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Abstract: In this paper we theoretically derive for the first time a matrix formalism for a coupled dual-core fiber optical parametric amplifier (FOPA). One of the most advantageous properties of this degenerate pump FOPA is the spectrally flat gain obtained when certain design parameters are met. This flat gain is obtained either in phase-sensitive (PS) or phase-insensitive (PI) operation of the dual-core FOPA. Properties such as maximum and minimum PS gain, along with maximum bandwidth and difference between maximum PS and PI gain are also investigated.

OCIS codes: (190.4410) Nonlinear optics, parametric processes; (060.4510) Optical communications.

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1. Introduction

Optical amplifiers are key signal processing devices used in an optical network in order to achieve a number of important tasks, such as amplifying the transmitter laser signal, increase receiver sensitivity and the most important: compensate for fiber loss [1, 2]. Erbium doped fiber amplifiers have been the preferred devices at least in what gives respect to compensation of fiber loss, however its limited capability to work in a broad range of frequencies [2] is certainly its most noticeable shortcoming. Parametric amplification offers a wide gain bandwidth and the possibility to work at any wavelength [2, 3]. Highly nonlinear fibers around a few hundred meters in length can be used as the amplifying medium using its $\chi^{(3)}$ nonlinear property, designating these amplifiers as fiber optical parametric amplifiers (FOPA).

Most of the literature devoted to FOPA just use a single mode single-core fiber in order to achieve amplification [1, 2]. However some literature has considered modulation instability (MI) in a dual-core system [4–12]. The term "parametric amplification" is used in some of these articles [5, 11, 12] although they are tightly connected to MI. Most of this literature deals with nonlinear directional couplers [4, 5, 7, 8, 11, 12] and some with fiber propagation [6–10]. Most of them studied the asymmetric case where the power in the dual-cores is different from each other [4–9]. An interesting study in the scope of MI, was done in [5], where flat gain was obtained (Fig. 2 and 3 of [5]). However the case studied was the asymmetric one, with cross phase modulation between the dual-cores included in the model of [5], which means that strongly coupled cores were considered, which is not the cases studied in this paper.

Nonlinear propagation in multi-core fibers has been studied extensively recently [13, 14], leading to investigation of the positive impact of linear crosstalk on the nonlinear penalties [14].

In this paper we present a new theoretical matrix formalism to study nonlinear propagation in a dual-core FOPA in the symmetric case, which is more closely related with the recent theoretical formalism of single-core FOPA [15]. An important property of dual-core FOPA, is the flat gain obtained if some system design parameters are met, promising performance similar to their dual pump single-core FOPA counterparts [16, 17]. In the case of a dual-core FOPA 4 waves are under consideration besides the pump waves, i.e., two signal waves (one per each core) and two idlers (one per each core), although only two of the four waves are necessary to achieve PS gain as we will demonstrate later on this paper. Any combination of waves are possible in order to achieve PS gain.

2. Theorectical description

We start this theoretical description with the two coupled nonlinear Schrodinger equations (NLSE) [12, 14]:

$$i\frac{dE_1}{dz} + \gamma|E_1|^2E_1 + CE_2 = 0 \tag{1}$$

$$i\frac{dE_2}{dz} + \gamma|E_2|^2E_2 + CE_1 = 0 \tag{2}$$

where E_1 and E_2 are the fields in core 1 and core 2 (see Fig. 1), C is the core coupling coefficient and γ is the nonlinear parameter. We assume 3 waves in each core, i.e.,

$$E_1 = u_{p_1}\exp\left(i\left(\omega_{p_1}t - \beta_{p_1}z\right)\right) + u_{s_1}\exp\left(i\left(\omega_{s_1}t - \beta_{s_1}z\right)\right) + u_{i_1}\exp\left(i\left(\omega_{i_1}t - \beta_{i_1}z\right)\right) \tag{3}$$

$$E_2 = u_{p_2}\exp\left(i\left(\omega_{p_2}t - \beta_{p_2}z\right)\right) + u_{s_2}\exp\left(i\left(\omega_{s_2}t - \beta_{s_2}z\right)\right) + u_{i_2}\exp\left(i\left(\omega_{i_2}t - \beta_{i_2}z\right)\right) \tag{4}$$

