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Noise in phase-(in)sensitive dual-core fiber parametric amplification

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Abstract: Flat and wide-bandwidth gain spectrum, along with phase-sensitive gain with no need to generate amplifier input idlers, can be achieved with a coupled dual-core fiber optical parametric amplifier. In this paper, we analyze the noise properties of such an amplifier. We achieve a 3 dB noise figure in the phase-insensitive case and a minimum of -6 dB noise figure in the phase-sensitive configuration. An alternative phase-sensitive configuration is also studied, that avoids the use of idlers at the input of the amplifier, leading to a 3 dB flat spectrum combined noise figure for the output signals. Pump transfer noise in phase-insensitive and phase-sensitive configurations is also studied along with the noise figure variation against the length of the amplifier.

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1. Introduction

Amplification in optical fiber communications has currently a bottleneck with nowadays commercial EDFA amplifiers achieving no more than 40 nm gain spectrum [1]. Moreover they are not suitable to work at wavelengths outside the C+L bands, limiting applications such as optical wireless communications (OWC), which work in a much broader spectrum (10nm to millimeter band). Included in this set of frequencies are the upcoming free-space optical (FSO) earth to satellite communications, satellite to satellite communications and visible light communications. Therefore in order to relax today's constraints, related with transmission capacity in fiber optical communication systems and to enable novel applications which require enhanced wavelength transparency such as OWC [2], new amplification technologies are required [3]. Recently the research community has been focusing on spatial division multiplexing to follow the constant demand for more bandwidth, however a recent review by Richardson appointed broadband amplifiers with up to 10-fold increase in amplification bandwidth, relative to the standard EDFA, as the first option to be considered in order to increase transmission capacity [4].

Optical parametric amplification (OPA) can meet the requirements for a broadband and wavelength transparent amplification. It has been demonstrated that fiber based OPA can offer over 360 nm gain bandwidth [5] which is 9 times more than the current commercially available technologies. It can also offer low noise figure (NF) amplifiers [6] which at the present state are important for ultra-long haul optical fiber communications. However this technology faces several challenges, i.e., it is intrinsically polarization dependent, despite possible undesirable use of polarization diversity and difficulties arise when using this technology in wavelength division multiplexing (WDM) systems, due to the generated higher order four-wave mixing (FWM) terms, which increase nonlinear crosstalk. Besides in order to reasonably justify the replacement of current amplification technology, performance parameters such as ultra-broadband and flat gain are important.

In this paper we analyze theoretically the noise performance of a recently proposed scheme that uses a four-mode parametric amplifier [7] in a dual-core fiber [8]. Such theoretical predictions can motivate the experimental realization of this scheme and the development of suitable fibers which are not currently available. However there are several advantages with this scheme. It can provide flat broadband gain spectrum, exponential gain dependence on pump power when the linear phase mismatch is equal to zero ($\Delta\beta = 0$), gain when the pump lies in the normal dispersion regime and phase-sensitive (PS) gain without the generation of idlers at the input, i.e., just two phase locked copies of the signal and the pump at the input of the amplifier are required. Non-degenerate dual-pump fiber optical parametric amplifiers (2P-FOPAs) can also provide flat gain over a wide spectrum. However the proposed scheme can achieve the same in a single pump configuration. Moreover the proposed scheme does not suffer from pump to pump Raman induced power transfer, such as the 2P-FOPA which can reduce the FWM efficiency and therefore reduces the gain. In addition our scheme is more spectrally efficient than the 2P-FOPA since no additional idlers are generated outside the gain spectrum of the amplifier. Another potential benefit is the possibility to interleave the odd and even numbered channels between the two cores of the amplifier with potential reduction of FWM crosstalk, maintaining the nice properties of flat and broad bandwidth gain spectrum. However the noise properties of such an amplifier have not been investigated so far. In this letter we investigate those properties.

