} \\ 0 & 0 & 0 & 0 & H_+/\sqrt{2} \\ 0 & 0 & 0 & 0 & iH_0/\sqrt{2} \\ 0 & 0 & 0 & 0 & H_0/\sqrt{2} \\ -iH_+/\sqrt{2} & H_+/\sqrt{2} & iH_0/\sqrt{2} & H_0/\sqrt{2} & 0 \end{pmatrix} \quad \text{(B.2)}
$$

$$
\Phi_0 = \begin{pmatrix} \phi_0^0/\sqrt{2} & 0 & i(\phi_0^- - \phi_0^+)/2 & (\phi_0^- + \phi_0^+)/2 & 0 \\ 0 & \phi_0^0/\sqrt{2} & (\phi_0^- + \phi_0^+)/2 & -i(\phi_0^- - \phi_0^+)/2 & 0 \\ i(\phi_0^- - \phi_0^+)/2 & (\phi_0^- + \phi_0^+)/2 & -\phi_0^0/\sqrt{2} & 0 & 0 \\ (\phi_0^- + \phi_0^+)/2 & -i(\phi_0^- - \phi_0^+)/2 & 0 & -\phi_0^0/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{(B.3)}
$$

$$
\Phi_+ = \begin{pmatrix} \phi_+^+/\sqrt{2} & i\phi_+^+/\sqrt{2} & i\phi_+^0/2 & \phi_+^0/2 & 0 \\ i\phi_+^+/\sqrt{2} & -\phi_+^+/\sqrt{2} & -\phi_+^0/2 & i\phi_+^0/2 & 0 \\ i\phi_+^0/2 & -\phi_+^0/2 & \phi_+^-/\sqrt{2} & -i\phi_+^-/\sqrt{2} & 0 \\ \phi_+^0/2 & i\phi_+^0/2 & -i\phi_+^-/\sqrt{2} & -\phi_+^-/\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{(B.4)}
$$

$$
E = \begin{pmatrix} \frac{\eta}{\sqrt{10}} & 0 & 0 & 0 & 0 \\ 0 & \frac{\eta}{\sqrt{10}} & 0 & 0 & 0 \\ 0 & 0 & \frac{\eta}{\sqrt{10}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\eta}{\sqrt{10}} & 0 \\ 0 & 0 & 0 & 0 & -2\sqrt{\frac{2}{5}}\eta \end{pmatrix} \quad \text{(B.5)}
$$

In this way with our conventions $\phi_m^{n*} = \phi_{-m}^{-n}$ the full matrix of pNGBs is real symmetric:

$$
\Pi = H + H^\dagger + \Phi_0 + \Phi_+ + \Phi_+^\dagger + E \,. \quad \text{(B.6)}
$$

– 22 –

The vacuum misalignment is described by the following unitary matrix obtained by exponentiating (half of) the Higgs v.e.v.

$$\Omega = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos\zeta & i\sin\zeta \\ 0 & 0 & 0 & i\sin\zeta & \cos\zeta \end{pmatrix}. \tag{B.7}$$

Ω preserves the custodial symmetry $SU(2)_D$ generated by $T_L^i + T_R^i$ and we write the non-linear realization of the pNGBs as a symmetric and unitary matrix

$$U = \Omega \exp(2i\Pi/f)\Omega^T. \tag{B.8}$$

All the fields in Π have zero v.e.v. and in the unitary gauge $H_+ = 0$ and $H_0 = h/\sqrt{2}$.

Notice that with our choice of basis, $\Omega = \Omega^T$. Had we chosen a more general δ_0, we would have obtained

$$U = \Omega \exp(2i\Pi/f)\delta_0\Omega^T = \Omega \exp(2i\Pi/f)\Omega\delta_0 \equiv \Sigma\delta_0 \tag{B.9}$$

where the last identity defines Σ. The matrix Σ has the advantage of making some formulas look more uniform in all three cases but the disadvantage of not transforming uniformly under $SU(5)$ and we chose not to use it. The covariant derivative is

$$D_\mu U = \partial_\mu U - igW_\mu^i(T_L^i U + U T_L^{iT}) - ig'B_\mu(T_R^3 U + U T_R^{3T}) \tag{B.10}$$

and in our convention can be written in terms of commutators. Finally, the kinetic term is

$$L_{\text{kin}} = \frac{f^2}{16}\int \text{tr}(D_\mu U^\dagger D^\mu U). \tag{B.11}$$

B.2 Notation for the SU(4)/Sp(4) coset

We pick the symplectic matrix

$$\epsilon_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{B.12}$$

The unbroken generators satisfy $T^i\epsilon_0 + \epsilon_0 T^{iT} = 0$. In particular, the generators of $SU(2)_L \times SU(2)_R$ are chosen to be

$$T_L^1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_L^2 = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_L^3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T_R^1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}, \qquad T_R^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}, \qquad T_R^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \tag{B.13}$$

and the pNGBs can be represented as

$$H = \begin{pmatrix} 0 & 0 & \frac{iH_0{}^*}{2} & \frac{iH_+}{2} \\ 0 & 0 & -\frac{iH_+{}^*}{2} & \frac{iH_0}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad E = \begin{pmatrix} \frac{\eta}{2\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{\eta}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{\eta}{2\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{\eta}{2\sqrt{2}} \end{pmatrix} \tag{B.14}$$

$$\Pi = H + H^\dagger + E. \tag{B.15}$$

Notice that $\Pi\epsilon_0 - \epsilon_0\Pi^T = 0$.