and $\omega_{p1,s1,i1,p2,s2,i2}$ are the frequencies of the pump, signal and idler in core 1 and core 2, respectively, where $2\omega_{p_1} = 2\omega_{p_2} = \omega_{i_1} + \omega_{s_1}$, $\omega_{i_1} = \omega_{i_2}$ and $\omega_{s_1} = \omega_{s_2}$. $\beta_{p1,s1,i1,p2,s2,i2}$ are the propagation constants of the pump, signal and idler in core 1 and core 2, respectively. The symbols $u_{p1,s1,i1,p2,s2,i2}$ are the signals of the pump, signal and idler in core 1 and core 2, respectively and are functions of z only. This results in the propagation of 6 fields oscillating at 3 different frequencies described by the following coupled equations

$$\frac{du_{p_1}}{dz} = iu_{p_1}\left(\beta_{p_1} + \gamma\left(2P_1 - |u_{p_1}|^2\right)\right) + i\gamma 2u^*_{p_1}u_{s_1}u_{i_1} + iCu_{p_2} \tag{5}$$

$$\frac{du_{s_1}}{dz} = iu_{s_1}\left(\beta_{s_1} + \gamma\left(2P_1 - |u_{s_1}|^2\right)\right) + i\gamma u^2_{p_1}u^*_{i_1} + iCu_{s_2} \tag{6}$$

$$\frac{du_{i_1}}{dz} = iu_{i_1}\left(\beta_{i_1} + \gamma\left(2P_1 - |u_{i_1}|^2\right)\right) + i\gamma u^2_{p_1}u^*_{s_1} + iCu_{i_2} \tag{7}$$

$$\frac{du_{p_2}}{dz} = iu_{p_2}\left(\beta_{p_2} + \gamma\left(2P_2 - |u_{p_2}|^2\right)\right) + i\gamma 2u^*_{p_2}u_{s_2}u_{i_2} + iCu_{p_1} \tag{8}$$

$$\frac{du_{s_2}}{dz} = iu_{s_2}\left(\beta_{s_2} + \gamma\left(2P_2 - |u_{s_2}|^2\right)\right) + i\gamma u^2_{p_2}u^*_{i_2} + iCu_{s_1} \tag{9}$$

$$\frac{du_{i_2}}{dz} = iu_{i_2}\left(\beta_{i_2} + \gamma\left(2P_2 - |u_{i_2}|^2\right)\right) + i\gamma u^2_{p_2}u^*_{s_2} + iCu_{i_1} \tag{10}$$

The total power in each core is given by P_1 and P_2

$$P_1 = |u_{p_1}|^2 + |u_{s_1}|^2 + |u_{i_1}|^2 \tag{11}$$

$$P_2 = |u_{p_2}|^2 + |u_{s_2}|^2 + |u_{i_2}|^2 \tag{12}$$

in addition the invariant

$$R = |u_s|^2 - |u_i|^2 = |u_{s_1}|^2 + |u_{s_2}|^2 - |u_{i_1}|^2 - |u_{i_2}|^2 \tag{13}$$

is the *Manley-Rowe invariant* [15]. Moreover in order to be in conformity with the cases studied in this paper we assume that:

