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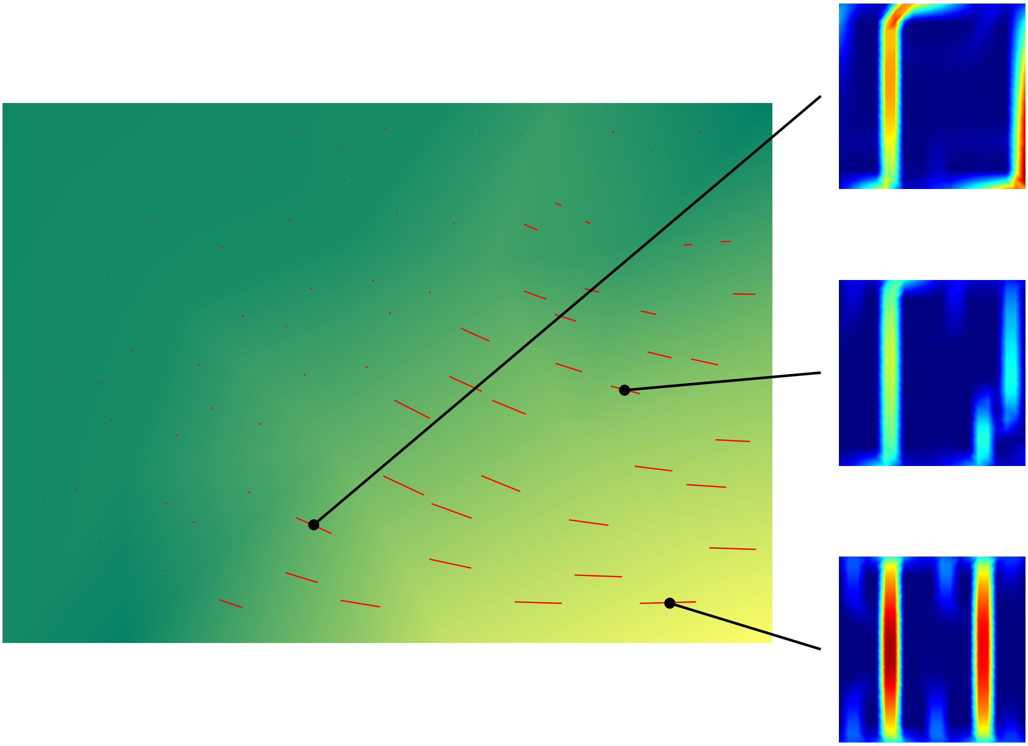

Multiscale Modelling of Reinforced Concrete

ADAM SCIEGAJ

Department of Architecture and Civil Engineering
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2018

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

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Cover:
Snapshot from an FE^2 analysis of a reinforced concrete deep beam showing the magnitude and direction (red lines) of the first principal strain. The distribution of the first principal strain at subscale is illustrated for three different RVEs.

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Abstract

Since concrete cracks at relatively low tensile stresses, the durability of reinforced concrete structures is highly influenced by its brittle nature. Cracks open up for ingress of harmful substances, e.g. chlorides, which in turn cause corrosion of the reinforcement. Crack widths are thus limited in the design codes, and accurate prediction methods are needed. For structures of more complex shapes, current computational methods for crack width predictions lack precision. Hence, the development of new simulation tools is of interest. In order to properly describe the crack growth in detail, cracking of concrete, constitutive behaviour of steel, and the bond between them must be accounted for. These physical phenomena take place at length scales smaller than the dimensions of large reinforced concrete structures. Thus, multiscale modelling methods can be employed to reinforced concrete.

This thesis concerns multiscale modelling of reinforced concrete. More specifically, a two-scale model, based on Variationally Consistent Homogenisation (VCH), is developed. At the large-scale, homogenised (effective) reinforced concrete is considered, whereas the underlying subscale comprises plain concrete, resolved reinforcement bars, and the bond between the two. Each point at the large-scale is associated with a Representative Volume Element (RVE) defining the effective response through a pertinent boundary value problem. In a numerical framework, the procedure pertains to a so-called FE^2 (Finite Element squared) algorithm, where each integration point in the discretised large-scale problem inherits its response from an underlying RVE problem. In order to properly account for the concrete–reinforcement bond action, the large-scale problem is formulated in terms of a novel effective reinforcement slip variable in addition to homogenised displacements.

In a series of FE^2 analyses of a plane problem pertaining to a reinforced concrete deep beam with distributed reinforcement layout, the influence of boundary conditions on the RVE, as well as the sizes of the RVE and the large-scale mesh, are studied. The results of the two-scale analyses with and without incorporation of the effective reinforcement slip are compared to fully-resolved (single-scale) analysis. A good agreement with the single-scale results in terms of structural behaviour, in particular load-deflection relation and average strain, is observed. Depending on the sub-scale boundary conditions, approximate upper and lower bounds on structural stiffness are obtained. The effective strain field gains a localised character upon incorporation of the effective reinforcement slip in the model, and the predictions of crack widths are improved. The two-scale model can thus describe the structural behaviour well, and shows potential in saving computational time in comparison to single-scale analyses.

