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Angular momentum and local gravitational instability in galaxy discs: does Q correlate with j or M?

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ABSTRACT

We introduce a new diagnostic for exploring the link between angular momentum and local gravitational instability in galaxy discs. Our diagnostic incorporates the latest developments in disc instability research, is fully consistent with approximations that are widely used for measuring the stellar specific angular momentum, $j_\star = J_\star/M_\star$, and is also very simple. We show that such a disc instability diagnostic hardly correlates with $j_\star$ or $M_\star$, and is remarkably constant across spiral galaxies of any given type (Sa–Sd), stellar mass ($M_\star = 10^{9.5}$–$10^{11.5}\,\mathrm{M}_\odot$), and velocity dispersion anisotropy ($\sigma_{z\star}/\sigma_{R\star} = 0$–1). The fact that $M_\star$ is tightly correlated with star formation rate, molecular gas mass (M_{mol}), metallicity (12 + log O/H), and other fundamental galaxy properties thus implies that nearby star-forming spirals self-regulate to a quasi-universal disc stability level. This not only proves the existence of the self-regulation process postulated by several star formation models, but also raises important caveats.

Key words: instabilities – stars: kinematics and dynamics – ISM: kinematics and dynamics – galaxies: ISM – galaxies: kinematics and dynamics – galaxies: star formation.

1 INTRODUCTION

Today, 35 yr after the pioneering work of Fall (1983), angular momentum is regarded as one of the most fundamental galaxy properties. Fall's scaling law $j_\star \propto M_\star^{2/3}$, which links the stellar specific angular momentum ($j_\star = J_\star/M_\star$) to the stellar mass ($M_\star$), has been confirmed and refined in a wide variety of contexts, and forms the basis of a new physical–morphological classification of galaxies (e.g. Romanowsky & Fall 2012; Obreschkow & Glazebrook 2014; Elson 2017; Lagos et al. 2017; Lapi, Salucci & Danese 2018; Posti et al. 2018; Sweet et al. 2018). Angular momentum is linked to global dynamical processes such as the formation and evolution of galaxies, and the gravitational instability of galaxy discs to bar formation (e.g. Mo, Mao & White 1998; Athanassoula 2008; Agertz & Kravtsov 2016; Sellwood 2016; Okamura, Shimasaku & Kawamata 2018; Zoldan et al. 2018).

There is recent evidence that angular momentum is also linked to local disc instability. Using the DARK SAGE semi-analytic model of galaxy evolution, Stevens et al. (2016) showed that disc instabilities are crucial for regulating both the mass and the spin of galaxy discs. Obreschkow et al. (2016) found that the mass fraction of neutral atomic gas in isolated local disc galaxies can be described by a hybrid stability model, which combines the H I velocity dispersion with the mass and specific angular momentum of the whole (gas+stars) disc. Such a stability model was used by Lutz et al. (2018) to analyse galaxies that are extremely rich in H I, and to associate their high H I content with their high specific angular momentum. Zasov & Zaitseva (2017) showed that the relation between atomic gas mass and disc specific angular momentum in late-type star-forming galaxies is equally well described by a simpler stability model controlled by the gas Toomre parameter. Zasov & Zaitseva (2017) also discussed the impact that radial variation in the gas velocity dispersion may have on their model, and the role that stars may play in that scenario. Swinbank et al. (2017) found that angular momentum plays a major role in defining the stability of galaxy discs at $z \sim 1$, and identified a correlation between the stellar specific angular momentum and the gas Toomre parameter. Other pieces of evidence are discussed by Lagos et al. (2017) and Swinbank et al. (2017).

In spite of such evidence, there is still no tight constraint on the link between angular momentum and local gravitational instability in galaxy discs. Note, in fact, that diagnostics like the gas Toomre parameter are highly unreliable indicators of gravitational instability. This concerns not only nearby spirals, where disc instabilities are driven by stars (Romeo & Mogotsi 2017), but also gas-rich galaxies at low and high redshifts, where turbulence can drive the disc into regimes that are far from Toomre/Jeans instability (Romeo, Burkert & Agertz 2010; Romeo & Agertz 2014).