The matrix describing vacuum misalignment and preserving the custodial symmetry is

$$\Omega(\theta) = \begin{pmatrix} \cos\frac{\zeta}{2} & 0 & -\sin\frac{\zeta}{2} & 0 \\ 0 & \cos\frac{\zeta}{2} & 0 & -\sin\frac{\zeta}{2} \\ \sin\frac{\zeta}{2} & 0 & \cos\frac{\zeta}{2} & 0 \\ 0 & \sin\frac{\zeta}{2} & 0 & \cos\frac{\zeta}{2} \end{pmatrix}, \tag{B.16}$$

in terms of which the non-linear realization can be expressed as an anti-symmetric and unitary matrix

$$U = \Omega \exp(2\sqrt{2}i\Pi/f)\epsilon_0\Omega^T. \tag{B.17}$$

Also in this case, the fields in Π have zero v.e.v. and in the unitary gauge $H_+ = 0$ and $H_0 = h/\sqrt{2}$. The covariant derivative reads as in the previous case (B.10) but the kinetic term is normalized differently

$$L_{\text{kin}} = \frac{f^2}{8} \int \text{tr}(D_\mu U^\dagger D^\mu U). \tag{B.18}$$

Even in this case one has the option of using $\epsilon_0\Omega^T = \Omega\epsilon_0$ and of introducing a matrix Σ through the identity $U = \Sigma\epsilon_0$ in an analogous way as for the previous coset, but we do not use it for the same reasons as above.

B.3 Notation for the SU(4) × SU(4)′/SU(4)$_D$ coset

In this case, the $\text{SU}(2)_L \times \text{SU}(2)_R$ subgroup is embedded in the unbroken $\text{SU}(4)_D$ by choosing

$$T_L^1 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_L^2 = \begin{pmatrix} 0 & -\frac{i}{2} & 0 & 0 \\ \frac{i}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad T_L^3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T_R^1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{pmatrix}, \qquad T_R^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} \\ 0 & 0 & \frac{i}{2} & 0 \end{pmatrix}, \qquad T_R^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}. \tag{B.19}$$

The pNGBs are parameterized as follows

$$
H = \begin{pmatrix} 0 & 0 & -\frac{1}{2}i\left(H_0^* + iH_0'^*\right) & -\frac{1}{2}i\left(H_+ + iH_+'\right) \\ 0 & 0 & \frac{1}{2}i\left(H_+^* + iH_+'^*\right) & -\frac{1}{2}i\left(H_0 + iH_0'\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} \frac{1}{2}\frac{\eta}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\frac{\eta}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\frac{\eta}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\frac{\eta}{\sqrt{2}} \end{pmatrix}
$$

$$
\Phi = \begin{pmatrix} \frac{1}{2}\phi_0 & \frac{1}{\sqrt{2}}\phi_+ & 0 & 0 \\ \frac{1}{\sqrt{2}}\phi_- & -\frac{1}{2}\phi_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}N_0 & \frac{1}{\sqrt{2}}N_+ \\ 0 & 0 & \frac{1}{\sqrt{2}}N_- & -\frac{1}{2}N_0 \end{pmatrix}, \quad \text{(B.20)}
$$

where $\phi_+^* = \phi_-$ and $N_+^* = N_-$. In the unitary gauge we have as usual $H_+ = 0$, $H_0 = h/\sqrt{2}$, having chosen to rotate the v.e.v. into the first of the two doublets. The neutral component of the second doublet is thus physical and can be written as a CP even plus CP odd part: $H_0' = (h' + iA')/\sqrt{2}$. Finally we set

$$
\Pi = H + H^\dagger + N + \Phi + E \tag{B.21}
$$

and, for the vacuum misalignment matrix we obtain

$$
\Omega(\theta) = \begin{pmatrix} \cos\frac{\zeta}{2} & 0 & \sin\frac{\zeta}{2} & 0 \\ 0 & \cos\frac{\zeta}{2} & 0 & \sin\frac{\zeta}{2} \\ -\sin\frac{\zeta}{2} & 0 & \cos\frac{\zeta}{2} & 0 \\ 0 & -\sin\frac{\zeta}{2} & 0 & \cos\frac{\zeta}{2} \end{pmatrix}. \tag{B.22}
$$

In this case, the non linear realization of the pNGBs is given by the unitary matrix

$$
U = \Omega \exp(2\sqrt{2}i\Pi/f)\Omega. \tag{B.23}
$$

The covariant derivative is obtained by the usual commutator

$$
D_\mu U = \partial_\mu U - igW_\mu^i[T_L^i, U] - ig'B_\mu[T_R^3, U] \tag{B.24}
$$

and the kinetic term is normalized as

$$
L_{\rm kin} = \frac{f^2}{8}\int {\rm tr}(D_\mu U^\dagger D^\mu U). \tag{B.25}
$$

C Additional three and four bosons couplings for the models in the text

Additional (i.e. other than those involving h) quartic couplings for $SU(4) \times SU(4)'/SU(4)_D$ are shown in (C.1) (in agreement with [34]).

