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Gao, Y., Driouchi, T., Bennett, D. (2018). Ambiguity aversion in buyer-seller relationships: A contingent-claims and social network
explanation. International Journal of Production Economics, 200: 50-67.
http://dx.doi.org/10.1016/j.ijpe.2018.02.004

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Accepted Manuscript

International Journal of Production Economics

# Ambiguity Aversion in Buyer-Seller Relationships: A Contingent-Claims and Social Network Explanation

Yongling Gao
Business School, Central University of Finance and Economics, Beijing, China

Tarik Driouchi
Department of Management, King's College London, London, UK

and

David Bennett
Department of Technology Management and Economics, Chalmers University of Technology, Gothenburg, Sweden
Aston Business School. Aston University, Birmingham, UK

**Ambiguity Aversion in Buyer-Seller Relationships:**
**A Contingent-Claims and Social Network Explanation**

**Abstract**

Negotiations between buyers and sellers (or suppliers) of goods and services have become increasingly important due to the growing trend towards international purchasing, outsourcing and global supply networks together with the high uncertainty associated with them. This paper examines the effect of ambiguity aversion on price negotiations using multiple-priors-based real options with non-extreme outcomes. We study price negotiation between a buyer and seller in a dual contingent-claims setting (call option holding buyer vs. put option holding seller) to derive optimal agreement conditions under ambiguity with and without social network effects. We find that while higher ambiguity aversion raises the threshold for commitment for the seller, it has equivocal effects on the buyer's negotiation prospects in the absence of network control. Conversely when network position and relative bargaining power are accounted for, we find the buyer's implicit price (or negotiation threshold) decreases (or increases) unequivocally with increasing aversion to ambiguity. Extending extant real options research on price negotiation to the case of ambiguity, this set of results provides new insights into the role of ambiguity aversion and network structures in buyer-seller relationships, including how they influence the range of negotiation agreement between buyers and sellers. The results also help assist managers in formulating robust buying/selling strategies for bargaining under uncertainty. By knowing their network positions and gathering background information or inferring the other party's ambiguity tolerance beforehand, buyers and sellers can anticipate where the negotiation is heading in terms of price negotiation range and mutual agreement possibilities.

**Keywords:** buyer-seller relationships; real options; supply networks; social networks; multiple-priors; ambiguity

## 1. Introduction

The relationships between buyers and sellers of goods and services have come under increasing scrutiny in the literature since the results and consequences of negotiations between them can be critical to the competitiveness and integrity of firms operating within international networks. Examples of relevant issues that have been investigated include trust (Schoenherr et al., 2015; Hemmert et al., 2016), transaction costs (Schneider et al., 2013; Abd Rahman et al., 2009), ethics and social responsibility (Goebel et al., 2012; Govindan et al., 2016). In this paper we examine the behavioural issue of *ambiguity*, which is a concern involving both sides during negotiations between buyers and sellers. As a type of uncertainty beyond probabilistic risk, ambiguity characterises commitment and transactional situations where future outcomes are not known with certainty or high confidence (Ellsberg, 1961; Ghosh and Ray, 1997). When faced with ambiguity buyers and sellers are unsure about their future prospects and are doubtful about the probabilities of future events and their subsequent realisations, displaying ambiguity aversion and pessimism (Hazen et al., 2012; Abdellaoui et al., 2015). This is more so in negotiation cases where commitment is irreversible and transactional arrangements are fraught with uncertainty on both sides. The ambiguity aversion bias of each party can distort pricing dynamics resulting in suboptimal relationships between buyers and sellers. Network positions and relative bargaining power are also key to these linkages. This paper studies the effect of ambiguity on price negotiations between buyers and sellers, with and without network control, using real options theory (Trigeorgis, 1996; Driouchi and Bennett, 2012; Charalambides and Koussis, 2017) and social network principles (Braun and Gautschi, 2006). In our research we use the term "seller" because it relates to commercial transactions where price is one of the main criteria used in negotiation. However, in the literature the terms “supplier” and “seller” are often used interchangeably within the same context of buying products and services (Oosterhuis et al, 2013; Esmaeilia and Zeephongsekul, 2010)

Several recent papers have been devoted to the study of the real option value of flexible decision making in buyer-seller relationships, optimal contracting and price negotiation (Li and Kouvelis, 1999; Kamrad and Siddique, 2004; Fotopoulos et al, 2008; Moon et al., 2011). Focusing on buyer-seller interaction and negotiation, Yao et al. (2010) and Jiang et al. (2008, 2010) show how each party’s real options determine contractual outsourcing arrangements under risk whereas Moon et al. (2011) examine the impact of risk-neutral optionality on negotiation performance. Moon et al. (2011) in particular present a bilateral negotiation model under risk-neutrality with optimal selling (buying) rules. They propose the idea of an implicit zone of possible agreement (IZOPA) and obtain negotiation agreement probabilities using real options (i.e. contingent-claims). They find that the negotiation range and probability of agreement between buyers and sellers are narrower in the presence of optionality than in its absence. What is missing from this growing literature, however, is an explicit recognition of the role of individual behaviour or miscalibration and network position in negotiation decisions and, especially, how ambiguity affects option-based price negotiation and its investment outcomes. Given that negotiation exercises are often influenced by ambiguity, behavioural factors and social network effects, it is important to account for negotiators’ beliefs, relational characteristics, psychology and uncertainty preferences (e.g. pessimism) in the decision making process.[1]

Our paper addresses this gap in research by investigating how negotiations between a buyer and seller are affected by their ambiguity and social network position (our ‘Extensions and additional results’ in Section 4 examines the case of multiple sellers). A search of the literature reveals that this is the first paper to integrate real options, ambiguity and social networks principles in bilateral negotiation and buyer-seller interaction. We contribute to extant literature on real options in buyer-seller relationships (e.g. Moon et al., 2011; Zheng and Negenborn, 2015) by providing novel decision-making and production economics insights into how ambiguity aversion and social network effects alter the relationships among uncertainty, real options and price negotiation outcomes. We also add to buyer-seller literature concerned with behaviour, ambiguity and information asymmetry (e.g. Esmaeilia and Zeephongsekul, 2010; Hazen et al., 2012; Schoenherr et al., 2015; Hemmert et al., 2016) by developing new theoretical propositions for empirical research. We analyse the effects of negotiators’ ambiguity aversion on their real options prospects, with and without network control, using a multiple-priors expected utility (MEU) with non-extreme outcomes (hereafter called NMEU) in continuous-time. Adjusting for uncertainty aversion in probabilistic appraisal, this utility specification is related to the maxmin expected utility (MEU) covered in recent ambiguity-based real options research, such as Nishimura and Ozaki (2007), Trojanowska and Kort (2010), and Moreno (2014). The MEU satisfies the dynamic consistency constraint and can reflect both the present value and

[1] The research findings of Ghosh (1994) and Zwick and Lee (1999) go along these lines and suggest that to enhance the descriptive power of negotiation models, risk preferences, information incompleteness and tolerance for ambiguity need to be included in the analysis.

option value effects dominating the timing of commitment but its assumption of complete pessimism might be considered too extreme in a number of cases. Our NMEU framework is also indirectly linked to the $\alpha$-maxmin expected utility ($\alpha$-MEU) used to study infrastructure projects (Gao and Driouchi, 2013), corporate investments (Schröder, 2011) and supplier contracting (Gao, 2017). The $\alpha$-MEU utility is useful in examining the impact of ambiguity attitudes on decision outcomes from the present value perspective but partly suffers from dynamic inconsistency and can result in timing thresholds for either extremely pessimistic or optimistic agents ($\alpha = 0$ or 1). This means that both the MEU and $\alpha$-MEU models are concerned with extreme attitudes towards ambiguity but ignore situations, such as bargaining and price negotiation, where unsure decision makers might still have some confidence in their probability judgments (i.e. realization of their risk-based estimates) while caring about the worst case scenario (i.e. uncertainty aversion).

Motivated by the above decision making issues, we rely on the NMEU heuristic to solve the optimal commitment and flexible timing problem for any level of ambiguity aversion while satisfying dynamic consistency. Our NMEU utility evaluates and combines the worst case in negotiators' minds with the standard probabilistic case. Separating risk from uncertainty (Ellsberg, 1961; Abdellaoui et al., 2015; Agliardi et al., 2015), we present conditions for negotiation agreement under ambiguity and incorporate negotiators' aversion to uncertainty and network position in the real options analysis to show how they affect investment outcomes and optimal agreement[2]. We deliberately do not investigate or discuss the role of risk aversion in the real options dynamics since these effects have been well documented in the literature (see e.g. Henderson and Hobson, 2002; Hugonnier and Morellec, 2007).

We extend uncertainty-neutral findings from recent studies, in particular those of Nagarajan and Bassok (2008), Moon et al. (2011) and Zheng and Negenborn (2015), and the Nash bargaining model considering social network effects by Braun and Gautschi (2006) to the case of ambiguity aversion. We contribute to extant literature on buyer-seller interaction (e.g. Bichescu and Fry, 2009; Birkeland and Tungodden, 2014) by examining the link between ambiguity aversion and mutual agreement while considering negotiators' flexibility and discretion regarding optimal investment choice and contract timing in the negotiation exercise. We find that in the absence of network effects, ambiguity and ambiguity aversion do not necessarily have symmetric effects on negotiation outcomes under the NMEU. This impact is reversed in the presence of network control. Thus, we add realism and generality to the analysis by explicitly allowing for miscalibration in the uncertain negotiation and accounting for the structural positions of buyers and sellers in the supply chain network, and show why standard risk-neutral or normative contingent-claims assessment might be incomplete for the appraisal of commitment situations where true uncertainty, cognition and vagueness determine outcomes and heterogeneous behaviour.

[2] In contrast to extant research on buyer-seller relationships, we do not use "risk" and "uncertainty" interchangeably in this paper. By ambiguity we refer to uncertainty, beyond probabilistic or measurable risk, as defined by Ellsberg (1961) and as discussed in Asano and Shibata (2011) and Nishimura and Ozaki (2007). Our paper is the first real option study to consider such dimension of uncertainty, and aversion towards it, in B2B buyer-seller interaction and price negotiation.

In addition, many real options studies on incomplete information tend to assume that the partial information about the uncertainty variables is generally symmetric (see also Grenadier, 2005; Nishihara and Shibata, 2008; Shibata and Nishihara, 2011; Feng et al., 2014; Grenadier et al., 2016). We consider this information to remain private in our setting and design incentives and signalling mechanisms for the buyer (seller) to elicit the true level of ambiguity aversion of his (or her) counterpart in the presence of information asymmetry. This is documented later in Section 4.3.

The paper is organized as follows. Section 2 presents the real options negotiation problem. Section 2.1 introduces notation, assumptions and our multiple-priors utility specification. Section 2.2 produces the policies for option exercise under ambiguity aversion. Section 2.3 identifies the negotiation's implicit zone of achievable agreement (IZOAA) or negotiation range, studying its optimal conditions, and the effects of ambiguity aversion and probabilistic ambiguity on the threshold for negotiation. In Section 3, we derive the Nash bargaining solution under ambiguity aversion by considering real options, structural autonomy and network positions in the price negotiation in the context of a negatively connected network. Section 4 extends our analysis to account for outside options (e.g. exiting or opting out from the negotiation), multilateral negotiation between one buyer (an assembler) and multiple sellers, and mechanism design and incentives under asymmetric information. The final section concludes with a summary of results and implications. Proofs and additional results are found in the Appendix and Supplementary Material.

## 2. The negotiation problem under ambiguity

### 2.1. Problem description and assumptions

We consider a (bilateral) price negotiation setting in which the decision to buy or sell goods within global supply networks has a long-term impact, incurs sunk costs and is at least partly irreversible. Due to uncertainty, there is a noticeable option value in delaying commitment and keeping options open (Trigeorgis, 1996; Roemer, 2004; Driouchi et al., 2010). Our model is based upon the IZOPA[3] under risk-neutrality of Moon et al. (2011) and Jiang et al. (2008). We extend these authors' findings to the case of ambiguity using a multiple-priors expected utility with non-extreme outcomes (e.g. Chateauneuf et al., 2007). This ambiguity-based utility specification is equivalent to a weighted average between a risk-neutral utility and the minimal outcome of a multiple-priors utility (i.e. worst case scenario) (see e.g. Chateauneuf et al., 2007; Fonseca and Rustem, 2012). This subjective utility should be more reflective of cognitive or behavioural biases affecting buyer-seller assessments than those of rational or uncertainty-neutral counterparts. In the bilateral price negotiation, the seller (called *she*) is uncertain about the costs of producing a certain good to be sold at a price $X$ to a buyer (called *he*) who is uncertain about the future revenues generated by investing in $X$. A typical representation/illustration of this situation would be the case of two supply chain actors negotiating over the price and potential distribution of a specific good or service. Revenues and costs are difficult to

[3] Their notion of the IZOPA is based on the concepts of negotiation range and contract zone in economics (e.g. Fundenberg and Tirole, 1983).

predict and follow lognormal diffusions with ambiguous drifts but unambiguous volatilities. Despite their ambiguity, both agents have some confidence in their probability judgments. Although interacting, parties do not have private information about counterparties' uncertain quantities. The case of slotting allowances/fees or new product introductions in retailing closely matches this price negotiation model. The buyer (e.g. retailer or wholesaler) holds a call option to exchange *X* for $S_2$ paying *X* in exchange for future revenues $S_2$. The seller (e.g. manufacturer) holds a put option to exchange operating costs $S_1$ for product/contract price *X*. The put (call) option is in the money when $X > S_1$ ($S_2 > X$). Buyers and sellers negotiate based on their individual attitudes towards ambiguity and their sentiment (ambiguity aversion or pessimism) regarding the future fluctuations of their stochastic variables (i.e. revenues for the buyer and costs for the seller, respectively). Section 3 adds a social network dimension to this problem. For the remainder of the paper, a seller's (buyer's) ambiguity aversion will refer to situations where the seller (buyer) is pessimistic about their operating costs (revenues).

Table 1. Variable definitions

| Notation | Definition |
|---|---|
| $i$ | $i$ = 1 and 2 denote the seller and buyer. |
| $X$ | Negotiation price that connects the buyer and seller. |
| $X_i$, $X_i^{nc}$ | $X_1$ ($X_2$) are the seller's (buyer's) implicit reservation prices. $X_i^{nc}$ is the implicit reservation price with network control for each party. |
| $S_i$ | $S_1$ and $S_2$ are the seller's costs and buyer's revenues. |
| $\mu_i$, $\sigma_i$ | $\mu_i$ and $\sigma_i$ are the growth rate and volatility of $S_i$. $\sigma_i > 0$. |
| $\kappa_i$ | $\kappa_i$ is the probabilistic ambiguity surrounding the drift term of $S_i$, $\kappa_i \geq 0$. When $\kappa_i = 0$, the corresponding geometric Brownian motion of $S_i(t)$ is denoted by $\tilde{S}_i(t)$. |
| $\rho_i$ | $\rho_1$ and $\rho_2$ reflect the degrees of ambiguity aversion regarding seller's expected costs $S_1$ and buyer's expected revenues $S_2$, respectively. $\rho_i \in [0,1]$. |
| $W_i(t)$ | The NMEU of $S_i(t)$ under ambiguity. |
| $\lambda_i$ | $\lambda_i$ is the NMEU-based ambiguity multiplier. It connects $S_i$ at time $t$ and the subjective expected value $W_i(t)$, where $i = 1, 2$. |
| $r$ | $r$ is the discount rate. |
| $\varepsilon$ | The correlation coefficient between $dB_1(t)^{\theta_1}$ and $dB_2(t)^{\theta_2}$. |
| $\xi_{lj}$ and $\gamma_{lj}$ | $\xi_{lj}$ ($\gamma_{lj}$) is $l$'s relative negotiation power with respect to $j$ in a negatively (positively) connected network. We use $\gamma$ to denote each seller's relative negotiation power vis-à-vis the buyer in a positively connected network in Section 4.2. |

$X$ , $X_i$ and $X_i^{nc}$ are the decision variables. Other variables in the table are exogenous and assumed to be constant. The following assumptions are adopted.

**Assumption 1.** The buyer's demand is assumed to be fixed and normalized to 1.

**Assumption 2.1.** In line with standard real options literature (Nembhard et al., 2005; Wu and Liou, 2011; De Waegenaere and Wielhouwer, 2011), costs and revenues follow two separate lognormal diffusions. Due to negotiators' lack of confidence in their probability estimates we consider parametric uncertainty in the drifts of the Brownian motions. This type of vagueness or probabilistic ambiguity determines the level of ambiguity aversion of each negotiating party.

