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Ramp metering for flow maximisation and emission reduction – a set-based multi-objective design approach

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Abstract

A set-theoretical approach is presented for a multi-objective control design of the local ramp metering problem. Two control objectives are specified: first, the optimization of traffic performance, by the minimization of total time spent. Second, the emission factor of CO_2 is minimized. The optimal state for traffic emission however lies in the unstable domain of the system. To resolve this inconsistency, the control problem is formalized in a set-theoretical context. For this purpose, the nonlinear METANET model is rewritten in a shifted coordinate frame in a parameter-varying, polytopic representation. Bounds on state-, input- and disturbance variables are expressed by convex polytopes. These sets are then used for the design of an interpolated H_∞ controller that is capable of improving traffic conditions according to the prescribed multi-objective criteria. Different control allocation methods are compared with non-linear model predictive control stategy, in order to illustrate the proposed methodology.

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Keywords: LPV systems, Hinf control, ramp metering, traffic emission modeling, multi-objective control

1. Introduction

The concept of sustainable development motivates to extend the control objectives of conventional, traffic-performance optimizing control strategies. The ultimate goal is to maintain a process that is optimal both in terms of travel times and environmental aspects. The literature of traffic control offers a variety of control approaches optimizing solely traffic performance (see e.g. [Hegyi et al., 2005, Kan et al., 2016] and the references therein), or emission performance (e.g. [Liu et al., 2012], [Alsabaan et al., 2013], [Csikós et al., 2014]). Recently, numerous approaches have emerged addressing the multi-objective control problem. [Zegeye et al., 2009] and [Groot et al., 2013] solve constrained optimization problems in a receding horizon manner. In [Ma, 2012] a genetic algorithm based optimization scheme is developed for the joint optimization of emission and flow level indicators.

We emphasize two aspects of the underlying problem: system dynamics are nonlinear and the control input is

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constrained. These characteristics generally imply nonlinear optimization techniques, which are purely numerical, hence do not provide insight of the solution. In addition, the lack of performance or robustness guarantees is another consequence of the applied online optimization.

Partially motivated by these facts, the existing results restrict to the case where the optima of all control objectives remain in the stable domain of the system. Nevertheless, this assumption is not straightforward: while the optimization of traffic performance calls for the tracking of the capacity maximizing critical density, certain pollutants may reach their optimum at traffic speeds that postulate the exceeding of the critical density and thus destabilizing the system. Consequently, a trade-off is observed and must be addressed accordingly. Current paper aims at solving this compromise by suggesting a general multi-objective design approach that does not need a restriction on the optimal values of the performances.

These problems have been recognized in systems and control theory and the need for a systematic framework for nonlinear systems gave birth to the Linear Parameter Varying methodology [Shamma et al., 1991], [Apkarian et al, 1995]. System nonlinearities are captured by scheduling parameters and consequently a linear (parameter dependent) structure is obtained. Accordingly, tractable analysis and synthesis tools for linear systems can be extended for nonlinear dynamics. Still, hard constraint handling remains one of the open challenges of LPV systems [Péni et al., 2009]. Among the possible approaches, a set-theory based solution is attractive due to its ability to involve physical considerations, as well as its compact numerical methods [Blanchini et al., 2008].

This paper suggests a control design for the local ramp metering problem. The work is motivated by the multi-objective problem for minimizing the total time spent in the network and the total emission of CO_2, the prescribed reference of which lies in the unstable domain of the system. In order to fulfil these goals the nonlinear traffic model is transformed into a polytopic system which serves as a basis for an interpolated H_∞ controller. The performance of the proposed controller is demonstrated in a case study through a comparison to the nonlinear model predictive control (NMPC) approach.

2. Preliminaries

2.1. Macroscopic modeling of motorway traffic

The traffic system is represented by a single motorway segment of length L containing λ lanes. Control is realized by a metered ramp allowing flow r to the main lane. Upstream flow q_{us}, mean speed of the upstream traffic v_{us}, ramp demand w, off-ramp flow s and downstream density ρ_{ds} are considered disturbances. System dynamics are modeled by the second order continuum model METANET introduced by [Papageorgiou et al., 1990]. The model is widely used for analyzing traffic phenomena, for identifying macroscopic parameters as well as for designing model-based observers and controllers [Papageorgiou et al., 2002], [Hegyi et al., 2005].

