Vehicle Dynamics Control for Active Safety Functions using Electrified Drivelines

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Vehicle Engineering and Autonomous Systems
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2018
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Cover:
AAM's Mercedes B-Class test vehicle equipped with an e-AAM 50 kW switchable torque vectoring electric rear axle undergoing winter testing in Arjeplog, Sweden for calibration of vehicle dynamics control software.

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Vehicle Dynamics Control for Active Safety Functions using Electrified Drivelines
Thesis for the degree of Doctor of Philosophy in Machine and Vehicle Systems
ADITHYA ARIKERE
Department of Mechanics and Maritime Sciences
Vehicle Engineering and Autonomous Systems
Chalmers University of Technology

ABSTRACT

Studies have shown that even considering pledges and commitments made by various governments and organisations, the growth in electrified vehicle sales is likely to be insufficient to reduce CO₂ emissions for successfully mitigating global warming. Some form of added incentive is needed that can help drive electrified vehicle sales in the open market. On the other hand, there is an increased need for traffic safety due to customer demand and the adoption of ambitious goals such as the Vision Zero. This thesis attempts to identify vehicle dynamic opportunities to improve vehicle safety that are enhanced or enabled by electrified drivetrains, thereby offering an opportunity to add value to electrified vehicles and make them more attractive to consumers.

As an example of low hanging fruit, the possibility of accelerating an electrified lead vehicle to mitigate the consequences of, or prevent being struck from behind is investigated. A hypothetical Autonomous Emergency Acceleration (AEA) system (analogous to the Automatic Emergency Braking (AEB) system) is envisioned and the safety benefit due to the same is estimated. It is seen that the AEA system offers significant opportunities for preventing or reducing injuries in rear-end collisions.

The possibility of using propulsion to improve safety in an obstacle avoidance scenario in the presence of oncoming traffic is also investigated. In order to better understand the manoeuvre kinematics, a point mass based optimal control analysis is done, in which a characteristic parameter is identified that correlates well with the need to increase or decrease speed in the manoeuvre for mitigating the risk of collision with the oncoming vehicle. After verification through experiments, an integrated motion controller is formulated, implemented and tested in a high-fidelity simulation environment. Results show that consistent reductions in collision risk to the oncoming vehicle can be achieved using the integrated controller. Specifically, the results show that the availability of electric drives consistently enabled reduced collision risk by allowing greater torque vectoring magnitudes and mitigating the deceleration side effect of differential braking. The integrated controller is then evaluated for robustness to steering effort in simulations followed by real-time implementation of the controller and testing using a Volvo XC90 test vehicle.

Intersection accidents are then investigated with regards to the possibility of crossing the intersection ahead of a bullet vehicle for collision avoidance. Optimal manoeuvres for the same are derived using analytical optimal control theory and it is seen that optimal manoeuvres can be represented as a maximisation of the tyre forces in a fixed global direction. Based on this finding, an integrated motion controller that exploits the precision of electric drives to accurately control tyre forces is implemented and tested. Simulation results show that collision risk can be reduced significantly over a passive vehicle even in limit scenarios where the tyre forces are saturated.

In summary, several vehicle dynamic opportunities for improving safety using electrified drivetrains are identified. Detailed investigation of select cases show that significant safety benefit potentially stands to be gained by appropriate control of electrified drivetrains in the accident scenarios. Consequently, a strong opportunity is seen for adding safety related value to electrified vehicles at little to no extra cost.

Keywords: electrified drivetrain, torque vectoring, speed control, active safety, vehicle dynamics, rear-end collisions, obstacle avoidance with oncoming traffic, intersection accidents, optimal control, driver assistance systems
The work presented in this thesis has been funded by the Fordonsstrategisk Forskning och Innovation (FFI) program of VINNOVA and by my employer, Trollhättan Technical Center (TTC) of American Axles & Manufacturing (AAM) (formerly e-AAM Driveline Systems) and I would like to sincerely thank them for their support. Two other partners in the project are Volvo Cars and ÅF. Volvo Cars supported the project with equipment, a test vehicle, test track time, validated models and of course supervision. Their support has been invaluable and I can't thank them enough for it. ÅF supported the project with administration and high level project management to ensure the project was on track and their support is gratefully acknowledged.

Next, I would like to thank my supervisors Bengt Jacobson, Matthijs Klomp and Derong Yang for their support and guidance. Bengt, for your insatiable curiosity, keeping track of the high-level objectives and for always taking time to review my papers and thesis at the last minute even through the weekends; Matthijs, for always pushing me and believing that the work being performed would lead to useful results, sometimes even when I didn't; and Derong, even though you were not officially my supervisor, your enthusiasm and the time you invested in my project, particularly for the experiments, has been invaluable: thank you! I would like to acknowledge and thank Mathias Lidberg, my erstwhile supervisor, for his support and guidance that I've received for the majority of my doctoral studies. And Gunnar Olsson for his high level feedback and keeping the overall project direction in check and taking care of the administrative stuff, project update reports, etc. Their experience and insight into anything vehicle related has been invaluable. I would like to thank Mathias and Gunnar in particular for initiating the project and for believing in me enough to employ me fresh out the master degree programme to do my doctoral studies.

On the industrial side, I would like to thank my current manager Matilda Hallnor and former managers Olle Larsson and Torbjörn Norlander for always taking care of all the administrative stuff, keeping me away from AAM work when I needed to focus on research and giving me AAM work when I suddenly wanted a change. A special thanks also to Anders Tysk who originally initiated and championed the project within AAM. A big thanks also to my colleagues at AAM for welcoming me into and making me feel part of the team even though I don't put in so much face time at the company. And thanks also for putting up with my lack of Swedish skills even after so long.

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And last but not least, a huge thank you to all my family and friends for the support and inspiration and putting up with my anti-social behaviour when I was busy with work (and sometimes even when I wasn't).

Adithya Arikere
Göteborg, Jan 2018
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_x, a_y$</td>
<td>Longitudinal and lateral acceleration in vehicle frame of reference</td>
</tr>
<tr>
<td>A</td>
<td>System state matrix</td>
</tr>
<tr>
<td>brkfl</td>
<td>Subscript representing the front left brake actuator</td>
</tr>
<tr>
<td>brkr</td>
<td>Subscript representing the front right brake actuator</td>
</tr>
<tr>
<td>brkl</td>
<td>Subscript representing the rear left brake actuator</td>
</tr>
<tr>
<td>brkr</td>
<td>Subscript representing the rear right brake actuator</td>
</tr>
<tr>
<td>B</td>
<td>Magic formula tyre model parameter</td>
</tr>
<tr>
<td>B</td>
<td>System input matrix</td>
</tr>
<tr>
<td>$c_0, c_1$</td>
<td>Tyre stiffness parameter at rated load and vertical load based non-linearity parameter for tyre stiffness</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Tyre stiffness parameter for tyre $i$ where $i$ is $f$ or $r$ representing the front or rear axle in the single track model or $fl, fr, rl$ or $rr$ representing each of the four tyres in the two track model</td>
</tr>
<tr>
<td>$c_{\phi, i}$</td>
<td>Roll damping of whole vehicle and at axle $i$</td>
</tr>
<tr>
<td>C</td>
<td>Magic formula tyre model parameter</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Linearised cornering stiffness of axle $i$</td>
</tr>
<tr>
<td>d</td>
<td>Global X-distance margin - distance between the host and the bullet vehicle at the end of the manoeuvre</td>
</tr>
<tr>
<td>$d_0$</td>
<td>Global X-distance between the host and the bullet vehicle at the beginning of the manoeuvre (obstacle avoidance with oncoming traffic scenario)</td>
</tr>
<tr>
<td>eng</td>
<td>Subscript representing the engine (on front axle)</td>
</tr>
<tr>
<td>E</td>
<td>Magic formula tyre model parameter</td>
</tr>
<tr>
<td>f</td>
<td>System differential equation function</td>
</tr>
<tr>
<td>F</td>
<td>Total force magnitude on particle</td>
</tr>
<tr>
<td>$F_{act}$</td>
<td>Actuator forces where $act$ is the actuator of interest and is one of ${eng, mtr, brkfl, brkr, brkl, brkr}$</td>
</tr>
<tr>
<td>$F_x, F_y$</td>
<td>Longitudinal and lateral forces in vehicle frame respectively</td>
</tr>
<tr>
<td>$F_{x, max}, F_{y, max}$</td>
<td>Maximum longitudinal and lateral forces in vehicle frame respectively</td>
</tr>
<tr>
<td>$F^g_x, F^g_y, M^g_z$</td>
<td>Total vehicle $x$ and $y$ forces and yaw moment in the vehicle frame respectively</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$F_{x,i}^g, F_{y,i}^g$</td>
<td>Longitudinal and lateral force of tyre $i$ in global reference frame</td>
</tr>
<tr>
<td>$F_{xi}$</td>
<td>Longitudinal force at axle $i$ in the single track vehicle model, where $i$ is front or rear</td>
</tr>
<tr>
<td>$F_{x,i,j}$</td>
<td>Longitudinal force at wheel $j$ of axle $i$ in the two track vehicle model, where $i$ is front or rear and $j$ is left or right</td>
</tr>
<tr>
<td>$F_{x,tgt}$</td>
<td>Desired total vehicle longitudinal force</td>
</tr>
<tr>
<td>$F_{XG}, F_{YG}$</td>
<td>Total vehicle X and Y forces respectively in global reference frame</td>
</tr>
<tr>
<td>$F_{yi}$</td>
<td>Lateral force at axle $i$ in the single track vehicle model</td>
</tr>
<tr>
<td>$F_{yi,max}$</td>
<td>Maximum lateral force available at axle $i$ in the single track model</td>
</tr>
<tr>
<td>$F_{y,i,j}$</td>
<td>Lateral force at wheel $j$ of axle $i$ in the two track vehicle model</td>
</tr>
<tr>
<td>$F_{z0}$</td>
<td>Rated load of tyre</td>
</tr>
<tr>
<td>$F_{zi}$</td>
<td>Vertical force at axle $i$ in the single track model</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of centre of gravity from the ground</td>
</tr>
<tr>
<td>$h'$</td>
<td>Height of centre of gravity over the roll axis</td>
</tr>
<tr>
<td>$h_i$</td>
<td>Roll centre height at axle $i$</td>
</tr>
<tr>
<td>$H, H_i$</td>
<td>Hamiltonian of vehicle and of tyre $i$</td>
</tr>
<tr>
<td>$I_{n \times n}$</td>
<td>$n$ by $n$ identity matrix</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>Roll moment of inertia</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>Yaw moment of inertia</td>
</tr>
<tr>
<td>$J, \hat{J}$</td>
<td>Original and augmented objective function</td>
</tr>
<tr>
<td>$k_{\phi}, k_{\phi,i}$</td>
<td>Roll stiffness of whole vehicle and at axle $i$</td>
</tr>
<tr>
<td>$K_u$</td>
<td>Understeer gradient</td>
</tr>
<tr>
<td>$L$</td>
<td>Wheelbase of host vehicle</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Distance from center of gravity to $i$-th axle</td>
</tr>
<tr>
<td>$l_{obs}$</td>
<td>Length of the obstacle</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the vehicle or point mass</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Sprung mass of the vehicle</td>
</tr>
<tr>
<td>$mtr$</td>
<td>Subscript representing the motor (on rear axle)</td>
</tr>
<tr>
<td>$M_{z,tgt}$</td>
<td>Desired vehicle yaw moment (from wheel longitudinal forces)</td>
</tr>
<tr>
<td>$M_{ZG}, M_{Z}, M_{Z}^d$</td>
<td>Global vehicle, actual and desired yaw moment</td>
</tr>
<tr>
<td>$p$</td>
<td>Set of terminal constraints</td>
</tr>
<tr>
<td>$q$</td>
<td>Input function for the particle model</td>
</tr>
</tbody>
</table>
\( t \) Time
\( t_f \) Duration of the manoeuvre
\( T_{\text{act}, F}, T_{\text{act}, R} \) Actual torques delivered at the front and rear axle
\( T_{\text{req}} \) Requested torque from the controller
\( u \) Control input vector
\( u_d \) Desired control input
\( -u, +u \) Min and max actuator position limits
\( -\dot{u}, +\dot{u} \) Min and max actuator rate limits
\( \mathcal{U} \) Set of admissible control inputs
\( v \) Virtual control input
\( v_0 \) Host vehicle initial velocity
\( v_b \) Bullet vehicle velocity
\( v_f, v_l \) Velocities of following and lead vehicle in the rear-end collision scenario
\( v_x, v_y \) Longitudinal and lateral velocities in vehicle frame
\( w \) Host vehicle width
\( W_u \) Weighting matrix for the true control inputs
\( W_v \) Weighting matrix for the virtual control inputs
\( x \) Vehicle state vector
\( x_i, y_i \) \( x \) and \( y \) distance of wheel \( i \) from the centre of gravity in vehicle reference frame
\( X, Y \) Global \( X \) and \( Y \) positions respectively
\( X_{b0} \) Initial \( X \)-distance between the host and bullet vehicles in the intersection accidents scenario
\( Y_b \) The initial lateral offset of the bullet vehicle to the host in the intersection accidents scenario
\( Y_{\text{tgt}} \) Target lateral displacement for the host vehicle in the obstacle avoidance with oncoming traffic scenario
\( \alpha_i \) Slip angle of tyre \( i \)
\( \alpha_{ij} \) Slip angle of tyre \( j \) of axle \( i \)
\( \beta \) Vehicle sideslip angle
\( \beta_f \) Front axle sideslip angle in the single track model
\( \beta_{fj} \) Sideslip angle of wheel \( j \) on the front axle
\( \gamma \) Control inputs for the particle model
δ  Steering wheel angle

η  Lagrange multipliers for terminal constraints

θ₀  Initial angle subtended by the host vehicle position and the centre of the road segment arc to the vertical in the intersection accidents scenario

λ  Lagrange multipliers for the system equations

λ  Lagrange multiplier for yaw moment in the MHA

μ,μᵢ  Road friction coefficient for the vehicle and for axle i in the single track model

μ₀,μ₁  Tyre-road friction coefficient at rated load and vertical load based friction non-linearity parameter

ν  Course angle

φ  Represents the force angle in global reference frame in the particle model and the MHA. Represents the roll angle in the two track vehicle model

φᵣᵣᵢ  Force angle for tyre i in global reference frame

ψ  Yaw angle

ω,ωᵢ  Yaw rate

ωₑₑₑₑ  Reference yaw rate (from reference model)