**Assumption 2.2.** The seller's costs ( $S_1$ ) and buyer's revenues ( $S_2$ ) follow ambiguous Brownian motions $B_1(t)$ and $B_2(t)$, which are defined on a probability space $(\Omega, \mathcal{F}_T, \mathbb{P})$. $(\mathcal{F}_t)_{0\le t\le T}$ is a standard filtration for $B_1(t)$ and $B_2(t)$. Ambiguity in the seller's costs ($S_1$) and buyer's revenues ($S_2$) is modelled by the set of priors $\mathcal{P}_i=\{Q_i^{\theta_i} \,|\, \theta_i = \theta_i(t) \in \Theta_i\}$. $Q_i^{\theta_i}$ is derived from the reference probability measure $Q_i$ using the density generator $\theta_i$ (see definitions in Chen and Epstein, 2002; Nishimura and Ozaki, 2007; Riedel, 2009). $\forall \theta_i \in \Theta_i$ are restricted to the non-stochastic range $K_i = [-\kappa_i, \kappa_i]$, where $\kappa_i$ ( $\kappa_i \ge 0$ ) stands for the probabilistic ambiguity surrounding the drift terms of the geometric Brownian motions used to model the seller's costs and buyer's revenues.[4] For any $\theta_i \in \Theta_i$ the Ito processes of $S_1$ and $S_2$ to the general sets $\mathcal{P}_1$ and $\mathcal{P}_2$ yield under ambiguity:

$$dS_i(t) = (\mu_i - \sigma_i\theta_i)S_i(t)dt + \sigma_i S_i(t)dB_i(t)^{\theta_i} \qquad \forall t \ge 0, \forall \theta_i \in \Theta_i, i = 1,2 \tag{1}$$

where $\mu_i - \sigma_i\theta_i$ is the expected growth rate of $S_i$ and $\sigma_i$ its volatility. Parameters $\mu_i$ and $\sigma_i$ are assumed to be constant. $\sigma_i > 0$. The drift term is affected by the ambiguity parameter $\theta_i$, $i = 1,2$. We assume the correlation coefficient between $dB_1(t)^{\theta_1}$ and $dB_2(t)^{\theta_2}$ to be $\varepsilon$. Let $\tilde{S}_i(\tau)$ denote the geometric Brownian motion under the benchmark probability measure $Q_i$. The expectation of $\tilde{S}_i(t)$ under risk-neutrality reflects the case of non-extreme outcomes for the uncertain decision maker.

**Assumption 3.** In their appraisal of economic prospects, the buyer and seller account for both the risk-neutral reward and the minimal/pessimistic outcome of a multiple-priors utility (i.e. combination of risk-neutral and worst case scenarios). Our NMEU specification combines the worst case in negotiators' minds with the risk-neutral outcome, thus adjusting for uncertainty aversion in probabilistic appraisal.

**Assumption 3.1.** For the seller, we use $\rho_1$ with $0 \le \rho_1 \le 1$ to denote her degree of ambiguity aversion or worst case appraisal regarding future operating costs. In line with extant real options and optimal stopping

[4] In practical terms, this implies that the exact rates of return on buyers' and sellers' commitment to a certain price or contractual arrangement are unknown to each party.

research, time horizon $T$ is assumed to approach infinity. The NMEU value of $S_1(t)$ with respect to $Q_1^{\theta_1}$ can be expressed as [5]:

$$W_1(t) = \rho_1 \sup_{\theta_1 \in [-\kappa_1, \kappa_1]} E^{Q_1^{\theta_1}} \left[ \int_t^{\infty} S_1(\tau) e^{-r(\tau - t)} d\tau \middle| \mathcal{F}_t \right] + (1-\rho_1) E\left[ \int_t^{\infty} \tilde{S}_1(\tau) e^{-r(\tau-t)} d\tau \middle| \mathcal{F}_t \right]$$
$$= \lambda_1 S_1(t) \tag{2}$$

where $\lambda_1 = \dfrac{\rho_1}{r - \mu_1 - \sigma_1\kappa_1} + \dfrac{1-\rho_1}{r-\mu_1}$, $\kappa_1 \geq 0$, $\lambda_1$ is the NMEU-based ambiguity multiplier which connects $S_1$ at time $t$ and the subjective value $W_1(t)$, $\lambda_1 \in R$. $r$ is the discount rate, $\mu_1 + \kappa_1\sigma_1 < r$. $E\left[ \int_t^{\infty} \tilde{S}_1(\tau) e^{-r(\tau-t)} d\tau \middle| \mathcal{F}_t \right]$ corresponds to the risk-neutral expectation of $S_1(\tau)$. When $\rho_1 = 1$, the NMEU value coincides with the maxmin heuristic of Gilboa and Schmeidler (1989) or the case of pure pessimism or extreme ambiguity aversion.

**Assumption 3.2.** For the buyer, we use $\rho_2$ ($\rho_2 \in [0,1]$) to denote his degree of ambiguity aversion about future revenues which reflects the weight attributed to the worst case for investment. Consequently under $\kappa_2$-ignorance, the NMEU value of $S_2(t)$ can be written as:

$$W_2(t) = \rho_2 \inf_{\theta_2 \in [-\kappa_2, \kappa_2]} E^{Q_2^{\theta_2}} \left[ \int_t^{\infty} S_2(\tau) e^{-r(\tau - t)} d\tau \middle| \mathcal{F}_t \right] + (1-\rho_2) E\left[ \int_t^{\infty} \tilde{S}_2(\tau) e^{-r(\tau-t)} d\tau \middle| \mathcal{F}_t \right]$$
$$= \lambda_2 S_2(t) \tag{3}$$

where $\lambda_2$ is the NMEU-based ambiguity multiplier of $S_2(t)$ incorporating buyer's attitude towards ambiguity, $\lambda_2 = \dfrac{\rho_2}{r - \mu_2 + \sigma_2\kappa_2} + \dfrac{1-\rho_2}{r-\mu_2}$. In the absence of ambiguity ($\rho_2 = 0$ or $\kappa_2 = 0$), $W_2(t)$ simplifies to a risk-adjusted perpetuity.

$\rho_1$ and $\rho_2$ consider the trade-off in negotiators' minds between the worst and risk-neutral scenarios and represent the degrees of ambiguity aversion of sellers and buyers towards price negotiation. Each parameter depends on individual ambiguity attitudes and is reflective of subjective beliefs about the accuracy of probability estimates. $\rho_i$ should help determine the direction of the negotiation process and its outcomes in terms of negotiation range and mutual agreement occurrence.

## 2.2. Ambiguity and buyer-seller real options

The seller's (e.g. a manufacturer) problem is to determine the optimal selling conditions to maximize her opportunity value under NMEU ambiguity $F_1(t)$:

[5] The derivation of the supremum of these costs can be easily obtained based on Nishimura and Ozaki (2007).

$$F_1(t)=\max_{t'\geq t}\left\{\rho_1\left[e^{-r(t'-t)}X-\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]\right]+(1-\rho_1)\left[e^{-r(t'-t)}X-E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]\right]\right\} \quad (4)$$

Then her put opportunity value $F_1(t)$ can be expressed as:

$$F_1(t)=\max_{t'\geq t}\left\{X-W_1(t),\, J(t')\right\} \quad (5)$$

where $J(t')=\max_{t'\geq t+dt}\left\{e^{-r(t'-t)}X-\left[\rho_1\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]+(1-\rho_1)E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]\right]\right\}$.

The seller's put option value under the NMEU ambiguity specification can be written as (see the recursive structure/properties of the option value and derivation of the solution in Appendix A):

$$F_1(W_1(t))=\begin{cases}A_1\lambda_1^{\beta_1}S_1(t)^{\beta_1} & \text{if} \quad S_1(t)>S_1^* \\ X-\lambda_1 S_1(t) & \text{if} \quad S_1(t)\leq S_1^*\end{cases} \quad (6)$$

where $S_1^*=\dfrac{\beta_1}{(\beta_1-1)\lambda_1}X$, $A_1=-\dfrac{\left(\lambda_1 S_1^*\right)^{1-\beta_1}}{\beta_1}$, $\lambda_1=\dfrac{\rho_1}{r-\mu_1-\sigma_1\kappa_1}+\dfrac{1-\rho_1}{r-\mu_1}$, $\beta_1=\dfrac{1}{2}-\dfrac{\zeta_1}{\sigma_1^2}-\sqrt{\left(\dfrac{1}{2}-\dfrac{\zeta_1}{\sigma_1^2}\right)^2+\dfrac{2r}{\sigma_1^2}}<0$,

$\zeta_1=\rho_1(\mu_1+\sigma_1\kappa_1)+(1-\rho_1)\mu_1$. $A_1$, $\lambda_1$ and $\beta_1$ are constants.

Different from the option value derived by Moon et al. (2011) under risk-neutrality, Eq. (6) accounts for the effect of ambiguity on decision making and shows how option exercising properties are affected by sellers' subjective beliefs $\lambda_1$, probabilistic ambiguity $\kappa_1$ and ambiguity aversion $\rho_1$. Ignoring such behavioural effects can result in biased investment triggers (e.g. premature commitment or late real option exercise) and suboptimal negotiation decisions if the negotiation process is characterized by vagueness and information incompleteness. $S_1^*$ is the critical trigger value of selling the good under ambiguity. The seller exercises the put option only when costs $S_1(t)\leq S_1^*$.

The buying opportunity value under ambiguity $F_2(t)$ is:

$$F_2(t)=\max_{t'\geq t}\left\{\rho_2\left[\inf_{\theta_2\in\Theta_2}E^{Q_2^{\theta_2}}\left[\int_{t'}^{\infty}S_2(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]-e^{-r(t'-t)}X\right]+(1-\rho_2)\left[E\left[\int_{t'}^{\infty}\tilde{S}_2(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]-e^{-r(t'-t)}X\right]\right\} \quad (7)$$

Using the same logic as above, the buyer's call option value under the NMEU specification can be written as:

$$F_2(W_2(t))=\begin{cases}A_2\lambda_2^{\beta_2}S_2(t)^{\beta_2} & \text{if} \quad S_2(t)<S_2^* \\ \lambda_2 S_2(t)-X & \text{if} \quad S_2(t)\geq S_2^*\end{cases} \quad (8)$$

where $S_2^*=\dfrac{\beta_2}{(\beta_2-1)\lambda_2}X$, $\lambda_2=\dfrac{\rho_2}{r-\mu_2+\sigma_2\kappa_2}+\dfrac{1-\rho_2}{r-\mu_2}$, $A_2=\dfrac{(\lambda_2 S_2^*)^{1-\beta_2}}{\beta_2}$, $\beta_2=\dfrac{1}{2}-\dfrac{\zeta_2}{\sigma_2^2}+\sqrt{\left(\dfrac{1}{2}-\dfrac{\zeta_2}{\sigma_2^2}\right)^2+\dfrac{2r}{\sigma_2^2}}>1$,

$\zeta_2=\rho_2(\mu_2-\sigma_2\kappa_2)+(1-\rho_2)\mu_2$. $S_2^*$ is the critical trigger value of buying the good under ambiguity.

The buyer will exercise the call option only when revenues $S_2(t) \geq S_2^*$. Otherwise, he will delay commitment until $S_2(t) \geq S_2^*$. Here again, exercise conditions and option value are influenced by the buyer's subjective beliefs $\lambda_2$ and ambiguity parameters $\rho_2$ and $\kappa_2$. Ignoring the interaction effects of these variables on option value can result in erroneous investment outcomes (e.g. premature exercise, late commitment or an impasse) if the negotiation is fraught with ambiguity. Option value and optimal exercise policies are affected by the buyer's ambiguity aversion and probabilistic ambiguity through individual and subjective factors $\beta_2$ and $\lambda_2$. Considering these cognitive factors in the uncertain price negotiation allows us to know how the worst case in the negotiator's mind affects her/his judgment about the timing of commitment and the likelihood of mutual agreement.

Using the above results (eqs. (6) and (8)), we next identify the implicit zone of achievable agreement (IZOAA) under ambiguity and its existence conditions, and study the effect of ambiguity aversion on the price negotiation range and the thresholds for mutual agreement. The IZOAA corresponds to the range of negotiation where buyers and sellers are likely to reach agreement and avoid impasse.

### 2.3. Ambiguity and the negotiation range

From eqs. (6) and (8), the seller (buyer) will agree to sell (buy) when $S_1(t) \leq S_1^* = \dfrac{X}{(1-1/\beta_1)\lambda_1}$ ($S_2(t) \geq S_2^* = \dfrac{X}{(1-1/\beta_2)\lambda_2}$). We refer to $X_1$ and $X_2$ as implicit reservation prices for the negotiating seller and buyer. For given $S_1(t)$ and $S_2(t)$:

$$X_1 = (1-1/\beta_1)\lambda_1 S_1(t) \leq X \tag{9}$$

$$X_2 = (1-1/\beta_2)\lambda_2 S_2(t) \geq X \tag{10}$$

The optimal buying and selling strategies are to sell when $X_1 \leq X$ and to buy when $X_2 \geq X$. Thus, the region $[X_1, X_2]$ stands for the IZOAA or negotiation range under ambiguity for buyers and sellers. It nests the risk-neutral IZOPA found in prior studies. Setting implicit reservation prices can help initiate a profitable relationship between parties and capture their expectations of costs and revenues under ambiguity aversion. This aids in establishing whether the negotiation is successful or not. Given $X_1$, a higher contract price $X$ will generate higher benefits for the seller. For the buyer to make a profitable investment, $X_2$ should exceed the contract price $X$. Proposition 1 summarizes how these implicit reservation prices are affected by changes in ambiguity aversion $\rho_i$ and probabilistic ambiguity $\kappa_i$, where $i = 1,2$.

***Proposition 1.*** **(The effect of ambiguity on implicit reservation prices)**

**An increase in the seller's ambiguity aversion $\rho_1$ (probabilistic ambiguity $\kappa_1$) increases her implicit reservation price $X_1$ when $\kappa_1 > 0$ ( $\rho_1 > 0$ ). The effect of the buyer's ambiguity aversion $\rho_2$ (probabilistic ambiguity $\kappa_2$ ) on his implicit reservation price $X_2$ is equivocal. If probabilistic ambiguity $\kappa_i$=0 (aversion $\rho_i$=0), changes in $\rho_i$ ($\kappa_i$) will not affect the implicit reservation price $X_i$, where $i = 1, 2$. (See the proof in Appendix B).**

$$\begin{cases} \dfrac{\partial X_1}{\partial \rho_1} > 0 & \text{if } \kappa_1 > 0 \\ \dfrac{\partial X_1}{\partial \rho_1} = 0 & \text{if } \kappa_1 = 0 \end{cases} ; \begin{cases} \dfrac{\partial X_1}{\partial \kappa_1} > 0 & \text{if } \rho_1 \in (0,1] \\ \dfrac{\partial X_1}{\partial \kappa_1} = 0 & \text{if } \rho_1 = 0 \end{cases} .$$

$$\frac{\partial X_2}{\partial \rho_2} \gtreqless 0 \text{ and } \frac{\partial X_2(\kappa_2 = 0)}{\partial \rho_2} = 0; \ \frac{\partial X_2}{\partial \kappa_2} \gtreqless 0 \text{ and } \frac{\partial X_2(\rho_2 = 0)}{\partial \kappa_2} = 0.$$

As shown in Figure 1, Proposition 1 implies that in the presence of ambiguity ($\kappa_1 > 0$) the seller will ask for a higher price (willingness-to-accept) for the good produced or to be delivered if she is more ambiguity averse about her future operating costs ($\partial X_1 / \partial \rho_1 > 0$). This result is logical and in accord with risk aversion and maxmin dynamics. Similarly, a positive relationship exists between the seller's probabilistic ambiguity $\kappa_1$ and her implicit reservation price $X_1$ under increasing uncertainty aversion ($\rho_1 > 0$). This suggests that despite the presence of optionality, the IZOAA can become wider or narrower under ambiguity with changing aversion and that the risk-neutral IZOPA is likely to overstate mutual agreement prospects if the seller is increasingly pessimistic about her costs. While individual behaviour is key to negotiation outcomes, risk-neutral analysis ignores its effects on optionality and neglects the multiplicity of the IZOAA under ambiguity.

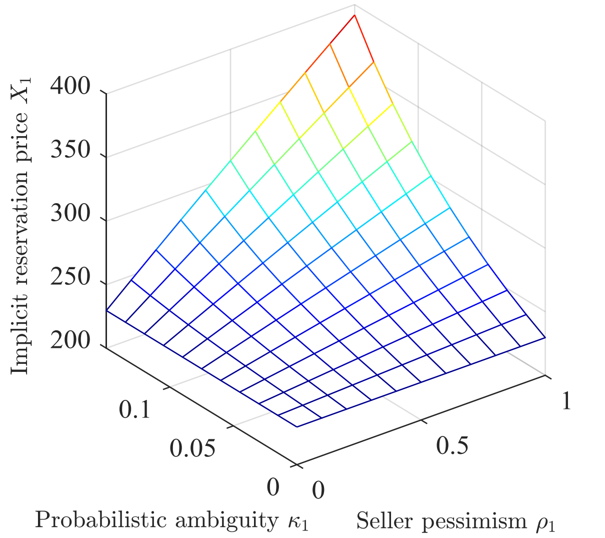



Fig. 1 Effects of seller's ambiguity on her implicit reservation price.
Here $(r, \mu_1, \sigma_1, S_1)$ = (0.08, 0.03, 0.15, 9)

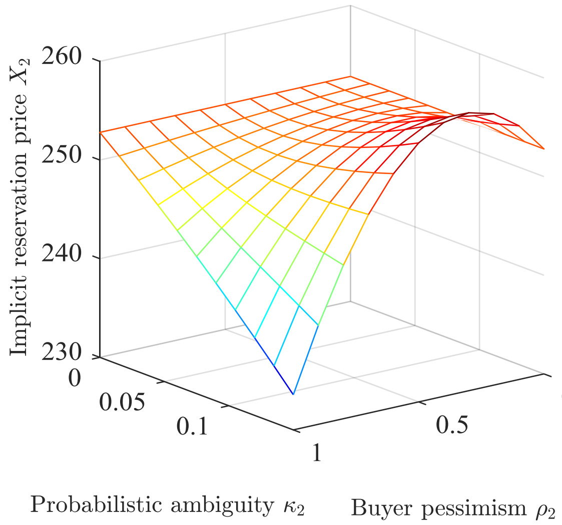



Fig. 2 Effects of buyer's ambiguity on his implicit reservation price.
Here $(r, \mu_2, \sigma_2, S_2)$ = (0.08, 0.04, 0.15, 25)

Proposition 1 states, on the other hand, that the buyer's ambiguity aversion $\rho_2$ has an equivocal and non-monotonic effect on the implicit reservation price $X_2$ under probabilistic ambiguity $\kappa_2$, as illustrated in Figure 2. This can be explained by the specific properties of $\beta_2$ in Eq. (8) and the potential interaction effects of $r$, $\mu_2$ and $\sigma_2$ on optimal timing dynamics. Such effects should be more pronounced under ambiguity. The resulting nonlinear association highlights the role of uncertainty (beyond just risk) on price negotiation outcomes and shows that the buyer's implicit price does not have to decrease with higher ambiguity aversion in the NMEU ambiguity specification. $\kappa_2$ also has an equivocal effect on the buyer's implicit reservation price $X_2$ under ambiguity aversion (i.e. when $\rho_2$ is greater than zero). In other words, an optimistic buyer can potentially decrease his implicit reservation price and delay commitment in the presence of ambiguity later than a more pessimistic buyer. This implies that the IZOAA will not necessarily be narrower with higher ambiguity aversion from the buyer. This underlines the asymmetric effect of NMEU-based ambiguity on the negotiation's prospects. Under the standard MEU specification, buyer's willingness-to-pay (WTP) would be negatively related to ambiguity aversion as predicted by Hazen et al. (2012) in the context of remanufacturing. Changes in ambiguity aversion $\rho_i$ (probabilistic ambiguity $\kappa_i$) do not affect the implicit reservation prices $X_i$ under risk-neutrality $\kappa_i = 0$ ($\rho_i = 0$), where $i = 1, 2$. The IZOPA is unique only in the absence of ambiguity.