For discrete sampling time T, state dynamics of the system is described by the following equations:

- The *conservation equation* is the basic dynamic equation describing the time evolution of traffic density:

$$\rho(k+1) = \rho(k) + \frac{T}{\lambda L}[q_{us}(k) - q(k) + r(k) - s(k)], \tag{1}$$

where ρ, v, q denote the traffic density, speed and flow of the modeled segment, respectively.

- The *equilibrium speed function* provides a static relationship between traffic density and traffic speed. This relationship holds in case of equilibrium flow.

$$V(\rho(k)) = v_{free} \exp\left(-\frac{1}{a}\left(\frac{\rho(k)}{\rho_{cr}}\right)^a\right), \tag{2}$$

where v_{free}, a, ρ_{cr} are constant model parameters.

- The *fundamental equation* comprises the relationship among the traffic variables:

$$q(k) = \lambda\, \rho(k)\, v(k). \tag{3}$$

- The dynamics of non-equilibrium flow can be modeled by using the *momentum equation*, describing the dynamics of traffic speed:

$$v(k+1) = v(k) + \frac{T}{\tau}\big(V(\rho(k)) - v(k)\big) + \frac{T}{L} v(k)\big(v_{us}(k) - v(k)\big) - \frac{\eta T}{\tau L}\frac{\rho_{ds}(k) - \rho(k)}{\rho(k) + \kappa} - \frac{\delta T}{\tau \lambda L}\frac{r(k)v(k)}{\rho(k) + \kappa}, \tag{4}$$

where τ, η, δ, κ are constant model parameters.

• The dynamics of the *ramp queue*, denoted by l can be modeled in the following form:

$$l(k+1) = l(k) + T(w(k) - r(k)). \tag{5}$$

The presented model can be viewed as a nonlinear process $x(k+1) = f(x(k),u(k),d(k))$ with controlled input $u = r$, state variables $x = [\rho\ v\ l]^T$ and disturbances $d = [q_{us}\ v_{us}\ \rho_{ds}\ s\ w]^T$.

2.2. Modeling of traffic emission

In the followings, the macroscopic description of traffic emission is summarized. For a detailed introduction of the concept, the reader is referred to [Csikós, 2015].

Assume that the vehicle composition is homogeneous and constant in time and its emission factor for pollutant p is represented by ef^p. Assuming that the average speed of traffic flow represents the speed of individual vehicles over the spatiotemporal sampling domain $L \times T$ in which the effect of acceleration can be neglected, the emission of vehicle individuals in each discrete sample can be given by average-speed emission models*. Then, the spatiotemporal distribution of traffic emission for pollutant p in the discrete framework is given by:

$$\varepsilon^p(k) = ef^p(v(k))q(k), \tag{6}$$

whereas the total emission in the spatiotemporal rectangle LxT of the discrete measurement can be approximated as

$$E^p(k) = ef^p(v(k))q(k)LT. \tag{7}$$

Considering (3), the macroscopic emission function of traffic is thus stated as a bivariate function of traffic density and traffic mean speed.

The exact modeling of ramp speed dynamics requires detailed information of the queue length and the flows both at the entrance and exit of the ramp (see Pasquale et al. 2015). Assuming that these measurements are not at disposal, a constant ramp speed v_{ramp} can be considered. The total emission of ramp traffic is thus given as

$$E^p_{ramp}(k) = ef^p(v_{ramp})v_{ramp}l(k)T. \tag{8}$$

2.3. Control objectives

In this section the control objectives of the considered multi-criteria control are formalized as cost functions. The control criteria are then interpreted as separate regulator problems for the state variables, the reference values of which are involved into the design of steady-states.