$$
\begin{aligned}
\mathcal{L} \supset\ & e^2 A_\mu A^\mu (N_- N_+ + \phi_- \phi_+ + H'_- H'_+) \\
& + \frac{e^2}{s_w c_w} Z_\mu A^\mu \big((c_{2w} - c_\zeta) N_- N_+ + (c_{2w} + c_\zeta)\phi_- \phi_+ + c_{2w} H'_- H'_+ \big) \\
& + \frac{e^2}{16 s_w^2 c_w^2} Z_\mu Z^\mu \big(2c_\zeta^2 h'h' + 2(c_{4w} + 3c_\zeta^2 - 4c_{2w}c_\zeta) N_- N_+ + 2(c_{4w} + 3c_\zeta^2 + 4c_{2w}c_\zeta)\phi_- \phi_+ \\
& \qquad\qquad\qquad - 2s_\zeta^2 N_+ \phi_- - 2s_\zeta^2 \phi_+ N_- - s_\zeta^2 N_0 N_0 + 2s_\zeta^2 N_0 \phi_0 \\
& \qquad\qquad\qquad - s_\zeta^2 \phi_0 \phi_0 + 2c_{2\zeta} A'A' - 2s_\zeta^2 \eta\eta + 4c_{2w}^2 H'_+ H'_- \big) \\
& + \frac{e^2}{8 s_w^2} W_\mu^- W^{+\mu} \big(2c_\zeta^2 h'h' - 4(1 - c_\zeta)c_\zeta N_- N_+ + 4(1 + c_\zeta)c_\zeta \phi_- \phi_+ + (1 - c_\zeta)(1 - 3c_\zeta) N_0 N_0 \\
& \qquad\qquad\qquad - 2s_\zeta^2 N_0 \phi_0 + (1 + c_\zeta)(1 + 3c_\zeta)\phi_0 \phi_0 + 2A'A' - 2s_\zeta^2 \eta\eta + 4c_\zeta^2 H'_+ H'_- \big) \\
& + \frac{e^2}{2 s_w} A_\mu W^{-\mu} \big(-(1 - c_\zeta) N_0 N_+ - (1 + c_\zeta)\phi_0 \phi_+ - iA'H'_+ + c_\zeta h'H'_+ \big) \\
& - \frac{e^2}{4 s_w^2 c_w} Z_\mu W^{-\mu} \big((1 - c_\zeta)(c_{2w} - c_\zeta) N_0 N_+ - s_\zeta^2 \phi_0 N_+ - s_\zeta^2 \phi_+ N_0 \\
& \qquad\qquad\qquad + (1 + c_\zeta)(c_{2w} + c_\zeta)\phi_+ \phi_0 + i(c_{2w} - c_{2\zeta}) A'H'_+ + 2c_\zeta s_w^2 h'H'_+ \big) \\
& - \frac{e^2}{8 s_w^2} W_\mu^- W^{-\mu} \big((1 - c_\zeta)^2 N_+ N_+ - 2s_\zeta^2 N_+ \phi_+ + (1 + c_\zeta)^2 \phi_+ \phi_+ - 2s_\zeta^2 H_+ H_+ \big) \\
& + \text{hermitian conjugates of the terms involving } AW^-,\ ZW^-,\ W^-W^-. \qquad\qquad \text{(C.1)}
\end{aligned}
$$

The anomalous cubic couplings for $SU(5)/SO(5)$ are shown in table 7. Each coupling should be multiplied by $e^2 \dim(\psi)/(48\pi^2 f)$.

Finally, we present in eq. (C.2) the additional (i.e. other than those involving h) quartic couplings for $SU(5)/SO(5)$.

Of course, the generation of masses by the potential introduces a mixing between these gauge eigenstates. This depends on the specific nature of the mass matrix and in many cases it could be handled by the mass insertion approximation. Throughout the paper we work with gauge eigenstates. Also note that one could use the Clebsch-Gordan coefficients to express the gauge eigenstates as eigenstates of the diagonal custodial symmetry group $SU(2)_D \subset SU(2)_L \times SU(2)_R$ as done in [46]. An even deeper difference with the model in [46] is that they used an additional $U(1)$ gauge field to induce vacuum-misalignment

Fields	Couplings	Fields	Couplings	Fields	Couplings
$\eta F_{\mu\nu}\tilde{F}^{\mu\nu}$	$3\sqrt{\frac{2}{5}}$	$\phi_0^0 F_{\mu\nu}\tilde{Z}^{\mu\nu}$	$6\sqrt{2}\,c_{2w}/s_{2w}$	$\phi_+^0 F_{\mu\nu}\tilde{W}^{-\mu\nu}$	$\frac{3i(c_\zeta-1)}{\sqrt{2}s_w}$
$\eta F_{\mu\nu}\tilde{Z}^{\mu\nu}$	$6\sqrt{\frac{2}{5}}\,c_{2w}/s_{2w}$	$\phi_0^0 Z_{\mu\nu}\tilde{Z}^{\mu\nu}$	$\frac{(-5c_{2\zeta}+6c_{4w}-1)}{2\sqrt{2}s_{2w}^2}$	$\phi_+^0 Z_{\mu\nu}\tilde{W}^{-\mu\nu}$	$-\frac{i(1-c_\zeta)(2c_\zeta+3c_{2w}-1)}{2\sqrt{2}s_w^2 c_w}$
$\eta Z_{\mu\nu}\tilde{Z}^{\mu\nu}$	$\frac{3(3c_\zeta^2+c_{4w})}{4\sqrt{10}c_w^2 s_w^2}$	$\phi_0^0 W_{\mu\nu}^+\tilde{W}^{-\mu\nu}$	$\frac{s_\zeta^2}{2\sqrt{2}s_w^2}$	$\phi_0^+ F_{\mu\nu}\tilde{W}^{-\mu\nu}$	$\frac{3i(c_\zeta+1)}{\sqrt{2}s_w}$
$\eta W_{\mu\nu}^+\tilde{W}^{-\mu\nu}$	$\frac{3(3c_{2\zeta}+5)}{4\sqrt{10}s_w^2}$	$\phi_+^- Z_{\mu\nu}\tilde{Z}^{\mu\nu}$	$\frac{s_\zeta^2}{\sqrt{2}s_{2w}^2}$	$\phi_0^+ Z_{\mu\nu}\tilde{W}^{-\mu\nu}$	$-\frac{i(1+c_\zeta)(2c_\zeta-3c_{2w}+1)}{2\sqrt{2}s_w^2 c_w}$
$\phi_0^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$	$3\sqrt{2}$	$\phi_+^- W_{\mu\nu}^+\tilde{W}^{-\mu\nu}$	$\frac{3s_\zeta^2}{2\sqrt{2}s_w^2}$	$\phi_+^+ W_{\mu\nu}^-\tilde{W}^{-\mu\nu}$	$-\frac{s_\zeta^2}{\sqrt{2}s_w^2}$