$$\beta_{p_1} = \beta_{p_2} \tag{14}$$

$$\beta_{s_1} = \beta_{s_2} \tag{15}$$

$$\beta_{i_1} = \beta_{i_2} \tag{16}$$

To get a more intuitive description we assume a solution in the small signal gain region where $|u_{p_1}|^2 = P_{p_1} = \frac{P_p}{2} \gg |u_{s1,i1}|^2$ and $|u_{p_2}|^2 = P_{p_2} = \frac{P_p}{2} \gg |u_{s2,i2}|^2$, $P_p = P_{p_1} + P_{p_2}$, where P_p, P_{p_1} and P_{p_2} are the total power of the pump, power of the pump in core 1 and power of the pump in core 2, respectively. Therefore the pump evolution through the length of the fiber can be approximated by $u_{p_1}(z) = u_{p_2}(z) = \sqrt{\frac{P_p}{2}}\exp\left(i\phi_0 + i\left(\beta_{p1,p2} + \frac{\gamma P_p}{2} + C\right)z\right)$ where ϕ_0 is the common initial phase of the pumps in core 1 and 2. Therefore we stress that the pumps should be phase locked and have equal power in order to be in conformity with the theory described in this paper. After inserting this into Eq. (5) and Eq. (8) we skip the phase dependence following the procedure described in [15]. By making the substitution $u_{s1,i1,s2,i2}(z) = e_{s1,i1,s2,i2}(z)\exp\left(i\phi_0 + i(\frac{P_p\gamma - \Delta\beta}{2} + \beta_{s1,i1,s2,i2} + C)z\right)$ we obtain a first order system that can be written in matrix form as:

$$\frac{d}{dz}\overrightarrow{E}(z) = iM\overrightarrow{E}(z) \tag{17}$$

$$M = \begin{pmatrix} K_0 & \frac{P_p\gamma}{2} & C & 0 \\ -\frac{P_p\gamma}{2} & -K_0 & 0 & -C \\ C & 0 & K_0 & \frac{P_p\gamma}{2} \\ 0 & -C & -\frac{P_p\gamma}{2} & -K_0 \end{pmatrix} \tag{18}$$

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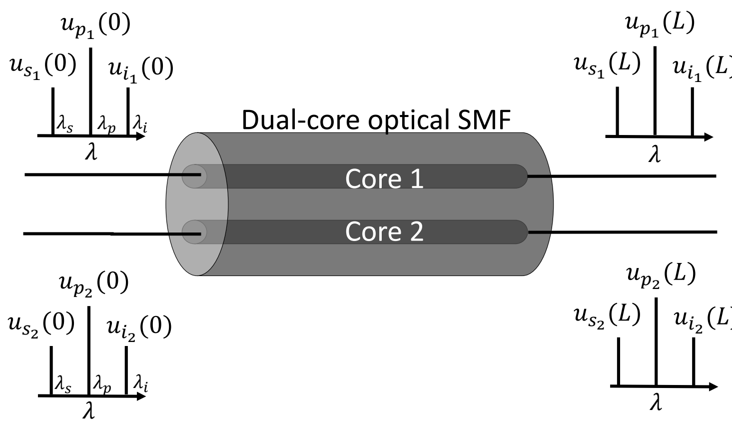

Fig. 1. Schematic of the model used in numerical computations.λ_s-wavelength of the signal, λ_p-wavelength of the pump, λ_i-wavelength of the idler.

where

$$K_0 = k - C \tag{19}$$

is the phase mismatch parameter of the dual-core fiber, $k = \frac{P_p \gamma}{2} + \frac{\Delta\beta}{2}$ and $\Delta\beta = \beta_3(\omega_p - \omega_0)(\omega_s - \omega_p)^2$, where β_3 is the third order dispersion parameter, ω_p, ω_0, ω_s are the pump, zero dispersion and signal angular frequency, respectively, and:

$$\vec{E}(z) = \begin{pmatrix} e_{s_1}(z) \\ e^*_{i_1}(z) \\ e_{s_2}(z) \\ e^*_{i_2}(z) \end{pmatrix}. \tag{20}$$

By solving Eq. (17) with initial conditions

$$\vec{E}(0) = \begin{pmatrix} e_{s_{10}} \\ e^*_{i_{10}} \\ e_{s_{20}} \\ e^*_{i_{20}} \end{pmatrix} \tag{21}$$

we obtain a solution of the type

$$\vec{E}(z) = K(z)\vec{E}(0) \tag{22}$$

where $K(z)$ is given by

$$\begin{pmatrix} A & B & D & E \\ -B & F & -E & G \\ D & E & A & B \\ -E & G & -B & F \end{pmatrix}, \tag{23}$$

with gain coefficients

$$g_1 = \sqrt{P_p{}^2 \gamma^2 - 4(K_0 + C)^2} = \sqrt{P_p{}^2 \gamma^2 - 4k^2} \tag{24}$$

$$g_2 = \sqrt{P_p{}^2 \gamma^2 - 4(K_0 - C)^2} = \sqrt{P_p{}^2 \gamma^2 - 4(k - 2C)^2} \tag{25}$$