We now turn to the effects of ambiguity aversion and ambiguity on the threshold for negotiation (i.e. joint options' exercise policy). Two parties involved in negotiation can reach agreement under ambiguity when the following condition is satisfied:

$$S_2(t)/S_1(t) \geq \delta_{KK} \tag{11}$$

where $\delta_{KK}=\frac{(1-1/\beta_1)\lambda_1}{(1-1/\beta_2)\lambda_2}$, $\delta_{KK}$ denotes the ambiguity-based negotiation threshold of the ratio[6] of the buyer's revenues ($S_2(t)$) to the seller's costs ($S_1(t)$) when call and put options are exercised. The threshold $\delta_{KK}$ represents the minimum profit space for the negotiation to succeed and agreement to occur. Proposition 2 summarizes how this threshold is affected by changes in ambiguity aversion $\rho_i$ and probabilistic ambiguity $\kappa_i$.

***Proposition 2.*** **(The effect of ambiguity on the negotiation threshold)**

**An increase in the seller's ambiguity aversion $\rho_1$ (probabilistic ambiguity $\kappa_1$) increases the negotiation threshold $\delta_{KK}$ if $\kappa_1>0$ ($\rho_1>0$). The effect of the buyer's ambiguity aversion $\rho_2$ (probabilistic ambiguity $\kappa_2$) on the negotiation threshold $\delta_{KK}$ is equivocal. When $\kappa_i$=0 ($\rho_i$=0), changes in $\rho_i$ ($\kappa_i$) do not affect the negotiation threshold $\delta_{KK}$, where $i=1,2$. (See the proof in Appendix B).**

$$\begin{cases}\frac{\partial\delta_{KK}}{\partial\rho_1}>0 & \text{if } \kappa_1>0\\ \frac{\partial\delta_{KK}}{\partial\rho_1}=0 & \text{if } \kappa_1=0\end{cases};\quad \begin{cases}\frac{\partial\delta_{KK}}{\partial\kappa_1}>0 & \text{if } \rho_1\in(0,1]\\ \frac{\partial\delta_{KK}}{\partial\kappa_1}=0 & \text{if } \rho_1=0\end{cases}.$$

$$\frac{\partial\delta_{KK}}{\partial\rho_2}\gtreqless 0 \text{ and } \frac{\partial\delta_{KK}(\kappa_2=0)}{\partial\rho_2}=0;\ \frac{\partial\delta_{KK}}{\partial\kappa_2}\gtreqless 0 \text{ and } \frac{\partial\delta_{KK}(\rho_2=0)}{\partial\kappa_2}=0.$$

Proposition 2 implies that under probabilistic ambiguity ($\kappa_1>0$), the threshold for joint options' exercise will be higher the higher the seller's ambiguity aversion ($\partial\delta_{KK}/\partial\rho_1>0$). Figure 3 illustrates this monotonic effect. The positive association also holds between the seller's probabilistic ambiguity $\kappa_1$ and the negotiation threshold $\delta_{KK}$ with higher uncertainty aversion. The negotiation process becomes more difficult if the seller is more pessimistic about her costs. On the other hand, and in line with the nonlinear effects highlighted in Proposition 1, higher ambiguity aversion from the buyer will not necessarily increase the negotiation threshold or likelihood of an impasse. This result differs from the one obtained by standard or uncertainty-neutral contingent-claims analysis and can be attributed to the asymmetric properties of the NMEU framework.

When $\kappa_1$=0 and $\kappa_2$=0, $\delta_{KK}$ reduces to the risk-neutral negotiation threshold $\delta_{RR}=\frac{(1-1/\beta_{1R})(r-\mu_2)}{(1-1/\beta_{2R})(r-\mu_1)}$,

[6] See Golan (2009) for an illustration of the negotiation threshold in employment contracts.

where $\beta_{1R}=\frac{1}{2}-\frac{\mu_1}{\sigma_1^2}-\sqrt{\left(\frac{1}{2}-\frac{\mu_1}{\sigma_1^2}\right)^2+\frac{2r}{\sigma_1^2}}<0$, $\beta_{2R}=\frac{1}{2}-\frac{\mu_2}{\sigma_2^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu_2}{\sigma_2^2}\right)^2+\frac{2r}{\sigma_2^2}}>1$.

Let $\delta_{KR}$ denote the negotiation threshold with $\kappa_1 \geq 0$ and $\kappa_2=0$ and $\delta_{RK}$ denote the negotiation threshold with $\kappa_1=0$ and $\kappa_2 \geq 0$. The joint effects of ambiguity aversion $\rho_i$ and ambiguity $\kappa_i$ on these thresholds are illustrated in Figures 3-4. Higher ambiguity aversion $\rho_1$ (greater probabilistic ambiguity $\kappa_1$) from the seller results in a higher threshold $\delta_{KR}$ when $\kappa_1>0$ ($\rho_1>0$) as shown in Figure 3. On the other hand, increasing aversion to uncertainty (greater probabilistic ambiguity) from the buyer does not induce a higher threshold $\delta_{RK}$. Figure 4 illustrates these equivocal effects. The role played by $r$, $\mu$ and $\sigma$, and their interactions, in optimal timing dynamics are more important for call prospects than for puts under NMEU ambiguity. When $\kappa_1=0$ or $\rho_1=0$ ($\kappa_2=0$ or $\rho_2=0$), $\delta_{KR}$ ($\delta_{RK}$) is not affected by individual behaviour as shown in Figures 3-4. Ignoring the interaction effects of ambiguity aversion and probabilistic ambiguity on real options dynamics in incomplete information settings, such as those of price negotiation, misses out the behavioural and subjective elements of buyer-seller interaction.

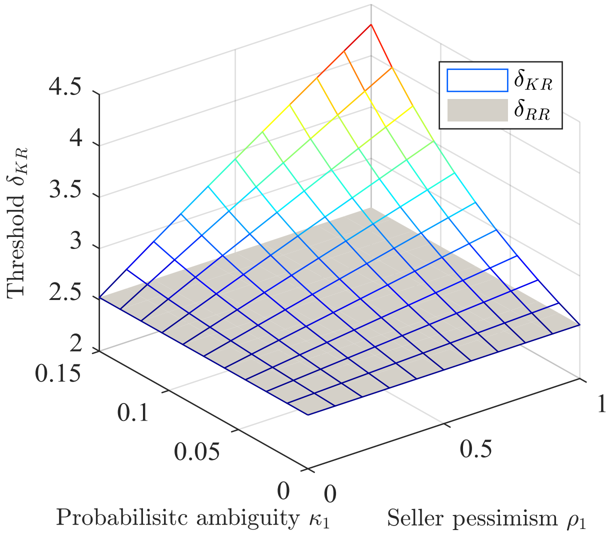



Fig. 3 Effects of seller's ambiguity on the negotiation threshold $\delta_{KR}$.
Here $(r,\mu_1,\sigma_1,\mu_2,\sigma_2)=(0.08,0.03,0.15,0.04,0.15)$

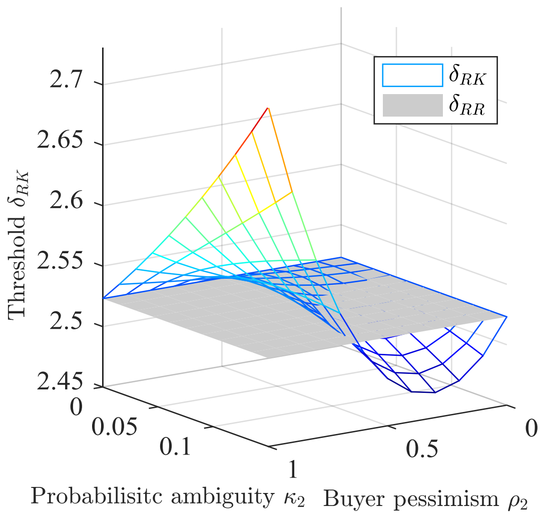



Fig. 4 Effects of buyer's ambiguity on the negotiation threshold $\delta_{RK}$.
Here $(r,\mu_1,\sigma_1,\mu_2,\sigma_2)=(0.08,0.03,0.15,0.04,0.15)$

The relationship between the threshold under ambiguity $\delta_{KK}$ and its risk-neutral counterpart $\delta_{RR}$ can be expressed as follows:

$$\begin{cases} \delta_{KK} \geq \delta_{RR} & \text{if} \quad \dfrac{(1-1/\beta_1)\lambda_1}{(1-1/\beta_2)\lambda_2} \geq \dfrac{(1-1/\beta_{1R})(r-\mu_2)}{(1-1/\beta_{2R})(r-\mu_1)} \\ \delta_{KK} < \delta_{RR} & \text{if} \quad \dfrac{(1-1/\beta_1)\lambda_1}{(1-1/\beta_2)\lambda_2} < \dfrac{(1-1/\beta_{1R})(r-\mu_2)}{(1-1/\beta_{2R})(r-\mu_1)} \end{cases} \tag{12}$$

Eq. (12) highlights the role of subjective beliefs, in the form of the NMEU-based ambiguity multiplier, and ambiguity aversion in shaping mutual agreement. For agreement to be reached under ambiguity, the threshold will generally differ from $\delta_{RR}$ confirming that rational option pricing assumptions can lead to inflexible and suboptimal outcomes if individual behaviour, miscalibration and subjective beliefs are not accounted for in the uncertain negotiation. The above dynamics and propositions are based on the realistic setup that neither the buyer nor the seller is knowledgeable about other party's cost and revenue patterns or ambiguity parameters. We address this issue of asymmetric information in the presence of ambiguity aversion and its decision making implications in Section 4.3. Appendix C covers how the subjective probability of agreement is affected by ambiguity.

Overall the above results demonstrate that although optionality is a key element of negotiation when making purchases within supply networks, the effects of ambiguity aversion and ambiguity on investment outcomes are equally important. Normative/rational predictions on the effects of uncertainty on implicit prices are indeed challenged when ambiguity, and aversion towards it, is part of the negotiation process. This is not surprising since psychology and the heterogeneity of individual attitudes towards uncertainty are known to significantly shape the direction of buyer-seller interaction (Ghosh, 1994; Moran and Ritov, 2002). Standard contingent-claims research on buyer-seller relationships and capital investment (e.g. Jiang et al., 2008; Driouchi et al., 2009 and Moon et al., 2011) neglects these so-called biases and frictional effects. We add realism to this literature by highlighting the moderating effects of ambiguity aversion and probabilistic ambiguity on real options dynamics and buyer-seller interaction. We find that under the NMEU ambiguity specification, uncertainty can have an asymmetric association with price negotiation outcomes. We also show that ambiguity aversion consistently influences negotiation performance when interacting parties, conscious of their options to delay commitment and negotiation agreement, are faced with ambiguity and information incompleteness.

While linear structures are useful for the analysis of bilateral negotiation and buyer-seller relationships, extant real options models omit to account for the relative positions and level of structural embeddedness of buyers and sellers in the supply chain network. As they influence both behaviour and strategy (Borgatti and Li, 2009; Kim et al., 2011), these factors should also play a role in negotiation dynamics. The next section adds a social network dimension to our negotiation problem under ambiguity.

## 3. The negotiation problem with network control and ambiguity

Building on the work of Braun and Gautschi (2006) on Nash bargaining solutions in social networks, we account for the bargaining power of each party, based on their network position and relational features, in the uncertain negotiation. Consider an exogenous network, with the set of nodes $\mathbb{N} = \ 1, 2, \cdots, n$ and $m$ mutual ties in which the seller and buyer are embedded in. In this network, bargaining and exchange relations always coincide and negotiators have their own "network control". In line with Braun and Gautschi

(2006), we adopt the following assumption.

**Assumption 4.** The seller's (or buyer's) relative negotiation power results from her (or his) network position in the network.[7]

The connectedness representation of the network is given by its $N \times N$ adjacency matrix $\mathbf{A}$. The main diagonal elements of this matrix are equal to zero, i.e., $a_{ll} = 0$, $l \in \mathbb{N}$. The relationship between members $l$ and $j$ is defined as follows: $a_{lj} = a_{jl} = \begin{cases} 1 & \text{if there is a mutual tie between member } l \text{ and } j \\ 0 & \text{otherwise} \end{cases}$, where $l \neq j$ and $l, j \in \mathbb{N}$. The binary variable $a_{lj}$ reflects whether $l$ is connected with $j$. The normalized adjacency matrix $\mathbf{A}$ is denoted by the relational matrix $\boldsymbol{R}$ with main diagonal elements $\alpha_{ll} = 0$ for all $l \in \mathbb{N}$. Its off-diagonal element $\alpha_{lj}$ is derived as follows: $\alpha_{lj} = a_{lj} \Big/ \sum_{k=1}^{n} a_{kj}$ for $l, j, k \in \mathbb{N}$, where $\alpha_{lj}$ denotes $l$'s level of "control" over $j$ in the network and $0 \le \alpha_{lj} \le 1$. The $l$th row of the matrix $\boldsymbol{R}$ reflects $l$'s control over others. $\alpha_{lj} = 1\,(\alpha_{lj} = 0)$ means $l$ has full (no) control over $j$.

To reflect how much power $l$ has over other network members, the mean of $l$'s control (i.e., $l$'s network control level) over other parties in the network is defined as:

$$c_l = \frac{1}{n_l} \sum_{k=1}^{n} \alpha_{lk} \tag{13}$$

where $n_l$ denotes $l$'s number of bargaining partners, $c_l$ is $l$'s network control level. $0 < c_l \le 1$.

We examine a network with negative connections in this section. The case with positive connections is covered in Section 4.2.[8] The connection between the seller and buyer is negative (positive) if the buyer's exchange of resources with the seller precludes (promotes) transfers from (with) others (Yamaguchi, 1996). According to Binmore (1985) and Braun and Gautschi (2006), $l$'s individual negotiation power (or market concentration) $\varpi_l$ can be defined as:

$$\varpi_l = \begin{cases} -1 \big/ \ln\left(\nu c_l\right) & \text{if } l \text{ faces a negatively connected relation} \\ -1 \big/ \ln\left(1 - \nu c_l\right) & \text{if } l \text{ faces a positively connected relation} \end{cases} \tag{14}$$

Then $l$'s relative bargaining power vis-à-vis $j$ in the network can be calculated by $\dfrac{\varpi_l}{\varpi_l + \varpi_j}$.

[7] For tractability, we study relative bargaining power effects based on network position and using social network dynamics. It should be noted that horizontal competition and cooperation can also affect bargaining power (Sheu and Gao, 2014; Leider and Lovejoy, 2016). For example, when sellers make substitute (complementary) products, they might end up having a lower/higher bargaining power over the buyer. Buyers might also benefit from collective bargaining because of individual purchasing, sourcing or competition (Li, 2012; Heese, 2015). We thank an anonymous referee for this suggestion.

[8] Similar to Braun and Gautschi (2006), we examine networks with either negative or positive relations. Negative (positive) connections are viewed as substitutable (complementary) (see e.g. Yamaguchi (2000) for mixed exchange networks with negative and positive connections).

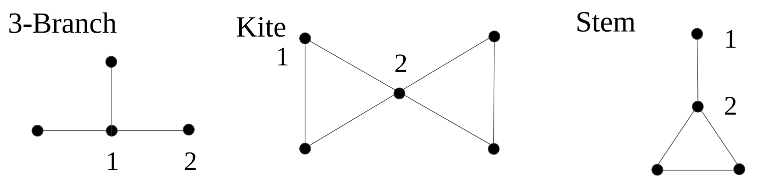
From Eq. (14), $l$'s relative bargaining power can be expressed as:

$$\begin{cases} \xi_{lj} = \dfrac{\ln(\nu c_j)}{\ln(\nu c_l) + \ln(\nu c_j)} & \text{if } l \text{ and } j \text{ negotiate in a negatively connected network} \\ \gamma_{lj} = \dfrac{\ln(1-\nu c_j)}{\ln(1-\nu c_l) + \ln(1-\nu c_j)} & \text{if } l \text{ and } j \text{ negotiate in a positively connected network} \end{cases} \quad \text{for } l \neq j \qquad (15)$$

where $\nu = \dfrac{m+n}{1+m+n}$, $\nu$ reflects the network-specific weight based on existing nodes and mutual ties.