This letter provides the astronomical community with a simple and reliable diagnostic for exploring this link in nearby spirals. Besides deriving such a diagnostic and comparing it with other stability parameters (Section 2), we illustrate its strength with an eloquent example, which tightly constrains the relation between

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disc stability level, stellar specific angular momentum and stellar mass (Section 3). This turns out to have wider implications, which we discuss together with our conclusions (Section 4).

2 DISC INSTABILITY DIAGNOSTIC

2.1 The route to $\langle\mathcal{Q}_\star\rangle$

To explore the link between angular momentum and local gravitational instability in nearby star-forming spirals, we need a reliable disc instability diagnostic. Contrary to what is commonly assumed, the gas Toomre parameter is not a reliable diagnostic: stars, and not molecular or atomic gas, are the primary driver of disc instabilities in spiral galaxies, at least at the spatial resolution of current extragalactic surveys (Romeo & Mogotsi 2017). This is confirmed by other investigations (Marchuk 2018; Marchuk & Sotnikova 2018; Mogotsi & Romeo 2018), and is true even for a powerful starburst+Seyfert galaxy like NGC 1068 (Romeo & Fathi 2016). The stellar Toomre parameter is a more reliable diagnostic, but it does not include the stabilizing effect of disc thickness, which is important and should be taken into account (Romeo & Falstad 2013). The simplest diagnostic that does this accurately is the Romeo–Falstad $\mathcal{Q}_N$ stability parameter for one-component ($N = 1$) stellar ($\star$) discs, which we consider as a function of galactocentric distance R:

$$\mathcal{Q}_\star(R) = Q_\star(R)\,T_\star\,, \tag{1}$$

where $Q_\star = \kappa\sigma_\star/\pi G\Sigma_\star$ is the stellar Toomre parameter (σ denotes the radial velocity dispersion), and $T_\star$ is a factor that encapsulates the stabilizing effect of disc thickness for the whole range of velocity dispersion anisotropy (σ_z/σ_R) observed in galactic discs:

$$T_\star = \begin{cases} 1 + 0.6\left(\dfrac{\sigma_z}{\sigma_R}\right)_\star^2 & \text{if } 0 \leq (\sigma_z/\sigma_R)_\star \leq 0.5\,, \\[2ex] 0.8 + 0.7\left(\dfrac{\sigma_z}{\sigma_R}\right)_\star & \text{if } 0.5 \leq (\sigma_z/\sigma_R)_\star \leq 1\,. \end{cases} \tag{2}$$

Observations do not yet constrain the radial variation of $(\sigma_z/\sigma_R)_\star$, hence that of $T_\star$ (Gerssen & Shapiro Griffin 2012; Marchuk & Sotnikova 2017; Pinna et al. 2018).

As $\mathcal{Q}_\star(R)$ is a local quantity, it cannot be directly related to the stellar specific angular momentum,

$$j_\star = \frac{1}{M_\star}\int_0^\infty R v_\mathrm{c}(R)\,\Sigma_\star(R)\,2\pi R\,\mathrm{d}R \tag{3}$$

(e.g. Romanowsky & Fall 2012). This equation tells us that $j_\star$ is the mass-weighted average of $Rv_\mathrm{c}(R)$, the orbital specific angular momentum. So it is natural to consider the mass-weighted average of $\mathcal{Q}_\star(R)$. Current integral-field-unit (IFU) surveys allow deriving reliable radial profiles of $\mathcal{Q}_\star$ up to $R \approx R_\mathrm{e}$, the effective (half-light) radius. This limit is imposed by the sparsity of reliable $\sigma_\star$ measurements for $R \gtrsim R_\mathrm{e}$ (Martinsson et al. 2013; Falcón-Barroso et al. 2017; Mogotsi & Romeo 2018). In view of these facts, we take the mass-weighted average of $\mathcal{Q}_\star(R)$ over one effective radius:

$$\langle\mathcal{Q}_\star\rangle = \frac{1}{M_\star(R_\mathrm{e})}\int_0^{R_\mathrm{e}} \mathcal{Q}_\star(R)\,\Sigma_\star(R)\,2\pi R\,\mathrm{d}R\,. \tag{4}$$

This ensures that $\langle\mathcal{Q}_\star\rangle$ and $j_\star$ have a similar relation to their local counterparts, which simplifies the following analysis.

