Wind Turbine Drive Train System Dynamics
Multibody Dynamic Modelling and Global Sensitivity Analysis

SAEED ASADI

Department of Mechanics and Maritime Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
Göteborg, Sweden 2018
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Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr. 4428
ISSN 0346-718X
Department of Mechanics and Maritime Sciences
Chalmers University of Technology
SE-412 96 Göteborg
Sweden
Telephone: +46 (0)31-772 1000

Cover:
Wind turbine drive train system model [1]

Chalmers Reproservice
Göteborg, Sweden 2018
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Thesis for the degree of Doctor of Philosophy in Solid and Structural Mechanics
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ABSTRACT

To facilitate the design and production of highly efficient and reliable wind turbine drive trains, this thesis deals with the mathematical modelling and analysis of drive train system dynamics. The drive train is considered as the subsystem of the wind turbine that transfers mechanical power from the rotor hub to the generator, and thereby plays an important role in the system dynamics and the efficiency of wind turbine operation.

The dynamics of wind turbines is complex and a critical area of study for the wind industry. The multidisciplinary nature of wind turbine design adds to the complexity of this task, as the subsystems of a wind turbine need to be tuned with respect to a common objective to achieve a cost effective, reliable and optimum structural and dynamic performance.

The overall performance of a drive train can be evaluated from different perspectives. In this thesis, mathematical model of drive train wind turbine for both direct and indirect drive train has been developed based on multibody dynamic modelling formalism. Afterwards, the dynamics behaviour of the drive train is evaluated by proposed objective functions referring to displacements, loads, fatigue damage indicators, and frequency responses. These objective functions are investigated for several wind operational scenarios such as normal operation, turbulent, vertical inclination cases.

The work also contributes to enhanced knowledge in the field with focus on the interaction between functional components and system dynamic response, faults modelling and detectability of defects in functional components such as bearings, and couplings in wind turbine drive trains. To have a better insight into wind turbine dynamics, the global sensitivity analysis (GSA) of the objective functions with respect to input structural parameters is considered. By introducing defects in functional components and investigating sensitivity indices, detectability of faults is proved. GSA also demonstrates the most influential input parameters to the output objective functions. The results of such analysis not only can narrow down the number of input variables for design problems, but also give understanding on which structural parameters are most important to have precise data for, ultimately designing more efficient drive trains in terms of cost and durability.

Keywords:
Wind turbine drive train dynamics, Model validation, Multibody dynamics system modelling, Floating reference frame, Global sensitivity analysis, Bearing defects detection, Gearbox modelling, Kinematic constraints
To my love, Parisa

Every warrior of the light had doubted that he is a warrior of the light. Every warrior of the light has failed in his spiritual obligation....... However, they endured all of this without loosing the hope to improve and THAT is why he or she are Warrior of the Light,

Paulo Coelho.
Preface

The work presented in this thesis has been performed from March 2013 to April 2018 at the Division of Dynamics at Chalmers University of Technology. This project is financed through the Swedish Wind Power Technology Center (SWPTC). SWPTC is a research center for design of wind turbines. The purpose of the centre is to support Swedish industry with knowledge of design techniques as well as maintenance in the field of wind power. The Centre is funded by the Swedish Energy Agency, Chalmers University of Technology as well as academic and industrial partners.

The main supervisor of this project is Docent Håkan Johansson, and I would like to acknowledge him for all the support and encouragement and for his guidance, kindness, fruitful discussions, positive energy he has provided during these years. He has been one of my best teachers in my life, and I have learned a lot from him. I would like to express my deepest gratitude to the project leader, Professor Viktor Berbyuk, for his guidance and encouragement, and decisive actions throughout the project. I would like to thank Jan Möller for his contribution in the laboratory. I also would like to thank Olle Bankeström, Anders Wickström, reference group members in the first phase of this project, for useful comments, cooperation and discussions they provided. I would like to thank Mikael Öhman for his assistance in C3SE center for cluster simulations, and his fruitful discussions during the thesis. I would also like to thank my colleagues for the nice working environment.

Last but not least, I would like to thank my mother and father and brother for their love and support. I also thank love of my life, Parisa, for her love, passion, enthusiasm, and kindness during our journey throughout these years. Finally, I’d like to say thank to my in-laws for their love and gentle kindness.

Gothenburg, May 2018

Saeed Asadi
This thesis consists of an extended summary and the following appended papers:

**Paper A**

**Paper B**
Asadi S., Johansson H. “Multibody dynamic modelling of a direct wind turbine drive train”, *Submitted to Wind Engineering*

**Paper C**
Asadi S., Johansson H. “Global sensitivity analysis of a direct wind turbine drive train”, *Submitted to Wind Energy*

**Paper D**
Asadi S., Johansson H. “Multibody dynamic modelling of a wind turbine indirect drive train with focus on gearbox modelling and motion”, *In preparation to be submitted to Wind Energy*

The appended papers were prepared in collaboration with the co-author. The author of this thesis was responsible for the major progress of the work in preparing the papers, i.e. took part in planning the papers, took part in developing the theory, performed all implementations and numerical calculations, and took part in writing the papers.
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Part I
Extended Summary

The outline of the thesis is detailed as follows: first a brief overview concerning the background and aims of the research. Then, the analysis tools used are introduced: multibody modelling of direct as well as indirect drive train (with kinematic constraints modelling the gearbox components), global sensitivity analysis, and high speed shaft test rig. The part ends with summary of the appended papers, conclusions and outlook to future research.