Equation (15) shows that equal network control levels result in similar negotiation power in each network. Also note that $\dfrac{\partial \xi_{lj}}{\partial c_l} > 0$ and $\dfrac{\partial \gamma_{lj}}{\partial c_l} < 0$. This indicates that $l$'s relative bargaining power vis-à-vis $j$ increases (decreases) with $l$'s network control in negatively (positively) connected networks if $\nu$ is unchanged .

Following Section 2, let $i$ = 1 and 2 denote our seller and buyer. The seller's relative negotiation power $\xi_{12}$ is defined based on her relational features and structural position in the network. Figure 5 illustrates examples of typical structures in which the seller and buyer might be embedded in (see Braun and Gautschi, 2006). For instance, the seller's network control level is 1 (1/3) in a 3-branch (stem) network indicating that she has complete (less) control over the buyer in this type of structure.




| 3-Branch | Kite | Stem |
| --- | --- | --- |
| $c_1 = 1, c_2 = 0.3333.$ | $c_1 = 0.375, c_2 = 0.5.$ | $c_1 = 0.3333, c_2 = 0.6667.$ |
| $n = 4, m = 3.$ | $n = 5, m = 6.$ | $n = 4, \quad m = 4.$ |
| $\xi_{12}^{\text{3-Branch}} = 0.90$ | $\xi_{12}^{\text{Kite}} = 0.42$ | $\xi_{12}^{\text{Stem}} = 0.30$ |

Fig. 5 Illustration of network control levels and relative negotiation powers in negatively connected networks

As shown by Braun and Gautschi (2006), the bargaining problem with network control between the seller and buyer can be written as: $I(t) = \max_X (\lambda_2 S_2(t) - X)^{1-\xi_{12}} (X - \lambda_1 S_1(t))^{\xi_{12}}$. Taking the first order of log $I(t)$ with respect to $X$, the contract price is determined as follows:

$$X = (1-\xi_{12})\lambda_1 S_1(t) + \xi_{12}\lambda_2 S_2(t) \qquad (16)$$

Equation (16) shows that the contract price with network control is equal to the weighted sum of the subjective values of $S_1(t)$ and $S_2(t)$ in the non-extreme maxmin expected utility (NMEU) framework.

For a given negotiation power, the buyer's option value is a function of his network control level and ambiguity. The buyer's timing option can be rewritten as:

$$U(S_1(t),S_2(t))=\left(1-\xi_{12}\right)\max_{t'\geq t}\left\{e^{-r(t'-t)}\left(\lambda_2 S_2(t')-\lambda_1 S_1(t')\right),0\right\} \tag{17}$$

For tractability, let $z=S_2(t)/S_1(t)$. $z$ is the ratio of buyer's revenues $S_2(t)$ to seller's costs $S_1(t)$ when put and call options are exercised under ambiguity. The buyer's call option value with social network effects and the corresponding optimal time to purchase can be derived as (see the proof in Supplementary Appendix E):

$$U(S_1(t),S_2(t))=\begin{cases}\left(1-\xi_{12}\right)\left(\lambda_2 S_2(t)-\lambda_1 S_1(t)\right) & \text{if } S_2(t)/S_1(t)\geq z^* \\ d_2\left(S_2(t)\right)^b\left(S_1(t)\right)^{1-b} & \text{if } S_2(t)/S_1(t)< z^*\end{cases} \tag{18}$$

where $z^*=\dfrac{\delta_2 b}{\delta_1\left(b-1\right)}=\dfrac{\lambda_1 b}{\lambda_2\left(b-1\right)}$, $\delta_1=1/\lambda_1$, $\delta_2=1/\lambda_2$, $\lambda_1=\dfrac{\rho_1}{r-\mu_1-\sigma_1\kappa_1}+\dfrac{1-\rho_1}{r-\mu_1}$, $\lambda_2=\dfrac{\rho_2}{r-\mu_2+\sigma_2\kappa_2}+\dfrac{1-\rho_2}{r-\mu_2}$, $\delta_1$ and $\delta_2$ are the convenience yields of the seller and buyer, $d_2=\dfrac{\left(1-\xi_{12}\right)\left(z^*\right)^{1-b}}{\delta_2 b}$, $b=\dfrac{1}{2}-\dfrac{\delta_1-\delta_2}{\sigma_2^2-2\varepsilon\sigma_1\sigma_2+\sigma_1^2}+\chi>1$, $\chi=\sqrt{\left(\dfrac{1}{2}-\dfrac{\delta_1-\delta_2}{\sigma_2^2-2\varepsilon\sigma_1\sigma_2+\sigma_1^2}\right)^2+\dfrac{2\delta_1}{\sigma_2^2-2\varepsilon\sigma_1\sigma_2+\sigma_1^2}}$.

The corresponding put option value with social network effects for the seller is:

$$R(S_1(t),S_2(t))=\begin{cases}\xi_{12}(\lambda_2 S_2(t)-\lambda_1 S_1(t)) & \text{if } S_2(t)/S_1(t)\geq z_1^* \\ d_1(S_2(t))^b(S_1(t))^{1-b} & \text{if } S_2(t)/S_1(t)< z_1^*\end{cases} \tag{19}$$

where $z_1^*=z^*=\dfrac{\delta_2 b}{\delta_1(b-1)}$, $d_1=\dfrac{\xi_{12}\left(z^*\right)^{1-b}}{\delta_2 b}$.

The ratio of buyer revenues $S_2(t)$ to seller costs $S_1(t)$, $z_1^*$ (that is equal to $z^*$) denotes the profit space threshold with network control. Though affected by ambiguity aversion, this behavioural threshold does not seem to account for social network effects. When $z$ is less than $z^*$, the total option value is too low for cooperation or mutual agreement to occur. When $z$ is larger than the threshold $z^*$, it is worth cooperating.

Propositions 3a-b consider the joint effects of ambiguity aversion and relative bargaining power, in terms of relationship characteristics and network position, on implicit reserve prices $X_1^{nc}(t)$ and $X_2^{nc}(t)$. These variables are more likely to be influenced by social network effects than the threshold.

***Proposition 3a.*** **(Implicit reservation prices with network control in negatively connected networks)**

**When the seller and buyer determine the negotiated share of cooperative profits under ambiguity, their reservation prices with social network effects are:**

$$X_1^{nc}(t)=\left[b-1+\frac{\ln(\nu c_2)}{\ln(\nu c_1)+\ln(\nu c_2)}\right]\frac{\lambda_1 S_1(t)}{b-1} \text{ and } X_2^{nc}(t)=\left(b-1+\frac{\ln(\nu c_2)}{\ln(\nu c_1)+\ln(\nu c_2)}\right)\frac{\lambda_2 S_2(t)}{b}.$$

Proposition 3a indicates that ambiguity aversion $\rho_i$ and probabilistic ambiguity $\kappa_i$ still affect the buyer's and seller's implicit reservation prices $X_i^{nc}(t)$ in the presence of social network effects. This is achieved through the option value parameter $b$ and ambiguity multiplier $\lambda_i$. We also find that network control level $c_i$ and network scale $m$ and $n$ influence price negotiation outcomes under ambiguity. This is in accord with social network theory predictions and leads to Proposition 3b.

***Proposition 3b.*** **(The effect of ambiguity on implicit reservation prices in negatively connected networks)**

**The seller's implicit reservation price considering network control $X_1^{nc}$ is increasing in her ambiguity aversion $\rho_1$ (probabilistic ambiguity $\kappa_1$). The buyer's implicit reservation price considering network control $X_2^{nc}$ is decreasing in his ambiguity aversion $\rho_2$ (probabilistic ambiguity $\kappa_2$). (See the proof in Supplementary Appendix F).**

$$\frac{\partial X_1^{nc}(t)}{\partial \rho_1} \geq 0, \ \frac{\partial X_1^{nc}(t)}{\partial \kappa_1} \geq 0, \ \frac{\partial X_2^{nc}(t)}{\partial \rho_2} \leq 0, \ \frac{\partial X_2^{nc}(t)}{\partial \kappa_2} \leq 0.$$

Recall: $X_1^{nc}(t) = (b - 1 + \xi_{12}) \dfrac{\lambda_1 S_1(t)}{b-1}$ and $X_2^{nc}(t) = (b - 1 + \xi_{12}) \dfrac{\lambda_2 S_2(t)}{b}$.

Proposition 3b confirms that while ambiguity aversion $\rho_i$ and probabilistic ambiguity $\kappa_i$ still affect the buyer's and seller's implicit reservation prices $X_i^{nc}(t)$ in the presence of social network effects, network control level $c_i$ and network scale $m$ and $n$ moderate the effects of ambiguity (aversion) on implicit negotiation prices, making the relationships between them unequivocally monotonic for both buyers and sellers. Ambiguity aversion is, hence, negatively (or positively) related to buyer implicit prices for willingness-to-pay (or willingness-to-accept) outcomes when network positions are known (the WTP finding is in line with Hazen et al. (2012) and their Hypothesis 1). This is different from the asymmetric finding without network control of Proposition 2.

For illustration, let us assume that the seller and buyer are in the 3-Branch, Kite and Stem network structures introduced above. Their specific positions are shown in Figure 5. Figures 6-7 highlight the effects of ambiguity aversion, probabilistic ambiguity and social network positions on price negotiation. Figure 6 shows that the seller's implicit reservation price with network control $X_1^{nc}(t)$ increases as her ambiguity aversion $\rho_1$ or probabilistic ambiguity $\kappa_1$ rises. This positive relationship holds in all three network structures. This is consistent with our findings without social network effects (i.e. Proposition 1). We additionally observe that higher relative bargaining power for the seller is associated with even higher reservation prices in all three network structures.

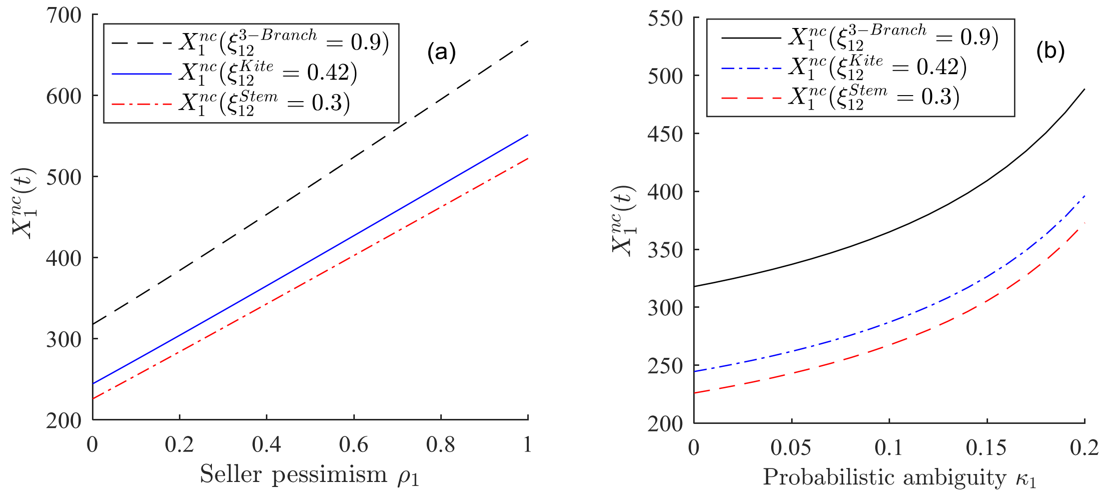



Fig. 6 Effects of seller's ambiguity on her implicit reservation price with network control. Here $S_1(t) = 9$, $r = 0.08$, $\mu_1 = 0.03$, $\mu_2 = 0.04$, $\varepsilon = 0.1$, $\sigma_1 = \sigma_2 = 0.15$, $\rho_2 = 0.5$, $\kappa_2 = 0.2$. In Fig.6.a, $\kappa_1 = 0.2$. In Fig.6.b, $\rho_1 = 0.5$.

On the other hand, a buyer with higher ambiguity aversion or higher probabilistic ambiguity would unequivocally decrease his implicit reservation price in the presence of bargaining power and network control as shown in Figure 7. This is different from the equivocal and nonlinear effects observed in Section 2 for the buyer. This implies that familiarity with the social network structure and understanding of relative bargaining powers provide information advantages to the buyer. The latter can use this information to decide his implicit reservation price unequivocally. The effect of ambiguity aversion becomes akin to that of risk aversion and maxmin MEU ambiguity when social network dynamics are accounted for. In other words, the asymmetric effect of NMEU ambiguity on the buyer's implicit price disappears in the presence of network control. This is explained by the profit sharing-based properties of eqs. (18-19), and by the dominant-negative effect of negotiation power on implicit prices. We indeed observe that higher relative bargaining power for the buyer is associated with lower reservation prices in all three network structures. This means that the narrowness of the IZOAA also depends on network structures.

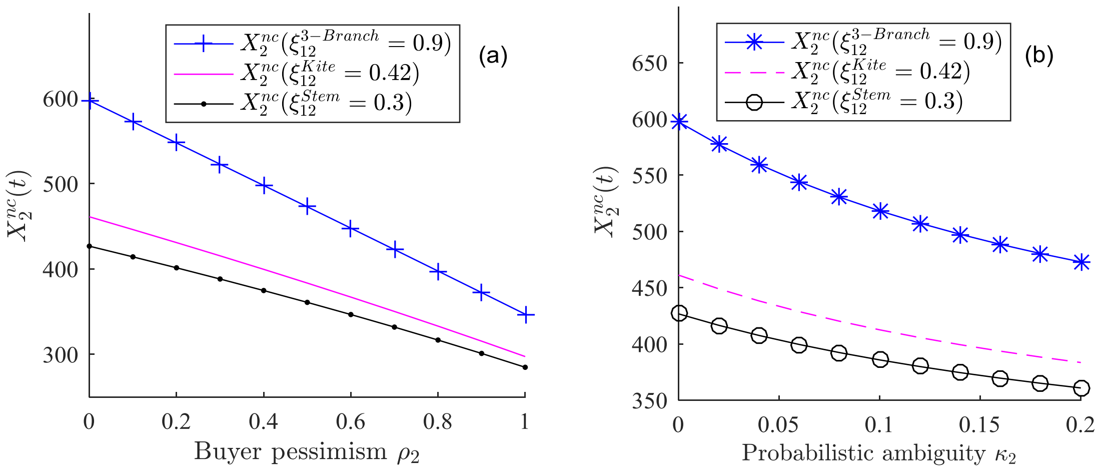



Fig. 7 Effects of buyer's ambiguity on his implicit reservation price with network control. Here $S_2(t) = 25$, $\rho_1 = 0.5$, $\kappa_1 = 0.2$. In Fig.7a, $\kappa_2 = 0.2$. In Fig.7b, $\rho_2 = 0.5$. Other parameter values are the same as in Fig. 6.

Adding new links or nodes to the network structure usually changes the network control levels and the network-specific weights. This increases the difficulty of studying their comparative statics analytically. We observed that higher network control levels tend to increase relative negotiation powers and can create pricing advantages in negatively connected networks (see Figures 5-7). Consider a supply chain network with $n \geq 4$ tiers (e.g. Waters, 2009). Suppose the tier number only affects the seller and buyer through their relative negotiation powers. If the seller and buyer are two of the most upstream entities in the network (see Figure 8a), their network control levels stay unchanged at $c_1$=0.5 and $c_2$=0.75. Thus, $\frac{\partial X_1^{nc}}{\partial \nu} < 0$, $\frac{\partial X_2^{nc}}{\partial \nu} < 0$. In this most upstream case, the wholesaler's relative bargaining power increases because of the addition of an intermediary (branch) in the network. Consequently, both the wholesaler (buyer) and the manufacturer (seller) decrease their implicit reservation prices. This is as if, due to a loss in relative bargaining power, the manufacturer is less ambiguity averse in this new structure.

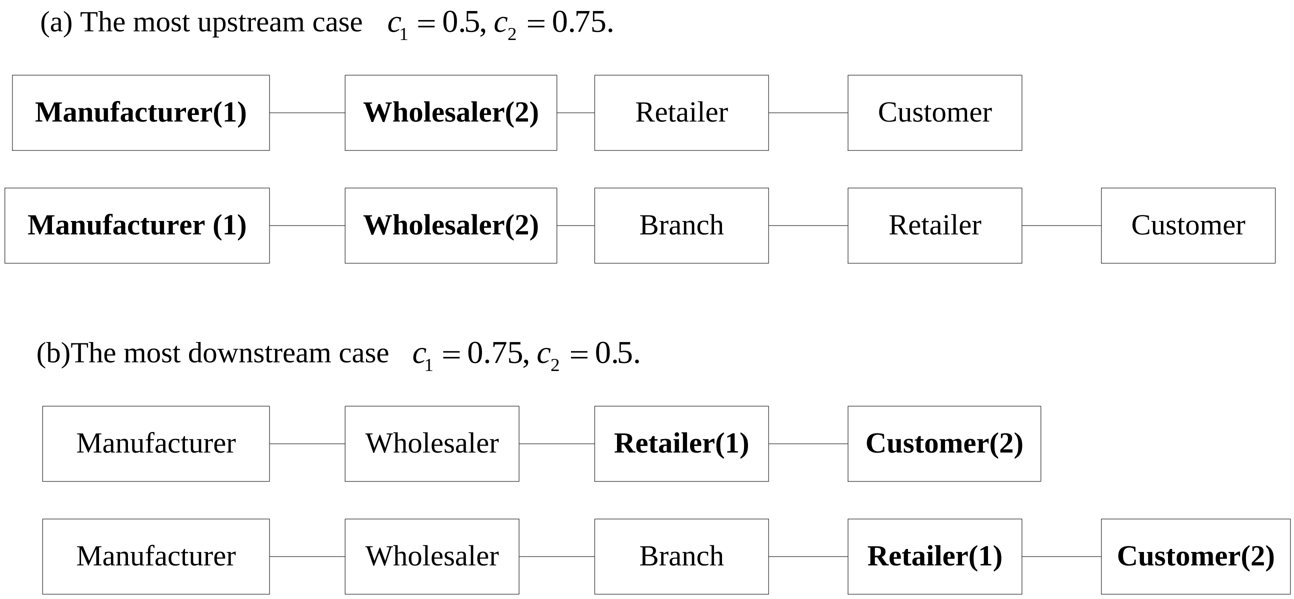



Fig. 8 Example of a narrow supply chain

Using the same logic as above, $\frac{\partial X_1^{nc}}{\partial \nu} > 0$, $\frac{\partial X_2^{nc}}{\partial \nu} > 0$ when the seller and buyer are two of the most downstream entities (see Figure 8b). Adding an intermediary in the network will lead to a higher (lower) relative bargaining power for the retailer (customer). Consequently, both the seller (manufacturing retailer) and buyer (customer) will increase their implicit reservation prices. This is as if the customer is relatively more ambiguity-seeking in this new structure. In the two cases, increasing network nodes and mutual ties strengthens the relative advantages of entities with higher network control levels despite the presence of ambiguity. Social network information might thus help to resolve some of the unknown uncertainty characterising the negotiation process.