Table 7. Anomalous couplings for SU(5)/SO(5), to be multiplied by $e^2\dim(\psi)/(48\pi^2 f)$.

instead of top coupling.

$$
\begin{aligned}
\mathcal{L} \supset\ & e^2 A_\mu A^\mu(\phi_-^0\phi_+^0 + \phi_0^-\phi_0^+ + 4\phi_-^-\phi_+^+) \\
& + \frac{e^2}{s_w c_w}Z_\mu A^\mu\big((c_{2w}-c_\zeta)\phi_-^0\phi_+^0 + (c_{2w}+c_\zeta)\phi_0^-\phi_0^+ + 4c_{2w}\phi_-^-\phi_+^+\big) \\
& + \frac{e^2}{16s_w^2 c_w^2}Z_\mu Z^\mu\big(-5s_\zeta^2\eta\eta + 2\sqrt{5}s_\zeta^2\eta\phi_0^0 - 2\sqrt{5}s_\zeta^2\eta\phi_-^- - 2\sqrt{5}s_\zeta^2\eta\phi_+^+ \\
& \qquad\qquad\qquad + 2s_\zeta^2\phi_0^0\phi_-^+ + 2s_\zeta^2\phi_0^0\phi_+^- - s_\zeta^2\phi_0^0\phi_0^0 \\
& \qquad\qquad\qquad + (6+10c_{2\zeta})\phi_-^+\phi_+^- + (2c_{4w}-8c_\zeta c_{2w}+6c_\zeta^2)\phi_+^0\phi_-^0 + 2s_\zeta^2\phi_+^0\phi_0^- \\
& \qquad\qquad\qquad + (2c_{4w}+8c_\zeta c_{2w}+6c_\zeta^2)\phi_0^+\phi_0^- + 2s_\zeta^2\phi_-^0\phi_0^+ + 16c_{2w}^2\phi_+^+\phi_-^-\big) \\
& + \frac{e^2}{16s_w^2}W_\mu^- W^{+\mu}\big(-10s_\zeta^2\eta\eta - 4\sqrt{5}s_\zeta^2\eta\phi_0^0 + (11+5c_{2\zeta})\phi_0^0\phi_0^0 + (12+4c_{2\zeta})\phi_-^+\phi_+^- - 8s_\zeta^2\phi_0^0\phi_-^+ \\
& \qquad\qquad\qquad - 8s_\zeta^2\phi_0^0\phi_+^- - 8s_\zeta^2\phi_0^+\phi_-^0 + (16+8c_\zeta+8c_{2\zeta})\phi_+^0\phi_-^0 - 8s_\zeta^2\phi_+^0\phi_0^- \\
& \qquad\qquad\qquad + (16-8c_\zeta+8c_{2\zeta})\phi_0^-\phi_0^+ + 16c_\zeta^2\phi_-^-\phi_+^+\big) \\
& - \frac{ie^2}{2s_w}A_\mu W^{-\mu}\big((1+c_\zeta)\phi_-^+\phi_+^0 - (1-c_\zeta)\phi_0^0\phi_+^+ - (1-c_\zeta)\phi_-^+\phi_0^+ + (1+c_\zeta)\phi_0^0\phi_0^+ \\
& \qquad\qquad\qquad - 3(1+c_\zeta)\phi_+^+\phi_-^0 + 3(1-c_\zeta)\phi_+^+\phi_0^-\big) \\
& - \frac{ie^2}{4s_w^2 c_w}Z_\mu W^{-\mu}\big(\sqrt{5}s_\zeta^2\eta\phi_+^0 + (1+c_\zeta)(1+c_{2w}-4c_\zeta)\phi_-^+\phi_+^0 + (1-c_\zeta)(c_\zeta-c_{2w})\phi_0^0\phi_+^+ \\
& \qquad\qquad\qquad - \sqrt{5}s_\zeta^2\eta\phi_0^+ + (1+c_\zeta)(c_{2w}+c_\zeta)\phi_0^0\phi_0^+ - (1-c_\zeta)(1+c_{2w}+4c_\zeta)\phi_0^+\phi_+^- \\
& \qquad\qquad\qquad - (1+c_\zeta)(1+3c_{2w}-2c_\zeta)\phi_+^+\phi_-^0 + (1-c_\zeta)(1+3c_{2w}+2c_\zeta)\phi_+^+\phi_0^-\big) \\
& + \frac{e^2}{8s_w^2}W_\mu^- W^{-\mu}\big((1-c_\zeta)^2\phi_+^0\phi_+^0 + (1+c_\zeta)^2\phi_0^+\phi_0^+ - 6s_\zeta^2\phi_0^+\phi_+^0 - 2\sqrt{5}s_\zeta^2\eta\phi_+^+ \\
& \qquad\qquad\qquad + 2(1+c_\zeta)^2\phi_-^+\phi_+^+ + 2(1-c_\zeta)^2\phi_+^-\phi_+^+ - 6s_\zeta^2\phi_0^0\phi_+^+\big) \\
& + \text{hermitian conjugates of the terms involving } AW^-,\ ZW^-,\ W^-W^-. \qquad\qquad \text{(C.2)}
\end{aligned}
$$