1 Background and Motivation

The need for alternatives to fossil fuels for electricity production, due to their finite supply and their negative effects on the environment, has driven the fast developments in the wind industry in recent years. The cost efficiency of a wind turbine can be scrutinized from different points of view. The maintenance cost and down time due to faults and failures occurring in drive train functional components (bearings, gearbox and coupling) are some of the most significant challenges [2, 3, 4, 5, 6]. In particular, wind turbine gearboxes have frequently required repair or replacement considerably shorter than the expected designed life of 20 years [7]. Nowadays the engineering challenge around drive train design for wind turbines is not only to enhance system reliability, but also to reduce the turbine top mass. These requirements together with the trend of up-scaling affect many system characteristics and parameters.

To further improve the cost efficiency of wind turbines, advanced engineering analysis in terms of reliability, diagnosis of faults, and design of larger turbines is in demand [8, 9, 10].

1.1 Wind Turbine Drive Train

The power output of a wind turbine varies with respect to wind speed and every wind turbine has a specific characteristic power performance curve. In Fig. 1.1, the power curve for a variable-speed 2 MW wind turbine is illustrated. With this curve it is possible to predict the maximum energy production, and it is therefore often used to compare different turbines. Typically, the turbine is operated with variable speed control up to the so-called rated wind speed (usually 10 - 12 m/s), where the power output and rotational speed is regulated by blade pitching.
Correspondingly, in Fig. 1.2 the rotor hub force and moment components as function of mean wind speed are illustrated. As shown in Fig. 1.2 (left), the thrust load is at its maximum around the rated wind speed, then it decreases for higher wind speeds due to the pitch control system. Also the rotor torque is illustrated in Fig. 1.2 (right), where it increases with wind speed, while held constant above rated wind speed.

The rotor torque is transferred to the generator through the power train. The wind turbine drive train system is here considered as the electromechanical subsystem comprising shafts, bearings, gearbox, shaft couplings, mounts, and other functional components (Fig. 1.3), that transfers mechanical power from the rotor hub to the electric power generator, and thereby plays an important role in a wind turbine dynamics [11, 12].

The main shaft is normally mounted on the bedplate by two main (spherical) bearings. Sometimes, the rear main bearing is integrated in the gearbox, so called ”3-point suspension”
design. The gearbox has a fixed gear ratio around 100 : 1 between the rotation of the low speed shaft (approximately 15 rpm) connected to the rotor, and high speed shaft (1500 rpm), connected to the generator, for a typical multi-MW wind turbine.

The design of a wind turbine gearbox is challenging due to the loading and environmental gearbox operating conditions. The rotor torque generates large moments and forces to the wind turbine drive train. It is crucial to ensure that the gearbox is designed to support the rotor torque, and is effectively isolated from the other hub force components. Otherwise internal gearbox components can become severely misaligned. This can lead to stress concentrations and failures.

In parallel, so-called ”direct drive” design without a gearbox is getting increased attention as shown in Fig. 1.3. In direct drive wind turbines, the rotor is attached to a main shaft supported by two bearings. This structure simplifies the nacelle system, increase reliability and efficiency by avoiding gearbox issues. However, the direct drive requires a much more complex generator (since low speed typically involves many poles) and power electronics (full invertor). A general trend towards direct drive systems has been evident in recent years, although, direct drive trains have not been yet the dominant market demand.

1.2 Failures in Functional Components of Drive Trains

As pointed out in Sec. 1, premature failures in wind turbines components’ (gearbox, bearings, etc) have influenced the financial payback, and is of major concern for the industry. For example, high speed bearing fails, gear teeth damages due to gear misalignment as well as bearing damages, predominantly on high speed shaft in terms of so-called White Etch Cracks (WEC), axial cracks in bearing raceways have become a major cause of premature gearbox failures recently [13]. According to the National Renewable Energy Laboratory (NREL) report [14], wind turbine drive train failure models are classified as follows: bending fatigue (due to inadequate material cleanliness or complete hardening on the tooth root), contact fatigue, rolling contact fatigue (RCF), wear (due to the tearing of asperities which can be alleviated by sufficient lubrication on tooth surfaces), grinding

---

**Figure 1.3:** Example of a indirect (left) and direct (right) drive concept, adopted from [1]
and case-core separation cracks (caused by improper heat treatment of gear materials), scuffing (when lubricant dries out unexpectedly). Contact fatigue life is assumed as drastic source of damage cause [15], and estimated by the sum of total number of load cycles required for the crack initiation and that required for the crack to propagate to the surface.