To illustrate the usefulness of equation (4), let us calculate $\langle\mathcal{Q}_\star\rangle$ for a galaxy model that is behind the simple, accurate, and widely used approximation $j_\star = 1.19\,R_\mathrm{e}v_\mathrm{c}$: an exponential disc having a constant mass-to-light ratio and rotating at a constant circular speed (e.g. Romanowsky & Fall 2012). For this galaxy model, $M_\star(R_\mathrm{e}) = \frac{1}{2}M_\star$ and $\kappa(R) = \sqrt{2}\,v_\mathrm{c}/R$ (see e.g. Binney & Tremaine 2008), which can be expressed in terms of $j_\star$ using the approximation above. The resulting $\langle\mathcal{Q}_\star\rangle$ is given by

$$\langle\mathcal{Q}_\star\rangle = 4.75\,\frac{j_\star\overline{\sigma}_\star}{GM_\star}\,T_\star\,, \tag{5}$$

where $j_\star$ is the total stellar specific angular momentum and $M_\star$ is the total stellar mass, while $\overline{\sigma}_\star$ is the radial average of $\sigma_\star(R)$ over one effective radius:

$$\overline{\sigma}_\star = \frac{1}{R_\mathrm{e}}\int_0^{R_\mathrm{e}} \sigma_\star(R)\,\mathrm{d}R\,. \tag{6}$$

Varying the radius over which $\mathcal{Q}_\star(R)$ and $\sigma_\star(R)$ are averaged has a remarkably weak effect on the numerical factor in equation (5): if one averages over $2R_\mathrm{e}$ (rather than R_e), then the numerical factor is 5.60 (rather than 4.75). Averaging over $2R_\mathrm{e}$ requires reliable $\sigma_\star$ measurements up to such radii, which are currently very sparse (Martinsson et al. 2013; Falcón-Barroso et al. 2017; Mogotsi & Romeo 2018) but will proliferate with the advent of second-generation IFU surveys using the multi-unit spectroscopic explorer. This is different from the case of $j_\star$ measurements, which have already entered the high-precision era (e.g. Obreschkow & Glazebrook 2014; Lapi et al. 2018; Posti et al. 2018).

2.2 $\langle\mathcal{Q}_\star\rangle$ versus other stability parameters

$\langle\mathcal{Q}_\star\rangle$ measures the local stability of galaxy discs in an averaged, mass-weighted sense. Since $\langle\mathcal{Q}_\star\rangle$ depends on mass and specific angular momentum, and since these quantities also affect the stability of galaxy discs against bar formation (Mo et al. 1998), $\langle\mathcal{Q}_\star\rangle$ must be related to the Efstathiou–Lake–Negroponte global stability parameter,

$$\epsilon_\mathrm{m} \equiv \frac{V_\mathrm{max}}{(GM_\mathrm{d}/R_\mathrm{d})^{1/2}}\,, \tag{7}$$

where V_max is the maximum rotation velocity, M_d is the mass of the disc, and R_d is the disc scalelength (Efstathiou, Lake & Negroponte 1982; Christodoulou, Shlosman & Tohline 1995). For the galaxy model that leads to equation (5), we get: $V_\mathrm{max} = v_\mathrm{c}$, $M_\mathrm{d} = M_\star$, and $R_\mathrm{d} = j_\star/2v_\mathrm{c}$ (e.g. Romanowsky & Fall 2012), hence

$$\langle\mathcal{Q}_\star\rangle \approx \epsilon_\mathrm{m}^2\,(10\,\overline{\sigma}_\star/v_\mathrm{c})\,T_\star\,. \tag{8}$$

In other words, $\langle\mathcal{Q}_\star\rangle$ can be viewed as ϵ_m^2 altered by two factors: the first one, $\approx(10\,\overline{\sigma}_\star/v_\mathrm{c})$, results from the different roles that random and ordered motions play in local and global gravitational instabilities; the second one, $T_\star$, represents the stabilizing effect of disc thickness, which depends on the velocity dispersion anisotropy (see equation 2).

