THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Effect of Partial Screening on Runaway-Electron Dynamics

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Cover:
Number density of bound electrons for the ionization states of argon, plotted on a logarithmic vertical scale as a function of atomic radius. The density increases with increasing number of bound electrons from Ar^{17+} on the left to Ar^{0} on the right.

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Abstract

One of the essential results of kinetic plasma physics is the runaway phenomenon: sufficiently large electric fields can accelerate a fraction of an electron population to relativistic energies. While such runaway electrons are fundamentally interesting objects of study in astrophysical settings, they are also of great practical relevance to fusion research. In the most developed fusion power production device, known as the tokamak, runaway electrons have the potential to cause severe damage to the first wall. Accordingly, runaway-electron mitigation is one of the critical issues in the design of a fusion power plant.

The most promising mitigation method to date is the injection of heavy atoms which only partially ionize and collisionally dissipate the energy of the runaway beam before it can collide with the wall. When the ions are partially ionized, their bound electrons screen out a fraction of the atomic charge, which directly affects the collisional scattering rates. However, accurate expressions for these collisional scattering rates between energetic electrons and partially ionized atoms have not been available previously, compromising modeling. In this thesis, we derive collisional scattering rates using a quantum-mechanical treatment, and study their effects on the kinetic runaway-electron dynamics. Using kinetic simulations, we find that the presence of partially ionized atoms significantly increases the dissipation rate of runaway electrons, compared to when the bound electrons completely screened the atomic nuclei. Moreover, we find that the increased scattering rates elevate the threshold electric field for runaway acceleration, but also enhance the avalanche growth rate at electric fields much larger than this threshold.

The results outlined in this thesis contribute to more accurate runaway-electron modeling and can lead to more effective mitigation schemes in the longer term. Experimental predictions of runaway mitigation however require that the kinetic model developed here be combined with the effect of spatial variation, which is a subject for future work.

Keywords: plasma physics, Fokker–Planck equation, fusion, tokamak, runaway electrons

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A L. Hesslow, O. Embréus, A. Stahl, T.C. DuBois, G. Papp, S.L. Newton and T. Fülöp,
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`http://arxiv.org/abs/1807.05036`

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Europhysics Conference Abstracts **41F**, O4.118 (2017).
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E O. Embréus, L. Hesslow, M. Hoppe, G. Papp, K. Richards and T. Fülöp,
Dynamics of positrons during relativistic electron runaway,
Submitted for publication in Journal of Plasma Physics.
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Contribution to the 45th EPS Conference on Plasma Physics, P5.4011 (2018).
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Contents

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Chapter 1

Introduction

The possibility of a power plant that takes its fuel from ordinary sea water has created a great interest in fusion energy since the middle of the last century. Today, fusion is considered an attractive possible ingredient of a sustainable power production system due to the prospect of a safe, CO_2-free and non-intermittent power source. Since the 1960's, fusion performance has increased remarkably; for example, one of the most commonly adopted performance measures – the *triple product* – has increased by four orders of magnitude in relation to the heat losses [1]. This performance increase was achieved in the currently most developed fusion device known as the *tokamak*: a ring-shaped magnetic cage where the particles are confined by a twisted magnetic field. Despite the progress, fusion research has also encountered several unforeseen challenges, and a functional power plant is still decades away.

One such challenge is *runaway electrons*. In the undesired event of a plasma-terminating *disruption*, the tokamak can turn into a racetrack where electrons make millions of laps per second while they are accelerated by an electric field. If the resulting beam of relativistic electrons hits the tokamak wall, it can locally melt significant amounts of the wall material [2]. The potential for damage in such an event is so large that not a single unmitigated runaway electron event is allowed in future, larger tokamak devices, such as ITER [3]. It is therefore essential to further develop runaway mitigation schemes, which requires the modeling of runaway-electron dynamics.