Keywords: multiscale, reinforced concrete, computational homogenisation, cracking, bond-slip

Rodzicom

Preface

The work in this thesis was carried out from August 2015 to February 2018 at the Division of Structural Engineering, Department of Architecture and Civil Engineering and the Division of Material and Computational Mechanics, Department of Industrial and Materials Science, Chalmers University of Technology. The research was financially supported by the Swedish Research Council (Vetenskapsrådet) grant no. 621-2014-5168. Some of the numerical simulations presented herein were performed on resources at the Chalmers Centre for Computational Science and Engineering (C3SE) provided by the Swedish National Infrastructure for Computing (SNIC).

Acknowledgements

I am most grateful to my excellent supervisors Professor Karin Lundgren, Professor Fredrik Larsson, Assistant Professor Filip Nilenius and Professor Kenneth Runesson. All four of you have, both individually and as a group, supervised me in the best possible way by sharing your experience, being always ready and willing to help, and by giving me encouragement and feedback. All of this has been invaluable to me. I look forward to a continued fruitful collaboration.

Moving on, I would like to acknowledge Dr. Mikael Öhman and Dr. Erik Svenning for their help with C++ and OOFEM. The stimulating discussions we have had about coding and multiscale modelling are greatly appreciated. Furthermore, I wish to express my gratitude to my dear friends and colleagues at the divisions of Dynamics, Material and Computational Mechanics and Structural Engineering (in alphabetical order), thanks to whom I have had a chance to enjoy the cordial working environment at both departments. I am grateful for all our interesting talks and all the moments of laughter and joy we have shared.

Finally, I would like to thank my parents and my brother for their love and support, especially at times of doubt. Without you, I would not be where I am today.

Gothenburg, February 2018

Adam Ścięgaj

Thesis

This thesis consists of an extended summary and the following appended papers:

Paper A	A. Sciegaj, F. Larsson, K. Lundgren, F. Nilenius and K. Runesson. "Two-scale finite element modelling of reinforced concrete structures: Effective response and subscale fracture development" *International Journal for Numerical Methods in Engineering*, in press. DOI: 10.002/nme.5776
Paper B	A. Sciegaj, F. Larsson, K. Lundgren, F. Nilenius and K. Runesson "A multiscale model for reinforced concrete with macroscopic variation of reinforcement slip" *Submitted for publication*

The appended papers were prepared in collaboration with the co-authors. The author of this thesis was responsible for the major progress of the work in the papers, i.e. took part in formulating the theory, led the planning of the papers, developed the numerical implementation, carried out the numerical simulations, and prepared the manuscript.

Contents

Part I
Extended Summary

1 Introduction

1.1 Background

Concrete is the most widely used construction material in the world. Beams, columns, foundations and slabs are just a few examples of essential structural concrete elements. However, due to the low tensile strength of plain concrete, the aforementioned structures are almost always reinforced in some way.

Even though concrete itself cracks at relatively low tensile stresses, such brittle behaviour does not entail structural failure since tension after cracking is carried by the reinforcement. Nevertheless, the durability of reinforced concrete structures is greatly influenced by the cracking of concrete, as the cracks allow harmful substances like chlorides to penetrate the cover, which eventually initiates corrosion of the reinforcement [46]. Hence, limiting crack width is an important part of structural design, and is therefore addressed in design codes, e.g. Eurocode 2 [8]. It is therefore important to be able to predict the evolving crack width in reinforced concrete structures accurately.

Crack width predictions can be performed using many different analytical and numerical methods. Even though they provide accurate results for structural members loaded in bending, their precision is lost when dealing with components loaded in shear, torsion, or with structures of complex shapes [7]. As an alternative to classical full-resolution modelling, where each individual constituent (concrete and reinforcement) is accounted for explicitly, multiscale methods can be employed. In particular, methods based on computational homogenisation [62, 40] are gaining popularity. In these models, the microstructure of the material is statistically represented in an appropriately sized volume element, often referred to as a Representative Volume Element (RVE). Although applied to plain concrete in a number of studies [42, 43, 46, 44, 45, 47, 61], multiscale modelling reinforced concrete is not commonly found in the literature.

1.2 Aim of research

The aim of the work is to develop techniques for multiscale modelling of reinforced concrete structures. To reach the aim, the following objectives are identified:

- establish a variationally consistent framework for multiscale analysis of cracking of reinforced concrete,
- implement an FE^2 (Finite Element squared) algorithm for established two-scale formulation,

- investigate the effect of different boundary conditions on the subscale problems on the Representative Volume Element (RVE),
- study the effect of incorporation of reinforcement slip as a macroscopic variable.

For all these objectives, it is imperative to examine how well the multiscale formulation simulates the structural behaviour in comparison to conventional fully-resolved (single-scale) analyses, i.e. to evaluate the performance of the multiscale formulation.