0ₘₙ  m by n null matrix

ΔFzx,i  Longitudinal load transfer at axle i

ΔFzy,ij  Lateral load transfer at wheel j of axle i
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2DS</td>
<td>World climate change scenario with a 50% chance of limiting expected global average temperature increase to 2°C</td>
</tr>
<tr>
<td>ABS</td>
<td>Anti-lock braking system</td>
</tr>
<tr>
<td>AEB</td>
<td>Automatic Emergency Braking</td>
</tr>
<tr>
<td>AEA</td>
<td>Automatic Emergency Acceleration</td>
</tr>
<tr>
<td>B2DS</td>
<td>World climate scenario with a 50% chance of limiting expected global average temperature increase to 1.75°C</td>
</tr>
<tr>
<td>CAN</td>
<td>Controller Area Network</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DYC</td>
<td>Direct Yaw Control</td>
</tr>
<tr>
<td>ECM</td>
<td>Engine Control Module</td>
</tr>
<tr>
<td>ESC</td>
<td>Electronic Stability Control</td>
</tr>
<tr>
<td>EU</td>
<td>European Union</td>
</tr>
<tr>
<td>FWS</td>
<td>Front wheel steering</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
</tr>
<tr>
<td>GHG</td>
<td>Greenhouse gas</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HC</td>
<td>Hydrocarbons</td>
</tr>
<tr>
<td>HMI</td>
<td>Human Machine Interface</td>
</tr>
<tr>
<td>IC/ICE</td>
<td>Internal Combustion/Internal Combustion Engine</td>
</tr>
<tr>
<td>IEA</td>
<td>International Energy Agency</td>
</tr>
<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>LTAP/OD</td>
<td>An intersection accident scenario called “Left turn across path - Opposite direction”. “Opposite direction” refers to the direction from which the bullet vehicle approaches the intersection</td>
</tr>
<tr>
<td>MHA</td>
<td>Modified Hamiltonian Algorithm</td>
</tr>
<tr>
<td>MPC</td>
<td>Model Predictive Control</td>
</tr>
<tr>
<td>NO&lt;sub&gt;x&lt;/sub&gt;</td>
<td>Nitrous oxides</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional, Integral and Derivative control. Also often used with only some of the components such as PI and PD</td>
</tr>
<tr>
<td>PM</td>
<td>Particulate matters</td>
</tr>
<tr>
<td>ppm</td>
<td>Particles per million</td>
</tr>
<tr>
<td>QCAT</td>
<td>Quadratic programming Control Allocation Toolbox</td>
</tr>
<tr>
<td>RTS</td>
<td>Reference Technology Scenario - a climate change scenario with a “business-as-usual” regulatory framework</td>
</tr>
<tr>
<td>THC</td>
<td>Total hydrocarbons</td>
</tr>
<tr>
<td>UK</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>UN</td>
<td>United Nations</td>
</tr>
<tr>
<td>UNEP</td>
<td>United Nations Environment Programme</td>
</tr>
<tr>
<td>US/USA</td>
<td>United States of America</td>
</tr>
<tr>
<td>V2I</td>
<td>Vehicle-to-Infrastructure communication</td>
</tr>
<tr>
<td>V2V</td>
<td>Vehicle-to-Vehicle communication</td>
</tr>
<tr>
<td>VDDM</td>
<td>Vehicle Domain Dynamic Module</td>
</tr>
<tr>
<td>WLS</td>
<td>Weighted Least Squares</td>
</tr>
<tr>
<td>YRC</td>
<td>Yaw Response Control</td>
</tr>
</tbody>
</table>
This thesis consists of an extended summary and the following appended papers:

**Paper A**

**Paper B**

**Paper C**

**Paper D**

**Paper E**

**Paper F**

In paper B, the author of this thesis was responsible for modelling, simulation and analysis of the results and compiling the manuscript. Lidberg contributed with supervision and reviews of the manuscript. The other co-authors contributed with discussion about expected safety benefit and potential risks from a bio-mechanical, system safety and driver interaction point of view.

In the remaining papers, the author of this thesis was responsible for modelling, simulation, experiments, analysis and writing the papers. The co-authors contributed with supervision and reviews of the manuscripts. The experiments were performed together with Matthijs Klomp and Derong Yang.
Other related publications by the author, but not included in the thesis:

**Paper I**

**Paper II**

**Paper III**
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Part I
Extended Summary
1 Introduction

1.1 Background

1.1.1 The emissions problem

The increasingly severe weather effects [10, 11] and urgent warnings from the Union of Concerned Scientists [12] and others over the last few decades have increased general public awareness regarding pollution, global warming, and diminishing oil reserves. As a result, there have been increasing calls from both the public and the governments on vehicle manufacturers to make cars that are more environmentally friendly and less dependent on fossil fuels. One of the side-effects of this is that legislation regarding emission and fuel efficiency requirements on new cars have been getting more and more stringent.

![Figure 1.1: Legislated Euro emission norms for passenger cars as a fraction of the Euro I standard. Note that before Euro III (2000), for gasoline cars, while the total HC+NO\textsubscript{x} was restricted there were no individual restrictions on THC or NO\textsubscript{x}. (HC=hydrocarbons, NO\textsubscript{x}=nitrous oxides, PM=particulate matters, CO=carbon monoxide, THC=total hydrocarbons). (data from [13])](image)

The United Nations (UN) estimated in a recent study that air pollution across Europe costs $1.6 trillion a year in deaths and diseases, which amounts to nearly one tenth of the region’s gross domestic product (GDP) [14]. Approximately 50% of this pollution (and consequently the damages and cost) is estimated to be caused by road transport [15]. In an effort to combat such pollution, emission norms are imposed on a regional basis and many emission regulations worldwide mandate maximum emission levels of less than 20% of that allowed in 1993 (for diesels, [13]). As an example, in fig. 1.1, the evolution of European emission norms (Euro I through Euro VI) for passenger cars is illustrated.
Fuel efficiency requirements have been imposed indirectly through restrictions on fleet average carbon dioxide (CO$_2$) emissions of new cars sold. While the average CO$_2$ emission has been falling in recent years, the EU has set an ambitious fleet average CO$_2$ emission target of 95 g/km in 2021. This represents approximately a 40% reduction over the 2007 emission levels of 158.7 g/km [16]. Figure 1.2 shows the average CO$_2$ emissions for the passenger car fleet as a whole and for different manufacturers. While manufacturers have largely been able to meet the 2015 target (130 g/km), meeting the 2021 target will likely be a challenge.

![Figure 1.2: Fleet CO$_2$ emissions. (data from [17])](image)

The combination of these stringent emission and efficiency requirements have led to governments and vehicle manufacturers investing large sums of money in research related to alternative fuel sources and in general, ways of reducing energy consumption. One of the methods to reduce energy consumption in vehicles that has been gaining prominence is drivetrain electrification.

While the numerous studies investigating the capabilities of electrified drivetrains suggest a strong potential to reduce greenhouse gas (GHG) emissions [18–20], electrified cars have not really captured the market due to a variety of reasons. Customers cite numerous reasons including high cost, range anxiety, lack of charging infrastructure, etc. Despite this however, electrification is increasing since it is one of the few promising ways to reduce fuel consumption.

In order to meet GHG emission targets, several governments and organisations have established targets for sales or penetration of electrified vehicles [21, 23] in the vehicle fleet. A study published in 2013 [24] shows that predictions made by several studies regarding the penetration of electrified vehicles in the passenger car fleet are too optimistic compared to reality. Other more limited studies [18, 19, 25, 26], while predicting a significant market penetration of electrified vehicles in different countries, show that we are nowhere near on track to meet the required electrified vehicle fleet penetration for an ultimately stabilising CO$_2$ concentration in the atmosphere of 450 ppm [23]. Figure 1.3 shows the required share of electrified vehicles sales for the 450 scenario and the predicted share assuming a "business-as-usual"
Figure 1.3: Predicted actual (Reference scenario) vs needed share (450 scenario) of electrified vehicles in global passenger vehicles sales for successfully mitigating global warming in the year 2030 as predicted by the International Energy Agency (IEA) in 2009 [21] (image reproduced from Paper A)

Figure 1.4: Deployment scenarios for the stock of electric cars to the year 2030. RTS=Reference Technology Scenario (similar to reference scenario in fig. 1.3), 2DS=2 °C scenario (similar to 450 scenario in fig. 1.3), B2DS=Beyond 2 °C scenario. (Data from [22])
regulatory framework as predicted by the International Energy Agency (IEA) in 2009. Since then, awareness of climate change and efforts to combat the same have increased significantly with major emission reduction targets and agreements being established. Figure 1.4 shows updated predictions of electric vehicle stock in the year 2030 based on these developments done by the IEA. Here, RTS refers to the Reference Technology Scenario which represents projections based on policies that have been announced or are currently under consideration, 2DS refers to a scenario with a 50% chance of limiting expected global average temperature increase to 2°C and B2DS refers to a scenario with a 50% chance of limiting expected global average temperature increase to 1.75°C.

The Paris Declaration scenario predictions assume that the signatories to the same stick to the commitments made in the agreement. The Paris Declaration aims to reduce greenhouse gas emissions in line with the need to limit global average temperature increase to no more than 2°C and is the strongest and largest agreement to combat climate change to date. However, there are several criticisms that can be levelled at the same. An independent study carried out by the United Nations Environment Programme (UNEP) in 2016 found that the emission cut targets in the Paris Declaration will result in over a 3°C temperature increase rather than the targeted 2°C [27]. Other studies found that the current pledges by the signatory countries are insufficient to meet the goals stated in the Paris Declaration and that many countries are failing to meet the pledges and some are not even enacting the policies that they had planned [28, 29]. Additionally, while the Paris Declaration represents a major step in combating climate change, it is still only a set of pledges and there exists no enforcement or penalisation mechanism to ensure the signatories actually stick to their pledges. Lastly, less than a year after the declaration, the USA - arguably the most influential economy in the world - has already filed a notice of withdrawal from the Paris Agreement [30].

It is clear therefore that, to drive the sales of electrified vehicles, purely relying on legislative reform or government intervention is insufficient. Some form of added incentive or value is needed that would help drive electrified vehicle sales in the free market. However, “added incentive or value” is a rather broad term. One way to narrow down what sort of “added value” is needed, is to look at the “gap areas” with respect to transportation and this leads us to the issue of safety.

1.1.2 The safety challenge

Due to urbanisation and the increasing mobility of the world population, there are now larger numbers of motorists in smaller areas. This increased traffic density not only exacerbates the emissions problem but also results in increased traffic conflicts and hence leads to higher number of accidents. Consequently, along with the increased demand for efficiency, there is also an increasing demand for traffic safety. Several countries and cities have therefore set targets for reducing fatalities in road accidents. For instance, Sweden has the Vision Zero goal which aims to eliminate fatalities in road accidents completely by 2020 [31] while the UK has similar ambitions [32]. Several cities in the US have also adopted the Vision Zero goal [33–36]. In a 2001 transport white-paper, the European Commission set a target of halving the fatalities on European roads by 2010. The EU failed to meet this target [37]. Furthermore, the road fatality statistics (fig. 1.5) show a vast spread in the performance of different countries in terms of safety and worryingly, have begun to stagnate over the last three years.
If we are to achieve the safety targets, it is clear that a lot more needs to be done. Any future approach for improved safety needs to take into account not only the new sensors and sources of information that will be available in the vehicles of the future, but also the capabilities enabled or enhanced by the new actuators available in the cars of tomorrow.

1.1.3 At the crossroads between emissions and safety

From the push for more fuel efficient vehicles, it appears that one of the new actuators that will be available in the cars of the future are electric drives. The rise of electrified vehicles seem to be inevitable given the stringent requirements on emissions and efficiency. However, as previously mentioned, while electrified vehicles appear to be the future, growth in their sales is too slow to be able to adequately reduce CO$_2$ emissions in the near future.

So, given that some form of added value is needed to drive electrified vehicle sales and that improved traffic safety will likely be an area of need in the future, the question that naturally arises is: \textit{can we add value to electrified vehicles by having new safety related functionality that is enabled or enhanced by electrified drivetrains?}

Adding such functionality would not only contribute towards the safety targets, but also make electrified vehicles more attractive to both consumers (due to improved safety, possibly lower insurance costs, etc), and to governments (since they now contribute to their safety goals) which might in turn incentivize the sales of such cars.

1.2 Research question

Given that the electric drives are completely different actuators based on an entirely different technology, they can also be expected (and are known to) have different and superior characteristics and behaviour. These superior characteristics can potentially be exploited to enhance
or implement novel functions that cannot be achieved with traditional internal combustion engine drivetrains. And based on the fact that a large portion of safety improvements in recent years have come about due to modern vehicle dynamics based active safety functions, the research questions that arise are as follows:

- How can the electric drive be used to improve vehicle dynamics?
- What are the traffic and/or accident scenarios in which the improved vehicle dynamics could be used for improved safety?
- How should the electric drive be used (in select scenarios) to improve safety?

1.3 Limitations

Several topics, although closely related or required for final realisation of functions described in this work are not investigated here. The ability of the electric drive to improve safety has been studied mainly from a vehicle dynamics point of view.

Idealising assumptions regarding actuator performance have been made in some cases and are mentioned where relevant. The environment sensing aspect (detection problem), although briefly discussed in some cases, has not been studied in detail. The decision making problem (which one of several possible interventions to perform) has been considered only to the extent required in different papers. The driver interaction and driver acceptance questions have also not been addressed in detail. The legal aspect of how to perform interventions while respecting the driver's wishes has not been discussed.

Functional safety analyses of the different functions, while an important step for the industrialisation of such functions, have not been performed in this work. While functional safety considerations can be expected to impact the performance of the realised safety systems in the near-term, they are not expected to change the maximum achievable performance since actuators and sensors will mature and improve in performance over time. On the other hand, the benefit predicted or estimated in this work is unlikely to change over time as they are based on basic principles of physics and vehicle dynamics which are well established.

Lastly, this work assumes that an electric drive is already available in the vehicle (can be fully electric vehicle, plug-in hybrid or normal hybrid). This project does not make a case for electrifying drivetrains in order to improve safety, but rather identifies opportunities for increasing safety given that an electric drive is already available.

1.4 Main contributions

The main scientific contributions of this work are:

- A non-exhaustive list of traffic scenarios where electric drives can potentially be expected to provide a safety benefit have been identified and listed. Also provided along with each scenario is a list of the types of control interventions that can be expected to be of use.
• In the rear-end collision scenario, a decision making algorithm for autonomous lead vehicle acceleration for collision mitigation has been formulated and presented. The potential safety benefit that can be expected in terms of velocity reductions from such interventions has been evaluated and quantified.

• The manoeuvre dynamics in the obstacle avoidance with oncoming traffic scenario has been analysed in detail and characteristic parameters that correlate strongly to the safety benefit potential that can be achieved with electrified drivetrains have been identified. These findings are also verified using open-loop driver-controlled real vehicle experiments.

• The potential safety benefit that can be expected with different actuator setups in the presence of restricted steering in the obstacle avoidance with oncoming traffic scenario has been evaluated and quantified.

• An integrated motion controller (controlling longitudinal and lateral or yaw dynamics) for mitigating the risk of collision with oncoming vehicles during evasive manoeuvres has been formulated and validated in simulations.

• The potential safety benefit that can be expected from two different variants of the integrated controller in evasive manoeuvres with oncoming traffic in the presence of restricted steering has been evaluated and quantified in simulation.

• A real-time closed-loop longitudinal acceleration controller for collision mitigation with oncoming vehicles during evasive manoeuvres has been implemented, tested and validated in experiments.