The gear tooth contact fatigue is caused by either surface or subsurface initiated cracking, caused by overheating of tooth surfaces due to insufficient lubrication [16]. The subsurface crack is initiated for properly lubricated gears in most cases. In [17, 18, 19], detailed information about the contact fatigue failures of gears, life estimation is given. The most common cause of gearbox failure is surface contact fatigue ([20] describes rolling contact fatigue, sliding-rolling contact fatigue and spalling). A description of four stages of fatigue damage from initial crack to failure is given in [21], which also provides an overview of the topic from a historical perspective together with suggestions for the concept of fatigue damage control. The crack initiation process can be modelled by the multi-axial high cycle fatigue criteria [22], which is influenced by contact stress and material fatigue parameters. To estimate the contact fatigue life of wind turbine drive train gear teeth, the maximum shear stress needs to be predicted accurately under various dynamic load condition [16]. For RCF, since a gear tooth experiences severe cyclic rolling and sliding contact as consequence of stochastic wind loads, fatigue prediction is not straightforward, thus contact fatigue becomes one of the major causes of unintended gearbox failure that prevents wind turbines from achieving the expected service life [23].

An accurate life estimation of functional components in drive trains is crucial for reliable operation of wind turbines. Among the failures of wind turbine functional components, the failure rate of gearboxes is higher [24, 25], therefore requires more studies and research to understand the cause of failures and possibly prevent them.

The effect of load level of wind turbines is demonstrated and occurring problems are discussed in [26]. Load simulations for multi-MW wind turbines usually are carried out for different design purposes. Often the mechanical loads on the wind turbine components caused by external forces are obtained from a specific commercial software such as Bladed [27] and Flex 4 [28], ViDyn [29], HAWC2 [30], that are based on multibody dynamics technique. In [31], since the focus lies mainly on the overall rotor dynamics and behaviour of the wind turbine, a detailed aerodynamic model with a simplified two-lumped-masses drive train model is used to present the dynamic behaviour of the drive train. This simplification is not sufficient to reveal the dynamic properties and internal loads of the complex drive trains with capacities up to 5 MW due to the large deflection and rotation of the wind turbine system.

Wind turbine drive trains undergo severe transient loading during start-ups, shutdowns, emergency brakes, and during grid connections. Load cases that result in torque reversals may be particularly damaging to bearings, as rollers may be skidding during the sudden relocation of the loaded zone.

There are however, some problems in the wind industry regarding to the wind loads: Wind loads are captured in wind farm level basis, not turbine by turbine basis. Also, they are used in the design stage, not during operation. Finally, Supervisory Control And Data Acquisition (SCADA) is not fully utilized, due to large amount of data produced.

Moreover, it is essential to develop effective condition monitoring techniques for
wind turbines [32, 33, 34], to provide data regarding the past and current conditions of the turbines, and to enable the optimal scheduling of maintenance tasks [35]. For the mechanical transmission system, monitoring and analysis of the vibration signals have been proven very effective, as it is easy to obtain the fault signature of a specific component in the frequency or time-frequency domains. However, it is difficult to obtain accurate vibration signals in the wind turbine under varying speed operation. A predictive monitoring scheme of wind turbines, allowing an early detection of faults, becomes essential to reduce maintenance costs and ensure continuity of production.

Since some faults in wind turbine drive trains are not yet well understood, there is a demand for experimental study to have better insight into faults sources. NREL (National Renewable Energy Laboratory) initiated the Gearbox Reliability Collaborative (GRC), that comprises analysis, modelling, condition monitoring, and development of a failure database to determine why wind turbine gearboxes do not always achieve their expected design life of 20 years [36]. The gear manufacturer ZF designed a test rig capable to 13.2 MW wind turbine drive train and developed a dynamic model for it using Simpack [37]. The work presents an experimental and simulation investigation into the dynamic response of modern wind turbines. However, little research has been conducted on drive train dynamics to prevent resonance in wind turbines according to rigid-flexible coupling dynamic theory and with experimental verification.

To prevent these failures, appropriate materials and careful surface treatments should be used. Gear shaft misalignment, which can be classified into a parallel misalignment and angular misalignment due to the manufacturing assembly error and/or the drive shaft deflection caused by an overhanging load of the wind turbine rotor blades, alters gear tooth contact stress distribution and can have significant impact on the gear contact fatigue damage [38, 39, 40].

In [41] an indirect torque control (ITC) technique has been investigated and the idea is that electromagnetic torque transients caused by grid faults and disturbances lead to severe gearbox fatigue. Fatigue loads for rotor and main gearbox bearings are calculated in [42] using a method that couples nonlinear Finite Element Method (FEM) and super element technique with a multibody approach [43]. An approach to predicting wind turbine gearbox flexibility for three generic gearbox configurations, based on assumptions of estimated failure rates, is given in [44]. A typical procedure for fatigue load data analysis for wind turbine gearboxes is given in [45]. Simulation of deterministic single domain could lead to an unrealistic load estimation due to stochastic nature of wind loads, and consequently an unrealistic value of the gear tooth fatigue life. In [46], a probabilistic pitting fatigue life prediction procedure is proposed, where the wind load uncertainty is defined solely by the probability density function (PDF) of the mean wind speed using the generalized gamma function and the random maximum contact pressure, obtained using the probabilistic analysis of the multibody drive train dynamics. The resonance phenomenon, when frequency of the external dynamic excitation approaches the natural frequencies of the structure, the system leads to failure [47, 48]. This is why the evaluation of natural frequencies and dynamic responses is one of the critical task in the development of high durable wind turbines [49].
1.3 Research Objectives