1.3 Scope and limitations

The main focus of this work is to develop multiscale modelling techniques for reinforced concrete. It is assumed throughout this work that concrete can be treated as a continuum, while the reinforcement bars can be modelled as beams. Thus, the concrete at mesoscale level with its heterogeneous microstructure (aggregates, cement paste and the interfacial transition zone) is not considered herein. First order homogenisation [28] is adopted consistently, i.e. is it assumed that the macroscopic fields vary linearly within the RVE. Lastly, this work encompasses two-dimensional modelling in plane stress. The modelling emphasis is put on structures with distributed reinforcement layout (uniformly reinforced in two directions), under quasi-static loading and prior to structural failure.

2 Cracking in reinforced concrete structures

Crack modelling in plain concrete is itself a challenging task, since the problem ranges from diffused microcracks to localized failure [60]. Presence of reinforcement further complicates the matter, as localized cracking does not mean failure of the structure. Due to stress transfer via the reinforcement bars, several cracks can form upon further loading of the structure. Hence, not only cracking of concrete and action of the reinforcement, but also the stress transfer between concrete and reinforcement, i.e. the bond action, must be considered in modelling [15].

2.1 Localised failure in concrete

The problem of localised failure in concrete well illustrated with a uniaxial tensile test, cf. Figure 2.1. When subjecting the test specimen to tensile stresses, microscopic cracks start to nucleate at the weak points in the material. Upon increase of the load, these incipient microcracks coalesce into a distinct fracture zone at the weakest section of the specimen. After reaching the tensile strength, the deformations further increase while unloading the material outside of the fracture zone. The microcracks eventually evolve into a macroscopic crack and upon separation of the material no further stress is transferred between the separated parts of the specimen.

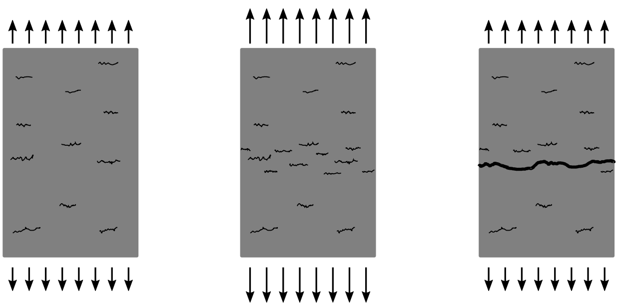
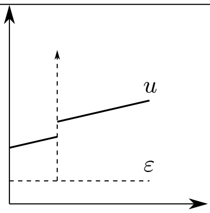
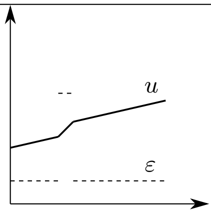
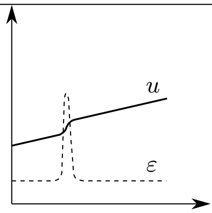


Figure 2.1: Fracture development in a concrete specimen subjected to tensile load.

The localised zone of microcracks can be represented in a few different ways, depending on the regularity of the displacement and strain fields, denoted $\boldsymbol{u}(\boldsymbol{x})$ and $\boldsymbol{\varepsilon}(\boldsymbol{x})$, respectively. In the classification, three main types of strain discontinuity can be distinguished: *strong discontinuity*, *weak discontinuity* and *continuous* strain field (i.e. no discontinuity), cf. [25] for an extensive review. An overview is presented in Table 2.1.

Strong discontinuity entails a jump in the displacement field, while the strain field comprises a regular part together with a part resembling a Dirac delta distribution. Cohesive zone models or discrete crack models [22], which define the traction-separation law are usually used to describe the constitutive behaviour of the crack. In the finite element setting, interface elements can be used to capture strong discontinuity, but the crack location and propagation path both need to be known in advance. Alternatively,

Table 2.1: *Overview of models for strain localisation.*

Strain regularity	Strong discontinuity	Weak discontinuity	Continuity
Kinematic representation			
Constitutive models	cohesive zone models/discrete crack models	smeared crack models	nonlocal/gradient enriched continua
Finite element implementation	interface elements, embedded discontinuity elements (ED-FEM), extended finite element method (XFEM)	standard elements, special elements with embedded localisation bands	standard elements, special enriched elements

elements with embedded discontinuity (ED-FEM) [41, 24] or the extended finite elements (XFEM) [4, 17], based on enrichment of the standard shape functions, can be used.

Weak discontinuities can be used to represent a localisation band of a finite thickness. In this case, the strain field shows a jump, while the displacement field remains continuous. The cracked solid is still represented as a continuum, where the inelastic effects are distributed within a band of finite size, often denoted the *crack band width* [3, 2]. The traction-separation law is replaced with stress-strain relation within this band, thus constituting a smeared crack model. Models based on the crack band width concept are popular in modelling of concrete structures, since they are compatible with standard finite elements and are relatively straight-forward to implement. Special elements with embedded localisation bands can also be used to approximate the weak discontinuities. A range of smeared crack models exists nowadays, e.g. rotating, fixed and multi-directional fixed crack models [53, 52]. Continuum damage models, which link the stress to the total strain can also be adapted to the smeared approach [27, 50]. In **Paper A** and **Paper B**, an isotropic damage model—the Mazars model [39, 38] along with certain modifications of the damage law [50]—was used to simulate the response of concrete under load. A schematic stress-strain response of concrete obtained with the mentioned model is illustrated in Figure 2.2. It is noteworthy that in order to assure mesh independence, the fracture energy of the concrete, G_{F}, must be preserved by calibrating the model to the chosen band width, h.