2. Theoretical background

We state the theoretical background that it is in the origin of the results presented later in this paper. We show in Fig. 1 the schematic of the dual-core FOPA, where $\lambda_{s,p,i}$ are the wavelengths of the signal, pump and idler, respectively and $e_s(0)$, $e_i(0)$ are the input signal and idler field

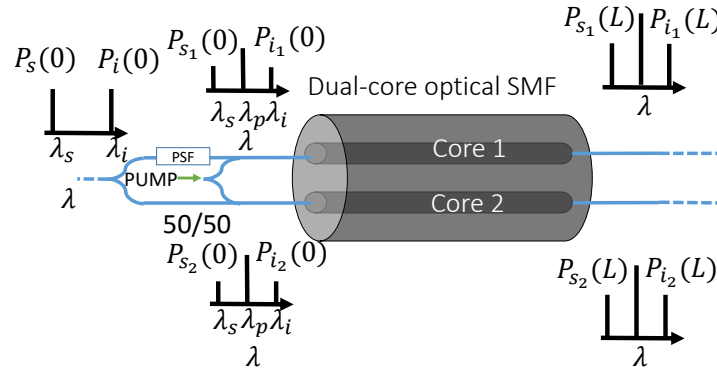


Fig. 1. Schematic of the dual core FOPA. L is the length of the amplifier. $P_{s1,2}$ is the power of the signal in core 1 and core 2, respectively. $P_{i1,2}$ is the power of the idler in core 1 and core 2, respectively. PSF- Phase shifter.

amplitudes. The dual core FOPA can be described by

$$\vec{E}(z) = K(z)\vec{E}(0), \quad (1)$$

where

$$\vec{E}(z) = \begin{pmatrix} e_{s1}(z) \\ e_{i1}^*(z) \\ e_{s2}(z) \\ e_{i2}^*(z) \end{pmatrix} \quad (2)$$

where s_1 and s_2 mean the signals in core 1 and 2, i_1 and i_2 mean the idlers in core 1 and 2, respectively. In the small signal limit $K(z)$ is given by [8]

$$K(z) = \exp(iMz) = \begin{pmatrix} A & B & D & E \\ B^* & A^* & E^* & D^* \\ D & E & A & B \\ E^* & D^* & B^* & A^* \end{pmatrix}. \quad (3)$$

where,

$$M = \begin{pmatrix} K_0 & \frac{P_p\gamma}{2} & C & 0 \\ -\frac{P_p\gamma}{2} & -K_0 & 0 & -C \\ C & 0 & K_0 & \frac{P_p\gamma}{2} \\ 0 & -C & -\frac{P_p\gamma}{2} & -K_0 \end{pmatrix} \quad (4)$$

(see Appendix for a complete derivation of the coefficients matrix M) with gain coefficients [8]

$$g_1 = \sqrt{P_p^2 \gamma^2 - 4(K_0 + C)^2} = \sqrt{P_p^2 \gamma^2 - 4k^2} \quad (5)$$

$$g_2 = \sqrt{P_p^2 \gamma^2 - 4(K_0 - C)^2} = \sqrt{P_p^2 \gamma^2 - 4(k - 2C)^2} \quad (6)$$

where $K_0 = k - C$, $k = P_p\gamma/2 + \Delta\beta/2$ and matrix elements [8]

$$A = |A|e^{i\phi_A} = \frac{\cosh(\frac{g_1 z}{2})}{2} + \frac{\cosh(\frac{g_2 z}{2})}{2} + \frac{(K_0 - C) \sinh(\frac{g_2 z}{2}) i}{g_2} + \frac{(K_0 + C) \sinh(\frac{g_1 z}{2}) i}{g_1} \quad (7)$$

$$B = |B|e^{i\phi_B} = \frac{P_p \gamma \sinh(\frac{g_1 z}{2}) i}{2 g_1} + \frac{P_p \gamma \sinh(\frac{g_2 z}{2}) i}{2 g_2} \quad (8)$$