## 4. Extensions and additional results

This section extends our previous modelling insights by considering outside options in the negotiation process (Section 4.1), sequential negotiation between one buyer and multiple sellers (Section 4.2), and mechanism and incentives design in the presence of asymmetric information (Section 4.3).

### 4.1. Ambiguity and the outside option

In Sections 2-3, we assumed that both the seller and buyer accept the price negotiation outcomes and commit to the contract if the optimal timing threshold is attained. However, the seller/buyer can also decide to exercise their outside options and exit the negotiation altogether. Here the outside option is viewed as the best alternative that a negotiator can go for if he or she withdraws unilaterally from the negotiation process (Binmore et al., 1986). The existence of outside options introduces new constraints to the problem as ambiguity also affects outside values (see e.g. Miao and Wang, 2011).

We adopt the outside option valuation model of Schröder (2011) and consider outside options for both sellers and buyers. Let the seller's (buyer's) outside option value $V_1$ ($V_2$) be random and follow a normal distribution $\mathcal{N}(u_1,\hat{\sigma}_1)$ ( $\mathcal{N}(u_2,\hat{\sigma}_2)$ ). The set of likelihood distributions $p_i \in \mathcal{R}_i(u_i,\hat{\sigma}_i) = \left\{\mathcal{N}(u_i,\hat{\sigma}_i) \middle| u_i \in \left[\bar{u}_i - y_i, \bar{u}_i + y_i\right]\right\}$ is defined to capture ambiguity in $i$'s outside option value. For simplicity, we assume these to be independent of the seller's costs and buyer's revenues $S_i$, where $y_i > 0$, $i = 1, 2$. We consider ambiguity in the mean of $u_i$ rather than the variance $\hat{\sigma}_i$. The scope of the mean $u_i \in [\bar{u}_i - y_i, \bar{u}_i + y_i]$ is defined based on $\kappa$-ignorance in continuous-time and $\varepsilon$-contamination (e.g. Nishimura and Ozaki, 2006; Kopylov, 2016), where the ambiguity level $y_i$ reflects how confident $i$ is in his/her probabilistic measure. Suppose the seller's (buyer's) ambiguity aversion $\rho_1$ ( $\rho_2$ ) is a trait that influences investment execution and outside option exercise. Then, the NMEU value of $V_i$ can be written as: $NMEU(V_i) = \rho_i \inf_{p_i \in \mathcal{R}_i} E^{p_i}[V_i] + (1-\rho_i)E\left[V_i\right] = \bar{u}_i - \rho_i y_i$, where $i = 1, 2$.

The NMEU version of outside option value $V_i$ differs from the outside option value in Schröder (2011) by considering the mean $\bar{u}_i$ , thus reflecting the influence of non-extreme prospects. $NMEU(V_i)$ decreases with ambiguity aversion $\rho_i$ and ambiguity level $y_i$. When the ratio of buyer's revenues ($S_2(t)$) to seller's costs ($S_1(t)$) reaches $z^*$, the seller and buyer maximize their utilities as follows:

$$\hat{R}(S_1(t), S_2(t)) = \max\left\{\xi_{12}\left(\lambda_2 S_2(t) - \lambda_1 S_1(t)\right), NMEU(V_1)\right\} \tag{20}$$

$$\hat{U}(S_1(t), S_2(t)) = \max\left\{\left(1-\xi_{12}\right)\left(\lambda_2 S_2(t) - \lambda_1 S_1(t)\right), NMEU(V_2)\right\} \tag{21}$$

Eqs. (20) and (21) indicate that negotiation agreement is reached if $\underline{\xi}_{12} \le \xi_{12} \le \bar{\xi}_{12}$ , where

$\underline{\xi}_{12} = \frac{NMEU(V_1)}{\lambda_2 S_2(t) - \lambda_1 S_1(t)}$, $\overline{\xi}_{12} = 1 - \frac{NMEU(V_2)}{\lambda_2 S_2(t) - \lambda_1 S_1(t)}$. This means the seller (buyer) will commit to the contract only if her (or his) negotiation power is not too low and the profit allocation policy is acceptable. Otherwise, the seller (or buyer) will opt out and exit the negotiation. To ensure $\overline{\xi}_{12} > \underline{\xi}_{12}$, the profit space $\lambda_2 S_2(t) - \lambda_1 S_1(t)$ should be strictly greater than the sum of outside option values $NMEU(V_1) + NMEU(V_2)$. Although relative bargaining power $\xi_{12}$ does not directly affect optimal investment timing, it does determine whether both parties should proceed with the contract when considering their outside options.

### 4.2. Sequential negotiation between one buyer and several suppliers

We next extend our bilateral negotiation problem to sequential multilateral negotiation situations involving one buyer and complementary sellers. Several papers have examined cases of suppliers or a group of sellers supplying complementary components to a downstream firm (e.g. Nagarajan and Bassok, 2008; Nagarajan and Soši´c, 2008; Granot and Yin, 2008; He and Yin, 2015). Herein, we incorporate ambiguity and social network dynamics in the negotiation framework of Nagarajan and Bassok (2008) and relax the fixed channel profit assumption characterising their sequential negotiation. As before, we account for ambiguity in the sellers' costs and buyer's revenues and their respective network position features. This enables us to examine the effect(s) of ambiguity aversion (and number of sellers) on profit allocation.

The buyer can be viewed as an assembler who buys one unit of complementary component from each seller and manufactures the final product. They are in a positively connected network in the sense that a deal between the buyer and one seller encourages the former to trade with other sellers. We consider a supply chain network with $n$ nodes consisting of a buyer and $n-1$ sellers, where $n \geq 2$. The buyer negotiates with the $h$th seller at stage $h$ using Nash bargaining solutions, where $h = 1, \cdots n-1$. From Eq. (15), $\gamma = \gamma_{12}$ denotes each seller's relative bargaining power in a positively connected network.[9] The buyer's relative bargaining power is $\eta = 1 - \gamma$.

The negotiation sequence is determined by the buyer. Each seller has her own subjective costs' expectations. The $h$th seller's NMEU-based ambiguity multiplier is $\lambda_1^h = \frac{\rho_1^h}{r - \mu_1^h - \sigma_1^h \kappa_1^h} + \frac{1-\rho_1^h}{r - \mu_1^h} > 0$. We add the superscript $h$ to denote the $h$th seller.

The total expected profit $\Pi(t)$ based on the NMEU specification can be written as: $\Pi(t) = \lambda_2 S_2(t) - \sum_{h=1}^{n-1} \lambda_1^h S_1^h(t)$, where $\Pi(t) > 0$. To simplify notation, we omit timing $t$ from $\Pi(t)$. Let $\Pi_h$ denote the total expected profit to be shared between the buyer and sellers $h, h+1 \cdots n-1$, where $\Pi_1 = \Pi$.

[9] An alternative to this would be to calculate relative bargaining power by considering revoking commitment at a certain cost (see e.g. Muthoo, 1996; Nagarajan and Bassok, 2008) so that the assumption of zero disagreement values becomes less restrictive.

After $n-1$ stages of negotiation, the buyer achieves his expected profit $\pi_B = \Pi_n$. Let $\Psi_h$ denote the set of feasible alternatives when the buyer negotiates with the $h$th seller at stage $h$. We use $\pi_h$ to represent the level of profit that the $h$ th seller achieves. In the first stage, the feasible set is defined by $\Psi_1 = \{(\Pi_2, \pi_1) : \Pi_2 + \pi_1 = \Pi\}$. At stage $h$, $\Psi_h = \{(\Pi_{h+1}, \pi_h) : \Pi_{h+1} + \pi_h = \Pi_h\}$. Then, the profit allocation rule between the buyer and the $h$th seller is determined through the generalized Nash bargaining solution: $\max_{(\Pi_{h+1}, \pi_h) \in \Psi_h} (\Pi_{h+1})^{1-\gamma} (\pi_h)^{\gamma}$. In line with Section 3, disagreement values of the buyer and sellers are assumed to be zero. The total surplus is split as follows: $\pi_h = \gamma \Pi_h$, $\Pi_{h+1} = (1-\gamma)\Pi_h$. The profit distribution between the buyer and sellers is, thus, obtained via backward induction. The buyer's expected profit $\pi_B$ and the $h$th seller's expected profit $\pi_h$ can be written as:

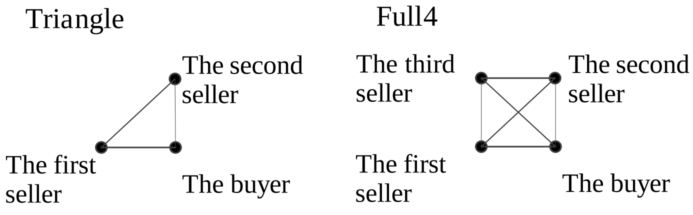
$$\pi_B = (1-\gamma)^{n-1} \left( \lambda_2 S_2(t) - \sum_{h=1}^{n-1} \lambda_1^h S_1^h(t) \right) \tag{22}$$

$$\pi_h = \gamma (1-\gamma)^{h-1} \left( \lambda_2 S_2(t) - \sum_{h=1}^{n-1} \lambda_1^h S_1^h(t) \right) \tag{23}$$

Eqs. (22) and (23) illustrate the distribution of profits based on the supply chain member number, network position and ambiguity in sellers' costs and buyer's revenues. Compared with the optimal profit allocation in Nagarajan and Bassok (2008), our solutions consider the role of probabilistic ambiguity and ambiguity aversion in the negotiation. These solutions also add to recent real options literature on price negotiation (e.g. Moon et al., 2011; Zheng and Negenborn, 2015). It is intuitive to see that both the buyer's and sellers' profits decrease with revenues- and costs-related ambiguity aversion.

From Eqs. (22) and (23), the "procurement" price for the $h$th seller $\tilde{X}_h$ and the total price paid by the buyer $\tilde{X}_B$ can be, respectively, expressed as: $\tilde{X}_h = \lambda_1^h S_1^h(t) + \gamma(1-\gamma)^{h-1}\Pi(t)$, $\tilde{X}_B = \lambda_2 S_2(t) - (1-\gamma)^{n-1}\Pi(t)$. The buyer will commit to higher prices if his ambiguity aversion is lower. The seller would ask for a higher price if her ambiguity aversion is higher.

Regarding the effects of multiple sellers, we follow Nagarajan and Bassok (2008) and discuss fixed versus adjustable negotiation sequences. In the fixed case, the buyer prefers fewer sellers as proved by Nagarajan and Bassok (2008) in their Theorem 4.1. We confirm this finding when considering network features. Note the buyer's expected profit $\pi_B$ is determined by $\Pi(t)$ and $(1-\gamma)^{n-1}$. A smaller number of sellers increases the total expected profit $\Pi(t)$ and the value of $(1-\gamma)^{n-1}$. For example, the term $(1-\gamma)^{n-1}$ equals 0.25 in a Triangle network, while it amounts to 0.125 in a Full-4 network in Figure 9. Consequently, the buyer benefits from a smaller number of sellers if the negotiation sequence is fixed. In the presence of a predefined negotiation sequence, the seller's expected profit decreases with her sequence $h$. For example, the first and second sellers' expected profits are, respectively, $0.5\Pi(t)$ and $0.25\Pi(t)$ in the Triangular network.




$c_2 = c_1^1 = c_1^2 = 0.5.$ $\quad$ $c_2 = c_1^1 = c_1^2 = c_1^3 = 0.333.$

$n = 3, m = 3.\ \gamma = 0.5.$ $\quad$ $n = 4, m = 6.\ \gamma = 0.5.$

Fig. 9 Examples of supply chains with a single buyer (assembler) and multiple suppliers (sellers)

If the negotiation sequence is adjustable, the buyer might encourage the sellers to pay for network positions in order to gain more profit share. Nagarajan and Bassok (2008) prove, in their Theorem 4.2, that when the sellers simultaneously compete for negotiation sequence and pay for their network positions[10], at every Nash equilibrium, the expected profit of each seller equals $\pi_{n-1}$ while the buyer's expected profit is $\left(1-\gamma(1-\gamma)^{n-2}(n-1)\right)\Pi(t)$. In such a setting, the buyer prefers to have more sellers (see Theorem 4.2 in Nagarajan and Bassok (2008)). We find that due to social network effects, the buyer does not necessarily benefit from a higher number of sellers. This is because a greater number of sellers decreases the total expected profit $\Pi(t)$ but might at the same time increase the $1-\gamma(1-\gamma)^{n-2}(n-1)$ term. For example, $1-\gamma(1-\gamma)^{n-2}(n-1)$ equals 0.5 in the Triangle network, while it is 0.625 in the Full-4 network in Fig. 9. As we study negotiation power from a social network perspective, the relationship between the number of sellers and the buyer's expected profits becomes equivocal if sellers compete and pay for negotiation position.

### 4.3. Asymmetric information and price negotiation under ambiguity

In the previous sections, we analysed investment timing and pricing decisions under ambiguity assuming that information was symmetric. However, information asymmetry is also known to influence buyer-seller interactions and their related transactional arrangements. There has been increasing interest surrounding issues of ambiguity aversion, asymmetric information and mechanism design in recent years (see Bodoh-Creed, 2012; Bose and Renou, 2014; Vierø, 2014; Wolitzky, 2016; Giraud and Thomas, 2017). We borrow from this literature to examine how asymmetric information affects our optimal timing and price negotiation outcomes under ambiguity. Our modelling builds on a rich and still growing stream of research on real options under incomplete information (e.g. Nishihara and Shibata, 2008; Shibata and Nishihara, 2011; Feng et al., 2014; Grenadier et al., 2016). We add to these studies by accounting for ambiguity and each party's private information about their own ambiguity aversion parameter $\rho_i$ (and option value parameter $b$) in the negotiation.

[10] Suppliers' coalitions and their stability are also discussed in Nagarajan and Bassok (2008).

Consider the buyer (principal) delegates the investment timing decision to the seller (agent) and determines the price contingent on the observable timing threshold. There are two types of sellers in the market in terms of their knowledge of their ambiguity aversion parameter $\rho_i$. We call the seller a high (in contrast to a low) type if her ambiguity aversion is $\rho_{1H}$ ($\rho_{1L}$) with $\rho_{1H} < \rho_{1L}$. This means $\lambda_{1H} < \lambda_{1L}$. The probability of any seller belonging to the high type category is $q_1$.

Let $\lambda_{1w}$ and $b_w$ denote $w$'s NMEU-based ambiguity multiplier and option value parameter, where : $b_w = \frac{1}{2} - \frac{\delta_{1w} - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} + \sqrt{\left(\frac{1}{2} - \frac{\delta_{1w} - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)^2 + \frac{2\delta_{1w}}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}}$, $\delta_{1w} = 1/\lambda_{1w}$, $\lambda_{1w} = \frac{\rho_{1w}}{r - \mu_1 - \sigma_1\kappa_1} + \frac{1-\rho_{1w}}{r - \mu_1}$, $w = H$ or $L$ denotes the high or low type, $\delta_2 = 1/\lambda_2$, $\lambda_2 = \frac{\rho_2}{r - \mu_2 + \sigma_2\kappa_2} + \frac{1-\rho_2}{r - \mu_2}$.