The topic of the present thesis is the study of wind turbine drive train system dynamics, with the aim to suggest tools for its analysis and to assess how operating conditions can contribute to failures in drive train components. A particular aim is to investigate how and in what ways variance-based global sensitivity analysis can be used in this context. To this end, the present thesis focus on the numerical modelling of the drive train and its analysis under loading relevant to wind turbine operating conditions (considering wind load, generator and bedplate excitation). More specifically, the following research objectives are addressed in this thesis:

- How can the high speed shaft subsystem be analyzed using Global Sensitivity Analysis (GSA)?
- How can GSA be used to evaluate methods to recognize the faults and failures in functional components (bearing, coupling, generator)?
- How can the direct drive wind turbine drive train be analyzed using GSA?
- What type of wind loads have most effect on the damage and dynamic performance of the wind turbine drive trains?
- How can observed gearbox motion be used to assess operating conditions of an indirect drive wind turbine?

As described in Sec. 1.2, the reason for many failures is unknown, or do not seem to be solely dependent on the sustained loading. However, it is here assume that the failures are accentuated by the sustained loading in the sense that an increased level of loading will lead to failure quicker, even if the root cause is something else (such as stray currents or degradation of lubrication). As a consequence, the precise degradation mechanisms leading to component failures are left outside the scope of present research.

1.4 Approach

The focus of the initial phase (PAPER A) of this work is the development of mathematical model for high speed shaft test rig as subsystem of a total wind turbine drive train, and application of GSA detectability of faults in functional components. The model development was carried out in conjunction with the design assembly of a drive train system test rig (Fig. 1.4) to study high speed shaft subsystem dynamic of an indirect drive wind turbine equiped with coupling of the shafts, between gearbox and generator.

The second phase (PAPER B - D) focus on modelling and analysis of the complete drive train.
Since wind turbines operate under a large number of operating conditions, there is a need to define measures (output functions) to quantify the drive train response. The key idea is then to analyze such measures with respect to different kinds of parameters, being structural parameters or parameters defining excitation. Since the models will be analyzed for a large number of variation of input parameters, it is chosen to use structural models developed in own code (based on multibody formulation in the spirit of Shabana [50]), rather than commercial geometry-based simulation tools (such as MSC ADAMS or SIMPACK). The model development includes the assessment of system dynamic performance such as bearing forces and damages, tip radial deflections. Example of such model development for indirect drive wind turbine can be found in [51]. In [52], the relation between blade root sensor data and main bearing forces are investigated, thus ultimately finding a relation between blade root flapwise bending moment and main bearing fatigue life. Also, a systematic evaluation of different wind operational scenarios has been considered. The wind operational scenarios include normal operation, turbulence, vertical inclination, all in a wide range of mean wind speeds. Ultimately, the investigation of the effect from structural and excitation parameters (wind loads, faults) is preformed within the Global Sensitivity Analysis. GSA quantifies the effects of these input parameters to the objective function within the Sobol variance based sensitivity indices. GSA was introduced as a viable method for analysis of drive train mechanics, with focus on fault detectability assessment [53].
2 Multibody Dynamic Modelling of a Wind Turbine Drive Train

Multibody dynamics models have been used for various engineering applications [43, 50, 54, 55], in order to perform the time domain transient analysis of gear system. In order to describe the dynamics of wind turbine drive train system, a proper structural model to describe the load transfer and flexibilities, together with all structural properties of the drive train components is required. The drive train is in this context considered as a system of interconnected flexible (or rigid) bodies, and can be treated under the framework developed for the dynamic analysis of multibody systems (MBS). For studies focusing on gearbox wear and torsion vibration (important in the design of control system and excitation from the electrical side), pure torsional models have been developed, where each body is limited to only torsional DOF (degrees of freedom). For instance, several studies have been carried out within the field of vibration of the planetary gear system in [56, 57, 58, 59]. Books related to mechanical vibrations and rotating machinery, are published [60, 61, 62], describing the analysis of dynamic drive train loads, where they start with the investigation of the drive train torque. In contrast, in order to consider the flexural vibrations and main bearing forces, the transverse forces, torques and degrees of freedom must be considered, which calls for a full multibody representation of the drive train. In this section, it is outlined how the multibody system dynamics formulations are employed in the context of wind turbine drive train analysis.

The most popular methods within flexible multibody dynamics modelling are floating frame of reference (FFR) formulation [63, 64, 65, 66, 67], absolute nodal coordinates formulation (ANCF) [68, 69], and corotational frame of reference (CFR) [70, 71]. Various drive train models are developed using multibody dynamics approaches for wind turbine applications [72, 73, 74, 75, 76], some of which are validated against experimental data or online measurement data.