$$D = |D|e^{i\phi_D} = \frac{\cosh(\frac{g_1 z}{2})}{2} - \frac{\cosh(\frac{g_2 z}{2})}{2} + \frac{(K_0 + C) \sinh(\frac{g_1 z}{2}) i}{g_1} - \frac{(K_0 - C) \sinh(\frac{g_2 z}{2}) i}{g_2} \quad (9)$$

$$E = |E|e^{i\phi_E} = \frac{P_p \gamma \sinh(\frac{g_1 z}{2}) i}{2 g_1} - \frac{P_p \gamma \sinh(\frac{g_2 z}{2}) i}{2 g_2}, \quad (10)$$

which are explicit functions of the coupling coefficient C , the linear phase mismatch $\Delta\beta = \beta_3(\omega_p - \omega_0)(\omega_s - \omega_p)^2$, the power of the pump P_p , the length of the fiber L , the nonlinear parameter γ , the wavelength of the pump $\lambda_p = 2\pi c/\omega_p$, the third order dispersion parameter β_3 , the zero-dispersion wavelength $\lambda_0 = 2\pi c/\omega_0$, whose values are stated in Table 1. The wavelength of the signal is $\lambda_s = 2\pi c/\omega_s$, c is the light velocity and $\phi_{A,D}$ are the phases of the matrix elements A, D .

We define the initial conditions of the case studies in this paper for the phase-insensitive (PI) and PS amplifier, respectively given by,

$$\vec{E}(0) = \begin{pmatrix} e_{s_{10}} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (11) \quad \vec{E}(0) = \begin{pmatrix} |e_{s_{10}}| e^{-i\phi_A} \\ |e_{s_{10}}| e^{-i\phi_B} \\ |e_{s_{10}}| e^{-i\phi_D} \\ |e_{s_{10}}| e^{-i\phi_E} \end{pmatrix}, \quad (12)$$

where $e_{s_{10}}$ is the initial value of the signal. In addition we will further in this paper study the noise performance of an alternative PS amplifier design with the input configuration

$$\vec{E}(0) = \begin{pmatrix} |e_{s_{10}}| e^{-i\phi_A} \\ 0 \\ |e_{s_{10}}| e^{-i\phi_D} \\ 0 \end{pmatrix}. \quad (13)$$

This yields a PS amplifier, where the idlers are not required in order to achieve PS gain.

In order to calculate the NF resultant from the amplified spontaneous emission noise (ASE) we used the following equations [9, Eq. (125),],

$$NF_{ASE_{s_1}} = 1 + (|B|^2 + |D|^2 + |E|^2)/|A|^2 \quad (14)$$

$$NF_{ASE_{i_1}} = 1 + (|A|^2 + |E|^2 + |D|^2)/|B|^2 \quad (15)$$

$$NF_{ASE_{s_2}} = 1 + (|E|^2 + |A|^2 + |B|^2)/|D|^2 \quad (16)$$

$$NF_{ASE_{i_2}} = 1 + (|D|^2 + |B|^2 + |A|^2)/|E|^2, \quad (17)$$

which give the PI ASE NFs of the 4 modes involved in the system given by Eq. (1).

The PI gain for each mode is,

$$G_{s_1} = |A|^2, G_{i_1} = |B|^2, G_{s_2} = |D|^2, G_{i_2} = |E|^2, \quad (18)$$

Table 1. Setup parameters (unless otherwise stated)