Thus, the high (low) type seller has her own private information about the NMEU-based ambiguity multiplier and option value parameter. In line with Section 3, let $z_H$ ($z_L$) represent the ratio of buyer's revenues $S_2(t)$ to seller's costs $S_1(t)$ when the high (or low) type seller undertakes the contract. Note $\left(z(t)/z_w\right)^{b_w}$ is akin to a discount function (Grenadier, 2005; Feng et al., 2014). Assume $z(t) < z_w$ indicates that the contract is not implemented immediately. Since $\lambda_{1w}$ and $b_w$ are the seller's private information, the buyer's objective is to maximize his option value by observing investment timing $z_w$ and buying the product or service at price $X_w$:

$$\max_{z_H, z_L, X_H, X_L} q_H \left(\frac{z(t)}{z_H}\right)^{b_H} \left(\lambda_2 z_H S_1(t) - X_H\right) + \left(1 - q_H\right)\left(\frac{z(t)}{z_L}\right)^{b_L} \left(\lambda_2 z_L S_1(t) - X_L\right) \tag{24}$$

subject to:

$$\left(\frac{z(t)}{z_H}\right)^{b_H} \left(X_H - \lambda_{1H} S_1(t)\right) \geq \left(\frac{z(t)}{z_L}\right)^{b_H} \left(X_L - \lambda_{1H} S_1(t)\right) \tag{25}$$

$$\left(\frac{z(t)}{z_L}\right)^{b_L} \left(X_L - \lambda_{1L} S_1(t)\right) \geq \left(\frac{z(t)}{z_H}\right)^{b_L} \left(X_H - \lambda_{1L} S_1(t)\right) \tag{26}$$

$$\left(\frac{z(t)}{z_H}\right)^{b_H} \left(X_H - \lambda_{1H} S_1(t)\right) \geq 0 \tag{27}$$

$$\left(\frac{z(t)}{z_L}\right)^{b_L} \left(X_L - \lambda_{1L} S_1(t)\right) \geq 0 \tag{28}$$

The terms $\lambda_2 z_H S_1(t)$ and $\lambda_2 z_L S_1(t)$ are the buyer's expected revenues if the seller belongs to the high type and low type categories, respectively. Constraints (25) and (26) mean that the high type seller is encouraged to undertake the contract at timing $z_H$ and the low type seller is induced to undertake the contract at timing $z_L$. Constraints (27) and (28) are the participation constraints.

Solving this principal–agent problem and assuming $z_L^* > \frac{b_L \lambda_{1L}}{(b_L - 1)\lambda_2}$ (see proofs in Appendix D), we find that when information about the seller's pessimism and ambiguity aversion is asymmetric, the buyer's optimal policy is as follows:

$\left(z_H^*, X_H^*, z_L^*, X_L^*\right) = \left( \frac{b_H \lambda_{1H}}{(b_H - 1)\lambda_2}, \left[ \lambda_{1H} + \left( \frac{z_H^*}{z_L^*} \right)^{b_H} \left( \lambda_{1L} - \lambda_{1H} \right) \right] S_1(t), z_L^*, \lambda_{1L} S_1(t) \right)$, where $z_L^*$ is a solution to $\left(1 - q_H\right) z_L^{\,b_H - b_L} \left( b_L \lambda_{1L} + \left(1 - b_L\right) \lambda_2 z_L \right) + q_H b_H z(t)^{b_H - b_L} \left( \lambda_{1L} - \lambda_{1H} \right) = 0$.

This shows there is a similar functional form between the timing trigger of the high type $z_H^*$ and the timing trigger under symmetric information $z^*$ in Eq. (18). Extant real options research (e.g. Nishihara and Shibata, 2008; Feng et al., 2014) documents that investment will usually be deferred if managers belong to the low type category. We confirm this, in our buyer-seller and price negotiation setting, in the presence of asymmetric information concerning option value parameters and the degree of ambiguity. This is further illustrated in Figure 10.a where timing threshold $z_H^*$ is smaller than $z_L^*$. We find an optimal incentives policy exists only if $z_L^* > \frac{b_L \lambda_{1L}}{(b_L - 1)\lambda_2}$. This implies that under ambiguity, the buyer can implement the incentive contract only if the low type seller's timing threshold is relatively high. When probabilistic ambiguity is nil, ambiguity aversion does not affect the seller's costs and there is a unique timing threshold (implicit price) as shown in Fig. 10.a (Fig. 10.b). Let $C_{1H}$ denote the high type seller's expected costs where $C_{1H} = \lambda_{1H} S_1(t)$. Her expected profits are shown in the grey area of Figure 10.b. These profits are determined by the costs difference between the high and low type sellers $\left( \lambda_{1L} - \lambda_{1H} \right) S_1(t)$ and the portion $\left( \frac{z_H^*}{z_L^*} \right)^{b_H} < 1$. This means the buyer covers part of the costs difference to encourage the high type seller to tell the truth. The incentives portion is, hence, affected by the ambiguity aversion of the buyer and that of each type of seller.

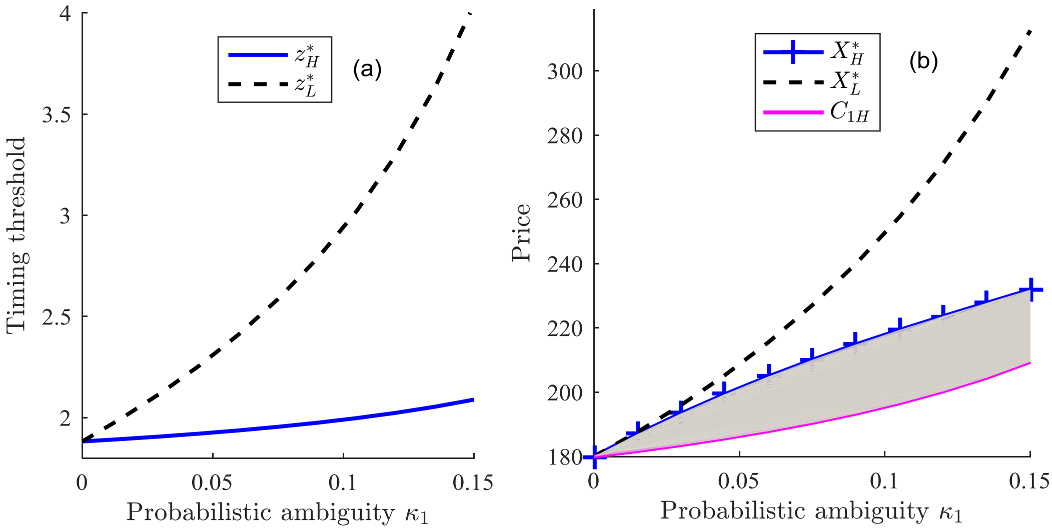



Fig. 10 Timing thresholds and prices under ambiguity and asymmetric information
$z(t)=1.3$, $q_H=0.5$, $\rho_{1H}=0.2$, $\rho_{1L}=0.9$. The other parameter values are the same as in Fig.6.

The seller can also act as a principal to induce the buyer to report his true type (good vs. bad). The good buyer has lower ambiguity aversion about his revenues than the bad buyer (see Appendix D). We, once again, find the functional form of the timing trigger for the good buyer $z_G^*$ to be similar to the symmetric $z^*$. $z_G^*$ is also smaller than $z_B^*$. The good buyer's expected profits are determined by revenues difference between the two types of buyers and the portion $\left(\frac{z_G^*}{z_B^*}\right)^{b_G}<1$, where $b_G$ is the good buyer's option value parameter.

In the two above principal-agent cases, the timing trigger for the high (good) type agent increases with her (his) ambiguity aversion about costs (revenues) and the principal's ambiguity aversion about revenues (costs). The timing trigger for the low type (bad) agent and implicit price of the high type (good) agent are nonlinear functions of the ambiguity aversion of the principal and that of each type of agent. The implicit price $X_L^*$ of the low type seller increases with her pessimism about costs $\rho_{1L}$. On the other hand, the implicit price $X_B^*$ of the bad buyer is a nonlinear function of his optimism about revenues $\rho_{2B}$ since $\rho_{2B}$ affects $X_B^*$ through the NMEU-based ambiguity multiplier $\lambda_{2B}$ and timing threshold $z_B^*$. The principal offers zero profit to the low (bad) type agent and positive profits to the high (good) type agent. These positive profits are contingent on the ambiguity aversion of the principal and that of each type of agent.

## 5. Conclusions

Contributing to behavioural operations and production management research on buyer-seller relationships (e.g. Esmaeilia and Zeephongsekul, 2010; Hazen et al., 2012; Hemmert et al., 2016), this paper examines the real options and social network dynamics of bilateral (and multilateral) negotiation under ambiguity by relying on a multiple-priors expected utility with non-extreme outcomes. Adjusting for uncertainty aversion in probabilistic appraisal, this utility combines the worst case in negotiators' minds with the risk-neutral case and provides flexible commitment thresholds for investment under ambiguity aversion. Besides extending risk-neutral insights from recent contingent-claims research, our results underline the moderating effects of individual behaviour and miscalibration on the process of price negotiation and its performance. We find that ambiguity aversion and ambiguity do not necessarily have symmetric effects on pricing outcomes. Specifically, an increase in seller's ambiguity aversion increases her implicit reservation price and negotiation threshold with and without network control. On the other hand, the buyer's ambiguity aversion affects his implicit reservation price and the threshold for negotiation (un)equivocally in the absence (presence) of social network effects. The seller's (buyer's) probabilistic ambiguity affects her (his) implicit reservation price and negotiation threshold in a similar direction as her (or his) ambiguity aversion. This is because ambiguity aversion and probabilistic ambiguity dominate the influence of the worst case heuristic on decision making in the same direction. We confirm that standard option analysis with a single prior can lead to restrictive pricing outcomes and might overstate mutual agreement prospects and the range for negotiation. We, additionally, show that knowledge of network positions and other social network effects still play an important role in negotiation performance in the presence of ambiguity. We also explore the case of one buyer and multiple sellers, examine the role of outside options, and consider the effect of information asymmetry in the various dynamics.

In terms of operations and production management implications, our proposed real options frameworks provide quantitative insights into how ambiguity aversion and social network effects influence the range of negotiation agreement between buyers and sellers, and help formalise - using real options theory - recent predictions by Hazen et al. (2012) on the role of ambiguity tolerance (and perceived quality) in the decision to purchase remanufactured products. We add to this literature by examining willingness-to-pay (WTP) and willingness-to-accept (WTA) decisions jointly and highlight the effect of social networks on the relationship between ambiguity aversion and price negotiation outcomes in the context of B2B situations.

Our results also help inform how probabilistic ambiguity and pessimism (or other attitudes towards uncertainty) generally affect negotiators' behaviour, real options payoffs and investment outcomes in buyer-seller relationships, social network structures and other practical decision making situations. By knowing their network positions and gathering background information or inferring the other party's ambiguity tolerance beforehand via cheap talk, buyers and sellers can anticipate where the negotiation is heading in terms of price negotiation range and mutual agreement possibilities despite the presence of ambiguity. This is especially useful for international operations and price negotiation situations that involve buyers and sellers from different countries. Knowing the cultural characteristics of a country, including its degree of uncertainty avoidance (e.g. Hofstede, 2001), can help international managers identify suppliers and customers who might be more uncertainty-seeking (averse) in the international network or else plan, in a contingent-manner and considering relative bargaining powers, for potentially lengthy and difficult negotiations. Extensions of this work could consider further game-theoretic interactions, quantity/quality dynamics and account for the effects of second moment 'uncertainty', learning, horizontal competition and cooperation on price negotiation outcomes and mutual agreement. Validating our frameworks using experimental principles can also provide interesting evidence on the emotional and perhaps irrational traits of price negotiation and highlight extra factors which could influence buyer-seller decision making in the presence of ambiguity and social network effects.

## Appendix A. Derivation of the seller's put option value under the NMEU

The selling opportunity value $F_1(t+dt)$ under ambiguity can be expressed as:

$$F_1(t+dt)=\max_{t'\geq t+dt}\left\{\rho_1 G(t')+(1-\rho_1)E\left[\tilde{G}(t')\middle|\mathcal{F}_{t+dt}\right]\right\} \tag{A.1}$$

where $G(t')=e^{-r(t'-t-dt)}X-\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t-dt)}d\tau\middle|\mathcal{F}_{t+dt}\right]$, $\tilde{G}(t')=e^{-r(t'-t-dt)}X-\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t-dt)}d\tau$.

The multiple-priors expected utility with non-extreme outcomes of $F_1(t+dt)$ is:

$$\begin{aligned}
&NMEU\left(e^{-rdt}F_1(t+dt)\middle|\mathcal{F}_t\right)\\
&=e^{-rdt}\max_{t'\geq t+dt}\Bigg\{\rho_1 e^{-r(t'-t-dt)}X-\rho_1\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\rho_1\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t-dt)}d\tau\middle|\mathcal{F}_{t+dt}\right]+(1-\rho_1)E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t-dt)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]\\
&+(1-\rho_1)e^{-r(t'-t-dt)}X-(1-\rho_1)E\left[\rho_1\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t-dt)}d\tau\middle|\mathcal{F}_{t+dt}\right]+(1-\rho_1)E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t-dt)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]\Bigg\}\\
&=\max_{t'\geq t+dt}\left\{e^{-r(t'-t)}X-\left(\rho_1^2M_{11}+\rho_1(1-\rho_1)M_{12}+\rho_1(1-\rho_1)M_{13}+(1-\rho_1)^2M_{14}\right)\right\}
\end{aligned} \tag{A.2}$$

where $M_{11}=\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\left[\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]$, $M_{12}=\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]$,

$M_{13}=E\left[\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]$, $M_{14}=E\left[E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]\middle|\mathcal{F}_t\right]$.

Considering the time-consistency (or rectangularity) of the set of priors, we have $M_{11}=\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]$. Since $E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]$ is a singleton set that is not affected by the density generators, $M_{12}$ is equal to $E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]$. Recall that $\sup_{\theta_1'\in\Theta_1}E^{Q_1^{\theta_1'}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_{t+dt}\right]$ denotes the expectation with respect to $Q_1^{-\kappa_1}$ conditional on $\mathcal{F}_{t+dt}$. This expectation stays unchanged at earlier times $t$. Then $M_{13}=\sup_{\theta_1\in\Theta_1}E^{Q_1^{\theta_1}}\left[\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]$. In line with the law of iterated expectations and recursive utility under risk, we obtain $M_{14}=E\left[\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]$. Then equation (A.2) becomes equivalent to:

$$\begin{aligned}
&NMEU\left(e^{-rdt}F_1(t+dt)\middle|\mathcal{F}_t\right)\\
&=\max_{t'\geq t+dt}\left\{e^{-r(t'-t)}X-\left[\rho_1\sup_{\theta_1\in\Theta_1}\left[E^{Q_1^{\theta_1}}\int_{t'}^{\infty}S_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]+(1-\rho_1)\left[E\int_{t'}^{\infty}\tilde{S}_1(\tau)e^{-r(\tau-t)}d\tau\middle|\mathcal{F}_t\right]\right]\right\}\\
&=J(t')
\end{aligned} \tag{A.3}$$

Eq. (A.3) implies that once the decision maker commits to wait at time $t$, he(she) does not change his (her) plan when time elapses.

Recall the NMEU-based ambiguity multiplier $\lambda_1$ is a constant in the infinite time horizon. According to Eq. (1), $dW_1(t) = (\mu_1 - \sigma_1\theta_1)W_1(t)dt + \sigma_1 W_1(t)dB_1(t)^{\theta_1}$. Since the seller's put opportunity value $F_1(t)$ depends on $W_1(t)$, we write $F_1\left(W_1(t)\right)$ for $F_1(t)$. According to Equations (A.3) and (5), the seller's put opportunity value can be defined as:

$$\begin{aligned} F_1\left(W_1(t)\right) &= \max_{t' \geq t}\left\{X - W_1(t),\ \ NMEU\left(e^{-rdt}F_1(t+dt)\middle|\mathcal{F}_t\right)\right\} \\ &= \max_{t' \geq t}\left\{X - W_1(t), (1-rdt)\left[F_1\left(W_1(t)\right) + NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right)\right]\right\} \\ &= \max_{t' \geq t}\left\{X - W_1(t), F_1\left(W_1(t)\right) + NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right) - rF_1\left(W_1(t)\right)dt\right\} \end{aligned} \tag{A.4}$$

where we approximate $e^{-rdt}$ using $(1-rdt)$ and rely on $F_1\left(W_1(t)\right) + NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right)$ to estimate $NMEU\left(F_1(t+dt)\middle|\mathcal{F}_t\right)$ (see Nishimura and Ozaki, 2007; Trojanowska and Kort, 2010).

The NMEU satisfies dynamic consistency as the seller's option value is defined recursively in Eq. (A.4).

In the waiting region, we have $NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right) = rF_1\left(W_1(t)\right)dt$. $NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right)$ can be expressed as follows:

$$\begin{aligned} &NMEU\left(dF_1\left(W_1(t)\right)\middle|\mathcal{F}_t\right) = NMEU\left[F_1'\left((\mu_1 - \sigma_1\theta_1)W_1(t)dt + \sigma_1 W_1(t)dB_1(t)^{\theta_1}\right) + \frac{1}{2}\sigma_1^2\left(W_1(t)\right)^2 F_1''dt\middle|\mathcal{F}_t\right] \\ &= \zeta_1 F_1' W_1(t)dt + \frac{1}{2}\sigma_1^2\left(W_1(t)\right)^2 F_1''dt \end{aligned} \tag{A.5}$$

where $F_1' = \dfrac{\partial F_1\left(W_1(t)\right)}{\partial W_1(t)}$, $F_1'' = \dfrac{\partial^2 F_1\left(W_1(t)\right)}{\partial W_1(t)^2}$, $\zeta_1 = \rho_1(\mu_1 + \sigma_1\kappa_1) + (1-\rho_1)\mu_1$.

This results in the following second-order ordinary differential equation:

$$\frac{1}{2}\sigma_1^2\left(W_1(t)\right)^2 F_1'' + \zeta_1 F_1' W_1(t) - rF_1\left(W_1(t)\right) = 0 \tag{A.6}$$

The seller's trigger is subject to the value-matching, smooth-pasting, and boundary conditions: $F_1(W_1^*) = X - W_1^*$, $F_1'(W_1^*) = -1$, $\lim_{W_1(t)\to\infty} F_1(W_1) = 0$. Thus, we obtain the seller's costs threshold and option value as expressed in Eq. (6).