2.1 Floating Frame of Reference (FFR) kinematics

In the FFR formulation, the distinction is made between the global (inertial) frame $XYZ$ and the local body-fixed frame $X'Y'Z'$. The position in the global frame $\mathbf{r}$ of an arbitrary material point on a flexible body can be represented as follows (cf. Fig. 2.1-a):

$$
\mathbf{r} = \mathbf{R} + \mathbf{A}(\theta) \quad \bar{\mathbf{u}} = \mathbf{R} + \mathbf{A}(\theta) \left( \bar{\mathbf{u}}_0 + \mathbf{S} \mathbf{q}_f \right)
$$

(2.1)

where $\mathbf{R}$ is the body-fixed coordinate system origin’s location, $\mathbf{A} = \mathbf{A}(\theta)$ is the transformation matrix which defines the body fixed (local) coordinate system $X'Y'Z'$ orientation with respect to the inertial frame $XYZ$ as function of the used orientation parameters $\theta$, and $\bar{\mathbf{u}}$ is the vector defining the arbitrary point location with respect to the body coordinate system. The position vector $\mathbf{u}$, which is defined in the local coordinate system $X'Y'Z'$, can be expressed in terms of the elastic coordinates ($\bar{\mathbf{u}} = \bar{\mathbf{u}}_0 + \bar{\mathbf{u}}_f$), where $\bar{\mathbf{u}}_0$ is the position vector of material point in the undeformed state, $\mathbf{S}$ is the shape function matrix, and $\mathbf{q}_f$ is the vector of the generalized elastic nodal coordinates, defined in the
local coordinate system $X'Y'Z'$. The vector of coordinates of a flexible body in the multibody system can be expressed in the following partitioned form:

$$ q_{rf} = [q_r, \; q_f] = [R, \; \theta, \; q_f] $$

(2.2)

In the context of a drive train system, low speed shaft can be considered as one body, while the high speed shaft and gearbox housing typically constitutes additional bodies within the multibody system, depending on the choice of detail in gearbox model, additional bodies relevant to intermediate shaft, planet gears, etc. can be considered. The details of the shape function matrix $S$ used in this investigation are presented in PAPER B. Fig. 2.1 defines the coordinate systems used in the FFR formulation:

![Figure 2.1: Coordinate System, FFR Formulation (left), and wind turbine drive train model with flexible main shaft and high speed shaft (right)](image)

The FFR formulation leads to exact modelling of the rigid body dynamics when the structures rotate as rigid bodies as demonstrated in [77]. Since the deformations of drive train components (i.e. each body) can be considered small, the FRF formulation is reasonable. In contrast, when deformations within the body becomes larger, which may be the case of long and flexible turbine blades, the ANCF or co-rotational formulations are more appropriate. The complete system of three bodies comprising the low speed shaft, high speed shaft, and gearbox housing ($q = \{q_{lss}, \; q_{hss}, \; q_{gb}\}$) are presented in PAPER D.
2.2 Gearbox Modelling with constraints

The gearbox and its housing interconnects the low speed shaft (LSS) with the high speed shaft (HSS) via gears, to transmit the aerodynamic torque from blades to the generator. In order to model gearbox, we have proposed a set of constraints, consisting of bearing locations as displacement constraints (holonomic), and contact position of the gears as a velocity constraint (nonholonomic). In Fig. 2.2, the bearing locations are denoted by points 1, 3 (LSS and gearbox housing interconnection). The main shaft connects the rotor hub with the gearbox. The main bearing and a second bearing inside the gearbox carry this main shaft and transfer all loads from the hub to the bed plate, except for the torque. The high speed shaft connects the generator to the gearbox denoted by points 2, 4 (HSS and gearbox housing interconnection). The gear contact position is denoted as point A.

![Figure 2.2: 1 stage Gearbox topology with the locations of the bearings and the gear contact point as constraints in XZ plane (left), and YZ plane (right)](image)

In the drive train model the spherical joints constraints are used to model (spherical) bearings. Two bodies, body $i$ and body $j$ can be assembled and connected by the following constraint equation:

$$ C_{i,j} = r_i - r_j = (R_i + A(\theta_i)\bar{u}_i) - (R_j + A(\theta_j)\bar{u}_j) = 0 \quad (2.3) $$

Some bearings are free, not carrying axial force, for which the constraint is modified as $N_1(r_i - r_j)$ and $N_2(r_i - r_j)$, for the two transverse directions, $N_1$ and $N_2$, respectively. For the contact point between the gear wheels, a velocity constraint is used. It is formulated such that the tangential velocity for the contact point in two separate coordinates related to the engaging gears must be equal.

$$ C_{i,j}^A = (\dot{r}_i^A - \dot{r}_j^A)^T \cdot (A_q \tilde{n}) = 0 \quad (2.4) $$

The motivation for this constraint Eq. (2.4) in contrast to a pure angular velocity constraint is that Eq. (2.4) eventually gives rise to a torque on the gearbox (as pointed out by Jorgensen [78]). The more detailed derivation of the constraints in Eq. (2.3) and Eq. (2.4) is presented in PAPER D.
2.3 Dynamic equations and numerical solution