For the *traffic performance*, the minimization of the total time spent (TTS) is aimed. The corresponding performance function is formalized as follows:

$$J_{TTS} = T\sum_{k=1}^{N_c}\rho(k)L + l(k) \tag{9}$$

where N_c denotes the time horizon of the analysis. As shown in [Papageorgiou and Kotsialos, 2002], TTS for the analyzed network, including ramp queues is minimized if $\rho_{opt}=\rho_{cr}$ and $l_{opt}=0$ are aimed.

Regarding the *emission performance*, current work focuses on minimizing the aggregated emission of pollutants with global effects (i.e. CO_2). Therefore, the control objective is the minimization of the following sum with arbitrarily long time horizon N_c:

$$J_{em} = T\sum_{k=1}^{N_c} ef^p(v(k))q(k)L + ef^p(v_{ramp})l(k) \tag{10}$$

* The emission factor function of the applied average-speed emission model COPERT [Ntziachristos et al., 2000] is given by the equation $ef^p(v) = \dfrac{\alpha^p + \gamma^p v + \varepsilon^p v^2}{1 + \beta^p v + \delta^p v^2}$, with the parameters α^p, β^p, γ^p, δ^p, ε^p of pollutant $p=CO_2$ given in the Appendix in Table 2.

Assuming the independence of the main lane and ramp dynamics (following the modeling assumptions of METANET) and that the cost term of ramp queue can be neglected relative to the one of the main lane, the following regulator problems can be stated for minimizing emission.

- The emission of the main lane (first term of (10)) for $N_c \to \infty$ is minimal if the emission factor function $ef(v)$ is minimized. Thus, the control objective can be stated as a regulator problem for the traffic mean speed where the reference v_{opt} is defined by

$$v^p_{opt} = \operatorname{argmin} ef^{\,p}(v) \tag{11}$$

Note, that a non-constant traffic composition entails the time-dependence of v^p_{opt}. In that case, a tracking problem needs to be formalized which is not in the scope of the paper.

- The second term of (10) represents the emission of the on-ramp, which can be minimized by l_{opt}.

3. LPV form of the model

Constrained control of nonlinear systems are generally a challanging task from both theoretical and practical perspectives. In order to apply systematic linear tools for nonlinear dynamics, the Linear Parameter Varying (LPV) framework emerged, where the main idea is to mimic linearity through appropriately chosen transformations. This section discusses briefly the main steps of the LPV reformulation of the underlying traffic dynamics. A more detailed derivation can be found in [Luspay et al., 2012], [Luspay et al., 2016].

3.1. Steady-state analysis

The first step of the derivation is the determination of the steady-state values for the nonlinear system (1)-(5). The discrete-time steady-state condition is:

$$x(k+1) = x(k) = x^*. \tag{12}$$

The above equation is satisfied by the triplet (x^*,u^*,d^*), which can be found by substituting the steady-state condition (12) to system dynamics (1), (4), (5) and solving the obtained algebraic equations. Generally, this is an underdetermined problem, as the number of free variables exceeds the number of equations. It is obvious that the steady-state ramp dynamics can be determined independently, hence for the two dynamic equations, regarding the main lane dynamics, seven variables need to be chosen: ρ^*, v^*, l^*, q_{us}^*, v_{us}^*, ρ_{ds}^*, r^*, s^* and w^*. Consequently, this represents a design freedom for the selection of these values. In the present application, the following considerations were made: i) the values for the uncontrolled boundary variables (q_{us}, v_{us}, ρ_{ds}, s, w) are chosen to represent stable traffic dynamics, ii) the aim of the control problem is to minimize the bias from the optimal states (ρ_{opt}, v_{opt}, l_{opt}).

As for the first step, the boundary variables, that account for the flow conditions and can be independently chosen, are fixed – i.e. the downstream traffic density, and the variables describing ramp flows, specifically:

- for the downstream density, critical density is assumed: $\rho_{ds}^*=\rho_{cr}$.
- zero off-ramp flow is considered, $s^*=0$.
- ramp flow is chosen as to obtain symmetrical bounds around the steady-state: $r^*=(r_{min}+r_{max})/2$.