Symbol	Value	Units
λ_p	1550	nm
λ_0	1549.98	nm
P_p	1.5	W
γ	11	$\text{W}^{-1} \text{km}^{-1}$
β_3	0.1	$\text{ps}^3 \text{km}^{-1}$
L	0.4	km
C	4.1	km^{-1}
$OSNR_{Pp}$	45	dB
P_{in}	-30	dBm

and the maximum PS gain (obtained with the input configuration (12)) is given by,

$$\begin{aligned}
 G_{PS_{s_1}} &= G_{PS_{i_1}} = G_{PS_{s_2}} = G_{PS_{i_2}} = G_{PSA} = \\
 &= |e_{s_1}(z)|^2 / |e_{s_{10}}|^2 \\
 &= \frac{|A|e^{i\phi_A} |e_{s_{10}}| e^{-i\phi_A} + |B|e^{i\phi_B} |e_{s_{10}}| e^{-i\phi_B} + |D|e^{i\phi_D} |e_{s_{10}}| e^{-i\phi_D} + E|e^{i\phi_E} |e_{s_{10}}| e^{-i\phi_E}|^2}{|e_{s_{10}}|^2} \\
 &= (|A| + |B| + |D| + |E|)^2.
 \end{aligned} \tag{19}$$

We shall derive the equation for the PS ASE NF. In order to successfully achieve that objective we start by the equation that determines the NF in a two mode amplifier [10],

$$NF = |e_{s_{10}}|^2 \frac{(|\mu|^2 + |\nu|^2)}{|e_{s_{10}}\mu - ie_{s_{10}}\nu|^2} = |e_{s_{10}}|^2 \frac{(|\mu|^2 + |\nu|^2)}{|e_{s_{10}}|^2 (|\mu| + |\nu|)^2} = \frac{(|\mu|^2 + |\nu|^2)}{(|\mu| + |\nu|)^2}. \tag{20}$$

where μ and ν are complex coefficients that determine the contribution of the signal and idler, respectively, to the output amplitude of the electrical field of the mode. It is not difficult to extend it to the case of a four-mode amplifier given that the denominator of (20) is just the output power of one mode. Taking into consideration (38) of [10] the numerator is related with the spectral density of the output noise. With this in mind we can write the PS ASE NF of the system represented in Fig. 1 and with input configuration given by (12) as

$$\begin{aligned}
 NF_{ASE_{r_j}} &= \\
 &= \frac{|e_{s_{10}}|^2 (|A|^2 + |B|^2 + |D|^2 + |E|^2)}{|e_{s_1}(z)|^2}
 \end{aligned} \tag{21}$$

where $r = s, i$ and $j = 1, 2$. The denominator gives $|e_{s_1}(z)|^2 = |e_{s_{10}}|^2 G_{PSA} = |e_{s_{10}}|^2 (|A| + |B| + |D| + |E|)^2$. Using (12) it becomes

$$NF_{ASE_{r_j}} = \frac{(|A|^2 + |B|^2 + |D|^2 + |E|^2)}{(|A| + |B| + |D| + |E|)^2}. \tag{22}$$

This is identical to $NF_{ASE_{r_j}} = NF_{ASE_{r_j}} \frac{G_{r_j}}{G_{PSA}}$. In the case of the system with input configuration