$\langle\mathcal{Q}_\star\rangle$ is not the only parameter that relates local disc stability to mass and specific angular momentum. The first attempt to do that was made by Obreschkow & Glazebrook (2014). Using dimensional analysis and physical insight, they defined a disc-averaged Toomre parameter as $\overline{Q} \propto \sigma_0\,j\,M^{-1}$, where σ_0 is a velocity dispersion scale. Obreschkow et al. (2016) redefined $\overline{Q}$ as $q \equiv j_\mathrm{disc}\,\sigma_\mathrm{HI}/(GM_\mathrm{disc})$ and referred to this hybrid quantity as a 'global' disc stability parameter. [1] The stability criterion also changed from $\overline{Q} \geq 1$ (Obreschkow et al. 2015) to $q \gtrsim 1/(\sqrt{2}\,\mathrm{e})$ or $q \gtrsim 0.4$ (Obreschkow et al. 2016),

[1] What Obreschkow et al. (2016) actually meant by 'global' was 'mass-weighted average'. In fact, q does not concern global disc stability against bar or spiral structure formation.

depending on the model. Although $\langle \mathcal{Q}_\star \rangle$ may look similar to $\overline{Q}$ and q, it is not. First of all, $\langle \mathcal{Q}_\star \rangle$ is a robustly defined parameter, which results from state-of-the-art diagnostics for detecting gravitational instabilities in galaxy discs (see Section 2.1). Secondly, $\langle \mathcal{Q}_\star \rangle$ depends on $\overline{\sigma}_\star$, which differs radically from σ_{HI} not only in value but also in meaning: disc instabilities in spiral galaxies are driven by stars, not by atomic gas (see again Section 2.1).

3 PRACTICAL USE OF $\langle \mathcal{Q}_\star \rangle$

3.1 Exploring the $\langle \mathcal{Q}_\star \rangle$ –$M_\star$–$j_\star$ correlation

Now that we have a reliable disc instability diagnostic, let us explore how $\langle \mathcal{Q}_\star \rangle$ correlates with $M_\star$ and $j_\star$. To do this, we make use of equation (5) and the following scaling relations:

(i) $\log j_\star = 0.52\,(\log M_\star - 11) + 3.18$, which has an rms scatter of 0.19 dex (Romanowsky & Fall 2012);

(ii) $\log \overline{\sigma}_\star = 0.45 \log M_\star - 2.77$, which has an rms scatter of 0.10 dex (Mogotsi & Romeo 2018).

These scaling relations are least-squares fits to accurate measurements of $j_\star$ [kpc km s^{-1}] and $\overline{\sigma}_\star$ [km s^{-1}] versus $M_\star$ [M$_\odot$], and are applicable in tandem to spiral galaxies of type Sa–Sd and stellar mass $M_\star \approx 10^{9.5}$–$10^{11.5}\,\mathrm{M}_\odot$. Contrary to $j_\star$ and $\overline{\sigma}_\star$, $T_\star$ is uncorrelated with $M_\star$. This follows from the facts that $(\sigma_z/\sigma_R)_\star$ is uncorrelated with Hubble type (Pinna et al. 2018) and Hubble type is strongly correlated with $M_\star$ (e.g. Conselice 2006). If we regard the $j_\star$–$M_\star$ and $\overline{\sigma}_\star$–$M_\star$ best-fitting relations as functional relations and the associated rms scatters as uncorrelated, then the expected $\langle \mathcal{Q}_\star \rangle$–$M_\star$ scaling relation is

$$\langle \mathcal{Q}_\star \rangle = 5.4 \left(\frac{M_\star}{\mathrm{M}_\odot} \right)^{-0.03} T_\star \tag{9}$$

and has an rms scatter of approximately 0.21 dex ($0.21 = \sqrt{0.19^2 + 0.10^2}$), i.e. an rms scatter of approximately a factor of 1.6. Inverting the $j_\star$–$M_\star$ relation, we can also infer $\langle \mathcal{Q}_\star \rangle$ as a function of $j_\star$:

$$\langle \mathcal{Q}_\star \rangle = 3.9 \left(\frac{j_\star}{1\ \mathrm{kpc\ km\ s^{-1}}} \right)^{-0.06} T_\star\,. \tag{10}$$