Secondly, for the remaining variables ρ^*, v^*, q_{us}^*, v_{us}^* an optimization problem is solved:

$$\begin{aligned} &\min_{[\rho^*,v^*,v^*_{us}]} (\rho^*-\rho_{opt})^2+(v^*-v_{opt})^2 \\ \text{subject to:}\quad & 0=\frac{T}{\lambda L}[q^*_{us}-\lambda\rho^* v^* + r^* - s^*] \\ & 0=\frac{T}{\tau}\left(V(\rho^*)-v^*\right)+\frac{T}{L}v^*\left(v^*_{us}-v^*\right)-\frac{\eta T}{\tau L}\frac{\rho^*_{ds}-\rho^*}{\rho^*+\kappa}-\frac{\delta T}{\tau\lambda L}\frac{r^* v^*}{\rho^*+\kappa} \end{aligned} \tag{13}$$

The steady-state condition (12) for the ramp queue dynamics imply $w^*=r^*$. Furthermore, for the steady-state value for the ramp queue, an arbitrary value can be chosen. Without the loss of generality and the sake of computation demands, a non-zero yet low l^* is accepted as steady-state value of the ramp queue. The calculated steady-states are given in Table 3 in the Appendix.

3.2. LPV form and set representations

Once the steady-states are determined, we are in the position to transform the nonlinear dynamics through a coordinate-frame change. Given the generic nonlinear description and the corresponding steady-states, we have

$x(k+1)=f(x(k),u(k),d(k))$ and $x^*=f(x^*,u^*,d^*)$ which imply:

$$x(k+1)-x^* = f(x(k),u(k),d(k)) - f(x^*,u^*,d^*). \tag{14}$$

Next, we introduce the centered variables to represent the deviation from the steady-state as follows: $\tilde{x}(k)=x(k)-x^*$, $\tilde{u}(k)=u(k)-u^*$ and $\tilde{d}(k)=d(k)-d^*$. Our aim is that (14) appears linear in the centered variables.

Before we proceed, we introduce the set representations of the new variables. One of the natural consequence of the coordinate change is the admissible domain of $\tilde{x}$, $\tilde{u}$ and $\tilde{d}$ can be given by compact, convex polytopes, including the origin as an interior point. Formally:

$$\mathsf{P}_{\tilde{x}}(H_{\tilde{x}},h_{\tilde{x}}) = \{\tilde{x} : H_{\tilde{x}}\tilde{x} \le h_{\tilde{x}}\} \tag{15}$$

and similarly $\mathsf{P}_{\tilde{u}}(H_{\tilde{u}},h_{\tilde{u}})$ and $\mathsf{P}_{\tilde{d}}(H_{\tilde{d}},h_{\tilde{d}})$. We refer to (15) as the half-plane representation.

Some of these variables are already linear in eqs. (1)-(5), while others can be factorized from the nonlinearities through a simple integral formula, as described in [Luspay et al., 2012], [Luspay et al., 2016]. Consequently, the following form is obtained:

$$\tilde{x}(k+1) = A(\tilde{x}(k))\tilde{x}(k) + B(\tilde{x}(k))\tilde{u}(k) + E(\tilde{x}(k))\tilde{d}(k). \tag{16}$$

Equivalency of the centered LPV dynamics of (17) with the original nonlinear system has been shown in [Luspay et al., 2012]. Note that $A(\tilde{x}(k))$, $B(\tilde{x}(k))$ and $E(\tilde{x}(k))$ are matrix functions with nonlinear dependency on the centered state variable $\tilde{x}(k)$. For numerical computation the LPV dynamics is transformed into a parameter varying polytopic form with the following algorithm:

1. Construct a finite $n\times m$ grid over the given polytope of $\mathsf{P}_{\tilde{x}}(H_{\tilde{x}},h_{\tilde{x}})$: $(\tilde{\rho}_i,\tilde{v}_j)$, $i=1,\dots,n$, $j=1,\dots,m$.
2. Evaluate the matrix functions over the discrete grid to obtain matrices $A_{i,j}$, $B_{i,j}$ and $E_{i,j}$.
3. Stack into a vector form the parameter-varying elements of the matrices and create a convex hull.
4. Determine the vertices of the convex hull and the corresponding LTI vertex systems A_f, B_f, E_f, $f=1,\dots,F$.