The runaway phenomenon is a basic result of plasma physics, and occurs in several contexts. The collisional friction force experienced by a fast plasma particle *decreases* with speed, so an electric field above a certain threshold – the critical electric field E_c – can accelerate particles to ultrarelativistic energies. This phenomenon is rather counterintuitive; translated into our every-day language, it would correspond to an ever-increasing speed when cycling with a constant force on the pedals! The runaway mechanism will occur in plasmas with electric fields above the critical field E_c [4], which includes several plasma systems: magnetic fusion devices [5], astrophysical plasmas [6], and in lightning initiation where this mechanism is believed to play a key role [7, 8].

In many plasmas where runaway electrons are observed, they interact with weakly ionized or neutral atoms. These *impurities* may be either atmospheric molecules, or the ions which are typically used to mitigate runaway electrons in magnetic fusion. Interaction with such impurities can drastically affect the dynamics of runaway electrons. If a slow electron interacts with a partially ionized ion, the interaction strength depends only on the net charge of the ion since the nucleus is *completely screened* by the bound electrons. In contrast, a fast electron can penetrate this cloud of bound electrons, which leads to *partial screening* of the nucleus. Since the interaction strength strongly depends on the degree of screening, this phenomenon has a significant impact on the runaway-electron dynamics. However, treatments of screening have previously been limited to simplified models, which either neglect quantum-mechanical effects [9, 10], or employ the approximate Thomas–Fermi theory to calculate the density of bound electrons around the ions [11, 12].

In this Licentiate Thesis, we therefore derive a more accurate collision operator for fast electrons in partially ionized plasmas. In Paper A, we employ density functional theory (DFT) simulations to obtain accurate quantum-mechanical scattering cross-sections, from which we construct an analytical model for the collision operator. Furthermore, we implement this model in a kinetic solver for the electron distribution function, and study the effects of screening on runaway dynamics. Paper C further develops this model and explores its effect on runaway electrons. These results represent the main contribution of this thesis and are applicable in and beyond tokamak research. Nevertheless, the primary application in mind is to runaways in tokamaks, and some results in Paper B are

specific to tokamaks. To put the contributions of this thesis into perspective, we now review the main mechanisms for runaway generation in tokamaks and the issues present in runaway modeling.

1.1 Generation of runaway electrons in tokamak experiments

Runaway dynamics is strongly affected by the mechanisms which produce the runaway-generating electric fields. Among systems which can sustain sufficiently large electric fields for runaway acceleration, the tokamak plasma is distinguished by its large inductance and its toroidal current. Both of these properties are closely linked to the formation of a large electric field in the tokamak.

Fortunately, the electric field required to produce the usual operational toroidal plasma current in a tokamak is below E_c, which means that runaway electrons are not observed during normal operation [5]. This is because the conductivity $\sigma \propto T^{3/2}$ is sufficiently large at high plasma temperatures that the toroidal current $I = \sigma A E$ (where A is the cross-section area) is large even at a low external electric field $E_\mathrm{ext} \ll E_\mathrm{c} \propto n$, where n is the electron density. Consequently, either a lower density or a decreased temperature (at a fixed plasma current) is required to obtain super-critical electric fields and generate runaways. This corresponds to two different scenarios: low-density discharges and tokamak disruptions.

For runaway generation at low plasma density, the current-driving electric field E_ext must exceed the critical electric field E_c, i.e. $E_\mathrm{ext}/E_\mathrm{c} \propto 1/(T^{3/2}n)$ must be large. This condition is most easily achieved during tokamak start-up, since a larger electric field is needed in order to ionize the gas into a plasma. In the early years of tokamak research, runaway electrons were routinely generated during the start-up phase [13], but in today's experiments, runaway generation during this phase can be reduced by applying additional heating during ionization. In addition, any runaway seed population formed during current ramp-up is suppressed by maintaining larger plasma densities [14].