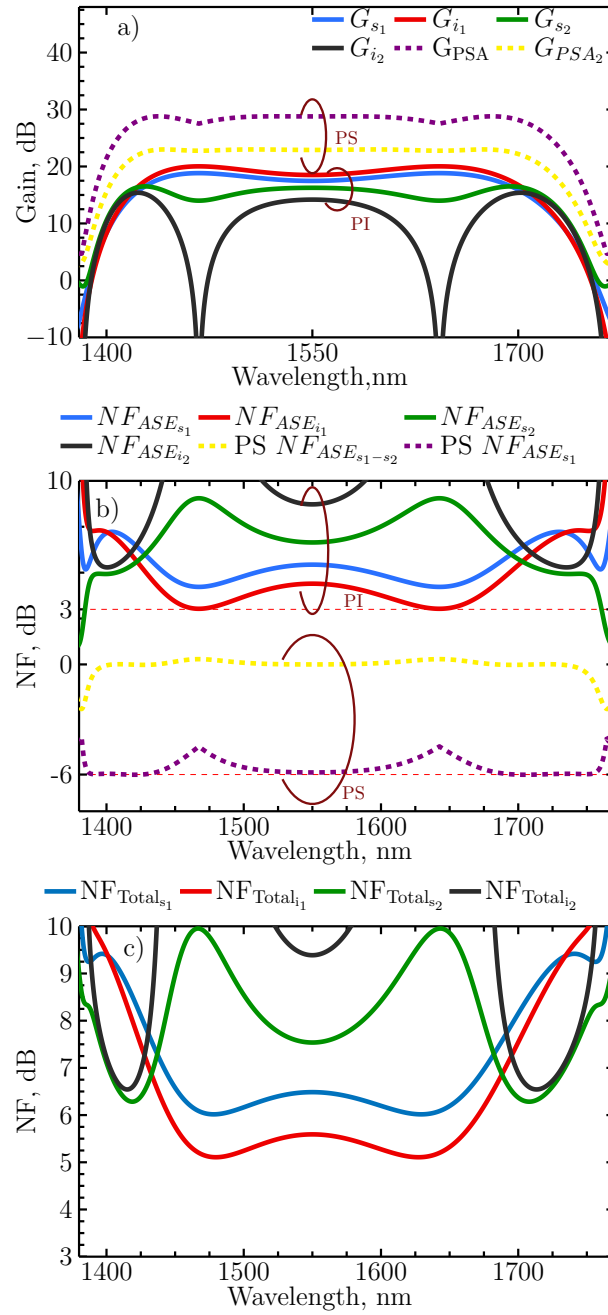


Fig. 2. Amplifier NF and gain spectrum. a) PI and PS gain, resultant from (18) and (19), respectively. b) PI and PS NF resultant from (14) to (17), (23) and (22), c) PI $NF_{Total_{rj}}$ resultant from (25) where $r = s, i$ and $j = 1, 2$ of (13) the PS ASE NF is

$$NF_{ASE_{s1-s2}} = \frac{(|A|^2 + |B|^2 + |D|^2 + |E|^2)}{G_{PSA2}} \quad (23)$$

where $G_{PSA2} = (|A| + |D|)^2$.

In addition to the ASE is the pump transfer noise (PTN) to the signals and idlers due to the ASE noise, generated by the erbium doped fiber amplifiers (EDFA), responsible for amplifying the pump wave. Therefore the pump will have a certain amount of noise and an optical signal to noise ratio given by $OSNR_{P_p}$ in Table 1. The PTN for the four modes is given by [11],

$$\Delta NF_{PTNr_j} = \frac{P_{in} \left(\frac{\partial G_{r_j}}{\partial P_p} \right)^2 P_p^2}{OSNR_{P_p} G_{r_j}^2 h\nu \Delta\nu}. \quad (24)$$

In the case of PS amplifier NF we must substitute in Eq. (24), $G_{r_j} = G_{PS_{r_j}}$. The total NF is given by,

$$NF_{Totalr_j} = NF_{ASE_{r_j}} + \Delta NF_{PTNr_j} \quad (25)$$

where $h\nu$ is the pump photon energy and $\Delta\nu$ is the 0.1 nm resolution of the $OSNR_{P_p}$ measurement.