• Collision avoidance at the “Left Turn Across Path - Opposite Direction” intersection accident scenario has been analysed and optimal acceleration manoeuvres for collision avoidance at the same have been derived through an analytical optimal control framework. An integrated controller that uses the optimal control result has been implemented in simulation and validated.

1.5 Thesis outline

This thesis is structured as follows:

• Chapter 1 provides the background for the project and outlines the motivations and the research questions.

• Chapter 2 outlines some of advantages of electric drives and how they translate to advantages at the vehicle dynamic and the control level.

• Chapter 3 provides some examples of use cases where electrified drivetrains can potentially be used for improving safety.

• Chapter 4 describes and discusses the tools, methods and models used and their applicability for the chosen tasks.
• Chapters 5 to 7 briefly introduce the accident scenarios (rear-end collision, obstacle avoidance with oncoming traffic and intersection accidents) which are considered in detail in the appended publications.

• Chapter 8 provides some discussion regarding the assumptions made and their impact on the and results presented in the thesis.

• Chapter 9 concludes this thesis and outlines potential areas for future work.

Figure 1.6 shows a summary of the papers included in this thesis and their content. Specifically, the accident scenarios under consideration, the main focus of each paper and their relationship to the other publications in this thesis are shown in the diagram.

Figure 1.6: Map of appended papers in this thesis, their content and their relationship to each other.
2 Electric drive advantages

This chapter captures some of the advantages offered by electric motors in comparison to its traditional counterparts (IC engines and brakes) at an actuator level and how these advantages translate to advantages at higher levels (vehicle dynamics, control and intervention opportunities).

2.1 At the actuator level

Before trying to determine how electrified drivetrains can be used to enhance safety, it might be useful first to review the advantages or benefits offered by electric drives over the internal combustion engine (ICE) which can be exploited for enhancing vehicle dynamics. Consequently, while arguably being among the most important advantages of electric drives, their efficiency and emissions related advantages are not discussed here.

One of the most important advantages of electric drives over IC engines relevant for vehicle dynamics functions is their response time and the reliability of the response. Shown in fig. 2.1 are the requested and actual delivered torques by the IC engine and the electric drive in a test vehicle with an electrified drivetrain. The test vehicle had a hybrid drivetrain which consisted of a gasoline IC engine driving the front axle and an electric motor driving the rear axle. The torque request consisted of a 2 s long step request with a 1000 Nm axle torque amplitude. Several runs were conducted with the request being sent to the ICE at the front or the electric drive at the rear axle (but not both at the same time) and the response of the respective actuator recorded. The speed at the start of the intervention was varied among 30 km/h, 50 km/h and 65 km/h to capture the actuator response under different conditions.

As can be seen from the figure, the response of the IC engine is highly inconsistent with the amplitude varying by a large amount. In contrast, with the electric drive, the response is very consistent with the torque amplitudes matching the requested torque almost exactly. Note that the test vehicle used was an early production prototype that had been used for function development and as a result had a hard rate constraint for the electric drive implemented in the Engine Control Module (ECM) software for durability and functional safety reasons.
such software limitations, much faster response times can be achieved as electric drives can have response times in the order of tens of milliseconds [39]. In contrast, in traditional IC engines, more than 200 ms may be required just to open the throttle actuator. Furthermore, if the engine is equipped with forced induction systems (turbochargers, superchargers) - as was the case with the test vehicle - the response time may be increased further and be inconsistent due to the need for the induction systems to spool up. In particular such engines are poor when starting from low engine load (typically the case at low speeds) as the turbochargers would not be fully spooled up. Finally, the need to shift gears in order to keep the engine in its desired operating range can add further to its response time. This is assuming that gear shifting is indeed done to keep the engine in its optimal operating range. In some cases, gear shifting might be eschewed in favour of a faster response time but sacrificing peak torque amplitude as is in some of the cases shown in fig. 2.1.

It is also worth keeping in mind that the torque plots shown are in fact estimates from the ECM and electric drive software themselves. And since electric drives are much better at sensing speed and torque, the torque plots for the motor are likely to be much more accurate compared to those of the IC engine. This improved sensing and estimation ability of the motor can be used in vehicle state and parameter estimation as shown in [40] and also makes them much easier and more precise to control which plays an important role in the consistency of their response. The improved controllability can also be used to enhance the performance of various interventions such as traction or slip control. In [41], the authors estimate that up to 7% reduction in braking distances can be achieved due to faster anti-lock braking (ABS) actuation alone. It also opens up new possibilities to perform interventions with a high degree of robustness and accuracy. For e.g., control of vehicle position is difficult with ICEs and brakes (but not impossible, especially at low speeds), but can be done much more easily even at high speeds using electric drives.

Another benefit of electric drives is that they are bi-directional, i.e., they can be used both for propulsion and braking. This combined with their fast response time and superior controllability means they can be used to perform interventions robustly by correcting for imperfections, drift or disturbances. It also makes it easy to perform simple corrections and obviates the need to manage the cooperation of multiple imperfect actuators to produce smooth actuation. For instance, in order to perform traction control during hard acceleration, it is necessary to combine the operation of the ICE and the brakes. However, due to their slow response, performing smooth traction control is difficult and typically results in jarring interventions. With electric drives however, such interventions can be performed very smoothly.

The continuous operational ability of electric drives is another major boon for active safety applications. This is in contrast to brakes which are relegated to short, last ditch, severe interventions as using them excessively can result in the brake system overheating and becoming ineffective and thereby causing a safety critical situation instead of resolving one. Electric drives on the other hand can be used for longer and smoother or even continuous interventions.

Electric drives offer a lot more advantages that can be exploited for enhanced functions. For a more detailed list of such benefits, please refer to the Electric drive advantages section in Chapter 1 of [42].
2.2 At the vehicle dynamic level

This section briefly describes how the benefits of electric drives described in the previous section translate to benefits at the vehicle dynamic level.

2.2.1 Longitudinal dynamics

In terms of longitudinal dynamics, the faster response time and consistency of the electric drive response allows higher bandwidth and lower phase shift for longitudinal acceleration response. For example, fig. 2.2 shows the longitudinal acceleration frequency response plots for an IC engine and an electrified drivetrain. Simplified models of longitudinal dynamics are used with a response time of 50 ms and 500 ms for the electrified drivetrain and the IC engine respectively. Note that only the response time effects are captured here and the impacts of inconsistent response from the IC engine are not seen here. More details regarding the models used and the results can be found in Chapter 2 of [42].

![Figure 2.2: Bode plots for longitudinal acceleration response for electrified and the IC engine based drivetrains](image)

This is particularly important for active safety interventions since they are mostly executed during a small time window shortly before a potential collision. Slow or inconsistent response during such interventions would risk missing the window of opportunity for an intervention. Moreover, since such interventions are often on-limit, the ability of electric drives to perform better slip and traction control (enabled by increased bandwidth and smaller phase shift) enhances the effectiveness of such interventions [41]. The combination of faster response, bi-directionality and better slip control also allows for accurate control of the vehicle longitudinal position which can be used to perform interventions that involve generating longitudinal displacements. Such interventions can be used to provide increased braking distances for bullet vehicles approaching from behind thereby reducing the severity or completely avoid being struck from behind [2].

2.2.2 Yaw dynamics

The electric drives can be combined with differential braking to apply yaw moments on the vehicle (brake based torque vectoring). The reliability and quick response of electric drives allow this coordination to be performed between it and the brakes. This ability to apply yaw moments can in turn be used to enhance the yaw dynamics of the vehicle. Figure 2.3 shows
the yaw rate response plots for three different cases of yaw dynamics control: the traditional Front Wheel Steering (FWS), Direct Yaw Control (DYC) where yaw moments are applied through wheel longitudinal forces instead of steering the wheels (i.e., zero steer angle), and Yaw Response Control (YRC) where yaw moments are applied through wheel longitudinal forces to improve the transient yaw response of a traditional FWS system but leave the steady state response unchanged. Idealised torque vectoring capability (ability to apply yaw moments without applying a net longitudinal force) is assumed when applying yaw moments through wheel longitudinal forces. More details about the models used and the results can once again be found in Chapter 2 of [42].

![Bode plots for yaw rate response for Front Wheel Steering (FWS), Direct Yaw Control (DYC) and Yaw Response Control (YRC) at 60 km/h](image)

As can be seen from fig. 2.3, the DYC has a similar bandwidth to that of FWS while the phase shift can be slightly reduced. Note that a comparison of their respective gains would be meaningless since the type of actuator inputs used in the two cases are different (steering angle vs longitudinal force). Effectively, this means that DYC can be used as a redundancy system for the steering wheel angle. This would be enormously useful not only for active safety systems but also for autonomous systems where the driver may be out of the loop. It can also be seen that YRC has a much wider bandwidth and very low phase shift compared to either FWS or DYC. This makes electric drives (with differential braking) particularly useful for driver assist functions wherein it can enhance the effectiveness of driver interventions. Hence when used for autonomous interventions, DYC has a performance similar to that of the driver (assuming the steering cannot be used by the function as well) and when used for assist interventions, YRC can significantly enhance the effectiveness of driver interventions.

It would also be possible to use multiple electric drives on an axle to achieve DYC and YRC but with higher performance due to faster response and the more precise and accurate control of electric drives over hydraulic brakes. Multiple electric drives on an axle also allow continuous yaw dynamics improvements to be performed, since unlike hydraulic brakes, electric drives do not overheat with continuous use. Alternatively, a switchable electric drive system such as [43] which uses a single motor that can switch between traction and torque vectoring modes can also be used for DYC, YRC or continuous yaw dynamic improvements. The same has been investigated previously in [44] among others.

Another important benefit with using electric drives to apply yaw moments is that it allows for a better decoupling of yaw and longitudinal dynamics. When using differential braking alone (or even steering to a certain extent) one has to necessarily put up with the deceleration side effect of braking. An electric drive can help offset this deceleration, thereby applying a pure yaw moment on the vehicle.
2.2.3 Global vehicle force

The possibility of applying positive tractive force on the wheels opens up additional ways of distributing longitudinal forces. This additional freedom could be useful in achieving an improved trade-off between global vehicle forces.

To illustrate this effect, consider the friction ellipse shown in fig. 2.4. In this case, if longitudinal forces \((F_x1, F_x2)\) are demanded and applied on a tyre by the controller, then the lateral forces \((F_y1, F_y2)\) can be interpreted as the maximum tyre lateral force available at the driver or the controller’s disposal. However, as can be seen, due to the digressive nature of the relationship between longitudinal and lateral force, the rate of loss of lateral force capacity \((F_{y,max} - F_y1)\) and \((F_{y,max} - F_y2)\) increases as the longitudinal force is increased \((F_{x1} \text{ and } F_{x2})\). This means that if the longitudinal force applied is doubled, the loss in lateral force capacity is more than doubled, i.e.,

\[
\frac{F_{y,max} - F_y2}{F_{y,max} - F_y1} > \frac{F_x2}{F_x1}
\]  

(2.1)

This has some strong implications for the distribution of longitudinal forces. For instance, consider the task of generating a yaw moment on the vehicle by applying longitudinal forces on the wheels of an axle. With differential braking, all the longitudinal force would have to be applied on one wheel whereas when propulsion is used as well, the forces can be distributed between both wheels leading to smaller longitudinal force magnitudes. And as seen from the friction ellipse and digressive nature of tyre forces, distributing the forces between the wheels equally results in a smaller loss in lateral force capacity of the axle. Effectively, this means that when propulsion is available, not only are greater torque vectoring magnitudes possible, but also more of the lateral force capacity of the tyres are available when interventions are performed.
Figure 2.5: Global vehicle force potential under hard steady-state cornering in the counter clockwise direction with the tyre lateral slips saturated
Figure 2.5 shows the global force capabilities (inspired by [45]) that can be achieved when using only brakes versus when using electric drives in addition to brakes during hard cornering with the lateral slips saturated. Here, the electric drives are assumed to be able to drive all four wheels and be able to apply a peak longitudinal force that is equivalent to 0.5μg of longitudinal acceleration. Note that the global force and moments have been normalized with the maximum forces and moments achievable.

Additionally, for this analysis, a few other assumptions are made. The steering angle is assumed to be zero (or small) and that it is not accessible by the controller. Consequently, the lateral slip of the tyres are fixed and cannot be influenced by the controller. We also assume a friction circle which is a simplification of the friction ellipse concept. Once again, more details regarding the modelling and analysis can be found in Chapter 2 of [42].

As can be seen, when propulsion is available, the global force capabilities of the vehicle are much larger. More importantly, it can be seen that when propulsion is available, the tradeoff between the different global forces is much better.

For instance, consider the case of applying a yaw moment on the vehicle under hard cornering. Marked in the $M_g^z$ vs $F_g^y$ plots of fig. 2.5 are the points corresponding to applying a moment of 0.4 on the vehicle. As can be seen, when only the brakes are used, it results in the global lateral force being reduced by half. When the electric motors are used on the other hand, only approximately 30% of the lateral force is lost. This means that when electric motors are used, not only are greater yaw moments possible, but the vehicle's lateral dynamic performance is not hampered as much when interventions are performed.

Similar effects can be seen in the trade-off between $M_g^z - F_g^x$ and $F_g^y - F_g^x$. It can be seen that near $M_g^z = 0.4$, when using brakes alone, a global negative longitudinal force of at least $F_g^x = -0.3$ results. With electric drives on the other hand, not only can the deceleration be offset, but a small positive longitudinal force can be applied. A similar effect is seen with $F_g^y - F_g^x$ wherein a much better trade off can be achieved when using electric drives than with brakes alone.

These improved global force trade-offs can be of large benefit in terms of safety. Since lots of active safety functions involve controlling the vehicle at the limits of its dynamic abilities, expanding the same can result in better vehicle dynamic performance and therefore better performance of the active safety functions.

2.3 At the control level

In this section, some of the major types of control interventions that can be performed with electrified drivetrains which are expected to be useful in safety critical scenarios are detailed. These control interventions can either be used independently or together as required in different accident scenarios to improve safety. Note also that each intervention type has been assigned a color coded abbreviation which is used in the following chapter to signify the control interventions expected to be of use in each accident scenario.
2.3.1 **Longitudinal speed control** [SPD]

In this type of control intervention, the primary control objective is the longitudinal speed of the vehicle. While longitudinal speed can be effectively controlled using traditional IC engine based drivetrains as well (as is the case with cruise control for example), it may not always be possible to do so well enough for use in active safety interventions. This is due to the fact that the time window of opportunity for many active safety interventions can be under a second which can be too short a duration for traditional drivetrains to be able to reliably deliver the requested torque.

2.3.2 **Longitudinal position control** [XPC]

Control of vehicle longitudinal position is the primary goal here. This control task is performed by translating the vehicle longitudinal position based objective to a lower level vehicle speed based objective. Due to this, once again, traditional IC engine based drivetrains may be difficult to use in such interventions.

In some cases, longitudinal position control can help avoid collisions completely while in others, it can help reduce the severity of an impact by providing more room for the bullet vehicle to perform interventions (e.g., rear-end collisions).