It is more difficult to prevent runaway generation during a disruption, which is an infrequent event where the plasma energy is suddenly lost,

typically due to instabilities, in what is known as a *thermal quench* [2, 15–17]. This temperature drop can decrease the conductivity $\sigma \propto T^{3/2}$ by several orders of magnitude, causing a *current quench*, which in turn induces an electric field $E_{\mathrm{ind}} \propto -dI/dt$. The induced electric field can often drastically exceed the critical electric field and may convert a significant fraction of the initial plasma current into a runaway beam of several hundred kiloamperes [18, 19].

The potential runaway-electron damage increases with the size of the device. The main reason is that the internal magnetic energy in the plasma scales as $W_{\mathrm{mag}} \propto I^2 R$ (where R is the major radius of the tokamak), and it is this energy that is partially converted into kinetic and magnetic energy of the runaway beam. Moreover, large devices confine high-energy particles better than small devices, which allows the electrons to reach higher energies before they are lost to the wall [13, 20]. Accordingly, runaway electrons are tolerable in today's tokamaks, whereas they will be unacceptable in larger reactor-size devices due to the risk of damage. In today's tokamaks, runaway electrons are therefore predominantly generated intentionally in dedicated experiments in order to develop reliable methods for runaway avoidance and mitigation. In these experiments, runaway electrons are usually created by either a disruption-causing impurity injection [21], or in a scenario similar to the early-phase runaway generation, by ramping up the current while constraining the density until the electric field exceeds the critical field [22].

Runaway generation is more problematic in future devices not only because of the increased possible damage, but also since a runaway beam is more easily formed. The reason is intrinsic to kinetic theory of plasmas: the runaway density can increase exponentially with time in an *avalanche* created by large-angle collisions between runaway electrons and slower electrons. While the plasma current in present machines allows for some avalanche multiplication, this effect could amplify a small runaway seed by a devastating factor of $\sim 10^{22}$ in ITER [23]. Experiments in small devices can therefore not investigate all aspects of the runaway problem in larger devices, which makes modeling crucial.

1.2 Modeling of runaway electrons in tokamaks

Since the runaway-electron generation mechanism fundamentally is a momentum-space effect, many of the features of runaway dynamics can be captured in a spatially homogeneous model. Momentum-space effects include collisions and radiation reaction, as well as acceleration by the electric field, which together describe both runaway generation and damping. Even if the problem is restricted to momentum space, the governing equations lead to complex dynamics; yet, the equations are simple enough for extensive analytical and numerical analysis. The momentum-space approach allows us to develop an intuition about the relative importance of competing effects in different parameter regimes. This is particularly useful for isolating and analyzing specific effects and is, therefore, the approach we take in this thesis to explore the effect of partial screening on runaway dynamics.

This thesis is restricted to momentum-space dynamics of runaway electrons. In making the connection to a tokamak, this approach amounts to assuming that the runaway electrons are located close to the magnetic axis (the circle at the center of the torus), and neglecting their radial transport as well as the spatial variation of the electromagnetic fields. Although the momentum-space restriction is sufficient to describe the most important generation and damping mechanisms, accurate predictions and experimental comparisons typically require spatial effects to be taken into account. Such spatial effects include radial transport and electric-field diffusion [24–27], the effects of the inhomogeneous magnetic field on particle orbits [28–30], as well as wave-particle interaction [31–34]. The spatial magnetic-field structure is particularly important during the disruption where the symmetric magnetic field structure breaks up and becomes chaotic, which can cause rapid losses of runaway electrons [35, 36]. Consequently, a complete understanding of runaway dynamics would require a solution of the evolution of the distribution in phase-space, including both momentum and spatial dependence.