It is clear by the above convex construction that the LPV dynamics can be obtained by suitable interpolation of the vertex systems through a parameter varying weighting functions. Consequently, the nonlinear dynamics is represented over the admissible operation domain by convex polytopes and LTI vertex systems. This representation forms the basis of the constrained control design, described in the following sequel.

4. Set-theory based control design

This section describes a set-theory based controller setup for the given traffic problem. In the previous section we derived the following representation:

$$\tilde{x}(k+1) = A(\alpha)\tilde{x}(k) + B(\alpha)\tilde{u}(k) + E(\alpha)\tilde{d}(k), \tag{17}$$

$$\alpha \in \Lambda, \Lambda = \left\{ \alpha = [\alpha_1 \dots \alpha_F] \mid \alpha^f \in \mathsf{R}^+, \sum_{f=1}^{F} \alpha^f = 1 \right\}, \tag{18}$$

$$[A(\alpha), B(\alpha), E(\alpha)] = \sum_{f=1}^{F} \alpha^f [A_f, B_f, E_f] \tag{19}$$

With the additional information on the admissible domain of the variables $\tilde{x} \in \mathsf{X}$, $\tilde{u} \in \mathsf{U}$ and $\tilde{d} \in \mathsf{D}$.

First of all, we recall some of our previous results in [Luspay et al., 2016], the maximal robust controlled invariant set for LPV systems [Blanchini et al., 2008], [Péni et al., 2009].

Definition 1 *The set* $\mathsf{S} \subseteq \mathsf{X}$ *is called robust controlled invariant set for system (17)-(19) if there exists a control* $u \in \mathsf{U}$ *such that for all* $\tilde{x}(0) \in \mathsf{S}$ *and for all allowable disturbance sequences* $\tilde{d} \in \mathsf{D}$, *the condition* $x(k) \in \mathsf{S}$ holds for all $0 \le k$.

The maximal robust controlled invariant set describes the generic region of applicability of constrained controllers (under the given assumptions on the disturbance). It is important to emphasize, that it is control-independent: there is no information involved on the computation of the control signal. Computational algorithm, numerical details and discussion can be found in [Luspay et al., 2016]. In the present study we further develope this concept by designing ramp metering controllers in a systematic fashion and incorporating set-theoretic tools.

Our control objective is to steer the system to the origin, in the centered coordinate frame, regardless of the external disturbances. For this purpose we intend to design feedback gains: $\tilde{u} = K\tilde{x}$, where K is determined on the basis of the polytopic system. We apply standard H_∞ type control design to minimize the effect of the disturbances on the specified performance channels z. Here, we simply set $z(k) = Q\tilde{x}(k)$, where Q expresses the relative importance of the state variables. This leads to the following Linear Matrix Inequality (LMI) optimization [Apkarian et al., 1995]:

$$\min_{P,Y} \gamma \quad \text{s.t.} \begin{bmatrix} \tilde{u}_{\max}^2 & Y \\ Y^T & P \end{bmatrix} \geq 0, \quad \begin{bmatrix} P & 0 & P^T A_f^T + Y^T B_f^T & P^T Q^T \\ * & \gamma^2 I & E_i^T & 0 \\ * & * & P & 0 \\ * & * & * & I \end{bmatrix} \geq 0, \quad f = 1,\ldots,F. \tag{20}$$

This is a convex optimization problem, involving F+1 number of constraints, i.e.: each grid-point and the corresponding vertex system is involved and an additional LMI is included to enforce control limitations. Once the optimization is solved the resulting feedback gain is available through $K=YP^{-1}$. The obtained γ value gives an upper bound on the closed-loop performance level, i.e. the worst case amplification from the external disturbance to the performance output [Apkarian et al., 1995].