Hereafter we will focus on equation (9), since $M_\star$ is a more classical observable than $j_\star$.[2]

equation (9) predicts that a two-orders-of-magnitude variation in $M_\star$, as observed across spiral galaxies of type Sa–Sd, 'collapses' into a <20 per cent variation in $\langle \mathcal{Q}_\star \rangle$:

$$M_\star = 10^{9.5}\text{–}10^{11.5}\,\mathrm{M}_\odot \quad \Longrightarrow \quad \langle \mathcal{Q}_\star \rangle \simeq 2.4\text{–}2.8\ T_\star\,. \tag{11}$$

The observed variation in $(\sigma_z/\sigma_R)_\star$ has a more significant impact, but the total expected variation in $\langle \mathcal{Q}_\star \rangle$ is still within a factor of two:

$$(\sigma_z/\sigma_R)_\star = 0\text{–}1 \quad \Longrightarrow \quad \langle \mathcal{Q}_\star \rangle \sim 2\text{–}4\,. \tag{12}$$

The prediction that $\langle \mathcal{Q}_\star \rangle$ has an expected value of ~2–4 for spiral galaxies of any given type, stellar mass, and velocity dispersion anisotropy is in remarkable agreement with high-quality measurements of the disc stability level in such galaxies (e.g. Westfall et al. 2014; Hallenbeck et al. 2016; Garg & Banerjee 2017; Romeo & Mogotsi 2017; Marchuk 2018; Marchuk & Sotnikova 2018). An

[2] Obreschkow et al. (2016) found that $q \propto M_{\mathrm{disc}}^{-1/3}$, but this cannot be compared with our $\langle \mathcal{Q}_\star \rangle$–$M_\star$ scaling relation since q and $\langle \mathcal{Q}_\star \rangle$ are conceptually different parameters (see Section 2.2).

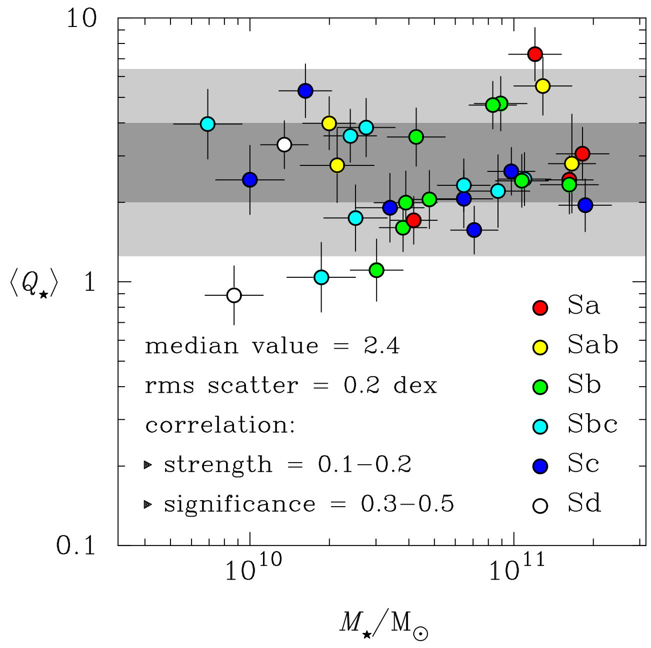

Figure 1. Our disc instability diagnostic, $\langle \mathcal{Q}_\star \rangle$, versus stellar mass, $M_\star$, for a sample of 34 nearby spiral galaxies of type Sa–Sd (colour-coded) from the CALIFA survey. The dark grey area shows the variation in $\langle \mathcal{Q}_\star \rangle$ predicted by equation (9), while the light grey area shows the rms scatter around this range of values predicted in Section 3.1. Statistical information about the data is given in summary form and simplified notation (see Section 3.2 for more information).

expected value of $\langle \mathcal{Q}_\star \rangle \sim 2$–4 is also meaningful from a theoretical point of view: it tells us that spiral galaxies are, in a statistical sense, marginally stable against non-axisymmetric perturbations (e.g. Griv & Gedalin 2012) and gas dissipation (Elmegreen 2011), although the precise value of the critical stability level is still questioned (Romeo & Fathi 2015).