2.3.3 **Occupant posture control** [OPC]

Here, the goal is to use an appropriately timed acceleration pulse to help adjust the posture of the occupants to reduce injury risk in an imminent collision. For instance, a quick burst of forward acceleration before an imminent rear end collision could potentially push the head back into the headrest thereby reducing the risk of whiplash injury.

Since electric motors can generate torques several times that of their rated torques for brief periods of time and can do so very quickly, they are well suited for this purpose. Furthermore, in this control task, not only the magnitude of acceleration, but also the timing, duration of the pulse and the jerk may be very important. Consequently, traditional IC engine based drivetrains are less suitable for this purpose.

2.3.4 **Yaw moment control** [YAW]

In this case, the goal is to control the yaw motion of the vehicle, which could either be to control the yaw acceleration, yaw rate or rarely, the yaw angle of the vehicle. Yaw rate and yaw angle control is mostly done by translating it to a lower level yaw acceleration control task. While this task can be accomplished by differential brakes, they necessarily slow the vehicle down as a side effect, which may not always be desirable. Furthermore, differential brakes have significant response times which make them less suitable for improving vehicle response in emergency manoeuvres.

2.3.5 **Lateral position control** [YPC]

While the vehicle's lateral position cannot be controlled directly, it can be controlled indirectly by controlling its yaw motion and in some cases, its longitudinal speed as well. At high speeds,
control of the vehicle's lateral position can be done by translating the task to a lower level yaw moment control task. At low speeds, both yaw moment and the vehicle longitudinal speed might need to be controlled. Lateral control at low speed is complicated by the fact that other effects such as scrubbing of the tyres, steering geometry, etc. become important which are difficult to account for. In this thesis, with regards to lateral position control, only high speed applications are dealt with. As in the case of yaw moment control, while this control task can be achieved with differential brakes, they are not very suitable for this purpose. Furthermore, since lateral position control typically requires precise and extensive actuation (as lateral position is a third order function of the applied yaw moment), they result in even more deceleration.

2.3.6 Longitudinal wheel slip control [SLP]

The control task is here to manage the tyre longitudinal slips so as to keep them within certain levels. Excessive longitudinal slip could lead to the tyre saturating in the longitudinal direction and losing lateral grip which could in turn lead to loss of control. Excessive slip also, in general, reduces the forces generated by the tyres and as a result decreases vehicle performance (both braking and cornering). While slip control can be effectively done with brakes alone, it has been shown that using electric drives for the same can lead to significant improvements [41].
3 Use cases for enhanced interventions

In this chapter, a short list of examples of different use cases for enhanced interventions using an electrified drivetrain is provided. More examples can be found in Paper A and an even more comprehensive list can be found in [42].

Before proceeding further, definitions (in the context of this thesis) of some important, commonly used terms are in order.

- **Accident scenario:** An outline of the scene which characterises a potential accident.

- **Manoeuvre:** The motion history of the vehicle in the accident scenario.

  This term is mostly used with reference to the host vehicle.

- **Intervention:** Any sort of action performed or input to the vehicle deviating from the initial condition or steady state.

  Can be performed by the driver, a controller or a combination of both.

  - **Driver intervention:** An intervention performed by the driver.

    For e.g., braking and/or steering to avoid an obstacle. Does not necessarily have to contribute towards improved safety.

  - **Control intervention:** An intervention performed by a controller.

    The interventions outlined in section 2.3 are examples of control interventions. These interventions have relatively low level control objectives (for e.g., control speed, control yaw rate, etc.) and are not specific to the accident scenario at hand.

- **Use case:** A combination of an accident scenario and a corresponding intervention which is expected to avoid or mitigate the collision in each case.

- **Function:** A strategic combination of one or more control interventions performed with the goal of improving safety in a certain accident scenario.

  Note that a function is a just an idea or strategy of how to perform interventions to improve safety and does not include the hardware or the specific implementation. For e.g., the concept of ABS (not the actual sensors, actuators, etc. that form the ABS) to control slip under severe braking is an example of a function.

- **System:** The practical realisation of a function including the hardware.

  For e.g., the ABS function along with the sensors, actuators and any other required hardware form the ABS system.

Each use case is briefly described in this chapter along with how an electrified drivetrain can enhance or enable an intervention to improve safety in each case. In the corresponding illustrations accompanying each use case (or a set of them if several use cases are very similar), the types of control interventions that are expected to be beneficial are marked using the color-coded abbreviations introduced in the previous chapter.

In the following sections, the **host vehicle** represents the vehicle of interest that has the electrified drivetrain whereas the **bullet vehicle** represents the threat which the host vehicle aims to avoid.
3.1 Accelerate to avoid rear-end collision

The case of a rear-end collision with an electrified lead vehicle (host) is shown in fig. 3.1. The availability of an electric drive in the lead vehicle opens up several intervention opportunities to improve safety in this scenario.

One of the possible ways to mitigate or even prevent the accident could be to accelerate the lead vehicle and thereby reduce the relative speed at impact. A beneficial side-effect is that it also provides more room for the bullet vehicle to brake and thereby amplifies the safety benefit of the bullet vehicle’s braking intervention. One could then envision a limited version of this intervention wherein the host vehicle is moved forward precisely by accelerating and then braking so that the vehicle speed is not increased at the end of this manoeuvre. This intervention may be useful, for instance, when the lead vehicle is stationary at a junction with a certain amount of usable free space in front of it.

Alternatively, the electric drive can be used to deliver a short, but sharp burst of acceleration with high jerk but with little increase in speed or displacement as this alone could reduce the risk of whiplash injuries for the occupants. The reason for this safety benefit is that the sudden and sharp acceleration pulse can potentially cause the heads of the occupants to be pushed back toward the head rests and this improvement in posture can lead to a reduced whiplash injury risk.

In all cases, slip control can enhance the effectiveness of the respective intervention. The interventions can also be combined in different ways to create enhanced versions of the same.

A similar case is considered and analysed in more detail in Paper B.

3.2 Evasive steering for frontal collision avoidance in the presence of oncoming traffic

When evasive steering is performed by the driver in order to avoid a frontal collision, there is a risk of collision with oncoming vehicles. In such a case, this risk can be reduced by appropriately performing yaw moment control to assist the steering while also controlling the speed to reduce the distance travelled as well as the time taken to complete the manoeuvre.

A specific case of this accident scenario has been considered and analysed in detail in Paper D, Paper E and Paper F.
3.3 Evasive steering and acceleration for avoiding T-bone collisions/pedestrians

In this scenario, the threat (bullet vehicle or pedestrian) has a constant (assumed) lateral velocity and encroaches on to the host vehicle lane. Assuming braking alone is insufficient to prevent the collision, it may be necessary to perform evasive steering as well. However, since the threat has a lateral velocity, the duration of the evasive manoeuvre becomes important: the longer the manoeuvre takes, larger is the encroachment of the threat into the host vehicle lane, and hence more severe is the evasive manoeuvre required from the host vehicle. Consequently, speed control becomes important in this manoeuvre.

Differential braking to assist the steering could be detrimental in this case since it would slow the vehicle down resulting in it taking a longer time to reach the threat and consequently requiring a more severe intervention. The ability to apply yaw moments without slowing the vehicle down (as can be done with torque vectoring) could be useful here. Control over speed, yaw moment (for stability, responsiveness), lateral position and tyre slips could be useful in this scenario.
3.4 Intersection accidents

A variety of similar intersection accidents are shown in fig. 3.4. While these cases mostly require the same types of interventions, they show up differently in the accident statistics and hence several variations of the same are shown distinctly here.

In all these cases, yaw moment, speed and slip control are required. While speed control is the crucial part that helps avoid the accident, due to the large curvature of the path being taken, speed control necessarily needs to be combined with yaw moment control and also slip control in order to ensure stability while performing this intervention.

A specific case of an intersection accident is considered and analysed in Paper C.

![Intersection accidents diagram](a) Intersection accident 1 (b) Intersection accident 2 (c) Intersection accident 3 (d) Intersection accident 4

Figure 3.4: Intersection accidents

3.5 Exit after give-way/stop sign

These cases, while similar to intersection accidents in terms of the types of interventions required, show up differently in crash statistics. Furthermore, the speeds involved in these collisions could be different from intersection accidents. The environmental detection aspect is also very different from intersection accidents in these cases.

As in intersection accidents, yaw moment, speed and slip control need to be performed to
(a) Host vehicle exits onto main road in front of bullet vehicle with small margin

(b) Host vehicle exits into roundabout in front of bullet vehicle with small margin

Figure 3.5: Exit onto road after give-way/stop sign

effectively improve safety in this scenario.
4 Methods, models and tools

In this chapter, the methods, models and tools used in this research work are presented along with a discussion about the assumptions used, their implication and their general applicability for the respective tasks.

4.1 Vehicle model

The simulation models of varying complexity that have been used in this work, along with the assumptions and their implications are presented in this section.

4.1.1 Point mass model

The point mass model (also called the particle model in some of the appended papers) is the simplest model used in this work. These simple 2 degree of freedom (DoF) models are used for preliminary analysis or for analytical work where the simplicity of the model often brings the kinematics of the manoeuvre into focus. Simplified models such as this do not capture most of the vehicle dynamic effects and findings from these are always followed up with verification in more detailed models.

The model can either be expressed in local or global coordinates with the latter being the one predominantly used in this work. The point mass model in global coordinates can be expressed as:

\[
\begin{align*}
\mathbf{f}(\mathbf{x}, \gamma, t) &= \mathbf{A} \mathbf{x} + \mathbf{q}(\gamma) \quad \forall \gamma \in \mathcal{U} \\
\mathbf{x} &= \begin{bmatrix} X & Y & \dot{X} & \dot{Y} \end{bmatrix}^T \\
\gamma &= \begin{bmatrix} F & \phi \end{bmatrix}^T \\
\mathbf{A} &= \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\
0_{2\times2} & 0_{2\times2} \\
\end{bmatrix} \\
\mathbf{q}(\gamma) &= \begin{bmatrix} 0_{2\times1} \\
\cos \phi \\
\sin \phi \end{bmatrix}
\end{align*}
\]

When the effects of different actuators (propulsion, braking) need to be investigated, they can be simulated by limiting the tyre forces as follows:

\[
\begin{align*}
-\mu g & \leq F_x \leq \mu g & \text{propulsion and braking} \\
-\mu g & \leq F_x \leq 0 & \text{braking only} \\
0 & \leq F_x \leq \mu g & \text{propulsion only} \\
0 & \leq F_x \leq 0 & \text{constant speed}
\end{align*}
\]
where $F_x$ is the longitudinal force in the vehicle reference frame and can be expressed as:

$$
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
0_{1 \times 2} & \cos v & \sin v \\
0_{1 \times 2} & -\sin v & \cos v
\end{bmatrix} q(\gamma)
$$

(4.3)

$$
\tan v = \frac{\dot{Y}}{\dot{X}}
$$

(4.4)

In Paper D, a mathematically equivalent model is used, but is formulated differently to make it computationally efficient for numerical optimisations.

### 4.1.2 Single track model

The single track model (also called the bicycle model) is commonly used either for early stage evaluation or verification in this work. Several variations of this model exist with differing levels of complexity, but the one predominantly used in this work is a 3 DoF non-linear vehicle model which is shown in fig. 4.1.

![Single track vehicle model](image)

Figure 4.1: Single track vehicle model

The equations of motion for the same can be written as follows:

$$
m(\dot{v}_x - v_y \omega_z) = F_{xf} \cos \delta - F_{yr} \sin \delta + F_{xr}
$$

(4.5a)

$$
m(\dot{v}_y + v_x \omega_z) = F_{yf} \cos \delta + F_{xf} \sin \delta + F_{yr}
$$

(4.5b)

$$
I_{zz} \dot{\omega}_z = (F_{yf} \cos \delta + F_{xf} \sin \delta) l_f - F_{yr} l_r
$$

(4.5c)

The slip equations for the wheels can be written as:

$$
\alpha_f = \delta - \text{atan}\left(\frac{v_y + l_f \omega_z}{v_x}\right)
$$

(4.6a)

$$
\alpha_r = -\text{atan}\left(\frac{v_y - l_r \omega_z}{v_x}\right)
$$

(4.6b)
Different tyre models can be used depending on the application. In this work, typically, a \( \tanh \) tyre model (eq. (4.8)) is used for optimal control analysis and a Magic formula tyre model (eq. (4.9)) is used for simulation. In all the single track and two track models used in this work, the longitudinal tyre forces \( (F_{x_f}, F_{x_r}) \) are used as inputs to the models, based on which the maximum available lateral force is calculated. This is then used to calculate the actual lateral force based on the slip angles.

\[
F_{yi,max} = \sqrt{(\mu_i F_{zi})^2 - F_{xi}^2}
\]  
(4.7)

where \( i \) can be \( f \) or \( r \) representing the front or rear axles respectively.

\[
F_{yi} = F_{yi,max} \tanh \frac{c_i \alpha_i}{\mu_i}
\]  
(4.8)
	tanh tyre model

\[
F_{yi} = F_{yi,max} \sin(C \text{atan}(B \alpha_i - E(B \alpha_i - \text{atan}(B \alpha_i))))
\]  
(4.9)
Magic formula tyre model

where,

\[
B_i = \frac{c_i}{(\mu_i C)}
\]  
(4.10)

\[
c_i = c_0 (1 - c_1 (F_{zi} - F_{z0}))
\]  
(4.11)

\[
\mu_i = \mu_0 (1 - \mu_1 (F_{zi} - F_{z0}))
\]  
(4.12)

While longitudinal load transfer can be considered in single track models, in this work, it is not taken into account.

In some parts of this work, simplified versions of the model outlined above have been used. For instance, in section 2.2.2, the yaw dynamics analysis uses a 2 DoF vehicle model with linear tyres. The longitudinal degree of freedom is removed in this case and the longitudinal velocity \( (v_x) \) is kept constant. The tyres are simply modelled as:

\[
F_{yi} = C_i \alpha_i
\]  
(4.13)

Figure 4.2 shows a comparison of the linear, \( \tanh \) and the Magic formula tyre models. As can be seen, the linear tyre model only matches the other tyre models at low levels of lateral force (also called the linear range of the tyres) and consequently can only be used at low levels of lateral acceleration. Due to its simplicity however, such a tyre model is useful for performing preliminary or analytical analyses as in section 2.2.2.

The \( \tanh \) model on the other hand matches the Magic formula until close to peak tyre force beyond which it saturates and diverges from the Magic formula model. The \( \tanh \) tyre model is useful in cases where the tyre slip angle is not expected or not intended to go beyond the peak tyre force slip angle. In numerical optimisation for instance, the \( \tanh \) model is preferred as it is numerically better behaved since it has no distinct tyre force maximum. Additionally, since it represents a one-to-one mapping of tyre force to slip angle, it also helps in preventing the optimisation from getting stuck in local optima.