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and matrix elements

$$A = \frac{\cosh\left(\frac{g_1 z}{2}\right)}{2} + \frac{\cosh\left(\frac{g_2 z}{2}\right)}{2} + \\ + \frac{(K_0 - C)\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{g_2} + \frac{(K_0 + C)\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{g_1} \tag{26}$$

$$B = \frac{P_p\,\gamma\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{2\,g_1} + \frac{P_p\,\gamma\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{2\,g_2} \tag{27}$$

$$D = \cosh^2\left(\frac{g_1 z}{4}\right) - \cosh^2\left(\frac{g_2 z}{4}\right) + \\ + \frac{(K_0 + C)\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{g_1} - \frac{(K_0 - C)\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{g_2} \tag{28}$$

$$E = \frac{P_p\,\gamma\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{2\,g_1} - \frac{P_p\,\gamma\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{2\,g_2} \tag{29}$$

$$F = \frac{\cosh\left(\frac{g_1 z}{2}\right)}{2} + \frac{\cosh\left(\frac{g_2 z}{2}\right)}{2} - \\ - \frac{(K_0 + C)\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{g_1} - \frac{(K_0 - C)\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{g_2} \tag{30}$$

$$G = \cosh^2\left(\frac{g_1 z}{4}\right) - \cosh^2\left(\frac{g_2 z}{4}\right) - \\ - \frac{(K_0 + C)\,\sinh\left(\frac{g_1 z}{2}\right)\,i}{g_1} + \frac{(K_0 - C)\,\sinh\left(\frac{g_2 z}{2}\right)\,i}{g_2}. \tag{31}$$

An alternative way to represent Eq. (17) is through its odd and even modes. If we write,

$$\frac{d}{dz}\begin{pmatrix} e_{s_1}(z) \\ e^*_{i_1}(z) \\ e_{s_2}(z) \\ e^*_{i_2}(z) \end{pmatrix} = \begin{pmatrix} K_0 & \frac{P_p\gamma}{2} & C & 0 \\ -\frac{P_p\gamma}{2} & -K_0 & 0 & -C \\ C & 0 & K_0 & \frac{P_p\gamma}{2} \\ 0 & -C & -\frac{P_p\gamma}{2} & -K_0 \end{pmatrix}\begin{pmatrix} e_{s_1}(z) \\ e^*_{i_1}(z) \\ e_{s_2}(z) \\ e^*_{i_2}(z) \end{pmatrix} \tag{32}$$

in the following form,

$$\frac{d}{dz}\begin{pmatrix} e_{s_1}(z) \\ e_{s_2}(z) \\ e^*_{i_1}(z) \\ e^*_{i_2}(z) \end{pmatrix} = \begin{pmatrix} K_0 & C & \frac{P_p\gamma}{2} & 0 \\ C & K_0 & 0 & \frac{P_p\gamma}{2} \\ -\frac{P_p\gamma}{2} & 0 & -K_0 & -C \\ 0 & -\frac{P_p\gamma}{2} & -C & -K_0 \end{pmatrix}\begin{pmatrix} e_{s_1}(z) \\ e_{s_2}(z) \\ e^*_{i_1}(z) \\ e^*_{i_2}(z) \end{pmatrix} \tag{33}$$

we can solve them by simultaneous diagonalization, by finding the eigenvalues of the diagonal blocks and the off diagonal blocks. Therefore the eigenvalues of the diagonal blocks are $(\tau - K_0)^2 \mp C^2 = 0 \Leftrightarrow \tau = K_0 \pm C$ and the double eigenvalue of the off diagonal blocks are $\delta = \frac{P_p\gamma}{2}$. Thus we can write Eq. (33) in the following form

$$\frac{d}{dz}\begin{pmatrix} e_o(z) \\ e^*_e(z) \end{pmatrix} = \begin{pmatrix} \tau & \delta \\ -\delta & -\tau \end{pmatrix}\begin{pmatrix} e_o(z) \\ e^*_e(z) \end{pmatrix} \tag{34}$$

Interesting to note that $\sqrt{(\delta^2 - \tau^2)} = \sqrt{\frac{{P_p}^2\gamma^2}{4} - (K_0 \pm C)^2}$, which are proportional to the corresponding gain coefficients g_1 and g_2.