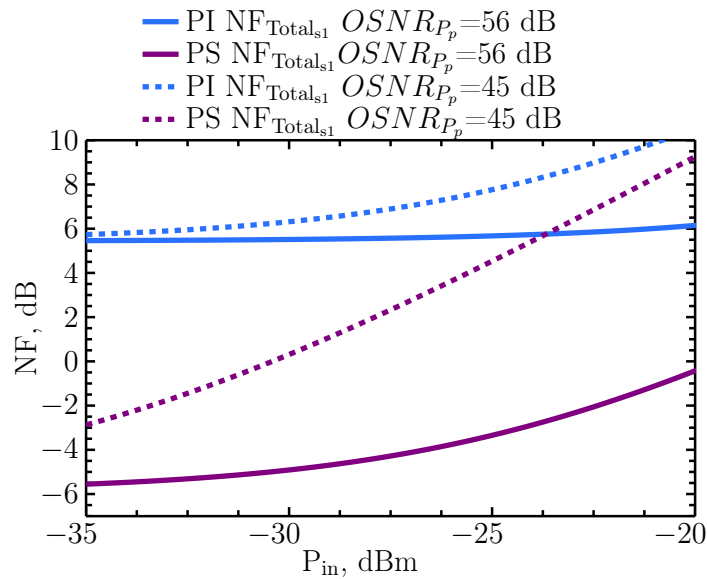


Fig. 3. PI and PS amplifier NF against power of the input signal P_{in} for signal wavelength $\lambda_s = 1550$ nm.

3. Results and discussion

In Fig. 2(a) we show the gain of the PI and PS amplifiers. We stress the broad bandwidth of the PI amplifier which can be wider if we set the $\lambda_0 = \lambda_p$. Although this will make the gain bandwidth unlimited and wavelength independent, since our model does not include higher order dispersion or zero-dispersion wavelength variations. It is worth to mention almost 12 dBs higher gain of the PS amplifier comparatively to the PI amplifier, similarly to the 4-mode FOPA in [7]. However if we consider the power of all the sidebands involved in the process, i.e., signals and idlers in core 1 and 2 we have a 6 dB combined gain. In Fig. 2(b) one can check that the PI NF of the modes s_1 and i_1 , i.e., $NF_{ASE_{s_1}}$ and $NF_{ASE_{i_1}}$ of the excited core (core 1) approach the quantum noise limit of 3 dB for a certain region of the spectrum. At about 1453 nm and 1657 nm there are two PI NF peaks in the spectrum of the mode i_2 , due entirely to the fact that $G_{i_2} = 0$ at these wavelengths as can be seen in Fig. 2 a). However the PI NF and the gain of the signals in the excited core is almost constant over the gain bandwidth of the amplifier, but not as constant as the

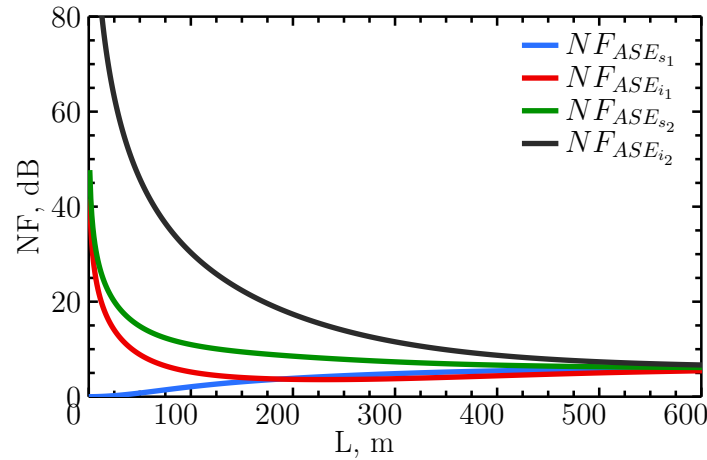


Fig. 4. PI NF against length of the fiber for signal wavelength $\lambda_s = 1550$ nm.