3.2 Non-correlation confirmed

To test the robustness of our results, we analyse a sample of 34 nearby spiral galaxies of type Sa–Sd from the Calar Alto Legacy Integral Field Area (CALIFA) survey, as listed in table 1 of Mogotsi & Romeo (2018). These are galaxies with accurate measurements of the epicyclic frequency κ (Kalinova et al. 2017; Mogotsi & Romeo 2018), stellar radial velocity dispersion $\sigma_\star$ (Falcón-Barroso et al. 2017; Mogotsi & Romeo 2018), stellar velocity dispersion anisotropy $\sigma_{z\star}/\sigma_{R\star}$ (Kalinova et al. 2017), stellar mass $M_\star$, and other galaxy properties (Bolatto et al. 2017). These are all the quantities needed to compute $\langle \mathcal{Q}_\star \rangle$ from equation (4), except for the stellar mass surface density $\Sigma_\star$, which has not been measured in many galaxies of our sample (Sánchez et al. 2016). Note, however, that $\Sigma_\star$ only enters the normalization factor $M_\star(R_{\mathrm{e}})$ in equation (4), which is close to $\frac{1}{2}M_\star$ (González Delgado et al. 2014). So we use this approximation, but compute the integral in equation (4) accurately by taking into account the Voronoi binning of CALIFA data (see Cappellari 2009 for a review). In simple words, we sum over Voronoi bins rather than over circular rings.

Fig. 1 illustrates that the resulting $\langle \mathcal{Q}_\star \rangle$ versus $M_\star$ is fully consistent with the predictions made in Section 3.1. $\langle \mathcal{Q}_\star \rangle$ has a median value of 2.4, which is within the expected range of values (~2–4), and has an rms scatter of approximately 0.2 dex, which is close to the expected one (0.21 dex). Fig. 1 also shows that there is no clear correlation between $\langle \mathcal{Q}_\star \rangle$ and $M_\star$. To quantify the strength and the significance of a possible $\langle \mathcal{Q}_\star \rangle$–$M_\star$ correlation, we present the results of three statistical measures and associated tests (see e.g. Press et al. 1992). We find that:

(i) Pearson's correlation coefficient $r = 0.17$, and its significance level $p_r = 0.32$;
(ii) Spearman's rank correlation coefficient $\rho = 0.11$, and its two-sided significance level $p_\rho = 0.53$;
(iii) Kendall's rank correlation coefficient $\tau = 0.12$, and its two-sided significance level $p_\tau = 0.31$.

These numbers speak clearly: $\langle \mathcal{Q}_\star \rangle$ hardly correlates with $M_\star$, as predicted in Section 3.1.

4 CONCLUSIONS

(i) If there is a direct link between angular momentum and local gravitational instability in nearby star-forming spirals, then it must involve $j_\star$ and $\mathcal{Q}_\star = Q_\star T_\star$. This is because stars ($\star$), and not molecular or atomic gas, play the leading role in the disc instability scenario, and because disc thickness has an important stabilizing effect ($T_\star$).
(ii) Since $j_\star$ is the mass-weighted average of a local quantity, $Rv_{\rm c}(R)$, and since $\mathcal{Q}_\star$ itself is a local quantity, an unbiased relation must involve $j_\star$ and $\langle \mathcal{Q}_\star \rangle$, the mass-weighted average of $\mathcal{Q}_\star$.
(iii) This Letter introduces a new disc instability diagnostic that satisfies the two requirements above, and which is simple and fully consistent with the widely used approximation $j_\star = 2R_{\rm d}V$ (see equation 5). Although conceptually distinct, our diagnostic is related to the Efstathiou–Lake–Negroponte global stability parameter via the degree of rotational support, V/σ, and the velocity dispersion anisotropy, σ_z/σ_R (see equation 8).
(iv) Making use of previously established scaling relations, we show that $\langle \mathcal{Q}_\star \rangle$ hardly correlates with $j_\star$ or $M_\star$: $\langle \mathcal{Q}_\star \rangle \propto j_\star^{-0.06} \propto M_\star^{-0.03}$ (see equations 9 and 10). This scaling relation results in a remarkably constant $\langle \mathcal{Q}_\star \rangle \sim 2$–4 across spiral galaxies of any given type (Sa–Sd), stellar mass ($M_\star = 10^{9.5}$–$10^{11.5}\,{\rm M}_\odot$) and velocity dispersion anisotropy ($\sigma_{z\star}/\sigma_{R\star} = 0$–1). These results are fully consistent with high-quality measurements of the disc stability level in such galaxies, and with theoretical estimates of the local stability threshold in galaxy discs. The robustness of our results is further confirmed by a detailed analysis of a sample of 34 nearby spirals from the CALIFA survey. Details are given in Section 3.