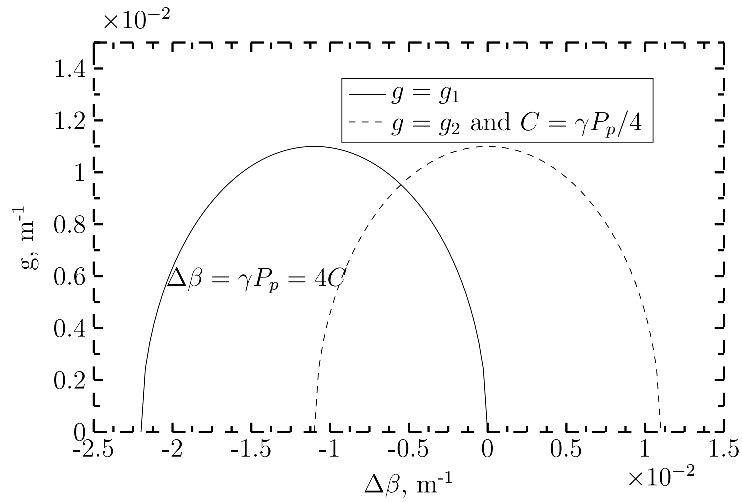

Fig. 2. Parametric gain coefficient g as a function of $\Delta\beta$.

It is noteworthy that vector $\overrightarrow{E}(z)$ obeys the *Manley-Rowe invariant* and $K(z)^H\,\sigma_1\,K(z) = \sigma_1 = const$ [15] which is equivalent to $R = \overrightarrow{E}^H(z)\sigma_1\overrightarrow{E}(z) = const$, where

$$\sigma_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{35}$$

and the superscript H denotes the hermitian conjugate.

3. Results and discussion

In this section we will discuss the above results, using the derived theoretical formalism of section 2. In order to achieve this objective we use the schematic of Fig. 1.

3.1. Phase-insensitive gain

We define the parametric gain in core 1 and 2 as

$$G_{1,2} = \frac{\left|u_{s_{1,2}}(L)\right|^2}{P_{s0}} \tag{36}$$

where $P_{s0} = |u_s(0)|^2/2 = \left|u_{s_1}(0)\right|^2 = \left|u_{s_2}(0)\right|^2$ and L is the length of the fiber. The initial conditions in the case of phase-insensitive (PI) gain are given as

$$\overrightarrow{E}(0) = e_{s_{10}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{37}$$

It can be demonstrated that our proposed dual-core FOPA has either exponential gain dependence on pump power when $\Delta\beta = 0$ and also has exponential gain dependence on pump power in the case of optimal mismatch, when $K_0 = \pm C$ where the maximum PI gain is given

Table 1. Setup Parameters

Symbol	Value	Units
λ_p	1550	nm
λ_0	1549.8	nm
P_p	1	W
γ	11	W^{-1} km^{-1}
β_3	1.10^{-40}	s^3m^{-1}
L	700	m