The equation of motion and set of kinematical constraints lead to the following form of Differential-Algebraic Equation (DAE) system of $n$ differential equations and $m$ algebraic equations

\[
\begin{align*}
\dot{M}\ddot{q} + C^T_q\lambda &= Q_v + Q_e, \\
C_q(q,t) &= 0
\end{align*}
\] (2.5)

where $M \in \mathbb{R}^{n \times n}$ is the mass inertia matrix. $q \in \mathbb{R}^n$ is a vector of the system generalized Cartesian coordinates. $C \in \mathbb{R}^{m \times 1}$ is the vector of the system constraint equations that describe mechanical joints and/or specified motion trajectories. $C_q \in \mathbb{R}^{m \times n}$ is the Jacobian matrix of the constraint equations of the $C(q,t) \in \mathbb{R}^m$ kinematic constraint equations. $\lambda \in \mathbb{R}^m$ are the Lagrange multipliers that are used to define the generalized constraint forces. $Q_v + Q_e \in \mathbb{R}^m$ are the vector of quadratic velocity forces and generalized external forces, respectively. They are both expressed in the global reference system.

Techniques for solving Eq. (2.5) are described in e.g. [79, 80, 81, 82]. Constraints are expressed at acceleration level, to form equations of motion from the DAE-system (Eq. (2.5)) into an ODE in the form given by ([79]):

\[
\begin{bmatrix}
M & C^T_q \\
C_q & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
Q_v + Q_e \\
Q_c
\end{bmatrix}
\] (2.6)

where the term $Q_c$ is denoting constraint forces. In the constraint contact formulation, the contact condition between tooth surfaces in contact is imposed on the equations of motion as constraint equations and the normal contact forces are evaluated by Lagrange multipliers associated with the contact constraint [83]. Moreover, a rigid contact assumption used in the formulation neglects variable mesh stiffness effect consideration. Thus, the elastic (penalty) contact approach is used in the analysis of multibody gear contact dynamics and the normal contact force is defined as a compliant force function of the penetration between the two surfaces in contact.

Since the constraints in principle introduce infinite stiffness into the global system, applying unconditional stable time integrators is vital. Fully implicit algorithms such as the Newmark algorithm are very useful when dealing specifically with flexible multibody dynamics [54]. In the aeroelastic multibody wind turbine code HAWC2 a Newmark algorithm is also applied [30].
Global Sensitivity Analysis

Global sensitivity analysis (GSA) is used as a methodology to analyse complex systems that can provide design insights. In this section, some basic concepts of the global sensitivity analysis formulation that is basis of PAPER A and PAPER C is presented. Generally, an arbitrary objective function \( OF \) of structural parameters \( X \) could be expressed in terms of a set of \( n \) design parameters \( X = [X_1, X_2, \ldots, X_n]^T \in \Omega \), through the respective deterministic function relationship \( OF(X) \) [84]. In GSA, the nominal structural parameters may not be known exactly in complex systems, whereby it is important estimate how the uncertainty in structural parameters \( X \) carry over to computed output \( OF \) of engineering interest. There are two sorts of sensitivity analysis: local and global.

In the local sensitivity, the design inputs' effects on the system response are calculated in terms of a partial derivative of an objective function (\( OF \)) with respect to the design parameter \( X_i \), which is varied around its nominal value \( X_i^* \). This method considers the variation of the objective function with respect to a single design parameter at a time. For a large scale nonlinear wind turbine dynamic model, the local sensitivity analysis may not be appropriate, since it does not cover all possible domains of the input variables. Thus, global sensitivity analysis should be considered instead.

In GSA, the considered parameters \( X \) are taken as random, thus the value \( OF(X) \) is also random. The heart of GSA is to retake the variance of \( OF \) to variance of \( X_i \). The primary \( (S_i) \) and total \( (ST_i) \) sensitivity indices are defined throughout the following relations [84, 85, 86]:

\[
S_i = \frac{E_i(V(OF|X_i))}{V(OF(X))} \quad (3.1)
\]

\[
ST_i = 1 - \frac{E_i(V(OF|\sim X_i))}{V(OF(X))} \quad (3.2)
\]

Here, \( E_i \) denotes the expected value of the function and, the subscripts \( i \) refers to \( X_i \) input parameter. The primary sensitivity index \( S_i \) gives an indication of how strong the direct influence of input parameters \( X_i \) is on the objective functions \( OF \) without any interactions with other variables. The total sensitivity index \( ST_i \) gives the total effect of the variance in variable \( X_i \) on the output variance of \( OF \) plus the effects from variance of the interaction of variable \( X_i \) with all the other variables in the system. The total sensitivity index gives an indication of the total influence of \( X_i \) on \( OF \) from its own direct effects along with its interaction with other variables [87, 88]. The differences between the primary and total sensitivity indices then give an indication of how important the interactions of \( X_i \) are with other variables in influencing \( OF \). If the main effect is small, whereas the total effect is large, then \( X_i \) does influence \( OF \) but only through interactive effects with other system inputs.

In order to compute the global sensitivity indices, multidimensional integrals have to be calculated, which are computationally expensive. Thus it is of priority to apply efficient algorithms to reduce the computational costs. The M-DRM method could approximate
the global sensitivity indices efficiently and accurately [87]. This approach has been applied in PAPER A for high speed shaft drive train and PAPER C for direct drive wind turbine, to solve the global sensitivity analysis with respect to input drive train structural parameters.