The third representation considers continuous displacement and strain fields, where the localisation is limited to a narrow band of larger strains with unbroken shift to smaller strains. Such strain fields can be captured by more advanced constitutive models based

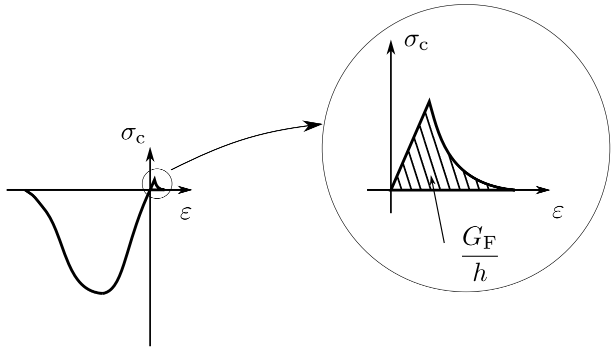


Figure 2.2: Stress–strain (σ_c–ε) relation for concrete adopted in a smeared crack model. Here, G_F corresponds to the fracture energy, and h to the chosen crack band width.

on nonlocal or gradient enriched continua [26, 51, 19]. Regular finite elements can be used, but the discretisation of the domain must be rather fine to capture the large strain gradients. Alternatively, special enrichments of the elements can be employed to allow for a coarser mesh.

2.2 Modelling of reinforcement bars

Both axial and transverse action of the reinforcement was considered in the present work. Thus, beam elements based on the Euler-Bernoulli beam theory were used. To account for plasticity, both **Papers A** and **B** employ an elasto-plastic constitutive model based on Von Mises yield criterion, which allows for linear isotropic strain hardening. In the model, the hardening is driven by the cumulative plastic strain. Figure 2.3 depicts a typical stress–strain response of steel considered by such a model.

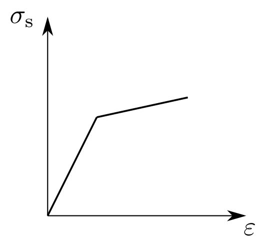


Figure 2.3: Illustration of the stress–strain (σ_s–ε) relation for steel using an elasto-plastic model with hardening.

2.3 Bond mechanism between reinforcement and concrete

As outlined in [1, 6], the bond between concrete and reinforcement is a combination of three mechanisms: chemical adhesion, friction, and mechanical interlocking. The contribution from chemical adhesion is small, and is lost as soon as a displacement difference (henceforth denoted *slip*) between steel and concrete builds up [15]. After that, stresses between reinforcement ribs and the neighbouring concrete develop. The inclined stresses between reinforcement ribs and concrete can be split into longitudinal and radial components, which are called *bond* and *splitting* stresses, respectively.

In modelling of reinforced concrete structures, the bond behaviour is often described by employing a bond-slip model, which relates the bond stresses to reinforcement slip. This is however a simplification, as the actual mechanical behaviour is more complex. It is noteworthy, that the presence of the splitting stresses is necessary for bond stress transfer after the loss of chemical adhesion. The splitting stresses can be lost in certain situations, e.g. when longitudinal splitting cracks penetrate the concrete around the reinforcement bar and no transverse reinforcement can carry the stresses. Also, the transverse contraction of the steel bar at yielding reduces the splitting stresses. In these cases, a splitting failure, which constitutes the lower bound of the bond capacity, can be observed. For a well-confined concrete around the reinforcement bars the splitting stresses can be transferred and a pull-out failure, constituting the upper bound of the bond capacity, can be observed. The steel/concrete interface law used in **Papers A** and **B** assumes a pull-out failure and is illustrated in Figure 2.4.

Numerically, the bond-slip behaviour of reinforced concrete has been modelled in a variety of ways. A popular choice is to resolve the steel/concrete interface with interface elements and utilize the bond-slip behaviour as its constitutive traction-separation law [35, 34, 36]. As an alternative, the finite elements used to discretise the problem domain can be appropriately enriched in order to reproduce the strain incompatibility between steel and concrete caused by the slip. For structures modelled with solid elements, element enhancements based on ED-FEM [24], XFEM [12], or the combination of both [23] have been studied. Other types of enrichments were also studied for beam elements [11, 16].

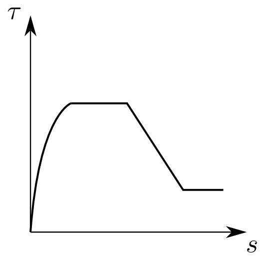


Figure 2.4: Example of bond-slip (τ–s) behaviour.