by $G_{1_{PI,max}} = \frac{{P_p}^2\gamma^2 z^2}{16} + \cosh\left(\frac{P_p\gamma z}{4}\right)^4$. In Fig. 2 we show the parametric gain coefficient g as a function of $\Delta\beta$ using the expression of Eqs. (24)–(25). The plots are given for $C = 0$ or equivalently $g = g_1$ and $C = \gamma P_p/4$ given by the curve $g = g_2$. $C = 0$ is a identical situation of the single-core FOPA and $C = \gamma P_p/4$ is an example value for the dual-core FOPA, proposed in this paper. One can distinguish both cases by the parametric gain coefficient g when $\Delta\beta = 0$, which is maximum in the shown case of $C = \gamma P_p/4$ and minimum when $C = 0$. The single-core FOPA has quadratic gain dependence on pump power when $\Delta\beta = 0$ while the dual-core FOPA has exponential gain dependence on pump power. This is a clear advantage for the dual-core FOPA scheme over the single-core FOPA. The non-degenerate double pump single-core FOPA is thus capable to present exponential gain dependence on pump power when $\Delta\beta = 0$ over a wide bandwidth [16, 17], however our proposed scheme can accomplish this in a degenerate configuration, avoiding the problem of Raman induced power transfer, that can reduce the four-wave mixing efficiency [18].

3.2. Phase-sensitive gain

In this subsection we will study the properties of the phase sensitive gain with 2 waves inserted at the input of the dual-core parametric amplifier. Therefore in the setup of Fig. 1 the waves u_{i_1} and u_{i_2} are omitted while the waves u_{s_1} and u_{s_2} remain. Later in subsection 3.3 we will discuss the maximum and minimum PS gain when 4 waves are taken into account. The initial conditions in the case of phase-sensitive (PS) gain when 2 waves (signal waves) are inserted at the input of the system are given as

$$\overrightarrow{E}(0) = e_{s_{10}} \begin{pmatrix} 1 \\ 0 \\ \exp(i\phi) \\ 0 \end{pmatrix}. \tag{38}$$

where the PS gain is obtained by inserting the signal $u_{s_1(0)}$ and $u_{s_2(0)}$ in core 1 and core 2, respectively, as shown in Fig. 1. We define the gain extinction ratio (GER) as given by the following expression

$$GER = \max_{-\pi\leq\phi\leq\pi} G_1 - \min_{-\pi\leq\phi\leq\pi} G_1 \tag{39}$$

where G_1 is given in dB. In Fig. 3(a) and 3(b) we show the gain G_1 (which for the present initial conditions is equal to G_2) versus λ_s and versus ϕ, respectively. Table 1 shows the parameters of the schematic of Fig. 1. The C parameter was chosen in order that $CL = \frac{\pi}{2}$, where L is the length of the fiber.

In our design when $\Delta\beta = 0$ flat and maximum exponential gain dependence on pump power can be obtained if the power of the pump is approximately equal to the critical power of a dual-core system $P_p \approx \frac{4C}{\gamma}$ [5]. Our proposed scheme can also provide gain when the pump lies in the normal dispersion regime, since if $\Delta\beta > 0$ this can still be compensated by the phase mismatch introduced by the term $-C$ in Eq. (19). Recalling the PS initial conditions of Eq. (38), it is shown in Fig. 3(a) that when $\phi = 0$ the gain spectrum G_1 (and G_2), is similar to the case of a single-core FOPA. This is due to the fact that accordingly to Eqs. (24)–(31) the gain is given by $G_{1,2} = |A + D|^2$ and the terms with g_2, that gives the unique properties of the dual-core FOPA, will vanish, while the terms with g_1 will remain. However when $\phi = \pi$ we have the opposite case, $G_{1,2} = |A - D|^2$ and the terms with g_1 will vanish, while the terms with g_2 will remain, yielding in exponential and flat gain when $\Delta\beta = 0$. When the frequency of the pump (f_p) is equal to the zero dispersion frequency (f_0) this results in broadband, flat, exponential gain dependence on pump power when $\phi = \pi$ and broadband, flat and quadratic gain dependence on pump power when $\phi = 0$. Figure 3(b) shows the gain against the phase shift ϕ presented in Eq. (38). The maximum difference between PS and PI gain is about 6 dB. This difference is dependent on the coupling coefficient C, however as we will discuss further, C must be chosen in order to have the best properties of the proposed dual core FOPA, such as flat and exponential gain dependence on pump power when $\Delta\beta = 0$. It is noteworthy that the 6 dB difference is