The applicability of a specific controller K can be characterized by the maximal robust invariant set of the closed loop system [Blanchini et al., 2008]. Generally the following tradeoff can be observed here. A controller with good control performance (low value of γ) generates small disturbance invariant set: only over a limited domain of operation can we guarantee improved disturbance rejection. On the other hand, controllers with lower performance result slower closed-loop response and consequently generate larger invariant sets. In [Péni et al., 2009] an interpolation-based controller is proposed, which combines the advantageous properties of the two types of controllers. Let us assume that m different controller had been designed: $(K_1,\gamma_1),(K_2,\gamma_2),\ldots,(K_m,\gamma_m)$, with different performance level: $0 < \gamma_1 < \ldots < \gamma_{m-1} < 1 < \infty$. Note that, the last controller provides the worst performance, but generates the largest invariant set. Then, the following augmented system can be defined:

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \vdots \\ \hat{x}_m(k+1) \end{bmatrix} = \begin{bmatrix} A(\alpha)+B(\alpha)K_1 & & \\ & \ddots & \\ & & A(\alpha)+B(\alpha)K_m \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \vdots \\ \hat{x}_m(k) \end{bmatrix} + \begin{bmatrix} E(\alpha)/m \\ \vdots \\ E(\alpha)/m \end{bmatrix} \tilde{d}(k). \tag{21}$$

The input-output equivalence between the extended system and the original one is shown and proved in [Péni et al., 2009]. Furthermore, it is shown that the interpolated control law:

$$\tilde{u}(k) = \sum_{i=1}^{m} K_i \hat{x}(k), \tag{22}$$

stabilizes system (17) and achieves the performance level of: $\hat{\gamma} \leq \sqrt{\sum_{i=1}^{m} \frac{\gamma_i^2}{m}}$. The control law (22) requires the knowledge of the extended state vector $\hat{x}(k)$ of the augmented system (21), which is done by offline and online optimizations.

• First a suitable storage function is computed for the system, under the quadratic form of:

$$\hat{V}(\hat{x}) = \hat{x}^T \hat{P} \hat{x}. \tag{23}$$

• Offline computation of the maximal robust invariant set of the extended system, represented by: $\mathsf{P}(\hat{H},\hat{h}) : \{\hat{x} : \hat{H}\hat{x} \leq \hat{h}\}$. Although it is an m x n_x dimensional problem, it is performed for the closed-loop (i.e. no control input involved) and hence computationally attractive.

• Online optimization: the state of the extended system is determined on the basis of the robust invariant set.

$$\hat{x}(k) = arg \min_{\hat{x}^T \in \mathrm{R}^{mnx}} \hat{x}\hat{P}\hat{x}, \text{ subject to: } \hat{H}\hat{x} \leq \hat{x}, \quad \tilde{x}(k) = \Pi\hat{x}. \tag{24}$$

The last condition connects the available real-time information on the system with the augmented one through the projection Π.

The above formulation implies the regulation for the steady-state (x^*,u^*,d^*) which is maintained if the initial state lies in the control invariant set and disturbance constraints are satisfied. In case of initial or boundary conditions outside the set-defining bounds (e.g. low-density conditions), the controller will not necessarily reach the set point, however will aim to approach it. As a summary: the controller will act as a normal regulator, but garanteeing to reaching it from the control invariant set.

5. Numerical results

In the sequel, the previously proposed interpolated H_∞ control policy is tested through a comparison to the nonlinear model predictive controller (NMPC, see [Grüne and Pannek, 2011], [Bellemans et al., 2006]). A major difference between the two controllers is that for the NMPC control the nonlinear system dynamics is used for dynamic recursion for a prediction horizon of N_c=5. The prediction horizon is chosen as suggested in [Hegyi et al., 2005]: during the horizon, the considered network is completed by the lowest considerable nominal speed of a vehicle. In our case, the lowest speed v=36 km/h is chosen). The NMPC is designed without considering the set-theoretic analysis, whereas for the interpolated H_∞ is based on the LPV form and the set representations.

The objective for both controllers is the joint minimization of the total time spent and the total emission of CO_2 under unstable traffic conditions by tracking the state triplet (ρ^*, v^*, l^*).