## Appendix B – Proofs of Propositions 1-2

Recall $X_i = \left(1 - \dfrac{1}{\beta_i}\right)\lambda_i S_i(t)$, $i = 1, 2$. Examining the effect of ambiguity aversion $\rho_i$ on the implicit reservation price $X_i$ :

$$\frac{\partial X_i}{\partial \rho_i} = \frac{\partial X_i}{\partial \beta_i}\frac{\partial \beta_i}{\partial \rho_i} + \frac{\partial X_i}{\partial \lambda_i}\frac{\partial \lambda_i}{\partial \rho_i} \tag{B.1}$$

where $i = 1, 2$, $X_i = \left(1 - \dfrac{1}{\beta_i}\right)\lambda_i S_i(t)$, $\beta_1 = \dfrac{1}{2} - \dfrac{\zeta_1}{\sigma_1^2} - \chi_1 < 0$, $\beta_2 = \dfrac{1}{2} - \dfrac{\zeta_2}{\sigma_2^2} + \chi_2 > 1$, $\chi_i = \sqrt{\left(\dfrac{1}{2} - \dfrac{\zeta_i}{\sigma_i^2}\right)^2 + \dfrac{2r}{\sigma_i^2}}$,

$\zeta_1 = \rho_1(\mu_1 + \sigma_1\kappa_1) + (1-\rho_1)\mu_1$, $\zeta_2 = \rho_2(\mu_2 - \sigma_2\kappa_2) + (1-\rho_2)\mu_2$, $\dfrac{\partial X_i}{\partial \beta_i} > 0$, $\dfrac{\partial X_i}{\partial \lambda_i} > 0$,

$$\frac{\partial\lambda_1}{\partial\rho_1}=\frac{1}{r-\mu_1-\sigma_1\kappa_1}-\frac{1}{r-\mu_1}\geq 0,\ \frac{\partial\lambda_2}{\partial\rho_2}=\frac{1}{r-\mu_2+\sigma_2\kappa_2}-\frac{1}{r-\mu_2}\leq 0.$$

The derivatives of $\beta_1$ with respect to $\rho_1$ and $\beta_2$ with respect to $\rho_2$ are:

$$\frac{\partial\beta_i}{\partial\rho_i}=\begin{cases}\dfrac{\beta_1\kappa_1}{\sigma_1\chi_1}\leq 0 & i=1\\ \dfrac{\beta_2\kappa_2}{\sigma_2\chi_2}\geq 0 & i=2\end{cases}\tag{B.2}$$

Considering $\lambda_1\leq\dfrac{1}{r-\mu_1-\sigma_1\kappa_1}$, we obtain the derivative of the seller's implicit reservation price ($X_1$) with respect to $\rho_1$:

$$\begin{aligned}\frac{\partial X_1}{\partial\rho_1}&=\frac{S_1(t)}{-\beta_1}\left\{(1-\beta_1)\left(\frac{1}{r-\mu_1-\sigma_1\kappa_1}-\frac{1}{r-\mu_1}\right)-\frac{\lambda_1\kappa_1}{\sigma_1\chi_1}\right\}\\ &\geq\frac{S_1(t)}{-\beta_1}\left\{(1-\beta_1)\left(\frac{1}{r-\mu_1-\sigma_1\kappa_1}-\frac{1}{r-\mu_1}\right)-\frac{\kappa_1}{\sigma_1\chi_1(r-\mu_1-\sigma_1\kappa_1)}\right\}\\ &=\frac{S_1(t)\kappa_1\left[(1-\beta_1)\sigma_1{}^2\chi_1-(r-\mu_1)\right]}{(-\beta_1)\sigma_1\chi_1(r-\mu_1-\sigma_1\kappa_1)(r-\mu_1)}\end{aligned}\tag{B.3}$$

Since $(1-\beta_1)\sigma_1{}^2\chi_1>2r$ and $\beta_1<0$, $\dfrac{\partial X_1}{\partial\rho_1}>0$ if $\kappa_1>0$ and $\dfrac{\partial X_1}{\partial\rho_1}=0$ if $\kappa_1=0$.

Using the same logic as above, the derivative of $X_2$ with respect to $\rho_2$ is:

$$\frac{\partial X_2}{\partial\rho_2}=\underbrace{\frac{\lambda_2\kappa_2}{\beta_2\sigma_2\chi_2}S_2(t)}_{\geq 0}+\underbrace{\left(\frac{1}{r-\mu_2+\sigma_2\kappa_2}-\frac{1}{r-\mu_2}\right)\left(1-\frac{1}{\beta_2}\right)S_2(t)}_{\leq 0}\gtreqless 0\tag{B.4}$$

Eq. (B.4) also shows that when $\kappa_2=0$, the derivative of $X_2(\kappa_2=0)$ with respect to $\rho_2$ is equal to zero.

Examining the effect of probabilistic ambiguity $\kappa_i$ on the implicit reservation price $X_i$:

$$\frac{\partial X_i}{\partial\kappa_i}=\underbrace{\frac{\partial X_i}{\partial\beta_i}\frac{\partial\beta_i}{\partial\kappa_i}}_{option-\text{value}\,effect}+\underbrace{\frac{\partial X_i}{\partial\lambda_i}\frac{\partial\lambda_i}{\partial\kappa_i}}_{present-\text{value}\,effect}\tag{B.5}$$

The derivatives of $\beta_1$ with respect to $\kappa_1$ and $\beta_2$ with respect to $\kappa_2$ are:

$$\frac{\partial\beta_i}{\partial\kappa_i}=\begin{cases}\dfrac{\beta_1\rho_1}{\sigma_1\chi_1}\leq 0 & i=1\\ \dfrac{\rho_2\beta_2}{\sigma_2\chi_2}\geq 0 & i=2\end{cases}\tag{B.6}$$

The derivative of the ambiguity multiplier $\lambda_i$ with respect to $\kappa_i$ can be expressed as:

$$\frac{\partial\lambda_i}{\partial\kappa_i}=\begin{cases}\dfrac{\rho_1\sigma_1}{(r-\mu_1-\sigma_1\kappa_1)^2}\geq 0 & i=1\\ \dfrac{-\rho_2\sigma_2}{(r-\mu_2+\sigma_2\kappa_2)^2}\leq 0 & i=2\end{cases}\tag{B.7}$$

Substituting $\frac{\partial \beta_1}{\partial \kappa_1}$ and $\frac{\partial \lambda_1}{\partial \kappa_1}$ in $\frac{\partial X_1}{\partial \kappa_1}$, we obtain the partial derivative of $X_1$ with respect to probabilistic ambiguity $\kappa_1$:

$$\begin{aligned}\frac{\partial X_1}{\partial \kappa_1} &= \frac{\rho_1 S_1(t)}{(-\beta_1)}\left\{(1-\beta_1)\frac{\sigma_1}{(r-\mu_1-\sigma_1\kappa_1)^2}-\frac{\lambda_1}{\sigma_1 \chi_1}\right\} \\ &\geq \frac{\rho_1 S_1(t)}{(-\beta_1)(r-\mu_1-\sigma_1\kappa_1)^2\sigma_1\chi_1}\left[(1-\beta_1)\sigma_1{}^2\chi_1-(r-\mu_1-\sigma_1\kappa_1)\right]\end{aligned} \tag{B.8}$$

Considering $(1-\beta_1)\sigma_1{}^2\chi_1 > 2r$ and $\beta_1 < 0$, we prove that $\frac{\partial X_1}{\partial \kappa_1} > 0$ if $\rho_1 \in (0,1]$ and $\frac{\partial X_1}{\partial \kappa_1}=0$ if $\rho_1=0$.

From Eqs. (B.6) and (B.7), we get:

$$\frac{\partial X_2}{\partial \kappa_2} = \frac{\rho_2 S_2(t)}{\beta_2 \sigma_2}\left\{\frac{\lambda_2}{\chi_2}-\frac{(\beta_2-1)\sigma_2{}^2}{r-\mu_2+\sigma_2\kappa_2{}^2}\right\} \gtreqless 0 \tag{B.9}$$

Eq. (B.9) shows that the effect of the buyer's probabilistic ambiguity $\kappa_2$ on his implicit reservation price $X_2$ is equivocal. When $\rho_2=0$, the derivative of $X_2(\rho_2=0)$ with respect to $\kappa_2$ is equal to zero. This proves Proposition 1.

Next, we examine the effect of ambiguity aversion on the negotiation threshold.

As $\delta_{KK} = \frac{X_1 S_2(t)}{X_2 S_1(t)}$, we have $\begin{cases}\frac{\partial \delta_{KK}}{\partial \rho_1} > 0 & \text{if } \kappa_1 > 0 \\ \frac{\partial \delta_{KK}}{\partial \rho_1} = 0 & \text{if } \kappa_1=0\end{cases}$ and $\begin{cases}\frac{\partial \delta_{KK}}{\partial \kappa_1} > 0 & \text{if } \rho_1 \in (0,1] \\ \frac{\partial \delta_{KK}}{\partial \kappa_1}=0 & \text{if } \rho_1 = 0\end{cases}$.

The effects of $\rho_2$ and $\kappa_2$ on $\delta_{KK}$ are equivocal:

$\frac{\partial \delta_{KK}}{\partial \rho_2} = -\frac{X_1 S_2(t)}{X_2{}^2 S_1(t)}\frac{\partial X_2}{\partial \rho_2} \gtreqless 0$ and $\frac{\partial \delta_{KK}}{\partial \kappa_2} = -\frac{X_1 S_2(t)}{X_2{}^2 S_1(t)}\frac{\partial X_2}{\partial \kappa_2} \gtreqless 0$. Note that $\frac{\partial \delta_{KK}(\kappa_2=0)}{\partial \rho_2} = 0$ and $\frac{\partial \delta_{KK}(\rho_2=0)}{\partial \kappa_2} = 0$. This proves Proposition 2.

## Appendix C- The subjective probability of negotiation agreement under ambiguity

Since the seller is concerned about her costs and the buyer cares about his revenues, the negotiation agreement probability should be determined by their expectations of these quantities. Extending the single prior analysis of Moon et al. (2011) to the case of uncertainty and NMEU ambiguity, we examine how changes in ambiguity aversion and probabilistic ambiguity affect negotiators' subjective likelihood of agreement $P_{KK}$. As defined in Eq. (1), $S_1(t)$ and $S_2(t)$ follow lognormal diffusions with ambiguous drifts and the two-dimension probability density function under our subjective probability measures is (assuming a possible correlation between costs and revenues):

$$g(S_1,S_2)=\frac{\exp\left\{-\frac{1}{2\left(1-\vartheta^2\right)}\left[\left(\frac{\log S_1-\mu_{1N}}{\sigma_{1N}}\right)^2-2\vartheta\frac{(\log S_1-\mu_{1N})(\log S_2-\mu_{2N})}{\sigma_{1N}\sigma_{2N}}+\left(\frac{\log S_2-\mu_{2N}}{\sigma_{2N}}\right)^2\right]\right\}}{2\pi\sigma_{1N}\sigma_{2N}\sqrt{1-\vartheta^2}S_1S_2} \tag{C.1}$$

where $\mu_{iN}$ is the ambiguity-adjusted drift rate and $\sigma_{iN}$ is the standard deviation of $\log S_i$ under the NMEU. $\mu_{1N}=(\zeta_1-\frac{1}{2}\sigma_1^2)t$ , $\zeta_1=\rho_1(\mu_1+\sigma_1\kappa_1)+(1-\rho_1)\mu_1$ , $\mu_{2N}=(\zeta_2-\frac{1}{2}\sigma_2^2)t$ , $\zeta_2=\rho_2(\mu_2-\sigma_2\kappa_2)+(1-\rho_2)\mu_2$ , $\sigma_{iN}=\sigma_i\sqrt{t}$ , for $i=1,2$ , $\forall t\geq 0$ , $\forall\theta_i\in\Theta_i$ . We write $S_1$ and $S_2$ for $S_1(t)$ and $S_2(t)$. The subscript $N$ implies $\log S_i$ follows the normal distribution. $\vartheta$ is the correlation between $S_1$ and $S_2$. Ambiguity appears in Eq. (C1) both through the numerator and denominator.

We identify the process followed by $S_1S_2$ (e.g., Dixit and Pindyck, 1994):

$$d(S_1S_2)=(\mu_1-\sigma_1\theta_1+\mu_2-\sigma_2\theta_2+\varepsilon\sigma_1\sigma_2)S_1S_2dt+(\sigma_1dB_1(t)^{\theta_1}+\sigma_2dB_2(t)^{\theta_2})S_1S_2 \tag{C.2}$$

where $\varepsilon$ is defined by $E[dB_1(t)^{\theta_1}dB_2(t)^{\theta_2}]=\varepsilon dt$.

Since the NMEU value and standard deviation of $S_i$ under ambiguity are given by $NMEU(S_i)=S_i(0)\exp\left(\zeta_i t\right)$, $std(S_i)=S_i(0)\sqrt{\exp(2\zeta_i t)\left[\exp(\sigma_i^2 t)-1\right]}$, we can write:

$$NMEU(S_1S_2)=S_1(0)S_2(0)\exp\left[(\zeta_1+\zeta_2+\varepsilon\sigma_1\sigma_2)t\right] \tag{C.3}$$

In line with Moon et al. (2011) but considering ambiguity in seller's costs and buyer's revenues, the correlation coefficient $\vartheta$ between $S_1$ and $S_2$ can be expressed as:

$$\begin{aligned}\vartheta&=\frac{NMEU(S_1S_2)-\left[NMEU(S_1)\right]\left[NMEU(S_2)\right]}{std(S_1)std(S_2)}\\&=\frac{\exp\left(\varepsilon\sigma_1\sigma_2 t\right)-1}{\sqrt{\left[\exp(\sigma_1^2t)-1\right]}\sqrt{\left[\exp(\sigma_2^2t)-1\right]}}\end{aligned} \tag{C.4}$$

Thus, the subjective probability of negotiation agreement under ambiguity $P_{KK}$ is obtained as follows:

$$\begin{aligned}P_{KK}&=P_{KK}(X\geq X_1\quad and\quad X\leq X_2;\quad t)=P_{KK}\left(S_1(t)\leq S_1^*\ and\ S_2(t)\geq S_2^*\right)\\&=\int_{-\infty}^{Y_1^*}\int_{Y_2^*}^{+\infty}\frac{\exp\left\{-\frac{1}{2(1-\vartheta^2)}\left[\left(\frac{Y_1-\mu_{1N}}{\sigma_{1N}}\right)^2-2\vartheta\frac{(Y_1-\mu_{1N})(Y_2-\mu_{2N})}{\sigma_{1N}\sigma_{2N}}+\left(\frac{Y_2-\mu_{2N}}{\sigma_{2N}}\right)^2\right]\right\}}{2\pi\sigma_{1N}\sigma_{2N}\sqrt{1-\vartheta^2}}dY_1dY_2\end{aligned} \tag{C.5}$$

where $Y_1=\ln\left(\frac{S_1(t)}{S_1(0)}\right)$ , $Y_2=\ln\left(\frac{S_2(t)}{S_2(0)}\right)$, $Y_1^*=\ln\left(\frac{S_1^*}{S_1(0)}\right)=\ln\left(\frac{X}{(1-1/\beta_1)\lambda_1S_1(0)}\right)$ and $Y_2^*=\ln\left(\frac{S_2^*}{S_2(0)}\right)=\ln\left(\frac{X}{(1-1/\beta_2)\lambda_2S_2(0)}\right)$. $S_1^*=\frac{X}{(1-1/\beta_1)\lambda_1}$ , $S_2^*=\frac{X}{(1-1/\beta_2)\lambda_2}$ .

The above (Eq. (C.5)) indicates that the subjective probability of negotiation success is not unique when considering parties' real options, ambiguity aversion and ambiguity, it once again depends on the value of $S_i$ at time 0, the ambiguity multiplier or subjective beliefs $\lambda_i$, the discount rate $r$ and the parameters of the Geometric Brownian motion(s) followed by $S_i$ as defined in Eq. (1). When probabilistic ambiguity ($\kappa_i$) is greater than zero, there is a negative relationship between ambiguity aversion ($\rho_i$) and the subjective negotiation agreement probability, where $i=1,2$. In the case of ambiguity averse negotiators ($\rho_i \in (0,1]$), the subjective probability of negotiation success decreases with increasing ambiguity ($\kappa_i$, $i=1,2$), reflecting a conservative approach towards negotiation.