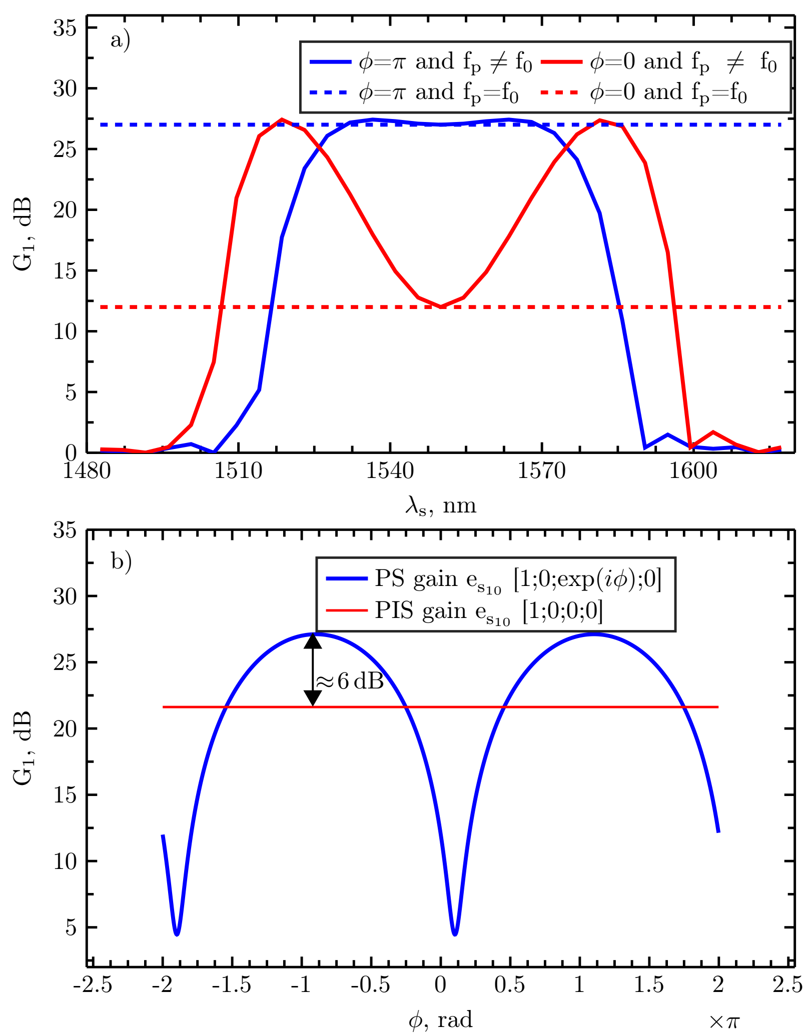

Fig. 3. Gain G_1 versus a) λ_s and b) ϕ. f_p-frequency of the pump and f_0-zero dispersion frequency.

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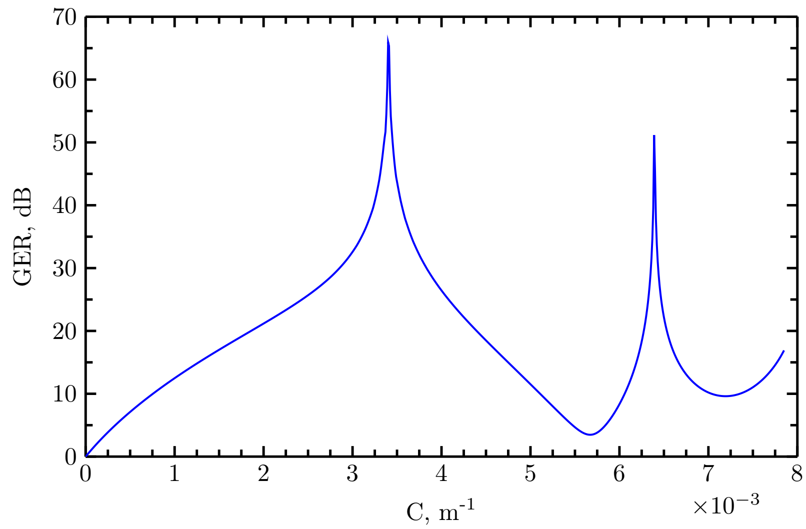