The Magic formula tyre model is the most detailed tyre model used in this work. The model shown in eq. (4.9) is a simplified version of the tyre model and is yet more detailed compared to the linear or \( \tanh \) models. The Magic formula model is used in detailed simulations where
Figure 4.2: Comparison of tyre models. Note that the force magnitudes are normalised. Data used: $c = 20, \mu = 1, F_{y,\text{max}} = 1, C = 1.5, E = 0$

...accuracy is important and also to capture behaviour related to yaw stability and on-limit dynamics. A much more detailed Magic formula tyre model based on [46] is used in Paper C.

When linear tyres are used with the single track model, the resulting model can be expressed in state-space form as follows:

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (4.14)

$$x = \begin{bmatrix} v_y \\ \omega_z \end{bmatrix}^T$$  \hspace{1cm} (4.15)

$$u = \delta$$  \hspace{1cm} (4.16)

$$A = -\begin{bmatrix} \frac{C_f + C_r}{mv_x} & \frac{C_f l_f - C_r l_r}{I_{zz}v_x} + v_x \\ \frac{C_f l_f - C_r l_r}{I_{zz}v_x} & \frac{C_f l_f^2 + C_r l_r^2}{I_{zz}v_x} \end{bmatrix}$$  \hspace{1cm} (4.17)

$$B = \begin{bmatrix} \frac{C_f}{m} \\ \frac{C_f l_f}{I_{zz}} \end{bmatrix}^T$$  \hspace{1cm} (4.18)

When a torque vectoring system is added, the input vector and the matrix $B$ can be extended as follows:

$$u = \begin{bmatrix} \delta \\ M_z \end{bmatrix}^T$$  \hspace{1cm} (4.19)

$$B = \begin{bmatrix} \frac{C_f}{m} & 0 \\ \frac{C_f l_f}{I_{zz}} & 1 \\ \frac{C_f l_f}{I_{zz}} & 1 \end{bmatrix}$$  \hspace{1cm} (4.20)

Here, $M_z$ is the input from the torque vectoring system which is assumed to be able to apply a pure yaw moment without applying a net longitudinal force. In the appended papers,
where a yaw rate reference is needed (for instance in the ESC), the above model is simplified even further by setting the state derivatives to zero in order to yield a steady-state single track vehicle model. In such a model, typically only the yaw rate is of interest which can be expressed as:

\[
\omega_z = \frac{v_x \delta}{L + K_u v_x^2} \quad (4.21)
\]

\[
K_u = \frac{m l_f}{L C_f} - \frac{m l_r}{L C_r} \quad (4.22)
\]

where, \(K_u\) is called the understeer gradient.

### 4.1.3 Two track model

The two track model is used for intermediate verification before moving on to third-party high-fidelity simulation environments. The two track model is an important step in verification since it allows most important vehicle dynamic effects to be captured and since it is a self-built model, allows various features to be arbitrarily switched off and on in order to analyse their impact on the results. In this work, a 3 and 4 DoF vehicle model is commonly used depending on whether it is used for numerical optimal control or for simulations. The wheel degrees of freedom are not considered in this model since it is typically not needed for most of the use cases considered in this work. The two track vehicle model used in this work is shown in fig. 4.3.

The equations of motion for the two track vehicle model can be expressed as:

\[
m(\dot{v}_x - v_y \omega_z) = (F_{x,f1} + F_{x,fr}) \cos \delta - (F_{y,f1} + F_{y,fr}) \sin \delta + F_{x,rl} + F_{x,rr} \quad (4.23a)
\]

\[
m(\dot{v}_y + v_x \omega_z) = (F_{y,f1} + F_{y,fr}) \cos \delta + (F_{x,f1} + F_{x,fr}) \sin \delta + F_{y,rl} + F_{y,rr} \quad (4.23b)
\]

\[
I_{zz} \ddot{\omega}_z = -w/2(F_{x,f1} - F_{x,fr}) \cos \delta + w/2(F_{y,f1} - F_{y,fr}) \sin \delta - w/2(F_{x,rl} - F_{x,rr}) + l_f(F_{y,f1} + F_{y,fr}) \cos \delta + l_f(F_{x,f1} + F_{x,fr}) \sin \delta - l_r(F_{y,rl} + F_{y,rr}) \quad (4.23c)
\]

A non-steered rear axle is assumed since rear wheel steering is not considered in this work at all. However, while the front wheels typically do not have the exact same steering angle, equal steer angles are nonetheless assumed for the front two wheels since they are not expected to make a significant difference, particularly at high speeds.

The slip angles for the four wheels in the case of the two track model are:

\[
\alpha_{f1} = \delta - \arctan \left( \frac{v_y + l_f \omega_z}{v_x - w/2 \omega_z} \right) \quad \alpha_{fr} = \delta - \arctan \left( \frac{v_y + l_f \omega_z}{v_x + w/2 \omega_z} \right) \quad (4.24)
\]

\[
\alpha_{rl} = -\arctan \left( \frac{v_y - l_r \omega_z}{v_x - w/2 \omega_z} \right) \quad \alpha_{rr} = -\arctan \left( \frac{v_y - l_r \omega_z}{v_x + w/2 \omega_z} \right) \quad (4.25)
\]

For the tyre models, either the \textit{tanh} or the \textit{Magic formula} tyre model as shown in eqs. (4.8) and (4.9) can be used. Typically, the \textit{tanh} model is used for numerical optimal control and the \textit{Magic formula} for simulations.
For the 4 DoF model, the roll degree of freedom is also modelled. The roll dynamics and the resulting load transfer resulting can be expressed as:

\[
(I_{xx} + m_s h'^2)\ddot{\phi} = m_s a_y h' - c_{\phi} \dot{\phi} - (k_{\phi} - m_s g h')\phi
\]  
(4.26)

\[
\Delta F_{zy,ij} = \frac{1}{w} \left( m a_y h_i \frac{L - l_i}{L} \mp k_{\phi,i} \phi \mp c_{\phi,i} \dot{\phi} \right) 
\]  
(4.27)

Here, \( \phi \) is the roll angle, \( I_{xx} \) is the roll moment of inertia, \( m_s \) the sprung mass, \( h' \) the height of the centre of gravity over the roll axis and \( c_{\phi} \) and \( k_{\phi} \) are the total roll damping and roll stiffness. In the load transfer equation, \( i \) and \( j \) stand for the axle (front or rear) and the side of the vehicle (left or right). The variables \( h_i, k_{\phi,i} \) and \( c_{\phi,i} \) represent the roll centre height, the roll stiffness and damping at the respective axles.

For the 3 DoF model, the roll degree of freedom is ignored and instead steady state lateral load transfer is considered. The equations for the same can be derived by setting the roll rate and roll acceleration to zero in eq. (4.26) and substituting the resulting expression for roll angle in eq. (4.27).

\[
\Delta F_{zy,ij} = \frac{m a_y}{w} \left( h_i \frac{L - l_i}{L} \mp \frac{k_{\phi,i} h'}{k_{\phi} - m g h'} \right) 
\]  
(4.28)

For simplicity, the sprung mass is assumed to be approximately equal to the total vehicle mass in the above equation.
The longitudinal load transfer is modelled as a steady state feature in all cases:

\[ \Delta F_{zx,i} = \mp \frac{mha_x}{2L} \]  

(4.29)

where \( i \) represents the axle (front or rear). The longitudinal load transfer is the same for the left and right wheels on an axle.

The total load transfer can then be expressed as a sum of the static load and the lateral and longitudinal load transfers:

\[ F_{z,ij} = mg \left( L - l_i \right) + \Delta F_{zx,i} + \Delta F_{zy,ij} \]  

(4.30)

### 4.1.4 CarMaker vehicle model

The vehicle model used in CarMaker is a validated Volvo XC90 vehicle model [47]. This vehicle model is used as a last step verification in simulation before moving on to experiments. The vehicle model is a highly detailed one with a large number of features and options. Detailed models of the subsystems such as steering, hydraulic brakes, suspension with kinematics and compliance, etc. are also used as part of the vehicle model. And while these allow for highly detailed and accurate simulations, they also make it difficult to work back from the results to establish the cause of different features in the results.

The vehicle model can also optionally be used with a closed Simulink based powertrain model. Additionally, the different subsystem models of the vehicle can be replaced with custom models as necessary. The tyre model used is a Magic formula tyre model based on [46]. Note that this tyre model is significantly more detailed than eq. (4.9) and is parameterised based on tyre test data.

### 4.2 Simulation environments

In this section the two major simulation environments that have been used in this work are presented.

#### 4.2.1 MATLAB/Simulink

Matlab is a general purpose scientific computing software that is perhaps the most commonly used tool in this work. Matlab has been used for general analysis, data processing, numerical optimal control (with additional software) and both small simulations (run entirely in Matlab) and large simulations (run with Simulink and/or CarMaker). In contrast to general purpose programming languages, Matlab’s language has less syntax and structure (don’t need to explicitly define variables, types, etc.) allowing for rapid prototyping. The Matlab language is also a scripting language, which means that programs do not need to be explicitly compiled by the user before being executed. This further reduces the effort for prototyping. However, this very flexibility can sometimes lead to errors in large programs that would have been avoided in stricter languages.
Simulink is a graphical programming environment focused toward modelling and simulation. Since Simulink runs on the Matlab platform, it inherits most of the advantages of Matlab itself but adds to it with features focused toward large scale modelling and simulation. For instance, Simulink supports specifying strict types for signals, allows libraries (equivalent to functions) to be created and reused, allows easy export and import of models to other formats, support for numerous third party tools to plug-in (e.g., CarMaker, CANoe), choice of numerous ordinary differential equation and differential algebraic equation solvers, allows generation of C code for models targeting specific processors, etc. Large simulations in this project are mostly run on Simulink, sometimes along with CarMaker. All controllers that are presented in this work have been implemented in Simulink.

The major disadvantage with both Matlab and Simulink is that, since they are general purpose software, features specific to vehicle dynamics (mainly models of vehicles, subsystems, manoeuvre specification, track layout specification, etc.) need to be implemented by the user. This effectively means that due to resource limitations, the accuracy and usability of such features tend to be limited compared to software tailor-made for such applications (e.g., CarMaker).

4.2.2 IPG CarMaker

IPG CarMaker is a high-fidelity vehicle dynamics simulation software that is used for final verification and validation in this project before real vehicle experiments. As mentioned in section 4.1.4, it has the advantage of having a detailed vehicle model with models of subsystems and common functions. In addition, it also enables easy and detailed specification of track layout, manoeuvres, load cases, test specification, etc. It also contains models for more common functions such as anti-lock brakes, stability control system, etc. Particularly useful for testing of active safety features, it allows easy and detailed specification of road traffic and other road users. Models of different sensors to detect traffic, vehicle states, traffic signs, road features, etc., are also available. Figure 4.4 shows a screenshot of a CarMaker simulation running the obstacle avoidance with the oncoming traffic scenario. The image shows CarMaker’s ability to incorporate different traffic elements into the scenario such as a stationary long obstacle (truck), an active road user (oncoming vehicle) and other elements like road shoulders, lane markings, verge posts at the sides of the road, etc.

CarMaker also has the ability to connect and interact with Simulink using the CarMaker for Simulink add-on which has been used extensively in this work. As mentioned, all controllers were implemented in Simulink which then communicated with CarMaker to apply torques and forces to the vehicle in a CarMaker simulation.

While CarMaker is useful as a final stage verification step, the large number of features and interacting effects that are modelled sometimes make it difficult to interpret the results as it becomes difficult to establish causality for different observed effects. As a result CarMaker is only used for final stage verification after the controller has been well tested and tuned in Simulink.
4.3 Control allocation

Control allocation is a tool commonly used to distribute control in over-actuated systems to meet a higher level objective. In other words, when there are more actuators than there are degrees of motion, the same effect or motion can be achieved in several (possibly infinite) ways. The task of achieving this motion while optimising a secondary goal (such as minimise control effort, maximise efficiency, etc.) is dealt with using control allocation.

An understanding of the salient features of control allocation can be obtained by comparing it to other control schemes.

PD, PI or PID control (collectively called PID henceforth) is one of the most commonly used control schemes due to their being simple to implement and its low computational requirements. While these have been used to perform difficult control tasks, their abilities are limited. They cannot, for instance, deal with allocating control to over-actuated systems. They cannot take into account the behaviour of the system to any extent as part of the control strategy. It should also be observed that the PID control only tries to minimise the error between a chosen and a reference signal. The choice of appropriate reference signals for optimal system performance is a task not performed by the PID controller.

Controllers like $H_{\infty}$ and LQR regulators on the other hand control the entire system to minimise a chosen performance objective. These controllers use optimisation techniques to choose an optimal control input to minimise a given performance objective. However, these controllers cannot handle over-actuated systems and hence cannot be used for the task considered in Paper C, Paper D and Paper E. Additionally, they both require the systems used to be linear. However, since longitudinal, lateral and yaw dynamics need to be considered for the control task, the vehicle model cannot be linearised. They also have strict limitations
on how the objective function can be formulated for the control which limits their utility for
the task at hand. Another disadvantage of this type of control is that, since they optimise the
system as a whole, the system behaviour is hidden and no insights about the system is gained.

A popular recent option for control of complex systems is Model Predictive Control (MPC).
While MPC can handle non-linear systems and over-actuated systems, they can be computa-
tionally intensive. While simpler versions of the MPC can be less computationally intensive,
they require a linear plant model and the objective function to be formulated in a specific
way (usually quadratic). Non-linear MPC is sometimes dealt with by linearising the plant
at each time step which, while making the optimisation efficient, adds to the computational
requirements by having to linearise the system every time step. Optimality and convergence
are also hard to guarantee in a non-linear MPC. Finally, once again, since the system as a
whole is optimised within the MPC, no insights into the system behaviour is gained.

In contrast, control allocation techniques perform a much more limited part of the control
task: they only distribute the desired control to a set of redundant actuators. The desired
control still has to be determined by other means; and in the case of this work, by using an
understanding of the vehicle and the manoeuvre dynamics to formulate a desired control that
improves safety in the scenario under consideration.

Different types of control allocation schemes exist. The control allocation schemes that
have been used in this work are presented in the following section.

4.3.1 Quadratic Programming Control Allocation

Control allocation in Paper D and Paper E have been done using the Quadratic Programming
Control Allocation Toolbox (QCAT) which is a control allocation library for Matlab/Simulink.
The toolbox is freely available for download from [48] and is free for use in research and
educational applications.

The QCAT toolbox contains implementations of several optimisation algorithms found in
the literature with different features and varying levels of efficiency. In the present work, the
Weighted Least Squares (WLS) formulation has been used which is solved using an active set
solver (called “WLS” in the toolbox). This formulation allows for taking into consideration
the actuator rate and amplitude limitations, in addition to allowing for weighting between the
actuator and the global forces. Another useful feature of the WLS formulation is that it allows
for a maximum number of iterations to be set which can be useful for applications involving
performance limitations.