2.4 Steel–concrete interaction in reinforcement concrete members

As mentioned in the previous sections, cracking of concrete does not entail structural failure in a reinforced concrete member. It is noteworthy, that gradients in bond stresses enable strain localisation in reinforced concrete [11]. As a result, the member continues to carry load after the concrete has cracked. This situation is schematically illustrated in Figure 2.5, where a cracked reinforced concrete member loaded in tension is shown together with the distribution of reinforcement slip, s, and the stress in concrete and steel, σ_{c} and σ_{s}, respectively.

Far away from the cracks (starting from the left side of the figure), the reinforcement can be assumed to be perfectly bonded with concrete, thus there is no difference in the deformation between the two materials and hence, no slip. Both concrete and steel carry a portion of the load, corresponding to the stiffness ratio. Moving towards the

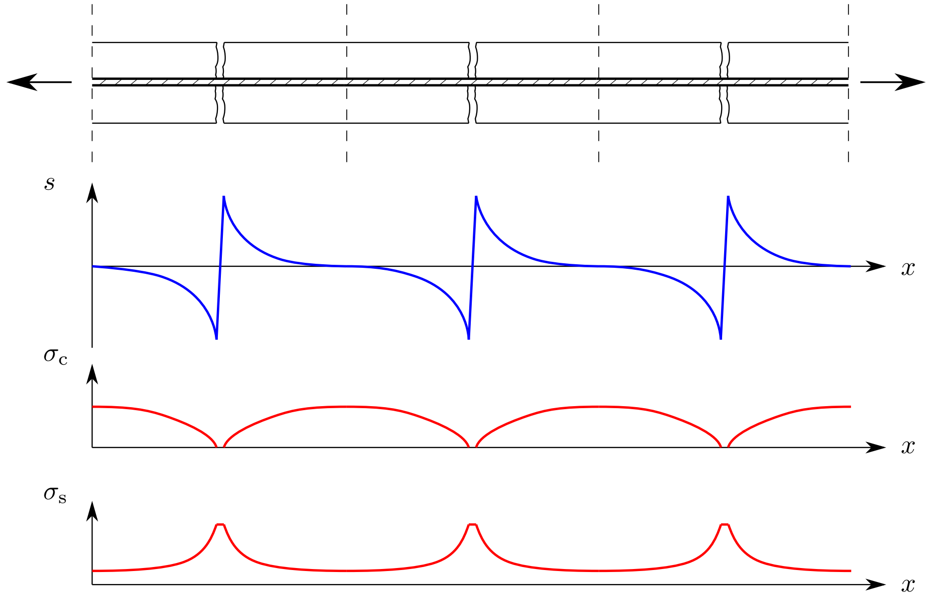


Figure 2.5: Variation of slip, s, concrete and steel stress, σ_c and σ_s, respectively, in a cracked reinforced concrete member in tension.

crack, a difference in the deformations between the two materials is present, and the slip builds up. In the vicinity of the crack the concrete unloads and the reinforcement carries additional load via bond action. This continues until the crack face is reached, where the discontinuity of the displacement in concrete results in relatively large value of the slip. Since the crack has separated the concrete, it cannot carry any stress. Thus, all the force is carried by the reinforcement. The situation is analogous on the other side of the crack—the only difference being the opposite sign of the slip, depending on the chosen sign convention. As we move further away from the crack face (to the right), the uncracked concrete starts taking up load, which is transferred from the reinforcement via bond stresses in the interface.

In the presented example, two things are noteworthy. First of all, the sum of the forces carried by steel and concrete is constant along the member. Second, the locations of the cracks can be identified from the slip (or bond stress) distribution; cracked concrete is indicated by large gradients of the slip, with the slip changing sign.

3 Computational Multiscale Modelling

3.1 Overview of multiscale methods

When modelling structures in full-scale using a macroscopic constitutive model, homogeneity of the material is often assumed. However, the actual physical phenomena within the material usually take place on a smaller scale, and can be properly taken into account by use of multiscale models. In spite of their usefulness and increasing popularity the computational effort is increased considerably, as material heterogeneities are included in the model of the structure (or its part). Depending on how the information is passed between the scales, multiscale methods can be divided into *hierarchical* an *concurrent* methods [5, 37], with the former group further subdivided into *coupled* and *uncoupled* hierarchical methods [59].

In concurrent methods, the fine-scale model is embedded in the coarse-scale model, enforcing equilibrium and compatibility along the micro-macro interface. Both of the models are then solved simultaneously, allowing for unrestrained exchange of information in both directions between the scales. Regarding structures, reinforced concrete frames and beams have been studied with this approach [56, 55, 54]. Although viable for frames and beams, structures that display distributed cracking, e.g. in presence of reinforcement grids, would eventually require fine resolution in the whole problem domain.