The multidimensional nature of the runaway problem – in combination with the wide separation in energy and time scale between thermal and relativistic particles – makes it computationally unaffordable to simultaneously capture all the relevant effects, which motivates the use of approximate methods. The most accurate models of momentum-space effects are today obtained by kinetic solvers in zero [37–39] or one spa-

tial dimension [40–44]. More accurate modeling of spatial effects can be achieved by coupling such kinetic solvers to transport solvers [45], or using reduced kinetic modeling, where kinetically determined runaway growth rates are implemented in tools which include spatial effects [26, 46]. More emphasis on the magnetic-field geometry, at the expense of kinetic accuracy, is obtained by treating runaway electrons as test particles and following the particle orbits in a pre-described magnetic field geometry [47], which can be obtained from, for example, magneto-hydrodynamic (MHD) codes in a post-disruption scenario [35, 48, 49]. Common to all of these approaches is the fact that they all have severe limitations, implying runaway-electron modeling is at a stage where no approach can account for all important effects; some of these effects cannot even be accurately modeled in isolation. At the same time, it is of the utmost importance for the success of the fusion program that runaway electron damage can be prevented in future devices, which makes modeling crucial.

1.3 Outline

This thesis is organized as follows. Chapter 2 presents an introduction to kinetic theory with particular emphasis on the collision operator. In chapter 3, we specialize the kinetic equation to the runaway problem. This chapter also includes a brief introduction to CODE [37, 38], which is the main numerical tool used here. In order to put the contribution from this thesis into context, the modifications due to partial screening are pointed out continuously in both chapters 2 and 3. Chapter 4 highlights the main findings of the appended papers concerning the effect of screening on the dynamics of runaway electrons. We find strongly increased dissipation rates of runaway electrons compared to previous estimates, which indicates a promising potential to mitigate damaging runaway behavior in tokamaks. We also discuss how these results can be built upon in future work.

Chapter 2

Kinetic theory of plasmas

The runaway phenomenon is fundamentally a kinetic effect depending on the balance between collisional friction and electric-field acceleration. Kinetic theory is therefore key to understanding runaway dynamics. Specifically, the collision operator is an essential object for the runaway mechanism and has a particularly important role in this thesis, which revolves around the collision operator in a partially ionized plasma. Accordingly, the aim of this chapter is to introduce the kinetic equation with particular emphasis on the collision operator.

2.1 The kinetic equation

The kinetic equation determines the evolution of the probability distribution of particles in phase-space, and takes the form

$$\frac{\partial f_a}{\partial t} + \frac{\partial}{\partial \mathbf{z}} \cdot (\dot{\mathbf{z}} f_a) = C^a\{f_a\}. \tag{2.1}$$

Here, $f_a(\mathbf{x}, \mathbf{p}, t)$ is the distribution function of species a evaluated at position $\mathbf{x}$ and momentum $\mathbf{p} = \gamma m \mathbf{v}$ (where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the Lorentz factor and $\mathbf{v}$ is the velocity). The distribution is normalized so that the particle number density is given by $n_a(\mathbf{x}, t) = \int f_a(\mathbf{x}, \mathbf{p}, t)\, d^3p$. Moreover, $\partial/\partial\mathbf{z}$ denotes the gradient operator with respect to the phase-space coordinates $\mathbf{z} = (\mathbf{x}, \mathbf{p})$, and $\dot{\mathbf{z}} = (\mathbf{v}, \mathbf{F}_a)$ is the time derivative of $\mathbf{z}$. In a plasma, the force is given by $\mathbf{F}_a = q_a(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where $\mathbf{E}$ and

B are the electric and magnetic fields respectively, and q_a is the charge of species a.

The distribution function f_a is a statistical object in the sense that it denotes a smooth function where the point-like contributions from individual particles have been ensemble averaged over many macroscopically equivalent particle configurations. Similarly, the forces acting on the distribution function are averaged, which removes the short-length-scale interaction between individual particles. This contribution to the dynamics is instead described by the collision operator $C^a\{f_a\}$ on the right-hand side of the kinetic equation (2.1), which gives the time rate of change in f_a due to collisions. The collision operator $C^a\{f_a\} = \sum_b C^{ab}\{f_a, f_b\}$ describes the effect on f_a due to collisions with all species b in the plasma. When the collision operator is discussed in the following section and elsewhere, we will use the shorter notation $C^{ab} \equiv C^{ab}\{f_a, f_b\}$.