The control allocation problem is then formulated as follows:

\[
u^* = \arg\min_u \left( \|W_u(u - u_d)\|^2 + \lambda \|W_v(Bu - v)\|^2 \right)\]

subj. to \[-u \leq u \leq +u\]

\[-\dot{u} \leq \dot{u} \leq +\dot{u}\]
where,
\[ u_d = 0_{6 \times 1} \]  
\[ B = \begin{bmatrix} 1 & 1 & -w/2 & w/2 & -w/2 & w/2 \\ 0 & 0 & -w/2 & w/2 & w/2 & 0 \end{bmatrix} \]  
\[ u = [F_{\text{eng}} F_{\text{intr}} F_{\text{brk},fl} F_{\text{brk},fr} F_{\text{brk},rl} F_{\text{brk},rr}]^T \]  
\[ v = [F_{x,tgt} M_{z,tgt}]^T \]

\( F_{x,tgt} \) and \( M_{z,tgt} \) are the global vehicle longitudinal force and moment demands from the longitudinal acceleration controller and the yaw stability or lateral controller. The weighting matrices \( W_u \) and \( W_v \) are used to establish priorities for the control actuators and the global forces respectively. \( W_u \) is set up taking into account the actuator rate limits and the usable grip at the tyres with the aim of minimising the combined actuator-tyre workload. \( W_v \) is set up by trial and error to achieve a suitable balance between distance margin maximisation and yaw stability. See Paper D for details regarding the setup of these matrices.

### Figure 4.5: Structure of the integrated controller for collision avoidance with the oncoming vehicle (figure reproduced from Paper E)

The structure of the specific controller used in Paper D and Paper E is shown in fig. 4.5. Here the “Control allocator” block represents the QCAT scheme. The QCAT toolbox used here has been modified from the original to extend its capabilities in terms of being able to accept time varying amplitude and rate limits and the weighting matrices whereas in the original, they are fixed.

All the major blocks in the control structure (Lateral control/ESC, Longitudinal Acceleration Control and Control allocator) are functionally independent from each other and can be replaced without requiring significant modifications to the rest of the controller.
4.3.2 Modified Hamiltonian Algorithm Controller

The Modified Hamiltonian Algorithm (MHA) controller is inspired by work in [49] and was originally presented in [50]. While the MHA is a more limited as a general control allocation scheme, it is a more powerful vehicle dynamic focused control allocator. In contrast to traditional control allocation schemes that can distribute control to an arbitrary number and types of actuators to satisfy an arbitrary number and types of control objectives, the MHA is focused on optimising tyre forces to maximise vehicle global force in a particular direction while maintaining yaw stability.

The controller is motivated by optimal control theory wherein the minimisation of an objective can be represented as a minimisation of the Hamiltonian. The MHA controller is well suited for interventions where an optimal manoeuvre can be represented as a maximisation of the vehicle global force in an optimal direction. The structure of the controller is shown in fig. 4.6.

Consider a three degree-of-freedom vehicle model in the global reference frame as follows with normalised global forces as inputs:

\[ \dot{x} = Ax + Bu \]  
(4.36)

\[ A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ I_{3\times3} & 0_{3\times3} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \]  
(4.37)

\[ x = [X \ Y \ \dot{\psi} \ \dot{X} \ \dot{Y} \ \dot{\psi}]^T \]  
(4.38)

\[ u = [F_{XG}/m \ F_{YG}/m \ M_{ZG}/I_{zz}]^T \]  
(4.39)

For an optimal control problem with such a system involving terminal time cost function,
the Hamiltonian can be written as:

\[ H = \lambda^T (Ax + Bu) \]  

(4.40)

According to Pontryagin's Maximum Principle [51], the minimisation of the objective function requires the minimisation of the Hamiltonian. Since only the second term of the Hamiltonian is influenced by \( u \), the minimisation can be reduced to a minimisation of:

\[ H_1 = \lambda^T Bu \]  

(4.41)

\[ = \lambda_4 F_{XG}/m + \lambda_5 F_{YG}/m + \lambda_6 M_{ZG}/I_{zz} \]  

(4.42)

This in turn can be rewritten in terms of the individual tyre forces:

\[ H_1 = \sum_i \lambda_4 \frac{F^g_{x,i}}{m} + \lambda_5 \frac{F^g_{y,i}}{m} + \lambda_6 \left( \frac{y_i F^g_{x,i} + x_i F^g_{y,i}}{I_{zz}} \right) \]  

(4.43)

\[ = \sum_i \left( \lambda_4 \frac{y_i}{m} + \lambda_6 \frac{x_i}{I_{zz}} \right) F^g_{x,i} + \left( \lambda_5 \frac{x_i}{m} + \lambda_6 \frac{y_i}{I_{zz}} \right) F^g_{y,i} \]  

(4.44)

\[ = \lambda^g_{x,i} F^g_{x,i} + \lambda^g_{y,i} F^g_{y,i} \]  

(4.45)

where, \( F^g_{x,i} \) and \( F^g_{y,i} \) are tyre forces in the global frame of reference and \( y_i \) and \( x_i \) are the moment arms from the wheel longitudinal and lateral forces respectively. Since the wheel forces are bounded, for minimisation of \( H_1 \), the absolute values of \( \lambda_x \) and \( \lambda_y \) do not matter; instead only their ratios matter which can be represented as follows:

\[ H_1 = \sum_i \cos \phi^i_{g,i} F^g_{x,i} + \sin \phi^i_{g,i} F^g_{y,i} \]  

(4.46)

Here, \( \phi^i_{g,i} \) is the direction in which the tyre force needs to be minimised. This can now be decomposed into individual wheel Hamiltonians that can be minimised independently assuming that the individual wheel Hamiltonians are insensitive to the applied braking forces at other wheels. While load transfer from applied forces changes the normal loads at other wheels, the dynamics of the vehicle and suspension affects tend to damp out rapid changes in wheel normal loads. The minimisation can then be reduced to independent minimisation of the wheel Hamiltonians:

\[ H_i = \cos \phi^i_{g,i} F^g_{x,i} + \sin \phi^i_{g,i} F^g_{y,i} \]  

(4.47)

The procedure for determining the target force angle can be seen in fig. 4.6. A force angle target is first determined using a particle model in an analytical optimal control framework. This is then adjusted for each wheel taking into account yaw stability requirements and then converted to wheel coordinates before being used for Hamiltonian minimisation. See Paper C for more details.

### 4.4 Optimal control

In this section, the optimal control methods that have been used in this work are presented and discussed.
4.4.1 Numerical optimal control

For numerical optimal control, a Matlab based software called PROPT from TOMLAB is used. With direct integration into and a syntax very similar to that of Matlab, PROPT allows rapid prototyping and evaluation of optimal control problems. In contrast, other optimal control software such as Optimica and jModelica run on different platforms which presents a challenge to those not already familiar with the programming language.

PROPT was used to solve complex optimal control problems that could not be solved analytically. Typically problems involving anything more complicated than a particle model (and sometimes even those) were solved using PROPT.

It was used often during the initial stages of scenario and/or manoeuvre analysis to better understand the dynamics involved. By determining optimal manoeuvres for a specific scenario, insights could be drawn about the scenario and the manoeuvres. It also allowed parameter studies to be performed by running large numbers of optimal control simulations quickly with varying scenario parameters and analysing the collected results to draw conclusions about the relationship between different parameters and the relevant performance objectives. Lastly, it was also used to establish a benchmark which was then used to evaluate the performance of different control strategies.

While numerical optimal control is computationally efficient for analysis purposes, it is still too slow to be used in closed-loop or real-time controller applications. Furthermore, performance - time required to find the optimal solution - is highly sensitive to the initial guess used. Lastly, convergence to the optimal solution is not guaranteed and depends heavily on the initial guess and the constraints.

4.4.2 Analytical optimal control

In this work, analytical optimal control is used for preliminary manoeuvre and scenario analysis using a particle model and also for generating the optimal force angle reference for the MHA in Paper C.

While analytical optimal control is limited in the complexity of the problems that it can solve, analytical optimal solutions can enable detailed and thorough analysis of the manoeuvre. Parameter analyses can be easily performed by simply varying the parameters in the solution itself. Determination of the numerical solution itself is very easy once the analytical solution has been derived since it involves simply evaluating an expression. And due to the very low computational requirements of determining the numerical solutions, they can now be determined on-line and hence can be used in closed-loop controllers.

The optimal control framework that is used in this work is described below and is defined as the minimisation of an objective of the form:

\[
J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t) \, dt
\]  

subject to system dynamics:

\[
\dot{x} = f(x(t), u(t), t)
\]
and final state constraint:

$$p(x(t_f), t_f) = 0$$  \hspace{1cm} (4.50)$$

with given initial conditions and final state $x(t_f)$ and time $t_f$ being free.

Based on this, the augmented objective (including the constraints) to be minimised can be written as:

$$\hat{J} = h(x(t_f), t_f) + \eta^T p(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t) + \lambda^T (f(x(t), u(t), t) - \dot{x}) dt$$  \hspace{1cm} (4.51)$$

We now define the Hamiltonian as:

$$H = g(x(t), u(t), t) + \lambda^T f(x(t), u(t), t)$$  \hspace{1cm} (4.52)$$

Finding the solution to the optimal control problem now reduces to simply finding a stationary point of the augmented objective $\hat{J}$. Hence, taking the derivative of the $\hat{J}$ with respect to all the variables and setting them to zero gives the solution as follows:

$$\frac{\partial \hat{J}}{\partial \lambda} = f(x, u, t) - \dot{x} = 0$$  \hspace{1cm} (4.53)$$

$$\frac{\partial \hat{J}}{\partial x} = \frac{\partial H}{\partial x} + \dot{\lambda}^T = 0$$  \hspace{1cm} (4.54)$$

$$\frac{\partial \hat{J}}{\partial u} = \frac{\partial H}{\partial u} = 0$$  \hspace{1cm} (4.55)$$

$$\frac{\partial \hat{J}}{\partial x(t_f)} = \left( \frac{\partial h}{\partial x} + \eta^T \frac{\partial p}{\partial x} - \lambda^T \right)_{t_f} = 0$$  \hspace{1cm} (4.56)$$

$$\frac{\partial \hat{J}}{\partial t_f} = \left( \frac{\partial h}{\partial t_f} + H + \eta^T \frac{\partial p}{\partial t_f} \right)_{t_f} = 0$$  \hspace{1cm} (4.57)$$

For details regarding the application of this method to the intersection accident scenario, please refer to Paper C.
5 Rear-end collisions: The low hanging fruit

With regards to being able to use electrified drivetrains for active safety interventions, the rear-end collision scenario is one of the simplest and yet most promising accident scenarios. This chapter describes this scenario (same as the one outlined in section 3.1) and the benefits that can be expected from a speed control intervention in this scenario.

The rear-end collision is one of the most common accident types that occur in the world, accounting for 29.7% of all accidents in the US in the year 2000. In the same year, approximately 2.2% of all licensed drivers in the US were involved in rear-end collisions and of those drivers involved in all types of crashes, 36% were involved in rear-end collisions alone [52]. Similarly, they accounted for 35% of all traffic fatalities and injuries in Japan in 2005 [53], 24% of all accidents in Germany [54] and 26% of all motor crashes resulting in insurance claims in the UK [55].

Due to the high incidence of these accidents, over the years, there has been a lot of effort to try and improve safety in this scenario. One of the outcomes of this is the Automatic Emergency Braking (AEB) system that is now available on the market. This system is fitted on the following vehicle and applies the brakes when it detects that a collision with a lead vehicle or obstacle is imminent. Several studies have been done investigating the effectiveness of this system and one such study which used real world crash data in its analysis found that up to 35% of all rear-end collisions could be avoided completely and 53% could be mitigated in severity using AEB [56].

However, given that rear-end collisions are one of most frequently occurring accidents, despite the high effectiveness of AEB, the remaining accidents that are not mitigated or prevented by AEB still account for a large number of accidents. These accidents could potentially be improved by a speed control intervention that accelerates the lead vehicle when a collision becomes imminent.

Analysis of accident statistics pertaining to rear-end collisions shows that electric drives are extremely well suited for an intervention in this scenario. In [57], the authors find that approximately 70% of rear-end collisions involve an impact speed of less than 30 km/h. Less than 15 km/h speed difference is seen in more than 70% of the cases according to [58]. Between
70-90% of rear-end collisions involve stationary lead vehicles [59, 60]. In summary, accident data shows that a majority of rear-end collisions involve low lead vehicle speeds and since electric drives deliver their peak torques at low speeds, this makes them suitable in this scenario. Furthermore, the small relative speed in most cases means that only a small speed increase is required in the lead vehicle which makes it easier to achieve and also less risky as an intervention.

Safety benefit can be expected from acceleration not only due to the reduced relative speed at impact, but also since, by moving the lead vehicle forward, it provides more distance for the following vehicle to brake. Furthermore, since electric vehicles can deliver their torques very quickly and can briefly supply torques several times that of their rated values, the resulting acceleration and jerk can be used to adjust the posture of the occupants’ heads to reduce whiplash injury risk.

These concepts and their expected safety benefit in the rear-end collision scenario are explained in more detail in Paper B.
6 Obstacle avoidance with oncoming traffic

This chapter describes the obstacle avoidance with oncoming traffic scenario (similar to the one outlined in section 3.2), how to use the electrified drivetrains to perform safety related interventions in this scenario and the benefit that can be expected from the same.

![Illustration of an obstacle avoidance with oncoming traffic scenario](image)

As shown in fig. 6.1, this scenario involves significantly coupled dynamics and hence both longitudinal as well as yaw dynamics need to be controlled.

6.1 Understanding the scenario and manoeuvre kinematics

Since this scenario requires relatively more complex interventions, it is important to first understand the dynamics of the manoeuvre involved and how the different manoeuvre parameters affect the interventions required. This is done in [7] where the parameters that characterise the manoeuvre with respect to the potential safety benefit that can be expected from electrified drivetrains are identified. Next, using the identified parameters, more detailed investigation is done to estimate the safety benefit that can be expected when electrified drivetrains are used for interventions. These investigations are done in an optimal control framework and in this initial analysis, optimal steering is assumed.

In [8], the impact of actuators with different capabilities (IC engine versus electrified drivetrains) on the distance margin achievable in the presence of restricted steering is analysed. Note that only stability control is performed here and no specific controller to increase the distance margin or reduce the risk of collision with the oncoming vehicle is used. Instead the impact of different actuators when used with a naive controller for yaw moment control on the distance margin is analysed. Specifically, the influence of electrified drivetrains’ ability to decouple yaw and longitudinal dynamics in yaw stability interventions performed during this manoeuvre is investigated.

A speed controller to reduce the risk of collision with oncoming vehicles in this scenario is first designed and presented in [9]. The distance margin improvements that can be achieved
with both IC engines and electrified drivetrains in the presence of restricted steering are investigated using high fidelity simulations in the IPG CarMaker environment.

6.2 Development of integrated controller for collision mitigation with oncoming vehicles

The knowledge gained about the accident scenario from [7–9] is refined, extended and validated through experiments in Paper D. Analysis is first done using a simplified particle model analysis and then using a large number optimal control simulations, the results of which are analysed through statistical analysis to identify parameters of interest. These findings are then validated in real-world experiments with a Volvo XC90 hybrid test vehicle. Note that no controller was used in these experiments; instead the driver was asked to follow different speed control strategies informed by the previous analysis and the resulting performance measures verified to match the hypothesis derived from the optimal control analysis.