In the case study a 1000s long rush hour scenario is analysed with changing demands both upstream and downstream of the controlled segment. Initial values are chosen as the steady-state triplet $(\rho(0),v(0),l(0) = (\rho,^* v,^* l^*)$. For the boundary conditions, two types of patterns are used:

- sinusoid excitation is given for the upstream demands: $q_{us}(t)=q_{us}^*+200\sin(50t)$; $v_{us}(t)=v_{us}^*+2\sin(50t+\pi)$; $w(t)=w^*+100\sin(100t)$,
- a step-function is given for the downstream density to represent a temporary bottleneck situation

$$\rho_{ds}(t) = \begin{cases} \rho_{ds}^* + 30 & t \in [300,600]\,s \\ \rho_{ds}^* & \text{otherwise} \end{cases}.$$

The simulation results are plotted in Fig. 1. In case of no control, the network starts to get congested around 300s, and by 450s the traffic speed reaches its minimum at 10km/h. By 500s the flow conditions become so weak that the main lane cannot accept vehicles from the ramp and a ramp queue begins to build up.

Both controllers are capable of suppressing the effect of the bottleneck and avoiding congestion. The two input signals along the simulation horizon show a similar shape, the NMPC having a faster response to the forming congestion. However, the interpolated H_∞ approach provides superior reference tracking. Regarding the aggregated performances (see Table 1), the proposed controller leads to better results both in terms of total time spent (10) and total emission of CO_2 (11).

According to the numerical studies, the following conclusions can be drawn. The set-theoretic approach offers systematic method for designing constrained controllers for nonlinear traffic systems. The resulting controller is more simple than the nonlinear optimization based solutions. Consequently, lower CPU times are obtained (see Table 1.).

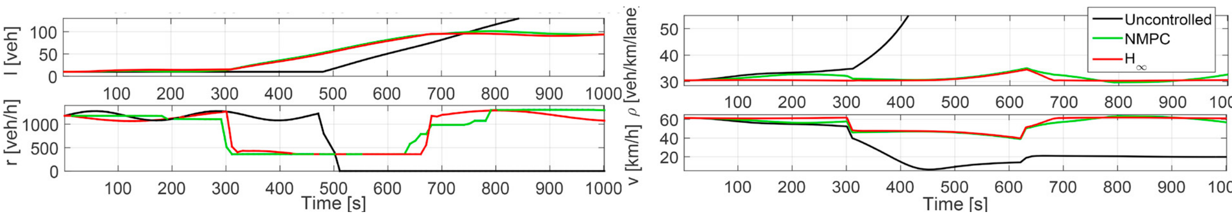

Figure 1: Dynamic demand - states and control input

Table 1. Aggregated performances

Performance	Uncontrolled	NMPC	Interpolated H_∞
Total time spent [veh h]	65.27	25.01	24.72
Total emission of CO_2 [veh g]	243007.89	173490.50	172804.08
CPU time/sample step [s]	-	0.2445	0.0112

6. Summary

A novel control design approach of the local ramp metering problem is proposed. The motivation of the research is the need for a multi-objective control design framework, in which such control objectives can be optimized that have an optimum in undesirable state regions (e.g. the optimum of certain pollutant emissions at low speeds).

For an effective solution of the control problem, the dynamic model is rewritten in a shifted coordinate frame with a parameter-varying, polytopic representation. The LMI based control design is computationally attractive, as it only solves a standard quadratic optimization in real-time. Note also, that information related to traffic variables and dynamics are compressed in the disturbance invariant set, appearing as linear constraint in the optimization (24). Simulation results prove that the proposed algorithm provides a robust controller with good performance.

Appendix A. Model parameters and steady states

Table 2. METANET model parameters

a	v_{free}	ρ_{cr}	τ	η	κ	δ	L	λ	r_{min}, r_{max}	T	v_{ramp}
2.44	116.34	24.26	0.0036	24.29	10.85	0.7	1	2	[360, 2000]	0.0028	50

Table 3. COPERT model paramers and steady state values

α^{CO_2}	β^{CO_2}	γ^{CO_2}	δ^{CO_2}	ε^{CO_2}	ρ^*	v^*	l^*	q_{us}^*	v_{us}^*	ρ_{ds}^*	$r^{*=w^*}$	s^*
401	0	-8.21	0	0.07	30.15	61.53	10	2567.52	75.56	24.26	1180	0

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