configuration provided by the input configuration (13) (yellow curve). In Fig. 2(b) we also show the PS $NF_{ASE_{s_1}}$. A flat bottom of -6 dB PS ASE NF is obtained due to the fact that this is a four mode PS amplifier [10]. This very low noise figure is the result of just considering the input signal to noise ratio of one sideband. However if we consider the input signal to noise ratio of the four sidebands involved in the process the input signal to noise ratio increases four times and the combined NF yields in 0 dB. We also show in Fig. 2(b) the PS NF of the system with the initial conditions of (13). In this case a flat bottom of 0 dB PS ASE NF is obtained. This has the potential to be an interesting configuration since 0 dB flat spectrum NF is obtained for the signals s_1 and s_2 . However the combined spectrally flat PS NF is 3 dB as mentioned in the abstract due to the superposition of the fields s_1 and s_2 at the input. In Fig 2(c) we show the total PI NFs which sum the noise contributions of the ASE and PTN. One can see the moderate sensitivity of the proposed amplifier configuration to PTN. We assume that both cores are pumped by the same EDFA and therefore the PTN is the same in both cores. In Fig. 3 we can see that the PI and PS curves with pump OSNR of 56 dB are relatively stable when considering the amplifier NF against P_{in} , while when the OSNR of the pump is 45 dB significant degradation occurs. In both cases, i.e., pump OSNR of 45 dB and 56 dB, the PS amplifier is more degraded by the PTN than the PI amplifier. In Fig. 4 the PI ASE NF is shown against the length of the amplifier. In Table 1 one can see that the PI ASE NF of the four modes stabilizes at about 6 dB for lengths of the amplifier larger than ≈ 500 m.

4. Conclusions

In conclusion we have studied the noise performance of a dual core FOPA. A minimum of 3 dB phase-insensitive (PI) noise figure (NF) is obtained when the pump is noiseless. In the phase-sensitive (PS) case a minimum of -6 dB is obtained due to the fact that this is a 4 mode PS amplifier. This yield a PS amplifier with a combined NF of 0 dBs, if we sum all the noise contributions of the signals and idlers involved in the amplification process. An alternative PS configuration has also been studied, that does not require the idlers to achieve PS gain. This leads to a noise figure of 0 dB for the signals s_1 and s_2 , which can raise interesting applications in the future. The pump transfer noise degrades considerably the functioning of the PS and PI amplifier, although it has a more negative impact on the PS amplifier. The pump OSNR shall be sufficiently high in order to avoid degradation of the amplifier NF. The PI amplified quantum noise NF of the amplifier stabilizes in 6 dB after overtaking a certain length of the fiber.

5. Appendix: derivation of coefficients matrix

We start by assuming that:

$$k = \gamma \frac{P_p}{2} + \frac{\Delta\beta}{2} \quad (26)$$

$$K_A = \gamma \frac{P_p}{2} - \frac{\Delta\beta}{2} \quad (27)$$

$$u_{s_1, i_1, s_2, i_2} = e_{s_1, i_1, s_2, i_2} \exp(i(K_A + \beta_{s_1, i_1, s_2, i_2} + C)z) \quad (28)$$

$$u_{p_1, p_2} = \sqrt{\frac{P_p}{2}} \exp\left(i\left(\gamma \frac{P_p}{2} + \beta_{p_1, p_2} + C\right)z\right) \quad (29)$$

$$\Delta\beta = -2\beta_{p_1, p_2} + \beta_{s_1, s_2} + \beta_{i_1, i_2} \quad (30)$$

$$\beta_{s_1} = \beta_{s_2} \quad (31)$$

$$\beta_{i_1} = \beta_{i_2} \quad (32)$$

where $\beta_{p_1, p_2, s_1, s_2, i_1, i_2}$ are the propagation constants of the pump, signal and idler in core 1 and 2, respectively. u is the electric field defined by [8]

$$\frac{du_{p_1}}{dz} = iu_{p_1}(\beta_{p_1} + \gamma(|u_{p_1}|^2)) + i\gamma 2u_{p_1}^* u_{s_1} u_{i_1} + iCu_{p_2} \quad (33)$$

$$\frac{du_{s_1}}{dz} = iu_{s_1}(\beta_{s_1} + \gamma(2|u_{p_1}|^2)) + i\gamma u_{p_1}^2 u_{i_1}^* + iCu_{s_2} \quad (34)$$