In hierarchical methods, different scales are present in the same part of problem domain, and are linked based on averaging theorems (homogenisation) or parameter identification. Uncoupled hierarchical methods generally allow for transfer of information only from the fine-scale to the coarse-scale model, with the fine-scale response computed *a priori* and stored for future use by the macroscopic model. Calibration of macroscopic constitutive models from subscale simulations (also referred to as upscaling), and classical homogenisation are important examples of methods falling within this category. For example, results from subscale analyses of imperfectly bonded steel bars in concrete were used in [31, 32] to calibrate a constitutive model for cracking reinforced concrete member.

Finally, in coupled hierarchical methods (also referred to as semi-concurrent methods) the information between scales passes in both directions. Typically, for a specific coarse-scale input, the fine-scale model is asked for the response, which is averaged and transferred back to the coarse-scale. Contrary to uncoupled hierarchical methods, no storage of the macroscopic phenomenological data is needed, since the fine-scale problem actually acts as an effective constitutive model. An example of an approach falling within this category is the FE^2 (Finite Element squared) method [14, 13], which is used to analyse reinforced concrete structures in multiscale manner in **Papers A** and **B**.

3.2 Computational homogenisation

The subscale unit cell, often denoted Representative or Statistical Volume Element (RVE or SVE, respectively) is supposed to represent the heterogeneity of the modelled structure or material. In practice, the shape of such computational cell is often taken as a square

(2D) or a cube (3D). The size of the cell is an important parameter an must fulfil certain criteria. Namely, the RVE must be large enough to correctly represent the substructure of the material (the phase with the largest dimension), but at the same time much smaller than the macroscopic structure. Furthermore, the virtual work in a large-scale point shall be equal to the volume average of virtual work performed on the RVE. This condition, commonly referred to as the Hill–Mandel macrohomogeneity condition [21] ensures energy equivalence across the scales. Since the subscale geometry is often complex, *computational homogenisation* [20, 28, 18] is used based on the finite element method.

In a process called *prolongation*, the macroscopic deformation is expanded in a Taylor series. The order of the Taylor expansion defines the order of homogenisation, i.e. in first order homogenisation only linear variation of the macroscopic field within the RVE is considered. Higher order homogenisation schemes have also been studied, cf. Kouznetsova et al. [29, 30]. In the case of first order homogenisation, the gradient of the macroscopic field is imposed on the subscale unit cell via appropriate boundary conditions. Even though there exists a range of suitable types of boundary conditions used in computational homogenisation today, the classical Dirichlet and Neumann type boundary conditions establish the basic assumptions. The two types of boundary conditions are illustrated for a square RVE in Figure 3.1.

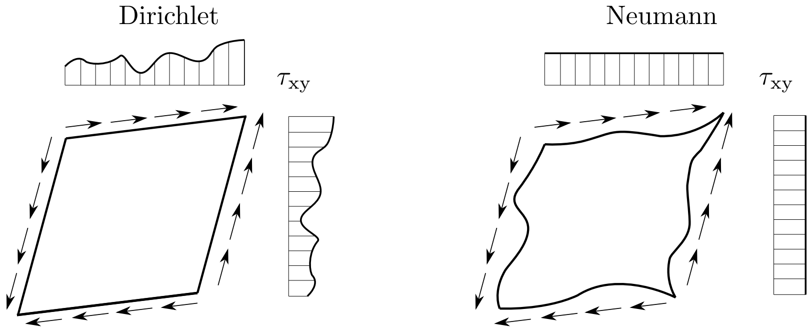


Figure 3.1: Dirichlet and Neumann assumptions on the Representative Volume Element. Deformation and distribution of the shear stress along the boundary of RVE is indicated.

In the Dirichlet assumption, the deformation at the boundary of the RVE varies linearly, i.e according to the imposed macroscopic deformation gradient. Considering the heterogeneous structure of the RVE, the resulting boundary tractions are not constant. Conversely, in the Neumann assumption the tractions are piecewise constant along the boundary, i.e. they are generated from a constant stress tensor. The corresponding displacements of non-homogeneous RVE vary at the boundary.

In case of more than one macroscopic field, or when distinct constituents build up the sub-structure of the RVE (e.g. reinforcement bars in concrete), the choice of boundary conditions can be even more complicated, as different assumptions might be used for different macroscopic fields or different materials. In **Paper A**, the reinforced concrete RVEs comprising the plain concrete solid, reinforcement bars and the steel/concrete interface were subjected to different combinations of boundary conditions, see Figure 3.2. Dirichlet and Neumann boundary conditions pertaining to uniform boundary displacement