References

[1] F. Englert and R. Brout, *Broken symmetry and the mass of gauge vector mesons*, [*Phys. Rev. Lett.* **13** (1964) 321](https://doi.org/10.1103/PhysRevLett.13.321) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.Lett.,13,321%22)].

[2] P.W. Higgs, *Broken symmetries and the masses of gauge bosons*, [*Phys. Rev. Lett.* **13** (1964) 508](https://doi.org/10.1103/PhysRevLett.13.508) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.Lett.,13,508%22)].

[3] G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, *Global conservation laws and massless particles*, [*Phys. Rev. Lett.* **13** (1964) 585](https://doi.org/10.1103/PhysRevLett.13.585) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.Lett.,13,585%22)].

[4] S. Weinberg, *A model of leptons*, [*Phys. Rev. Lett.* **19** (1967) 1264](https://doi.org/10.1103/PhysRevLett.19.1264) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.Lett.,19,1264%22)].

[5] D.B. Kaplan and H. Georgi, SU(2) × U(1) *breaking by vacuum misalignment*, [*Phys. Lett.* **B 136** (1984) 183](https://doi.org/10.1016/0370-2693(84)91177-8) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Lett.,B136,183%22)].

[6] D.B. Kaplan, *Flavor at SSC energies: a new mechanism for dynamically generated fermion masses*, [*Nucl. Phys.* **B 365** (1991) 259](https://doi.org/10.1016/S0550-3213(05)80021-5) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Nucl.Phys.,B365,259%22)].

[7] S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. I*, [*Phys. Rev.* **177** (1969) 2239](https://doi.org/10.1103/PhysRev.177.2239) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.,177,2239%22)].

[8] C.G. Callan Jr., S.R. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. II*, [*Phys. Rev.* **177** (1969) 2247](https://doi.org/10.1103/PhysRev.177.2247) [[INSPIRE](https://inspirehep.net/search?p=find+J+%22Phys.Rev.,177,2247%22)].

[9] R. Contino, *The Higgs as a composite Nambu-Goldstone boson*, in Proceedings of the *Theoretical Advanced Study Institute in Elementary Particle Physics*, Boulder U.S.A., 1–26 Jun 2009 [[arXiv:1005.4269](https://arxiv.org/abs/1005.4269)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1005.4269)].

[10] G. Panico and A. Wulzer, *The composite Nambu-Goldstone Higgs*, [*Lect. Notes Phys.* **913** (2016) 1](https://doi.org/10.1007/978-3-319-22617-0) [[arXiv:1506.01961](https://arxiv.org/abs/1506.01961)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1506.01961)].

[11] J. Barnard, T. Gherghetta and T.S. Ray, *UV descriptions of composite Higgs models without elementary scalars*, [*JHEP* **02** (2014) 002](https://doi.org/10.1007/JHEP02(2014)002) [[arXiv:1311.6562](https://arxiv.org/abs/1311.6562)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1311.6562)].

[12] G. Ferretti, *UV completions of partial compositeness: the case for a* SU(4) *gauge group*, [*JHEP* **06** (2014) 142](https://doi.org/10.1007/JHEP06(2014)142) [[arXiv:1404.7137](https://arxiv.org/abs/1404.7137)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1404.7137)].

[13] L. Vecchi, *A "dangerous irrelevant" UV-completion of the composite Higgs*, [arXiv:1506.00623](https://arxiv.org/abs/1506.00623) [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1506.00623)].

[14] G. Ferretti and D. Karateev, *Fermionic UV completions of composite Higgs models*, [*JHEP* **03** (2014) 077](https://doi.org/10.1007/JHEP03(2014)077) [[arXiv:1312.5330](https://arxiv.org/abs/1312.5330)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1312.5330)].

[15] F. Caracciolo, A. Parolini and M. Serone, *UV completions of composite Higgs models with partial compositeness*, [*JHEP* **02** (2013) 066](https://doi.org/10.1007/JHEP02(2013)066) [[arXiv:1211.7290](https://arxiv.org/abs/1211.7290)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1211.7290)].

[16] D. Marzocca, A. Parolini and M. Serone, *Supersymmetry with a pNGB Higgs and partial compositeness*, [*JHEP* **03** (2014) 099](https://doi.org/10.1007/JHEP03(2014)099) [[arXiv:1312.5664](https://arxiv.org/abs/1312.5664)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1312.5664)].