$$\frac{du_{i_1}}{dz} = iu_{i_1}(\beta_{i_1} + \gamma(2|u_{p_1}|^2)) + i\gamma u_{p_1}^2 u_{s_1}^* + iCu_{i_2} \quad (35)$$

$$\frac{du_{p_2}}{dz} = iu_{p_2}(\beta_{p_2} + \gamma(|u_{p_2}|^2)) + i\gamma 2u_{p_2}^* u_{s_2} u_{i_2} + iCu_{p_1} \quad (36)$$

$$\frac{du_{s_2}}{dz} = iu_{s_2}(\beta_{s_2} + \gamma(2|u_{p_2}|^2)) + i\gamma u_{p_2}^2 u_{i_2}^* + iCu_{s_1} \quad (37)$$

$$\frac{du_{i_2}}{dz} = iu_{i_2}(\beta_{i_2} + \gamma(2|u_{p_2}|^2)) + i\gamma u_{p_2}^2 u_{s_2}^* + iCu_{i_1} \quad (38)$$

It is assumed that:

$$P_p \gg |u_{s_1, i_1, s_2, i_2}|^2 \quad (39)$$

Starting from (34) to (35) and making the substitution of (28) it follows that (the same procedure can be applied to (37) and (38)),

$$\begin{aligned} \frac{de_{s_1}}{dz} \exp(i(K_A + \beta_{s_1} + C)z) &= ie_{s_1} \exp(i(K_A + \beta_{s_1} + C)z)(k - C) + \\ &+ i\gamma \frac{P_p}{2} \exp((i(K_A + \beta_{s_1} + C))z) e_{i_1}^* + \\ &+ iCe_{s_2} \exp(i(K_A + \beta_{s_1} + C)z), \end{aligned} \quad (40)$$

$$\begin{aligned} \frac{de_{i_1}^*}{dz} \exp(-i(K_A + \beta_{i_1} + C)z) &= (-ie_{i_1}^* \exp(-i(K_A + \beta_{i_1} + C)z)(k - C)) - \\ &- \left(i\gamma \frac{P_p}{2} \exp((i(-K_A - \beta_{i_1} - C))z) e_{s_1}\right) - \\ &- iCe_{i_2}^* \exp(-i(K_A + \beta_{i_1} + C)z) \end{aligned} \quad (41)$$

we can now eliminate the phase factors

$$\frac{de_{s_1}}{dz} = ie_{s_1}(k - C) + i\gamma \frac{P_p}{2} e_{i_1}^* + iC e_{s_2} \quad (42)$$

$$\frac{de_{i_1}^*}{dz} = \left(-ie_{i_1}^*(k - C) \right) - \left(i\gamma \frac{P_p}{2} e_{s_1} \right) - iC e_{i_2}^* \quad (43)$$

and finally obtain the coefficients matrix

$$\frac{de_{s_1}}{dz} = ie_{s_1}K_0 + i\gamma \frac{P_p}{2} e_{i_1}^* + iC e_{s_2} \quad (44)$$

$$\frac{de_{i_1}^*}{dz} = -ie_{i_1}^*K_0 - i\gamma \frac{P_p}{2} e_{s_1} - iC e_{i_2}^* \quad (45)$$

$$\frac{de_{s_2}}{dz} = ie_{s_2}K_0 + i\gamma \frac{P_p}{2} e_{i_2}^* + iC e_{s_1} \quad (46)$$

$$\frac{de_{i_2}^*}{dz} = -ie_{i_2}^*K_0 - i\gamma \frac{P_p}{2} e_{s_2} - iC e_{i_1}^* \quad (47)$$

$$M = \begin{pmatrix} K_0 & \frac{P_p\gamma}{2} & C & 0 \\ -\frac{P_p\gamma}{2} & -K_0 & 0 & -C \\ C & 0 & K_0 & \frac{P_p\gamma}{2} \\ 0 & -C & -\frac{P_p\gamma}{2} & -K_0 \end{pmatrix} \quad (48)$$

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