The application of GSA to the developed mathematical models can be used to reach the following aims:

- Investigate fault detectability: By calculating the sensitivity of objective functions with respect to parameters representing faults, considering uncertainties of other parameters, the detectability of that fault in system response was determined. This could contribute to development of systems for early detection of faults in functional components of wind turbine components, such as bearings and couplings.

- Determine effect from parameters uncertainty: System simulations and fatigue life predictions are typically carried out for nominal structural parameters. However, since the actual value of some of these parameters are uncertain, it is of interest to determine to what extent this parameter uncertainty carry over to quantities of engineering interest, such as predicted component life. This study is not limited to structural parameters, but can also be used for parameters defining external excitation or forces.

- Efficient optimization: In many cases it is desired to carry out structural optimization to improve the drive train design. From GSA the subset of structural parameters that have prominent effect on the objective function can be determined, which simplifies the optimization.

- Provide design insights: The analysis of an engineering structure requires long and careful examination of all results. In some cases, the relation between structural parameters and predicted outcome is not evident, and unexpectedly large (or small) sensitivity indices can reveal unintended model behavior that calls for further study.
4 Wind operational scenarios

In this section, the different wind operational scenarios and their characteristics are described.

![Wind operational scenarios diagram](image)

Figure 4.1: Wind turbine operating in a wind box (a), Wind shear profile (variation of wind speed above ground). Turbulence (red curve) added to mean wind field (b), Vertical inclination (c)

The wind field is often characterized by mean wind speed, wind shear and turbulence. As illustrated in Fig. 4.1, the wind field is in turbine simulations according to IEC 61400-1 [89] considered as a wind speed profile with a random fluctuation added. The profile, wind shear, suggest increasing wind speeds at higher altitude. Commonly the mean wind speed is given at a reference height, often turbine hub height. A common assumption is the following power law relation between altitude $z$ and wind speed $V$

$$V(z) = V(z_r) \left( \frac{z}{z_r} \right) ^ \alpha$$  \hspace{1cm} (4.1)

where $z_r$ is the reference height. Added to the wind profile is a random fluctuation, which is how turbulence is accounted for. The turbulence intensity, the magnitude (root mean square) of the wind speed fluctuation divided by mean wind speed, is a measure of how unsteady the wind field is. Since the fluctuation is random, a realization of the wind field is created by using some probabilistic function. To ensure statistically representative results a number of realization of wind fields are made.

This wind uncertainty implies two aspects. One is the difficulty in validating computational models, since the wind field is intrinsically unknown, model predictions cannot be confirmed by turbine measurements other than in some statistical sense. The other is that turbines analyses must be carried out for a large set of different operating conditions (all relevant wind speeds, with several realizations each of the turbulent wind field) to ensure the validity and limitations of the conclusions drawn.

A situation of particular interest is the case of vertical inclination. It applies to the
situation where the wind tower is on top of the hill and the wind profile has a sharp velocity gradient with respect to height. This creates uneven loads across the wind turbine blades from tip to root, it may effect the internal dynamic response of the functional components. One of the studies of the current work has been focused on this issue and investigation of internal dynamics of bearings and gearbox with respect to vertical inclination angle in different mean wind speeds.

5 Summary of Appended Papers

- **Paper A:**
  Global sensitivity analysis of high speed shaft subsystem of wind turbine drive train.

  This work considers variance based global sensitivity analysis of wind turbine drive train structure. Here, we investigate the sensitivity indices for developed models in different operational scenarios related to the wind turbine applications. A mathematical model considers both torsional vibration by introducing a simplistic motor model and shaft flexibilities in term of bending within Euler-Bernoulli beam theory. Dynamic analysis was carried out to investigate the transient and steady state and during shut down responses of the drive train system. The variance based global sensitivity indices are introduced and the Gaussian quadrature integrals are employed to evaluate the contribution of input structural parameters correlated to the objective functions. For each operational scenario, the most effective parameters have been recognized for high speed shaft drive train. The study examines the primary and total sensitivities of the objective functions to each input parameter. In particular, the GSA can provide better understanding of wind turbine drive train system dynamics with respect to different structural parameters, ultimately useful tool when designing more efficient drive trains. Finally, the proposed GSA methodology demonstrates the detectability of faults in different components. Ultimately, the following conclusions could be stated in this paper:

  - Mathematical model development for high speed shaft drive train, and validation against measurement data.
  - Application of GSA for full and separate models, and analyse the sensitivity indices with and without faults have proved that the fault detections by investigation of sensitivity indices are feasible. This demonstrate the applicability of faults and failures detection by recognizing different sensitivity contributions within GSA.

- **Paper B:**
  Multibody dynamic modelling of a direct wind turbine drive train.