Our results have wider implications. It is well known that $M_\star$ is tightly correlated with star formation rate, molecular gas mass ($M_{\rm mol}$), metallicity (12 + log O/H), and other fundamental galaxy properties (e.g. Conselice 2006; Nagamine et al. 2016; Lapi et al. 2018). The fact that $\langle \mathcal{Q}_\star \rangle$ varies very weakly with $M_\star$ thus implies that nearby star-forming spirals self-regulate to a quasi-universal disc stability level. This is conceptually similar to the self-regulation process postulated by several star formation models, which assume $Q = 1$ throughout the disc (see Section 1 of Krumholz et al. 2018 for an overview). Note, however, that there are two significant differences. First of all, the key quantity is basically $\mathcal{Q}_\star$ and not the gas Toomre parameter $Q_{\rm g} = \kappa\sigma_{\rm g}/\pi G\Sigma_{\rm g}$. In fact, $Q_{\rm g}$ varies by more than one order of magnitude in nearby star-forming spirals (see fig. 5 of Romeo & Wiegert 2011). Second, $\mathcal{Q}_\star$ is well above unity and is approximately constant ($\sim$2–4) only in a statistical sense. In fact, $\mathcal{Q}_\star$ can vary by more than a factor of two even within an individual spiral galaxy (see fig. A.14 of Grebović 2014). New-generation star formation models must take these two facts into account, and a significant step forward has just been taken (Krumholz et al. 2018).

Finally, the practical use of $\langle \mathcal{Q}_\star \rangle$ extends beyond the eloquent example illustrated in this letter. Since angular momentum and local gravitational instability are key ingredients in the formation and evolution of galaxy discs (e.g. Lagos et al. 2017; Krumholz et al. 2018), $\langle \mathcal{Q}_\star \rangle$ can indeed be used in a variety of contexts. One such application could be to constrain the relation between angular momentum, galaxy morphology, and star formation more tightly than now, which is a primary goal in galactic angular momentum research (e.g. Obreschkow & Glazebrook 2014; Obreschkow et al. 2015; Lagos et al. 2017; Swinbank et al. 2017). This requires reliable measurements of the disc stability level, which $\langle \mathcal{Q}_\star \rangle$ has been shown to provide.