[17] R. Nevzorov and A.W. Thomas, E_6 *inspired composite Higgs model*, [*Phys. Rev.* **D 92** (2015) 075007](https://doi.org/10.1103/PhysRevD.92.075007) [[arXiv:1507.02101](https://arxiv.org/abs/1507.02101)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1507.02101)].

[18] N. Bizot and M. Frigerio, *Fermionic extensions of the standard model in light of the Higgs couplings*, [*JHEP* **01** (2016) 036](https://doi.org/10.1007/JHEP01(2016)036) [[arXiv:1508.01645](https://arxiv.org/abs/1508.01645)] [[INSPIRE](https://inspirehep.net/search?p=find+EPRINT+arXiv:1508.01645)].

[19] V. Sanz and J. Setford, *Composite Higgses with seesaw EWSB*, *JHEP* **12** (2015) 154 [arXiv:1508.06133] [ɪɴSPIRE].

[20] A. De Simone, O. Matsedonskyi, R. Rattazzi and A. Wulzer, *A first top partner hunter's guide*, *JHEP* **04** (2013) 004 [arXiv:1211.5663] [ɪɴSPIRE].

[21] A. Thamm, R. Torre and A. Wulzer, *Future tests of Higgs compositeness: direct vs indirect*, *JHEP* **07** (2015) 100 [arXiv:1502.01701] [ɪɴSPIRE].

[22] J. Barnard and M. White, *Collider constraints on tuning in composite Higgs models*, *JHEP* **10** (2015) 072 [arXiv:1507.02332] [ɪɴSPIRE].

[23] D. Croon, B.M. Dillon, S.J. Huber and V. Sanz, *Exploring holographic composite Higgs models*, arXiv:1510.08482 [ɪɴSPIRE].

[24] M.J. Schlaffer, *Boosted searches for new physics at the LHC*, DESY-THESIS-2015-036 [ɪɴSPIRE].

[25] C. Englert, R. Rosenfeld, M. Spannowsky and A. Tonero, *New physics and signal-background interference in associated* $pp \to HZ$ *production*, *Europhys. Lett.* **114** (2016) 31001 [arXiv:1603.05304] [ɪɴSPIRE].

[26] M.A. Luty and T. Okui, *Conformal technicolor*, *JHEP* **09** (2006) 070 [hep-ph/0409274] [ɪɴSPIRE].

[27] M.J. Strassler, *Nonsupersymmetric theories with light scalar fields and large hierarchies*, hep-th/0309122 [ɪɴSPIRE].

[28] R. Rattazzi, V.S. Rychkov, E. Tonni and A. Vichi, *Bounding scalar operator dimensions in 4D CFT*, *JHEP* **12** (2008) 031 [arXiv:0807.0004] [ɪɴSPIRE].

[29] V.S. Rychkov and A. Vichi, *Universal constraints on conformal operator dimensions*, *Phys. Rev.* **D 80** (2009) 045006 [arXiv:0905.2211] [ɪɴSPIRE].

[30] C. Vafa and E. Witten, *Restrictions on symmetry breaking in vector-like gauge theories*, *Nucl. Phys.* **B 234** (1984) 173 [ɪɴSPIRE].

[31] O. Matsedonskyi, *On flavour and naturalness of composite Higgs models*, *JHEP* **02** (2015) 154 [arXiv:1411.4638] [ɪɴSPIRE].

[32] G. Cacciapaglia et al., *Anarchic Yukawas and top partial compositeness: the flavour of a successful marriage*, *JHEP* **06** (2015) 085 [arXiv:1501.03818] [ɪɴSPIRE].

[33] G. Panico and A. Pomarol, *Flavor hierarchies from dynamical scales*, arXiv:1603.06609 [ɪɴSPIRE].

[34] T. Ma and G. Cacciapaglia, *Fundamental composite 2HDM:* SU(N) *with 4 flavours*, *JHEP* **03** (2016) 211 [arXiv:1508.07014] [ɪɴSPIRE].

[35] ATLAS collaboration, *Search for resonances decaying to photon pairs in* $3.2\,fb^{-1}$ *of pp collisions at* $\sqrt{s} = 13\,TeV$ *with the ATLAS detector*, ATLAS-CONF-2015-081 (2015).

[36] CMS collaboration, *Search for new physics in high mass diphoton events in proton-proton collisions at* $\sqrt{s} = 13\,TeV$, CMS-PAS-EXO-15-004 (2015).

[37] A. Belyaev et al., *Singlets in composite Higgs models in light of the LHC di-photon searches*, arXiv:1512.07242 [ɪɴSPIRE].

[38] B. Bellazzini, R. Franceschini, F. Sala and J. Serra, *Goldstones in diphotons*, *JHEP* **04** (2016) 072 [arXiv:1512.05330] [ɪɴSPIRE].

[39] Z.-y. Duan, P.S. Rodrigues da Silva and F. Sannino, *Enhanced global symmetry constraints on ε terms*, *Nucl. Phys.* **B 592** (2001) 371 [hep-ph/0001303] [ɪɴSPIRE].

[40] E. Katz, A.E. Nelson and D.G.E. Walker, *The intermediate Higgs*, *JHEP* **08** (2005) 074 [hep-ph/0504252] [ɪɴSPIRE].