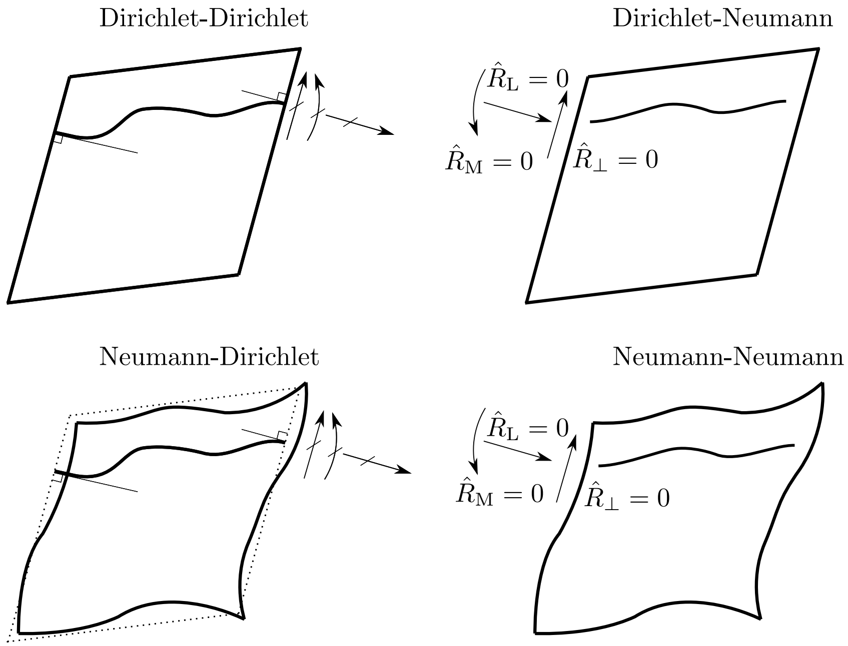


Figure 3.2: Combinations of boundary conditions on the reinforced concrete RVE used in **Paper A**.

and constant boundary traction, respectively, were considered for the concrete. Similarly, Dirichlet and Neumann boundary conditions pertaining to prescribed boundary displacement and vanishing sectional forces were employed for the reinforcement. In **Paper B**, the macroscopic displacement and slip fields were imposed on the RVE via Dirichlet-Dirichlet boundary conditions, whereby the boundary displacements were prescribed independently for the concrete and reinforcement.

Homogenisation is the process defining how the subscale quantities are extracted, condensed and transferred back to the large-scale upon solution of the RVE boundary value problem, becoming the "effective" material quantities. It is noteworthy that the classical Dirichlet and Neumann boundary conditions under certain assumptions provide upper and lower (Voigt and Reuss [21]) bounds on the RVE response, respectively. Throughout this work, the Variationally Consistent Homogenisation was used for derivation of the effective properties. This framework is based on the decomposition of the solution into the macrocopic and microscropic parts, and local averaging of integrals, see e.g. Larsson et al. [33] and Öhman et al. [48] for detailed descriptions.

3.3 FE^2 method

In the FE^2 (Finite Element squared) method, the problem domain is split into the large-scale and subscale domains. At the large-scale, the structure is represented with a homogeneous material with "effective" properties, which are obtained upon computational homogenisation of the response of the subscale unit cell. The primary field and its gradient are computed at the integration points of large-scale finite elements. These are then imposed on the RVE via chosen boundary conditions, and the boundary value problem is solved at each integration point. The subscale quantities are then averaged, and the "effective" large-scale work conjugates are obtained at the locations of the integration points. The internal force vectors are then computed for the large-scale elements, and the analysis continues. Hence, the RVE problem acts as a constitutive driver, providing the homogenised response for a given macroscopic input. The internal algorithmic loop at a large-scale integration point is schematically illustrated in Figure 3.3. Even though FE^2 is computationally expensive, it is well suited for parallel computation, as all RVE problems are uncoupled and can be solved independently. In application to plain concrete, FE^2 has been used to model e.g. diffusion phenomena [45] and localisation phenomena [49, 58, 43].

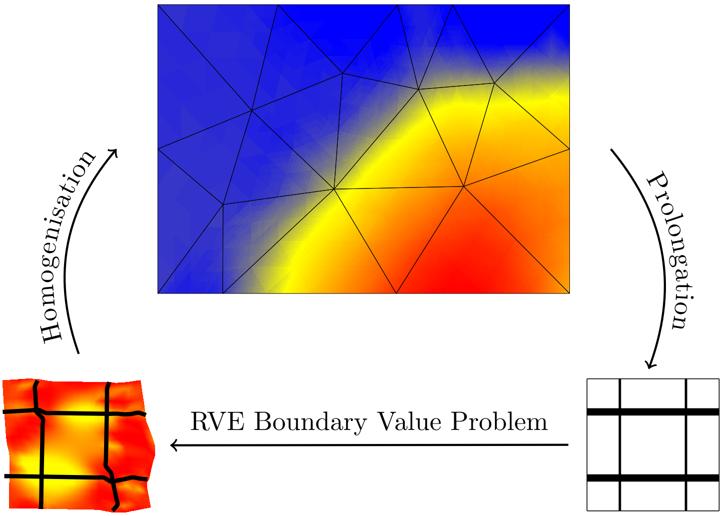


Figure 3.3: Illustration of the computational homogenisation scheme. Macroscopic field is prolonged and imposed on the RVE via chosen boundary conditions. The RVE problem is solved, and the effective quantities are homogenised from the RVE output.