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REFERENCES

Agertz O., Kravtsov A. V., 2016, ApJ, 824, 79
Athanassoula E., 2008, MNRAS, 390, L69
Binney J., Tremaine S., 2008, Galactic Dynamics, 2nd edn, Princeton University Press, Princeton
Bolatto A. D. et al., 2017, ApJ, 846, 159
Cappellari M., 2009, preprint (arXiv:0912.1303)
Christodoulou D. M., Shlosman I., Tohline J. E., 1995, ApJ, 443, 551
Conselice C. J., 2006, MNRAS, 373, 1389
Efstathiou G., Lake G., Negroponte J., 1982, MNRAS, 199, 1069
Elmegreen B. G., 2011, ApJ, 737, 10
Elson E. C., 2017, MNRAS, 472, 4551
Falcón-Barroso J. et al., 2017, A&A, 597, A48
Fall S. M., 1983, in Athanassoula E., ed., Proc. IAU Symp. 100, Internal Kinematics and Dynamics of Galaxies. Reidel, Dordrecht, p. 391
Garg P., Banerjee A., 2017, MNRAS, 472, 166
Gerssen J., Shapiro Griffin K., 2012, MNRAS, 423, 2726
González Delgado R. M. et al., 2014, A&A, 562, A47
Grebović S., 2014, Gravitational Instability of Nearby Galaxies: Dwarfs vs. Spirals., MSc thesis, Chalmers University of Technology, Gothenburg, Sweden
Griv E., Gedalin M., 2012, MNRAS, 422, 600
Hallenbeck G. et al., 2016, AJ, 152, 225
Kalinova V. et al., 2017, MNRAS, 469, 2539
Krumholz M. R., Burkhart B., Forbes J. C., Crocker R. M., 2018, MNRAS, 477, 2716
Lagos C. d. P., Theuns T., Stevens A. R. H., Cortese L., Padilla N. D., Davis T. A., Contreras S., Croton D., 2017, MNRAS, 464, 3850
Lapi A., Salucci P., Danese L., 2018, ApJ, 859, 2
Lutz K. A. et al., 2018, MNRAS, 476, 3744
Marchuk A. A., 2018, MNRAS, 476, 3591
Marchuk A. A., Sotnikova N. Y., 2017, MNRAS, 465, 4956
Marchuk A. A., Sotnikova N. Y., 2018, MNRAS, 475, 4891
Martinsson T. P. K., Verheijen M. A. W., Westfall K. B., Bershady M. A., Schechtman-Rook A., Andersen D. R., Swaters R. A., 2013, A&A, 557, A130
Mo H. J., Mao S., White S. D. M., 1998, MNRAS, 295, 319
Mogotsi K. M., Romeo A. B., 2018, preprint (arXiv:1804.10119)

Nagamine K., Reddy N., Daddi E., Sargent M. T., 2016, Space Sci. Rev., 202, 79
Obreschkow D., Glazebrook K., 2014, ApJ, 784, 26
Obreschkow D. et al., 2015, ApJ, 815, 97
Obreschkow D., Glazebrook K., Kilborn V., Lutz K., 2016, ApJ, 824, L26
Okamura T., Shimasaku K., Kawamata R., 2018, ApJ, 854, 22
Pinna F., Falcón-Barroso J., Martig M., Martínez-Valpuesta I., Méndez-Abreu J., van den Ven G., Leaman R., Lyubenova M., 2018, MNRAS, 475, 2697
Posti L., Fraternali F., Di Teodoro E. M., Pezzulli G., 2018, A&A, 612, L6
Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, Numerical Recipes in Fortran: The Art of Scientific Computing, Cambridge University Press, Cambridge
Romanowsky A. J., Fall S. M., 2012, ApJS, 203, 17
Romeo A. B., Agertz O., 2014, MNRAS, 442, 1230
Romeo A. B., Falstad N., 2013, MNRAS, 433, 1389
Romeo A. B., Fathi K., 2015, MNRAS, 451, 3107
Romeo A. B., Fathi K., 2016, MNRAS, 460, 2360
Romeo A. B., Mogotsi K. M., 2017, MNRAS, 469, 286
Romeo A. B., Wiegert J., 2011, MNRAS, 416, 1191
Romeo A. B., Burkert A., Agertz O., 2010, MNRAS, 407, 1223
Sánchez S. F. et al., 2016, Rev. Mex. Astron. Astrofis., 52, 171
Sellwood J. A., 2016, ApJ, 819, 92
Stevens A. R. H., Croton D. J., Mutch S. J., 2016, MNRAS, 461, 859
Sweet S. M., Fisher D., Glazebrook K., Obreschkow D., Lagos C., Wang L., 2018, ApJ, 860, 37
Swinbank A. M. et al., 2017, MNRAS, 467, 3140
Westfall K. B., Andersen D. R., Bershady M. A., Martinsson T. P. K., Swaters R. A., Verheijen M. A. W., 2014, ApJ, 785, 43
Zasov A. V., Zaitseva N. A., 2017, Astron. Lett., 43, 439
Zoldan A., De Lucia G., Xie L., Fontanot F., Hirschmann M., 2018, preprint (arXiv:1803.08056)

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