[41] P. Lodone, *Vector-like quarks in a "composite" Higgs model*, *JHEP* **12** (2008) 029 [arXiv:0806.1472] [ɪɴSPIRE].

[42] B. Gripaios, A. Pomarol, F. Riva and J. Serra, *Beyond the minimal composite Higgs model*, *JHEP* **04** (2009) 070 [arXiv:0902.1483] [ɪɴSPIRE].

[43] G. Cacciapaglia and F. Sannino, *Fundamental composite (Goldstone) Higgs dynamics*, *JHEP* **04** (2014) 111 [arXiv:1402.0233] [ɪɴSPIRE].

[44] M. Schmaltz, D. Stolarski and J. Thaler, *The bestest little Higgs*, *JHEP* **09** (2010) 018 [arXiv:1006.1356] [ɪɴSPIRE].

[45] H. Georgi and D.B. Kaplan, *Composite Higgs and custodial* SU(2), *Phys. Lett.* **B 145** (1984) 216 [ɪɴSPIRE].

[46] M.J. Dugan, H. Georgi and D.B. Kaplan, *Anatomy of a composite Higgs model*, *Nucl. Phys.* **B 254** (1985) 299 [ɪɴSPIRE].

[47] N. Arkani-Hamed, A.G. Cohen, E. Katz and A.E. Nelson, *The littlest Higgs*, *JHEP* **07** (2002) 034 [hep-ph/0206021] [ɪɴSPIRE].

[48] L. Vecchi, *The natural composite Higgs*, arXiv:1304.4579 [ɪɴSPIRE].

[49] J. Mrazek et al., *The other natural two Higgs doublet model*, *Nucl. Phys.* **B 853** (2011) 1 [arXiv:1105.5403] [ɪɴSPIRE].

[50] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *A custodial symmetry for* $Zb\bar{b}$, *Phys. Lett.* **B 641** (2006) 62 [hep-ph/0605341] [ɪɴSPIRE].

[51] P. Sikivie, L. Susskind, M.B. Voloshin and V.I. Zakharov, *Isospin breaking in technicolor models*, *Nucl. Phys.* **B 173** (1980) 189 [ɪɴSPIRE].

[52] S.R. Coleman and E.J. Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, *Phys. Rev.* **D 7** (1973) 1888 [ɪɴSPIRE].

[53] T. DeGrand, Y. Liu, E.T. Neil, Y. Shamir and B. Svetitsky, *Spectroscopy of* SU(4) *gauge theory with two flavors of sextet fermions*, *Phys. Rev.* **D 91** (2015) 114502 [arXiv:1501.05665] [ɪɴSPIRE].

[54] T. DeGrand, Y. Liu, E.T. Neil, Y. Shamir and B. Svetitsky, *Spectroscopy of* SU(4) *lattice gauge theory with fermions in the two index anti-symmetric representation*, PoS(LATTICE2014)275 [arXiv:1412.4851] [ɪɴSPIRE].

[55] T. DeGrand, *Lattice tests of beyond standard model dynamics*, *Rev. Mod. Phys.* **88** (2016) 015001 [arXiv:1510.05018] [ɪɴSPIRE].

[56] D.K. Hong, S.D.H. Hsu and F. Sannino, *Composite Higgs from higher representations*, *Phys. Lett.* **B 597** (2004) 89 [hep-ph/0406200] [ɪɴSPIRE].

[57] D.D. Dietrich, F. Sannino and K. Tuominen, *Light composite Higgs from higher representations versus electroweak precision measurements: predictions for CERN LHC*, *Phys. Rev.* **D 72** (2005) 055001 [hep-ph/0505059] [ɪɴSPIRE].

[58] D.D. Dietrich and F. Sannino, *Conformal window of* SU(N) *gauge theories with fermions in higher dimensional representations*, *Phys. Rev.* **D 75** (2007) 085018 [hep-ph/0611341] [INSPIRE].

[59] M. Golterman and Y. Shamir, *Top quark induced effective potential in a composite Higgs model*, *Phys. Rev.* **D 91** (2015) 094506 [arXiv:1502.00390] [INSPIRE].

[60] K. Agashe, R. Contino and A. Pomarol, *The minimal composite Higgs model*, *Nucl. Phys.* **B 719** (2005) 165 [hep-ph/0412089] [INSPIRE].

[61] ATLAS collaboration, *Observation of a new particle in the search for the standard model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett.* **B 716** (2012) 1 [arXiv:1207.7214] [INSPIRE].

[62] CMS collaboration, *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys. Lett.* **B 716** (2012) 30 [arXiv:1207.7235] [INSPIRE].

[63] J. Wess and B. Zumino, *Consequences of anomalous Ward identities*, *Phys. Lett.* **B 37** (1971) 95 [INSPIRE].

[64] E. Witten, *Global aspects of current algebra*, *Nucl. Phys.* **B 223** (1983) 422 [INSPIRE].

[65] Ö. Kaymakcalan, S. Rajeev and J. Schechter, *Non-Abelian anomaly and vector-meson decays*, *Phys. Rev.* **D 30** (1984) 594 [INSPIRE].

[66] H. Georgi and M. Machacek, *Doubly charged Higgs bosons*, *Nucl. Phys.* **B 262** (1985) 463 [INSPIRE].

[67] J. Alwall et al., *The automated computation of tree-level and next-to-leading order differential cross sections and their matching to parton shower simulations*, *JHEP* **07** (2014) 079 [arXiv:1405.0301] [INSPIRE].