  The paper considers the direct wind turbine drive train modelling based on multibody dynamic and application of floating frame of reference, in order to describe the dynamic performance. The system model response has been investigate for several mean wind speeds ([6, 11, 18] m/s), and showed a quite good agreement with online system simulation tool ViDyn. The developed mathematical model consists
of two simplified (only rigid DOF) and full model (with flexible DOF). The flexible modes includes the first two bending modes, first torsional mode, and first axial mode of the main shaft based on Euler-Bernouli beam theory. Operational scenarios consist of wind loads with different mean wind speeds, different turbulence intensity factors, and different vertical inclination angles of the hub. Ultimately, the objective functions (main shaft bearings’ damage indices, radial deflection fields at the hub and the generator denoting both ends of the main shaft) are evaluated within the operational scenarios. Ultimately, the following items has been covered in the paper:

- Model development of direct wind turbine drive train based on multibody dynamic modelling formalism.
- Validation of the mathematical model versus online system simulation tool ViDyn, and compare the system dynamic response of full and simplified (no flexible modes) mathematical models, and ViDyn models
- A study of the system model response in terms of the main shaft front and rear bearing forces, and damage indices, and tip radial deflections, for different turbulence intensity factors ($I_{ref} = [0.05, 0.12, 0.20]$), and vertical inclination angles ($\alpha = [0, 10, 20]^\circ$), and mean wind speeds ([5, 6, ..., 20] m/s) for 10 wind field realization ($N_{real} = 20$).

• Paper C:
**Global sensitivity analysis of a direct wind turbine drive train.**
In this work we consider the application of global sensitivity analysis (GSA) to the developed model in PAPER B. In this regard, two types of GSA has been studied. Firstly, with respect to structural parameters. Secondly, the excitation parameters characterizing the wind loads is studied. The results of GSA in both cases are evaluated within several wind operational scenarios, namely turbulence intensity factors, vertical inclinations, for different mean wind speeds. Ultimately, the following items has been covered in the paper:

- GSA with respect to excitation parameters revealing the importance of yawing torque and overhang torque on the drive train dynamics
- GSA of direct drive train with respect to structural parameters revealing the importance of bearing location and stiffness on the drive train dynamics
- The investigation of sensitivity indices was carried out at different operational scenarios namely normal, turbulent, vertical inclination, all in different mean wind speeds ([6, 11, 18] m/s).

• Paper D:
**Multibody dynamic modelling of a wind turbine indirect drive train with focus on gearbox modelling and motion.**
In this paper, we consider the model development of a indirect wind turbine drive train with focus on gearbox modelling based on assumption of kinematic constraints within multibody dynamic formalism. The low speed shaft (LSS) and high speed shaft are interconnected to each others under the assumption of kinematic constraints.
These constraints consist of the holonomic constraints denoting the bearing locations of the shafts inside the gearbox, and a non-holonomic constraint simulating the gear contact of both shafts. In this study, the movement of the gearbox housing is predicted to evaluate optical measurements of turbine motion. Ultimately, the following items has been covered in the paper:

- Model development of indirect wind turbine drive train based on multibody dynamic modelling formalism.
- A model for 1 stage gearbox modelling based on kinematic constraints, involving bearing locations constraints, and velocity constraints for the gear contact in order to consider the reaction torque acting on the gearbox housing.
- An overview is given of how operation conditions affect the motion for a point on the gearbox.

6 Conclusion and outlook

6.1 Contribution of the thesis

The thesis describes the theoretical and experimental study of high speed shaft drive train as subsystem of a wind turbine (PAPER A, [90, 91]), as well as multibody dynamic modelling of direct and indirect wind turbine drive trains (PAPER B-D). The main contributions are listed as follows:

- The development of a mathematical model for a high speed shaft subsystem test rig to investigate detectability of faults.
- Evaluation of operating conditions of a wind turbine direct drive train model, and the application of GSA to investigate the sensitivity with respect to structural parameters as well as excitation parameters.
- A model of indirect drive train to predict gearbox motion for the evaluation of data from optical measurement system.

6.2 Suggestions for future work

The current work results can contribute in the understanding how wind loads may affect wind turbine drive train component life. The current methodology and model development together with wind loads which is of essence when evaluating the profitability of certain wind turbine design at an intended site, can assess the dynamic response and ultimately the fatigue life of bearings and gearbox.
Some further research could be done within the wind turbine drive train system dynamics as listed below:

- Carry out GSA with respect to wind parameters such as wind shear, turbulence factor, wind speed as input $X$ in GSA (the same procedure, but starting from scratch).
- Determine and approximate an autocorrelation function for the hub forces, which can be used for reliability analysis as well as input for GSA (Karnhunen-Loeve theory).
- Perform indirect wind turbine multibody dynamic modelling with kinematic constraints with more elaborate gearbox model including multi-stage gearbox with intermediate shafts.
- Rotor blade pitch control is crucial for achieving high energy production efficiency as well as preventing rotor blades from damages under stochastic extreme wind loads. More study is needed in the future to shed light on the effect of the pitch control design and parameters on the drive train fatigue damages.
Bibliography


Part II
Appended Papers A–D
Global Sensitivity Analysis of High Speed Subsystem of a Wind Turbine Drive Train
Multibody Dynamic Modelling of a Direct Drive Wind Turbine
Paper C

Global Sensitivity Analysis of a direct Drive Wind Turbine
Multibody dynamic modelling of a wind turbine indirect drive train with focus on gearbox modelling and motion