Once the hypothesis was validated, an integrated speed controller for collision mitigation with oncoming vehicles is formulated and implemented in simulation. The same is then evaluated in a high fidelity IPG CarMaker simulation environment with a validated Volvo XC90 vehicle model. Consistent increases in distance margin were recorded with the integrated controller irrespective of the drivetrain used and larger increases were recorded with the electrified drivetrain.

6.3 Robustness of integrated controller to steering effort

As previously mentioned, this manoeuvre involves significant lateral dynamics and as a result, requires a steering intervention as well. However, drivers cannot always be guaranteed to perform this steering intervention optimally and hence Paper E investigates the potential safety benefit that can be expected when the steering intervention is restricted.

Two variants of the integrated controller are evaluated: one where the speed controller is integrated with a traditional ESC and another where the speed controller is integrated with a lateral controller that assists the driver in performing the evasive manoeuvre. Simulations were performed in the IPG CarMaker environment by varying the driver model parameters to obtain different steering profiles. Analysis of the results showed that both variants of the integrated controller improved the distance margin, with the lateral controller based integrated controller performing significantly better than the other. The improvement in robustness to steering effort resulting from the integrated controller means that the impact of low driver skill on the outcome of the manoeuvre is mitigated to some extent by the controller.

More details, results and analysis of these are presented in Paper E.

6.4 Experimental verification of integrated controller

In Paper F, a real-time implementation of the integrated controller is done and the same tested in experiments using a Volvo XC90 hybrid test vehicle. Due to practical limitations, only the
speed control part of the integrated controller was implemented while the on-board ESC was left to perform the stability control.

To be able to control the propulsion actuators on the vehicle, modifications were made to the vehicle hardware. A schematic of the modified CAN network used in the vehicle is shown in fig. 6.2.

![Figure 6.2: Schematic of the modified CAN network](image)

As shown, to be able to send torque requests to the Engine Control Module (ECM), the propulsion and the chassis CAN networks were cut close to the ECM and a Vector VN8910 system placed in between to act as a gateway. When an intervention is to be performed, torque requests from the Vehicle Domain Dynamic Module (VDDM) to the ECM are overridden and the torque request from the controller is sent instead. The VN8910 unit is also connected to an Oxford RT3000 GPS and INS system from which vehicle global position and other dynamic signals are used by the controller. Finally, the VN8910 is connected to a laptop running a Vector CANoe dashboard from where the controller can be stopped or started, parameters tuned, etc.

Figure 6.3 shows the results from one set of experiments that were run for a specific case of the scenario. The track layout used is shown in one of the panels of the figure. A host vehicle initial speed of 55 km/h, bullet vehicle speed of 90 km/h and a friction of approximately 0.8 was observed. Note that due to limitations in the torque interface, torque could be requested from either the IC engine or the electric motor at one time but not both. Additionally, a maximum torque request limit of 1500 Nm was enforced. Finally, some safety features in the engine control module that limited drivetrain torque could not be disabled which meant that the controller torque request was not always satisfied. This effect was more dominant when requesting torques from the rear axle.

For comparison, the driver was also asked to perform the manoeuvre by letting go of the accelerator pedal at the beginning of the manoeuvre (“Thr. off”) and by manually accelerating
Figure 6.3: Paths, velocity, track layout, steering wheel angle, actual and requested torques and distance margin plots for one of the cases tested. The horizontal lines at the top right portion of the path plots show the trajectories and the final positions of the bullet vehicles as the corresponding host vehicles return to the original lane. In the torque request plots, for the cases of “Ctrl. Fr” and “Ctrl. Rr”, positive and negative values indicate torque request for the engine and the motor respectively from the controller. For the other cases, the torque request is the driver requested torque from the accelerator pedal position. In the distance margin plots, × represents a failed run (hitting one or more cones).
through the manoeuvre (“Accelerate”). For the “Accelerate” case, the driver was instructed to accelerate to the extent the driver felt comfortable and confident that they could successfully complete the manoeuvre without knocking over any cones. These cases served as reference cases for comparison with the controller.

As can be seen from the figure, both versions of the controller significantly increase the distance margin over the “Thr. off” case. It can be seen that control of rear axle torque performs as well or even better than front axle torque control despite the fact that the delivered torque is cut off early by the ECM. Finally, it can be seen that with driver acceleration, the distance margin increases are even higher. The cause for this is apparent from the front and rear axle actual torque plots: torque delivered on the front axle alone is more than twice the torque magnitude requested by the controller. However, it can also be seen that in this case, there are a lot more failed runs indicating that even with a fully aware driver (no surprise factor), controlling the vehicle speed to mitigate oncoming vehicle collision risk while performing an emergency evasive manoeuvre is too much for a driver to handle.

Detailed results for the other cases that were tested, including analyses and methodology are presented in Paper F.
7 Intersection Accidents - Collision Avoidance by Crossing the Intersection Ahead of the Bullet Vehicle

Like the rear-end collision, intersection accidents are one of the most common accident types in the world accounting for nearly 40% of all accidents in the US and Europe [61, 62]. When looking only at fatal crashes, they account for approximately 25% and 20% of all traffic fatalities in the US [63] and Europe [64] respectively.

However, unlike rear-end collisions, intersection accidents consist of numerous different sub-categories characterised by factors such as type of intersection (T-junction, 4-way, Y-junction, etc.), direction from which bullet vehicle approaches, signalised vs non-signalised, etc. These factors significantly change the dynamics involved, the threat detection, decision making, etc. In order to limit the scope of work, in Paper C, a specific variant of intersection accidents called “Left Turn Across Path - Opposite Direction” (LTAP/OD) is identified based on previous research and chosen for investigation. Figure 7.1 shows an illustration of the same. The accident scenario involves a host vehicle turning left across the path of an oncoming bullet vehicle, which is at a certain lateral offset to the host.

![Figure 7.1: Illustration of an LTAP/OD intersection accident that is considered for investigation](image)

While previous research on this scenario has dealt with collision avoidance through braking, steering, coordination with other vehicles with the help of vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication, etc., no work so far has dealt with comprehensive online vehicle motion control, particularly with regards to the possibility of assisting the driver if he/she decides to avoid the collision by crossing the intersection ahead of the bullet vehicle.
In Paper C, the LTAP/OD scenario is investigated first through an analytical particle model optimal control framework to determine the optimal manoeuvre that maximises the distance to the bullet vehicle at the time of crossing. It was found that the optimal manoeuvre can be represented as a maximisation of the vehicle global force in a fixed direction. This finding was then verified with more detailed optimal control simulations with a non-linear two-track vehicle model. It was seen not only that the particle model result was valid for the two-track model, but that the optimal intervention could be simplified to maximising the individual tyre forces in the optimal direction independently.

A Modified Hamiltonian Algorithm (MHA) controller that uses the analytical result and maximises the individual tyre force in the optimal direction was then implemented in simulation for collision avoidance in the LTAP/OD scenario. The controller was then evaluated in the IPG CarMaker simulation environment using a validated Volvo XC90 vehicle model. The results showed that when the MHA is used, the distance margin to the bullet vehicle can be increased by more than 1.5 m in this specific case of the scenario. Figure 7.2 shows a snapshot of the active vehicle with the MHA (solid car) and the passive vehicle (faded car) at the moment of crossing the intersection. As can be seen, the active vehicle manages to cross the intersection well ahead of the oncoming vehicle.

![Figure 7.2: Screenshot of the host vehicle avoiding the bullet vehicle. The shaded host vehicle is the passive vehicle without an intervention. (image reproduced from Paper C)](image)

More details about the scenario, its analysis, the controller and the simulation results can be found in Paper C.
8 Discussion

In this chapter, the impact of the assumptions made on the results presented, the potential applications of the results and the methods developed in this work are detailed.

8.1 Impact of assumptions

Since the focus of this work is vehicle dynamics, to limit the scope of work, several simplifying assumptions have been made. These assumptions and their impact on the results are detailed below.

8.1.1 Sensors and information

For the purposes of this discussion, the sensors and information that is used in this work can be split into two major categories: vehicle dynamic and environmental.

In all work that are part of this project, except Paper C, availability of vehicle dynamic states and signals are assumed based on what can reasonably be assumed to be available in a production vehicle. For instance, signals like vehicle longitudinal speed, yaw rate, lateral acceleration, etc., are assumed to be available, but not lateral velocity or tyre slip angle, etc. One major exception is the tyre-road friction level which is assumed to be available in [7–9]. However, in Paper D and Paper E, even this is not assumed. Instead, a maximum driver-acceptable friction level is estimated based on the part of the evasive manoeuvre that precedes control activation and this is instead used as the available friction in the controller. Other signals like vehicle position and yaw angle are assumed to be available, however, these are only needed in relation to the road and other traffic and hence is assumed to be derivable from environmental sensors information.

In Paper C, the tyre-road friction and the vehicle lateral velocity or sideslip angle is also assumed to be available. Since the manoeuvre is on-limit, it might be possible to estimate the friction on-line. Estimation of sideslip angle, while tricky, can be done on-line as is done in several ESC systems. Moreover, since the intervention is of short duration (≈1.5 s), assuming the vehicle starts from a straight ahead position, dead-reckoning can be used to estimate the sideslip angle for the duration of the manoeuvre. While not covered in this work, previous work has covered the estimation of these from the measurable vehicle states [65]. Another major piece of data assumed to be available in Paper C is the tyre model. While this data is difficult to obtain, it is a one-time requirement that needs to be input into the controller when it is designed. For gradual changes in tyre properties due to wear, temperature, etc., it might be possible to continuously adapt parameters in the tyre model through on-line estimation of tyre properties [66]. Tyre changes are more tricky to deal with; in such cases, either the tyre model needs to be updated along with the tyre change or perhaps the controller can revert to a “fallback” tyre model when it detects that the tyre model does not match the actual tyre behaviour.

Throughout this thesis, all required environmental information from sensors or other sources have been assumed to be readily available. While this may not be true in the current generation of vehicles, due to the advent of advanced active safety, cooperative and autonomous
systems, a vast array of sensors and information sources might become available in the cars of the future. Since it is very hard to predict exactly which sensors or information will become available or the properties of that information (accuracy, reliability, etc) we make the simplifying assumption now of perfect information to establish a basis for what is possible. It would be possible later on to adjust the estimates based on the actual accuracy and reliability of information.

The results presented here regarding the potential of electrified drivetrains in various scenarios can also act as an incentive to add or enhance the fidelity of sensors or information in order to enable or achieve as much of the safety benefit as possible. The results can also be used to establish requirements on sensor and information sources for use in such safety interventions.

8.1.2 Actuator performance
Reasonable assumptions have been made regarding actuator performance in Paper B with most values pertaining to the same having been taken from other scholarly or state-of-the-art papers. In [7–9], most actuators are assumed to have optimal or high performance and this assumption is highly unfair to the electric drive since the other actuators have significantly worse performance in reality. The assumptions have been made however to ensure that the results are robust to any possible advancement in the respective technologies which may improve actuator performance in the future. Additionally, the use of idealising assumptions allows us to use the results to generate requirements on actuator performance. In Paper D and Paper E however, realistic assumptions about actuator rate and amplitude limits have been made in the controller itself based on actual observed actuator performance from experiments. In Paper C, while no actuator assumptions are made in the controller, there exist dynamics of the driveline components in the vehicle model (but not of the powertrain actuators themselves).

In the rest of the thesis, the assumptions made and their impact are mentioned where relevant. In general, improvement in actuator performance would reduce the benefit offered by electrified drivetrains over traditional ones. However, IC engine performance is unlikely to improve to an extent so as to be usable in an active safety intervention in the future. This is largely due to the downsizing trend which involves turbocharging and while this reduces emissions, it also increases their response times. Brakes on the other hand could improve in performance over time; however electric drives are still likely to be faster and have the advantage of being able to supply driving torques as well.

8.1.3 Human factors
The human factors issue has mostly not been addressed in this thesis even though it is an important part of active safety functions. While this definitely needs to be addressed in any active safety function, these are not deal-breakers by themselves. Instead they put restrictions on how the results presented in this thesis can be used.

For instance, for an Autonomous Acceleration System (AEA) presented in Paper B, a warning system similar to those used in AEB systems would be unsuitable. Since the threat is now behind the host vehicle, the new warning system would need to be designed to help direct the driver’s attention to the rear-view mirror. This can have a significant effect on the
driver's response time and change the effectiveness of the warning, but the vehicle dynamics in this scenario remain unchanged. Consequently, autonomous systems which would need little to no interaction with the driver would be unaffected, whereas driver assist systems would be affected a little and warning systems would be heavily affected by the human factors issue.

Driver interaction in the obstacle avoidance with oncoming traffic scenario has been partially investigated in the form of restricted steering. But other factors such as how a warning system should be designed, how would a driver actually respond in the presence of a warning or an acceleration intervention particularly when there is a surprise factor involved, etc., need to be investigated. Appropriate design of a warning system here could potentially have a large impact on the safety outcome in this scenario. Particular attention may also need to be paid to lateral and steering feedback control as this could be used to nudge the driver toward choosing the optimal course of action for the best scenario outcome.

Similarly, driver interaction in the intersection accidents will also need to be studied further. However, since the assumption in Paper C is that the driver intends to accelerate, the driver intention here is fixed, and the driver interaction investigation will need to be focused more on cooperative motion control of the vehicle, involving aspects such as driver comfort, steering feedback, etc.

8.2 Applications

The potential applications for the predominantly vehicle dynamic results and analysis presented in this thesis are detailed below.

8.2.1 Driver interaction

One of the major factors that affect the quality of driver interaction is the delay between the driver making a request and that request being satisfied. Due to the nearly instant response of electric drives, they offer a strong opportunity for enhancement of driver interaction. Since most current generation differential brakes have significant response times, their ability to enhance the driver interaction is limited.

Differential brakes are particularly unsuited for driver interaction enhancement in the yaw or lateral dynamics domain due to the undesirable deceleration side-effect of differential brakes and relegates them for use only in extreme situations. When coupled with an electrified drivetrain however, which can compensate for the deceleration, the two can be used effectively to enhance driver-vehicle interaction as it can be used to significantly reduce response time as shown in section 2.2 and also to improve safety.

The possibility of controlling or influencing the driver vehicle interaction opens up new possibilities with regards to guiding the driver toward safer behaviour when necessary. The same can be used during handover situations - for instance, when an autonomous function hands over control of the vehicle to the driver, it might be necessary to control the driver interaction to let the driver gradually get back in control of the vehicle.
8.2.2 Warning systems

New driver warning systems can be envisioned which use the results presented in this work to estimate when the vehicle approaches a point beyond which the actuator set available in the vehicle would be unable to help, and use that to issue warnings and adjust their timings.