[68] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 — a complete toolbox for tree-level phenomenology*, *Comput. Phys. Commun.* **185** (2014) 2250 [arXiv:1310.1921] [INSPIRE].

[69] M. Frigerio, A. Pomarol, F. Riva and A. Urbano, *Composite scalar dark matter*, *JHEP* **07** (2012) 015 [arXiv:1204.2808] [INSPIRE].

[70] M. Kim, S.J. Lee and A. Parolini, *WIMP dark matter in composite Higgs models and the dilaton portal*, arXiv:1602.05590 [INSPIRE].

[71] A. Joseph and S.G. Rajeev, *Topological dark matter in the little Higgs models*, *Phys. Rev.* **D 80** (2009) 074009 [arXiv:0905.2772] [INSPIRE].

[72] L. Calibbi, G. Ferretti, D. Milstead, C. Petersson and R. Pöttgen, *Baryon number violation in supersymmetry: n-n̄ oscillations as a probe beyond the LHC*, *JHEP* **05** (2016) 144 [arXiv:1602.04821] [INSPIRE].

[73] B. Gripaios, *Composite leptoquarks at the LHC*, *JHEP* **02** (2010) 045 [arXiv:0910.1789] [INSPIRE].

[74] G. Cacciapaglia et al., *Composite scalars at the LHC: the Higgs, the sextet and the octet*, *JHEP* **11** (2015) 201 [arXiv:1507.02283] [INSPIRE].

[75] CMS collaboration, *Search for top-quark partners with charge 5/3 in the same-sign dilepton final state*, *Phys. Rev. Lett.* **112** (2014) 171801 [arXiv:1312.2391] [INSPIRE].

[76] T. DeGrand and Y. Shamir, *One-loop anomalous dimension of top-partner hyperbaryons in a family of composite Higgs models*, *Phys. Rev.* **D 92** (2015) 075039 [arXiv:1508.02581] [INSPIRE].

[77] C. Pica and F. Sannino, *Anomalous dimensions of conformal baryons*, arXiv:1604.02572 [INSPIRE].

[78] J.E. Kim, *A composite invisible axion*, *Phys. Rev.* **D 31** (1985) 1733 [INSPIRE].

[79] E. Farhi and L. Susskind, *Technicolor*, *Phys. Rept.* **74** (1981) 277 [INSPIRE].

[80] R.F. Dashen, *Chiral* SU(3) × SU(3) *as a symmetry of the strong interactions*, *Phys. Rev.* **183** (1969) 1245 [INSPIRE].

[81] B. Döbrich, J. Jaeckel, F. Kahlhoefer, A. Ringwald and K. Schmidt-Hoberg, *ALPtraum: ALP production in proton beam dump experiments*, *JHEP* **02** (2016) 018 [arXiv:1512.03069] [INSPIRE].

[82] E949 collaboration, V.V. Anisimovsky et al., *Improved measurement of the* $K^+ \to \pi^+ \nu \bar{\nu}$ *branching ratio*, *Phys. Rev. Lett.* **93** (2004) 031801 [hep-ex/0403036] [INSPIRE].

[83] D. Cadamuro and J. Redondo, *Cosmological bounds on pseudo Nambu-Goldstone bosons*, *JCAP* **02** (2012) 032 [arXiv:1110.2895] [INSPIRE].

[84] G. 't Hooft, *Symmetry breaking through Bell-Jackiw anomalies*, *Phys. Rev. Lett.* **37** (1976) 8 [INSPIRE].

[85] G. Veneziano, U(1) *without instantons*, *Nucl. Phys.* **B 159** (1979) 213 [INSPIRE].

[86] E. Witten, *Current algebra theorems for the* U(1) *Goldstone boson*, *Nucl. Phys.* **B 156** (1979) 269 [INSPIRE].

[87] K. Howe, S. Knapen and D.J. Robinson, *Diphotons from an electroweak triplet-singlet*, arXiv:1603.08932 [INSPIRE].

[88] G. Cacciapaglia and A. Parolini, *Light 't Hooft top partners*, *Phys. Rev.* **D 93** (2016) 071701 [arXiv:1511.05163] [INSPIRE].

[89] G. 't Hooft et al., in Proceedings of *Recent developments in gauge theories*, Cargèse France, 26 Aug–8 Sep 1979 [*NATO Sci. Ser.* **B 59** (1980) 1] [INSPIRE].

[90] F. Sannino, *Conformal windows of* Sp(2N) *and* SO(N) *gauge theories*, *Phys. Rev.* **D 79** (2009) 096007 [arXiv:0902.3494] [INSPIRE].

[91] T.A. Ryttov and F. Sannino, *Conformal house*, *Int. J. Mod. Phys.* **A 25** (2010) 4603 [arXiv:0906.0307] [INSPIRE].

[92] J.L. Cardy, *Is there a c-theorem in four dimensions?*, *Phys. Lett.* **B 215** (1988) 749 [INSPIRE].

[93] H. Osborn, *Derivation of a four-dimensional c-theorem for renormaliseable quantum field theories*, *Phys. Lett.* **B 222** (1989) 97 [INSPIRE].

[94] I. Jack and H. Osborn, *Analogs of the c-theorem for four-dimensional renormalisable field theories*, *Nucl. Phys.* **B 343** (1990) 647 [INSPIRE].