When revenues and costs functions are independent, the subjective probability of negotiation agreement simplifies to the following closed-form solution under ambiguity $P_{KKI}$ (confirming the interaction effects of uncertainty aversion and ambiguity on negotiation performance):

$$P_{KKI} = P_K(S_1(t) \le S_1^*) P_K(S_2(t) \ge S_2^*) = \Phi\left(\frac{Y_1^* - (\zeta_1 - \frac{1}{2}\sigma_1^2)t}{\sigma_1\sqrt{t}}\right)\left(1 - \Phi\left(\frac{Y_2^* - (\zeta_2 - \frac{1}{2}\sigma_2^2)t}{\sigma_2\sqrt{t}}\right)\right) \tag{C.6}$$

where $P_K(S_2(t) \ge S_2^*) = \int_{Y_2^*}^{+\infty} \frac{e^{-\frac{1}{2}\left(\left[Y_2 - (\zeta_2 - \frac{1}{2}\sigma_2^2)t\right] / \sigma_2\sqrt{t}\right)^2}}{(2\pi\sigma_2^2 t)^{1/2}} dY_2$, $P_K(S_1(t) \le S_1^*) = \int_{-\infty}^{Y_1^*} \frac{e^{-\frac{1}{2}\left(\left[Y_1 - (\zeta_1 - \frac{1}{2}\sigma_1^2)t\right] / \sigma_1\sqrt{t}\right)^2}}{(2\pi\sigma_1^2 t)^{1/2}} dY_1$

**Appendix D- Asymmetric information and price negotiation**

We formulate the Lagrangian by considering the incentive compatibility constraint of the high type agent and the participant constraint of the low type agent as the follows (Grenadier and Wang, 2005; Shibata, 2009):

$$\begin{aligned} K_1 &= q_H\left(\frac{z(t)}{z_H}\right)^{b_H}\left(\lambda_2 z_H S_1(t) - X_H\right) + \left(1 - q_H\right)\left(\frac{z(t)}{z_L}\right)^{b_L}\left(\lambda_2 z_L S_1(t) - X_L\right) \\ &+ e_1\left[\left(\frac{z(t)}{z_H}\right)^{b_H}\left(X_H - \lambda_{1H}S_1(t)\right) - \left(\frac{z(t)}{z_L}\right)^{b_H}\left(X_L - \lambda_{1H}S_1(t)\right)\right] + e_2\left[\left(\frac{z(t)}{z_L}\right)^{b_L}\left(X_L - \lambda_{1L}S_1(t)\right)\right] \end{aligned}$$

The first order condition of $K_1$ with respect to $X_H$ and $X_L$ indicates that $e_1 = q_H$ and $e_2 = 1 - q_H + q_H\left(\frac{z_L}{z(t)}\right)^{b_L - b_H}$. Recall $z_L > z(t)$ and $b_L > b_H$ based on $\frac{\partial b}{\partial \rho_1} \ge 0$ in Supplementary Appendix F. Then we know that $e_2 > 1$. From the Kuhn-Tucker condition, constraints (25) and (28) are binding. We obtain the solution $X_H^* = \left[\lambda_{1H} + \left(\frac{z_H^*}{z_L^*}\right)^{b_H}\left(\lambda_{1L} - \lambda_{1H}\right)\right]S_1(t)$ and $X_L^* = \lambda_{1L}S_1(t)$.

This implies $X_H^* > \lambda_{1H} S_1(t)$ and constraint (27) does not bind. The first order conditions of $K_1$ with respect to $z_H$ and $z_L$ mean $z_H^* = \dfrac{b_H \lambda_{1H}}{(b_H - 1)\lambda_2}$ and $z_L^*$ is the solution to $(1-q_H) z_L^{b_H - b_L} (b_L - 1)\lambda_2 \left[\dfrac{b_L \lambda_{1L}}{(b_L - 1)\lambda_2} - z_L\right] + q_H b_H z(t)^{b_H - b_L} (\lambda_{1L} - \lambda_{1H}) = 0$. Note $\lambda_{1L} > \lambda_{1H}$ and $b_L > 1$. To ensure the existence of solution, we assume that $z_L^* > \dfrac{b_L \lambda_{1L}}{(b_L - 1)\lambda_2}$. To satisfy constraint (26), $X_H$ should be smaller than $\lambda_{1L} S_1(t)$. Recall $\dfrac{\partial z^*}{\rho_1} \geq 0$ from Supplementary Appendix F. As $\rho_{1H} < \rho_{1L}$, we know that $z_H^* < z_L^*$ and $(\lambda_{1L} - \lambda_{1H})\left(1 - \left(\dfrac{z_H^*}{z_L^*}\right)^{b_H}\right) \geq 0$. Then constraint (26) is satisfied.

Consider the seller (principal) delegates the investment timing decision to the buyer (agent). There are two types of buyers. The probability of any buyer belonging to the good type category is $q_2$. We call the buyer good (bad) if his ambiguity aversion about revenues is $\rho_{2G}$ ($\rho_{2B}$) with $\rho_{2G} < \rho_{2B}$. This indicates that $\lambda_{2G} > \lambda_{2B}$. Let $\lambda_{2s}$ and $b_s$ denote the NMEU-based ambiguity multiplier and option value parameter of type $s$ agent, where $b_s = \dfrac{1}{2} - \dfrac{\delta_1 - \delta_{2s}}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} + \sqrt{\left(\dfrac{1}{2} - \dfrac{\delta_1 - \delta_{2s}}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)^2 + \dfrac{2\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}}$, $\delta_{2s} = 1/\lambda_{2s}$, $\lambda_{2s} = \dfrac{\rho_{2s}}{r - \mu_2 + \sigma_2\kappa_2} + \dfrac{1-\rho_{2s}}{r - \mu_2}$, $s = G$ or $B$. Assume that $z(t) < z_s$. As $b_s$ and $\lambda_{2s}$ are the buyer's private information, the seller asks for price $X_s$ based on the observable contract timing $z_s$ as follows:

$$\max_{z_G, z_B, X_G, X_B} q_2 \left(\frac{z(t)}{z_G}\right)^{b_G} \left(X_G - \lambda_1 S_1(t)\right) + (1-q_2)\left(\frac{z(t)}{z_B}\right)^{b_B} \left(X_B - \lambda_1 S_1(t)\right),$$

subject to:

$$\left(\frac{z(t)}{z_G}\right)^{b_G} \left(\lambda_{2G} z_G S_1(t) - X_G\right) \geq \left(\frac{z(t)}{z_B}\right)^{b_G} \left(\lambda_{2G} z_B S_1(t) - X_B\right)$$

$$\left(\frac{z(t)}{z_B}\right)^{b_B} \left(\lambda_{2B} z_B S_1(t) - X_B\right) \geq \left(\frac{z(t)}{z_G}\right)^{b_B} \left(\lambda_{2B} z_G S_1(t) - X_G\right)$$

$$\left(\frac{z(t)}{z_G}\right)^{b_G} \left(z_G \lambda_{2G} S_1(t) - X_G\right) \geq 0$$

$$\left(\frac{z(t)}{z_B}\right)^{b_B} \left(z_B \lambda_{2B} S_1(t) - X_B\right) \geq 0$$

Using the same logic as above and when $z_B^* > \dfrac{b_B \lambda_1}{(b_B - 1)\lambda_{2B}}$, the incentive policy under asymmetric information concerning the buyer's type can be defined by $(z_G^*, X_G^*, z_B^*, X_B^*) = \left( \dfrac{b_G \lambda_1}{(b_G - 1)\lambda_{2G}}, \left[ \lambda_{2G} z_G^* - \left( \dfrac{z_G^*}{z_B^*} \right)^{b_G} (\lambda_{2G} - \lambda_{2B}) z_B^* \right] S_1(t), z_B^*, \lambda_{2B} z_B^* S_1(t) \right)$, where $z_B^*$ is the solution to $(1 - q_2)\lambda_{2B}(b_B - 1) z_B^{b_G - b_B - 1} \left[ \dfrac{b_B \lambda_1}{(b_B - 1)\lambda_{2B}} - z_B \right] + (b_G - 1) q_2 z(t)^{b_G - b_B} (\lambda_{2G} - \lambda_{2B}) = 0$ . From Supplementary Appendix F, we have $\dfrac{\partial z^*}{\partial \rho_1} \geq 0$ and $\dfrac{\partial z^*}{\partial \rho_2} \geq 0$. Then we know that $\dfrac{\partial z_H^*}{\rho_{1H}} \geq 0$ and $\dfrac{\partial z_H^*}{\rho_2} \geq 0$, $\dfrac{\partial z_G^*}{\rho_1} \geq 0$ and $\dfrac{\partial z_G^*}{\rho_{2G}} \geq 0$.

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## SUPPLEMENTARY APPENDIX E – Proofs of eqs. (18)-(19)

The seller's and buyer's convenience yields are defined as $\delta_1 = 1/\lambda_1$ and $\delta_2 = 1/\lambda_2$. They generalize the convenience yields under risk of Dixit and Pindyck (1994) and under maxmin ambiguity by Trojanowska and Kort (2010). The value of the option to invest depends on both $S_1(t)$ and $S_2(t)$. We write $U$ for the buyer's option value $U(S_1, S_2)$. Consider a portfolio consisting of one unit of the option, $q_1$ units short in the output and $q_2$ units short in capital. Using Ito's lemma:

$$d(U - q_2 S_2 - q_1 S_1) = (U_2' - q_2)dS_2 + (U_1' - q_1)dS_1 + \frac{1}{2}(U_{22}''\sigma_2^2 S_2{}^2 + 2U_{21}''\varepsilon\sigma_1\sigma_2 S_2 S_1 + U_{11}''\sigma_1^2 S_1{}^2)dt \tag{E.1}$$

where $U_{22}'' = \dfrac{\partial^2 U}{\partial S_2{}^2}$, $U_{21}'' = \dfrac{\partial U}{\partial S_2}\dfrac{\partial U}{\partial S_1}$, $U_{11}'' = \dfrac{\partial^2 U}{\partial S_1{}^2}$.

Let $q_2 = U_2'$ and $q_1 = U_1'$. Then, the capital gain from the portfolio can be written as:

$$\frac{1}{2}(U_{22}''\sigma_2^2 S_2{}^2 + 2U_{21}''\sigma_1\sigma_2 S_2 S_1 + U_{11}''\sigma_1^2 S_1{}^2)dt \tag{E.2}$$

Considering the convenience yields on output and capital $\left(q_2\delta_2 S_2 + q_1\delta_1 S_1\right)dt$ and equating the sum of Eq. (E.2) and this component to the risk-free return $r\left(U - q_2 S_2 - q_1 S_1\right)dt$, we get:

$$\frac{1}{2}(U_{22}''\sigma_2^2 S_2{}^2 + 2U_{21}''\varepsilon\sigma_1\sigma_2 S_1 S_2 + U_{11}''\sigma_1^2 S_1{}^2) + \left(r - \delta_2\right)U_2' S_2 + \left(r - \delta_1\right)U_1' S_1 = rU \tag{E.3}$$

As $z = S_2(t)/S_1(t)$, $U = S_1 f(z) = S_1^{1-b} d_2 S_2{}^b$ where $f(z) = d_2 z^b$. From Eq. (E.1), we obtain:

$$\frac{1}{2}b(b-1)(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2) + \left(\delta_1 - \delta_2\right)b - \delta_1 = 0 \tag{E.4}$$

We get Eqs. (18)-(19) as solutions subject to the following boundary, value-matching and smooth-pasting conditions:

$$f(0) = 0 \tag{E.5}$$

$$f(z^*) = (1-\xi_{12})\left(\frac{z^*}{\delta_2} - \frac{1}{\delta_1}\right) \tag{E.6}$$

$$f'(z^*) = \frac{1-\xi_{12}}{\delta_2} \tag{E.7}$$

## SUPPLEMENTARY APPENDIX F – Proof of Proposition 3b

Taking the first order derivative of $b$ with respect to $\rho_1$:

$$\frac{\partial b}{\partial \rho_1} = \left(\frac{1}{\lambda_1^2}\frac{\partial \lambda_1}{\partial \rho_1}\frac{1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)\left(\frac{b-1}{\chi}\right) \geq 0 \tag{F.1}$$

Taking the derivative of $X_1^{nc}(t)$ with respect to $\rho_1$:

$$\begin{aligned}\frac{\partial X_1^{nc}(t)}{\partial \rho_1} &= \frac{-\xi_{12}}{\left(b-1\right)^2}\frac{\partial b}{\partial \rho_1}\lambda_1 S_1(t) + \left(1 + \frac{\xi_{12}}{b-1}\right)\frac{\partial \lambda_1}{\partial \rho_1}S_1(t) \\ &= \left[1 + \frac{\xi_{12}}{b-1}\left(1 - \frac{1}{\chi\lambda_1\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)}\right)\right]\frac{\partial \lambda_1}{\partial \rho_1}S_1(t)\end{aligned} \tag{F.2}$$

Let $\ell = \left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)\chi\lambda_1$. If $\ell < 1$, as $\xi_{12} \in (0,1)$, then:

$$\frac{\partial X_1^{nc}(t)}{\partial \rho_1} > \left[1 + \frac{1}{(b-1)}\left(1 - \frac{1}{\ell}\right)\right]\frac{\partial \lambda_1}{\partial \rho_1} S_1(t) = \frac{b\ell - 1}{(b-1)\ell}\frac{\partial \lambda_1}{\partial \rho_1} S_1(t) \tag{F.3}$$

Let $b = \phi + \chi$, where $\phi = \frac{1}{2} - \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}$, $\chi = \sqrt{\phi^2 + \frac{2\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}}$. From $\delta_1 = 1/\lambda_1$, the term $b\ell - 1$ can be written as:

$$\begin{aligned} & b\chi\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)\lambda_1 - 1 \\ & = \left(\phi^2 + \frac{2\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} + \phi\sqrt{\phi^2 + \frac{2\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}}\right)\lambda_1\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right) - 1 \end{aligned} \tag{F.4}$$

Recall $\frac{\partial \lambda_1}{\partial \rho_1} \geq 0$, $\ell > 0$ and $b > 1$. From Eqs. (F.3) and (F.4), if $\phi \geq 0$ then the term $b\ell - 1 > 0$ and $\frac{\partial X_1^{nc}(t)}{\partial \rho_1} > 0$.

Since $\left(\phi^2 + \frac{\delta_1}{\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)}\right)^2 - \phi^2\left(\phi^2 + \frac{2\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right) > 0$, the term $b\ell - 1 > 0$ and $\frac{\partial X_1^{nc}(t)}{\partial \rho_1} > 0$ if $\phi < 0$. This means $\frac{\partial X_1^{nc}(t)}{\partial \rho_1} > 0$ when $\ell < 1$. From Eq. (F.2), when $\ell \geq 1$, $\frac{\partial X_1^{nc}(t)}{\partial \rho_1} \geq 0$. Therefore: $\frac{\partial X_1^{nc}(t)}{\partial \rho_1} \geq 0$.

Since $\frac{\partial b}{\partial \kappa_1} = \frac{(b-1)}{\lambda_1^2\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)\chi}\frac{\partial \lambda_1}{\partial \kappa_1}$ and $\frac{\partial \lambda_1}{\partial \kappa_1} \geq 0$, using the same logic, the derivative of $X_1^{nc}(t)$ with respect to $\kappa_1$ is $\frac{\partial X_1^{nc}(t)}{\partial \kappa_1} = \left\{1 + \frac{\xi_{12}}{b-1}\left[1 - \frac{1}{\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)\lambda_1\chi}\right]\right\}\frac{\partial \lambda_1}{\partial \kappa_1} S_1(t) \geq 0$.

The derivative of $b$ with respect to $\rho_2$ is:

$$\frac{\partial b}{\partial \rho_2} = \left(\frac{-1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\frac{\partial \lambda_2}{\partial \rho_2}\right)\left(\frac{b}{\lambda_2^2\chi}\right) \geq 0 \tag{F.5}$$

Recall $X_2^{nc}(t) = \left(1 - \frac{1-\xi_{12}}{b}\right)\lambda_2 S_2(t)$. The derivative of $X_2^{nc}(t)$ with respect to $\rho_2$ is as follows:

$$\frac{\partial X_2^{nc}(t)}{\partial \rho_2} = \frac{(1-\xi_{12})}{b^2}\frac{\partial b}{\partial \rho_2}\lambda_2 S_2(t) + \left(1 - \frac{(1-\xi_{12})}{b}\right)\frac{\partial \lambda_2}{\partial \rho_2} S_2(t) = \Omega\frac{\partial \lambda_2}{\partial \rho_2}\frac{S_2(t)}{b} \tag{F.6}$$

where $\Omega = b - (1-\xi_{12})\left(1 + \frac{1}{\lambda_2\chi\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)}\right)$.

Since $\xi_{12} \in (0,1)$, $\Omega > \underline{\Omega} = b - \left(1 + \frac{1}{\lambda_2\chi\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)}\right)$. Then $\underline{\Omega}$ can be expressed as:

$$\underline{\Omega} = \frac{1}{\chi}\left(\chi^2 - \left(\frac{1}{2} + \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)\chi - \frac{1}{\lambda_2\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)}\right) = \frac{1}{\chi}\left(\underline{\Omega}_a - \underline{\Omega}_b\right) \tag{F.7}$$

where $\underline{\Omega}_a = \frac{1}{4} + \left(\frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)^2 + \frac{\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}$, $\underline{\Omega}_b = \left(\frac{1}{2} + \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2}\right)\chi$.

Recall $\chi > 0$. If $\frac{1}{2} + \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} < 0$, then $\underline{\Omega}_b < 0$ and $\underline{\Omega} > 0$. If $\frac{1}{2} + \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} > 0$, then $\underline{\Omega}_b > 0$. Let $\mathrm{H} = \frac{1}{4} + \left( \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} \right)^2$.

We get:

$$\begin{aligned} \underline{\Omega}_a^2 - \underline{\Omega}_b^2 &= \left( \mathrm{H} + \frac{\delta_1}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} \right)^2 - \left( \mathrm{H} + \frac{\delta_1 - \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} \right)\left( \mathrm{H} + \frac{\delta_1 + \delta_2}{\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2} \right) \\ &= \frac{\delta_2^2}{\left(\sigma_2^2 - 2\varepsilon\sigma_1\sigma_2 + \sigma_1^2\right)^2} > 0 \end{aligned} \tag{F.8}$$

Thus, $\underline{\Omega}_a > \underline{\Omega}_b$. Consequently, $\underline{\Omega} > 0$ and $\Omega > 0$.

Since $\frac{\partial \lambda_2}{\partial \rho_2} = \frac{1}{r - \mu_2 + \sigma_2\kappa_2} - \frac{1}{r - \mu_2} \le 0$, $b_2 > 1$ and $\chi > 0$, $\frac{\partial X_2^{nc}(t)}{\partial \rho_2} \le 0$.

Similarly, as $\frac{\partial \lambda_2}{\partial \kappa_2} \le 0$, $\frac{\partial X_2^{nc}(t)}{\partial \kappa_2} = \Omega \frac{\partial \lambda_2}{\partial \kappa_2} \frac{S_2(t)}{b} \le 0$.