For instance, for forward collision warning at high speeds, typically the system needs to wait until evasive steering is no longer a viable option for collision avoidance before a warning is issued. Such systems typically do not account for the possibility that there may be an oncoming vehicle in the adjacent lane which would limit the possibility of performing evasive steering. However, if an oncoming vehicle were to be detected, using the results presented in Paper D and Paper E regarding the manoeuvre kinematics in the obstacle avoidance with oncoming traffic scenario, the risk of collision with the oncoming vehicle can be estimated. Using this estimate, decisions can then be made regarding the viability of an evasive steering manoeuvre. If it can be determined that there is a high risk of collision with an oncoming vehicle if the host vehicle moves to the adjacent lane, there would no longer be any need to wait for evasive steering to become unviable anymore and the warning can be given earlier.

Similarly, for intersection collisions, the possibility of acceleration to avoid a collision can be considered in the timing of warning systems or assist systems which can be used to warn the driver against performing potentially infeasible interventions.

8.2.3 Assistance systems

The same factors mentioned in section 8.2.2 can be used for assistance systems as well since most assistance interventions are preceded by a warning phase. The driver interaction aspects mentioned in section 8.2.1 can also be used in the assistance phase to enhance the effectiveness of the intervention. Additionally, estimates regarding collision risk can be used to determine the extent and type of assistance to be delivered and also to determine if an intervention by the driver is in fact a collision avoidance intervention and how the intervention needs to be supported.

For instance, in the obstacle avoidance with oncoming traffic scenario, an estimate of the risk of collision with an oncoming vehicle can be used to determine whether to assist the driver in overtaking the obstacle by maintaining or increasing speed (if demanded by the driver) or to mitigate a possible collision with an oncoming vehicle by reducing speed.

For the intersection accident scenario in Paper C, since the investigation is done using optimal control, the resulting manoeuvres represent the physical limit case and hence can potentially serve as motivation to override the driver in some cases. Furthermore, since the assumption in Paper C is that of driver assistance to begin with, the results and methods presented there are wholly suitable for designing assistance systems.

8.2.4 Autonomous systems

Once again, the applications mentioned in sections 8.2.2 and 8.2.3 can be applied for autonomous active safety systems as well. The applications for this research have been largely captured in chapter 3, [42] and the appended papers. However, the intervention opportunities identified in section 2.3 can still be used in other accident scenarios as necessary to improve
safety. Based on the improved potential of electrified drivetrains as shown, further novel intervention opportunities can be envisioned for use in safety scenarios.

### 8.2.5 Cooperative systems

Although electrified drivetrains expand the dynamic capabilities of the vehicle, the very same factor could make it difficult to implement active safety systems since these now have to account for the increased opportunities that are available not only to the host vehicle but also to the bullet vehicle and other traffic participants. With cooperative systems however, such concerns could be laid to rest since the vehicles would then be able to exchange relevant information and synchronise their interventions to maximise safety. Cooperative systems also mitigate the issue of sensor and information quality and availability that is mentioned in section 8.1.1 and reduce the requirements on sensors.

### 8.3 Utility of methods

In a broader context, while the presented results prove that electric drives can be used to implement enhanced safety functions, the presented methods themselves are more general in utility. The methods can be used to develop functions not only for electrified vehicles, but also for vehicles with traditional powertrains and for scenarios other than those that have been presented in this thesis.

Numerical optimal control has been used extensively to study accident scenarios and evaluate potential manoeuvres for collision avoidance in Paper C, Paper D and Paper E. Results from the same have been used to gain deeper understanding of the dynamics involved in the manoeuvres and used to design controllers for driver assistance. Similar methods can be applied to other scenarios in order to design and evaluate optimal manoeuvres which can then be used as templates to design driver assistance functions in those scenarios.

Using the understanding of the manoeuvre dynamics gained from optimal control, in Paper D, a longitudinal acceleration controller is formulated. An evaluation of the proposed controller’s performance showed that even with a point mass model with simplified dynamics being used as the basis for controller development, performance close to that of optimal control can be achieved by carefully choosing assumptions and application of insights gained from manoeuvre analysis. Such a method could potentially be used to design driver assistance functions for a wide range of scenarios using simplified models and carefully applying knowledge of the manoeuvre dynamics.

While control allocation is a commonly used method, in Paper D and Paper E it has been used to arbitrate between longitudinal (longitudinal acceleration control) and yaw (ESC) dynamics to increase distance margins without loss of stability. This goes against the industry standard practise of keeping the ESC as a low level controller that can override most other functions. Paper D and Paper E show that control allocation methods can be used in other use cases as well to arbitrate between different functions to achieve a better safety outcome instead of simply letting the ESC override all other functions.

In Paper C, previously presented analytical optimal control methods have been extended to analyse the LTAP/OD scenario which shows the versatility and potential of such methods.
It has also been used to generate a force angle reference for use in a lower level controller. The efficiency of such method allows for on-line generation of optimal reference trajectories that can be used to perform driver assistance. Additionally, a relatively novel control allocation method called Modified Hamiltonian Algorithm (MHA) is used which further proves the capabilities of the MHA as a versatile and powerful vehicle dynamic focused control allocation scheme. The combination of particle model optimal control for reference generation and the MHA for low level control is a proven strategy that can be used to perform autonomous or driver assist interventions in other scenarios as well.

The presented results, in general, hopefully not only proves that electrification can be used for improved safety, but can also be used to implement other enhanced functions. For the most part, electric drives have been used as drop-in replacements for IC engines until now without significantly exploiting their enhanced capabilities. But now that their capability as a powerful actuator for active safety functions have been shown, hopefully, their capabilities will be exploited to implement other novel functions as well.
9 Conclusions and future work

In this chapter, the salient conclusions drawn from this research work are presented, followed by potential avenues for future work on this topic.

9.1 Conclusions

Adding customer value to electrified vehicles through enhanced or novel active safety functions that cannot be achieved with traditional IC engines could be a major way not only to make electrified vehicles more attractive to consumers, but also to governments and regulatory agencies trying to reduce traffic accidents. However, there are many open questions around the possibility, feasibility and the extent to which such functionalities can be achieved. This work aims to answer some of these questions.

The question regarding the possibility of using electric drives for enhanced or novel functions has been answered by showing the advantages offered by electrified drivetrains over IC engines in terms of expanded vehicle dynamic capabilities (Chapter 2) and how they can be used for novel or improved interventions for safety (Chapter 3). More advantages offered by electric drives, how they translate to advantages at higher levels and how they can be used in different accident scenarios are listed in Paper A and also in [42]. The listed advantages and use cases are not by means exhaustive and more use cases may be discovered, particularly as the way vehicles are used changes with the advent of autonomous and cooperative systems.

Questions regarding the feasibility and extent to which enhanced functions would be beneficial are more appropriately answered on a case-by-case basis. To this end, three accident scenarios, namely the rear-end collision, the obstacle avoidance with oncoming traffic and the intersection accident scenario have been investigated in detail and the safety potential that can be expected with electrified drivetrains (in vehicle dynamic terms) in each of these scenarios have been quantified.

In Paper B, the rear-end collision, one of the most common types of accidents, is analysed in detail. An analysis of the accident statistics shows that rear-end collisions predominantly involve stationary or low lead vehicle speeds, low impact speeds and small speed differences which make electric drives well suited for interventions here, since they deliver their peak torque at low speeds. A decision making scheme to determine when to accelerate was then formulated based on similar principles as those used for Automatic Emergency Braking (AEB). Simulations with simplified models using this decision making scheme showed that acceleration alone could reduce the impact speeds by up to 15 km/h and when combined with braking on the following vehicle, impact speed reductions of up to 75 km/h could be achieved. This large speed reduction in the combined case is achieved due to the fact that lead vehicle acceleration not only decreases the relative speed but also provides increased braking distance for the following vehicle. Based on this knowledge, another intervention was designed which involved displacing the lead vehicle forward and coming back to rest at the end of intervention. Evaluation of this intervention showed that speed reduction improvements of up to 20 km/h could be achieved.

In Paper C, an analysis of the intersection accident scenario, specifically, the “Left Turn Across Path - Opposite Direction” scenario, is presented. The possibility of assisting the driver
in collision avoidance by crossing the intersection ahead of the oncoming vehicle is analysed. Optimal manoeuvres for collision avoidance by crossing the intersection are determined through an analytical particle model optimal control framework. It was seen that the optimal manoeuvres could be represented as a maximisation of the vehicle global force in a fixed direction. Verification with a two-track non-linear vehicle model found that the optimal manoeuvre can be simplified as a maximisation of the individual tyre forces in a fixed global direction. Based on these findings, an integrated motion controller is implemented and tested in a high fidelity simulation environment. Simulation results show that the driver can be assisted effectively to avoid the collision with as much as 1.5 m more distance margin when compared to the passive vehicle even in an on-limit case, i.e., the distance margin improvement is achieved mainly through optimisation of the direction of the individual tyre forces and not by increasing the magnitude of the forces themselves. Even more improvement in distance margin can be expected in non-limit scenarios where the tyre force magnitudes can be increased by the controller.

Paper D and Paper E deal with the obstacle avoidance with oncoming traffic scenario in detail. Paper D starts with an analytical study of the accident scenario to identify the important parameters that characterise the manoeuvre. These findings were then verified through a large number of optimal control simulations, the results of which were analysed with statistical tools. The finding was also verified through open-loop driver-controlled experiments with a Volvo XC90 vehicle. Based on the findings, an integrated controller to assist the driver in collision mitigation with oncoming vehicles while performing evasive manoeuvres is formulated and presented. The integrated controller is then evaluated in a few selected variants of the obstacle avoidance with oncoming traffic scenario and it was seen that distance margin improvements of up to ≈4 m could be achieved.

In Paper E, two variants of the integrated controller are considered for evaluation: one where the speed controller is integrated with a traditional ESC and another where it is integrated with a lateral controller designed to assist the driver in performing the evasive manoeuvre. The controllers are evaluated in select variants of the scenario in the presence of restricted steering. Simulations in a high-fidelity environment showed that both controllers increase the robustness with respect to steering effort over a traditional ESC-only control strategy. Specifically, the variant with the lateral controller performed noticeably better not only in increasing the distance margin but also in the distance it takes to perform the avoidance manoeuvres.

Finally, in Paper F, a real-time implementation of the integrated controller has been done and tested in experiments with a Volvo XC90 test vehicle. For comparison, several runs with the driver adopting different speed control strategies were also performed. The results show that the integrated controller consistently increased the distance margin compared to a reference case where the driver lifted off the accelerator pedal at the beginning of the manoeuvre (throttle off). And while the distance margin improvements were greater when the driver accelerated manually, it was seen that they also resulted in a lot more failed runs where the driver ran over one or more cones that marked out the track. This indicates that the task of speed control is difficult to perform in such an emergency manoeuvre despite the lack of a “surprise” factor which a real driver would likely face.

In general, the results from the different analyses show that electrified drivetrains offer a strong opportunity to improve safety in these scenarios. The results also highlight the
importance of being able to control the speed or at least not affect it (if not demanded by the driver) in safety critical scenarios. Another feature highlighted in the results is the importance of being able to decouple yaw and longitudinal control interventions. When yaw moment interventions can be done without affecting the longitudinal dynamics, not only can it be used to improve vehicle response and stability in critical scenarios, it can also be used for steering redundancy.

In summary, several vehicle dynamic opportunities for improving safety using electrified drivetrains were identified. Detailed investigations of select cases showed that significant safety benefit potentially stands to be gained by appropriate control of electrified drivetrains in the accident scenarios. Consequently, a strong opportunity is seen for adding safety related value to electrified vehicles at little to no extra cost.

9.2 Future work

Before the results can be used in production vehicles however, several vehicle dynamic and non vehicle-dynamic aspects need to be investigated and questions answered.

The human factors aspect, i.e., how would the drivers of the host and bullet vehicles react to acceleration of the host vehicle, needs to be considered. For instance, a sudden unexpected acceleration could cause the driver to panic and brake hard or steer away toward some other threat. To avoid such outcomes, appropriate warning systems that provide the driver with relevant and timely information need to be designed and implemented taking into account driver behaviour and HMI design considerations.

The interaction between the safety functions on the host and the bullet vehicles or other traffic participants also needs to be investigated. For instance, in the rear-end accident scenario, it is assumed that the following vehicle either performs a braking intervention until the vehicle halts or at least that there exists a dead-band of intervention activation so that when the lead vehicle accelerates, the braking intervention on the following vehicle is not terminated. However, it remains to be investigated how this interaction can be optimised or at least to ensure that the interaction does not result in a worse outcome. Similarly, in both the obstacle avoidance and intersection accident scenarios, investigation needs to be done to determine if and what safety functions are triggered in the bullet vehicle. For instance, in the obstacle avoidance scenario, automatic emergency braking (AEB) could, in principle, be triggered by acceleration of the host vehicle. The triggering thresholds for such safety functions need to be considered so as to prevent such interventions in the host vehicle.

The decision making and the interpretation of driver input (is the driver trying to make an avoidance manoeuvre or just performing an ordinary lane change?) is another important aspect which will need to be solved. Decision making for the rear-end collision scenario is covered and discussed in Paper B. However, for the obstacle avoidance in the presence of oncoming traffic and the intersection accident scenarios, the task of determining if and when what intervention to perform is yet to be solved. The controllers presented in Paper C and Paper D serve to improve the situation once the intervention is started but do not decide if or when an intervention is to be initiated. One of the factors that needs to be considered while performing decision making is driver intent. However, the interpretation of driver intention, particularly in an emergency situation when the driver might be panicked, is still an open
question.

While the work presented here has quantified the benefit that can be expected in a vehicle dynamic sense, it needs to be translated to an actual safety benefit, i.e., what percentage of a certain accident type can the safety function avoid, how big a reduction in severe injuries or fatalities can the function achieve, etc? For this purpose, accident epidemiology studies need to be done using various accident databases to quantify or translate the benefit quantified in this work to a more relevant safety benefit that can be used to motivate implementation of these functions.

From a vehicle dynamics point of view on the other hand, several opportunities exist for future work.

In the obstacle avoidance with oncoming traffic scenario, the robustness of interventions in the presence of moving obstacles or accelerating or braking bullet vehicles needs to be analysed. Additionally, the integrated controller can be extended to consider such factors as moving obstacles or accelerating bullet vehicles. The benefit that can be expected with different actuator limitations (motor power, torque, front-wheel drive vs. all wheel drive, etc) under different scenario conditions (low friction, high vehicle speed, etc.) needs to be quantified which in turn can be used to derive actuator requirements.

A robustness and sensitivity analysis of the controllers needs to be performed with respect to the accuracy of the data that they use. For instance, the robustness of the controller for the intersection accident to inaccurate tyre data needs to be investigated and quantified. The on-line estimation of different vehicle states and parameters that are used by the controllers is another opportunity for future work.

While a real-time implementation of the integrated controller for collision mitigation with oncoming vehicles while performing evasive manoeuvres has been tested in Paper F, a more comprehensive validation still needs to be performed where the both the propulsion actuators and the brakes can be controlled simultaneously. Experimental validation of the collision avoidance controller for intersection accidents still needs to be performed.
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