4 Summary of appended papers

Paper A: Two-scale finite element modelling of reinforced concrete structures: effective response and subscale fracture development

A single-scale model of reinforced concrete describing the response of concrete, steel reinforcement, and the interface between them, was used as a basis for development of a two-scale model. A first-order variationally consistent homogenisation scheme was utilized to derive the pertinent large-scale and subscale problems. It was assumed that only the concrete displacement possesses an independent macroscopic component, i.e. the variation of the reinforcement slip is considered only locally at the RVE level. Dirichlet and Neumann boundary conditions were formulated for both concrete (pertaining to uniform boundary displacement and constant boundary traction, respectively) and reinforcement (pertaining to prescribed boundary displacement and vanishing sectional forces, respectively) in the unit cell. Hence, a total of four combinations were studied: Dirichlet-Dirichlet(DD), Dirichlet-Neumann(DN), Neumann-Dirichlet(ND) and Neumann-Neumann(NN), see Figure 3.2. Numerical examples comprised tensile tests of the reinforced concrete RVE, and FE^2 analyses of a reinforced concrete deep beam, the results of which were compared to a fully-resolved (single-scale) analysis. The DD and NN boundary conditions provided upper and lower bounds, respectively, on the initial effective stiffness of the material. Although the model yielded promising results in terms of the structural behaviour for the DD and ND boundary conditions, excessive softening could be observed for the DN and NN boundary conditions, which signified their potential infeasibility in modelling of reinforced concrete structures. The DD combination of boundary conditions was shown to be most reliable in the FE^2 analyses. Although the crack widths were underestimated by the two-scale formulation, a good match was observed between the models in terms of the deformed shape, force-deflection relation and average strain for the studied reinforced concrete deep beam.

Paper B: A multiscale model for reinforced concrete with macroscopic variation of reinforcement slip

Variationally consistent homogenisation was utilized to construct a two-scale model of reinforced concrete. In contrast to **Paper A**, the reinforcement slip was considered a macroscopic variable. The pertinent large-scale problem was defined in terms of finding the macroscopic displacement and reinforcement slip fields. First-order homogenisation was used to construct the subscale problem, where the subscale displacement field of reinforcement was a resultant of both macroscopic displacement and reinforcement slip fields. Appropriate Dirichlet-Dirichlet (DD) boundary conditions were formulated, and the way of computing the effective work conjugates was outlined. Numerical examples included reinforcement pull-out tests from the reinforced concrete RVE, and FE^2 analyses of a reinforced concrete deep beam, the results of which were compared to both fully-resolved and two-scale analyses neglecting the macroscopic slip variation. It was shown that the bond-slip law for the interface can be recovered from the effective bond-slip response of the RVE, either directly or indirectly, depending on the length of the reinforcement bars. Even though the incorporation of the macroscopic slip variable had little influence on the structural force-deflection curves, it resulted in more localised macroscopic strain fields. As a result, the crack widths computed by the two-scale solution were in better agreement with the fully-resolved solution than the crack width computed by the two-scale formulation disregarding macroscopic slip variable. The inherent scale mixing was captured as a result of the enrichment of the problem by a macroscopic field and its gradient. Hence, it was shown that not only the size of the RVE, but also the macroscopic mesh size plays an important role in the FE^2 analyses.

5 Concluding remarks and future work

In the present work, a two-scale model of reinforced concrete was developed along the lines of Variationally Consistent Homogenisation. At the large-scale, reinforced concrete was treated as a homogeneous (effective) material, while plain concrete, reinforcement bars, and interface between them were considered at the subscale. Suitable boundary conditions on the subscale Representative Volume Element (RVE) were proposed, and the model was implemented in an FE^2 (Finite Element squared) algorithm. In its current state of development, the model was limited to two-dimensional plane stress problems under quasi-static loading. Moreover, only structures prior to structural failure with distributed reinforcement layout were considered. The multiscale model provided approximate upper and lower bounds on structural stiffness, and was in good agreement with single-scale solution in terms of general structural behaviour, such as the deformed shape, load–deflection curve and average strain. Furthermore, the capability of the model to simulate the evolving crack width was improved upon the incorporation of a macroscopic reinforcement slip variable. In summary, the developed two-scale model can describe both global and local structural behaviour well and shows potential in saving computational time in comparison to single-scale analyses.

As an outlook for future developments, it is of interest to study how the subscale strain localisation transfers back to macroscale. In this setting, it might be expedient to employ continuous–discontinuous homogenisation [42, 43, 9] or other types of boundary conditions better suited to treat subscale strain localisation [57, 10]. Furthermore, it is noted that since the long term goal of the project is to extend the multiscale modelling framework to reinforced concrete structures, other types of structures should be studied. In particular, reinforced concrete beams and slabs are of interest; this will bring the multiscale formulation to beam and plate/shell elements. Such extension is vital if large-scale reinforced concrete structures, like e.g. bridges are to be modelled. Moreover, it is of interest to extend the model to 3D, which could include additional effects not foreseen by the 2D simplification. In order to improve the modelling of fracturing concrete at the subscale, aggregates, cement paste and the interfacial transition zone should be resolved at the RVE level. Finally, in order to ascertain the engineering capabilities of two-scale modelling, comparison of the results with experimental data will be valuable.

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