Fig. 4. GER as a function of the coupling coefficient C with $\Delta\beta = 0$.

obtained when there are no idlers generated at the input of the dual core FOPA and therefore they are not required to have the 6 dB difference between PI and PS gain, usually obtained in a single-core phase-sensitive amplifier. Looking to Eqs. (5)–(10), we can intuitively see that this is due to the fact that the signal u_{s_1} interacts not just linearly through the coupling coefficient C with u_{s_2} but also nonlinearly through the generation of the idlers in both cores. This is on the basis of the additional generated PS gain. In Fig. 4 we demonstrate that maximum GER, which may be important for phase regeneration applications, is achieved for a specific coupling coefficient. This value depends on the power of the pump among other parameters.

3.3. Maximum and minimum phase-sensitive gain

PS gain occurs when at least two of the four waves, are introduced in the dual-core FOPA, being one of them the signal wave. The maximum PS gain is given by $G_{1,2_{PS,max}} = |A + B \pm D \pm E|^2 = \cosh\left(P_p \gamma z\right)$ when $K_0 = \mp C$, respectively. It must be stressed that when $G_{1,2_{PS,max}} = |A + B - D - E|^2$ all the terms with g_1 will vanish, and therefore it is possible to have exponential gain dependence on pump power when $\Delta\beta = 0$, while when $G_{1,2_{PS,max}} = |A + B + D + E|^2$ all the terms with g_2 will disappear and the spectrum shape will look as a single-core FOPA. It can be demonstrated that the minimum PS gain is obtained when $K_0 = P_p \gamma/2$. Therefore

$$G_{1,2_{PS,min}} = |A - B + D - E|^2 = \\ = 1 - \frac{P_p\,\gamma \sin\left(z\sqrt{C^2 + P_p\,\gamma\,C}\right)^2}{\left(C + P_p\,\gamma\right)}. \tag{40}$$

For a fair comparison between the maximum PS gain and the maximum PI gain, we must take into consideration that the ratio between the former and the later depends on the coupling coefficient. Therefore one shall choose the configuration that gives the best properties of the dual core FOPA, which is when we can obtain both flat and exponential gain dependence on pump power with $\Delta\beta = 0$. In this situation the maximum gain is obtained when we maximize g_2, which

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is when $C = P_p\gamma/4$ and the maximum PS gain is obtained by $G_{1,2_{PS,max}} = |A + B - D - E|^2$. Therefore in this conditions the ratio between $G_{1,2_{PS,max}}$ and $G_{1_{PI,max}}$ is given by

$$\frac{G_{1,2_{PS,max}}}{G_{1_{PI,max}}} = \frac{\cosh(P_p\gamma z)}{\frac{{P_p}^2\gamma^2 z^2}{16} + \cosh\left(\frac{P_p\gamma z}{4}\right)^4} \approx \frac{\cosh(P_p\gamma z)}{\cosh\left(\frac{P_p\gamma z}{4}\right)^4} \leq 8, \quad (41)$$

therefore 9 dBs between the maximum PS and maximum PI gain can be obtained with the proposed configuration. To note that the maximum difference between maximum PS gain and maximum PI gain is obtained for $P_p\gamma z \gtrsim 10$.

4. Conclusions

In this paper we proposed a theoretical formalism to study the dual-core FOPA. The dual-core FOPA presents several attractive characteristics over the known single-core FOPA, such as exponential gain dependence on pump power when $\Delta\beta = 0$, gain in the normal dispersion regime and flat gain around the frequency of the pump. Besides the dual-core FOPA works in a degenerate configuration which avoids certain problems of the non-degenerate double pump single-core FOPA such as Raman induced power transfer over distinct frequencies, which reduces the four-wave mixing (FWM) efficiency, due to unequal power of the pumps along the fiber length. We present in this paper the expressions for maximum and minimum PS gain and include the results for phase sensitivity due to the coupling coefficient C, showing that maximum phase sensitivity is achieved for a single value of core coupling coefficient.

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