A rigorous derivation of the kinetic equation is outside the scope of the present text. Such derivation is associated with several subtle issues including precise definitions of the averaging process, particularly in the collision operator [50]. Nevertheless, the kinetic equation can be intuitively understood in a simple manner as a continuity equation with an additional term from the collision operator, which describes the effect of the microscopic fields.

Without collisions, the distribution function would obey the continuity equation, i.e. $\partial f_a/\partial t + (\partial/\partial \mathbf{z}) \cdot (\dot{\mathbf{z}} f_a) = 0$. In a Hamiltonian system, this continuity equation leads to Liouville's theorem, stating that the distribution function is conserved along the trajectories of a system [51]. With the dissipation introduced by collisions, the system is, however, no longer Hamiltonian which invalidates Liouville's theorem. As a consequence, the motion of individual particles is not deterministic, but is described by a stochastic differential equation known as the Langevin equation [51].

2.2 The Fokker–Planck collision operator

The form of the collision operator depends on the range of the inter-particle forces. Short-range forces decrease rapidly with inter-particle distance r, and include molecular forces which fall off as $1/r^6$ or $1/r^7$ [50].

Short-range interaction is dominated by large-angle two-body collisions, which are described by the Boltzmann collision operator. The Boltzmann operator can be understood as the rate at which species a scatters from $\mathbf{p}_1$ into $\mathbf{p}$, minus the rate of the opposite scattering process. Its general form is [52]

$$C^{ab}_{\text{Boltz}} = \int \frac{d\sigma_{ab}}{d\Omega} g_\varnothing \left[f_a(\mathbf{p}_1) f_b(\mathbf{p}_2) - f_a(\mathbf{p}) f_b(\mathbf{p}') \right] d^3p' d\Omega, \tag{2.2}$$

where $g_\varnothing = \sqrt{(\mathbf{v} - \mathbf{v}')^2 - (\mathbf{v} \times \mathbf{v}')^2/c^2}$ is the Møller relative speed and $d\sigma_{ab}/d\Omega$ is the differential cross section for collisions in which the momentum of species a changes from $\mathbf{p}$ to $\mathbf{p}_1$, and $\mathbf{p}' \to \mathbf{p}_2$ for species b.

In contrast to gases of molecules, plasma particles mainly interact via long-range forces, namely Coulomb forces. Coulomb forces fall off as the inverse square of the inter-particle distance $1/r^2$. The distinguishing feature of inverse-square forces is that the interaction is dominated by small-angle deflections to the particle trajectories, which are described by the Fokker–Planck operator.

The Fokker–Planck operator may be obtained from a small-angle expansion of the Boltzmann operator which describes two-body collisions, but it can also be derived independently using methods from statistical mechanics [50]. The Fokker–Planck collision operator between species a and b is given by

$$C^{ab}_{\text{FP}} = -\nabla_k \left(f_a \langle \Delta p^k \rangle_{ab} \right) + \frac{1}{2} \nabla_k \nabla_l \left(f_a \langle \Delta p^k \Delta p^l \rangle_{ab} \right), \tag{2.3}$$

where the term $\langle \Delta p^k \rangle_{ab}$ represents the average change in the kth component of the momentum of the incoming electron during a collision, while $\langle \Delta p^k \Delta p^l \rangle_{ab}$ describes the average change in the tensor $p^k p^l$. Here, ∇_k refers to the momentum-space gradient operator. These average momentum changes are given by

$$\langle \Delta p^k \rangle_{ab} = \int d\mathbf{p}' f_b(\mathbf{p}') \int \frac{d\sigma_{ab}}{d\Omega} g_\varnothing \Delta p^k d\Omega, \tag{2.4}$$

$$\langle \Delta p^k \Delta p^l \rangle_{ab} = \int d\mathbf{p}' f_b(\mathbf{p}') \int \frac{d\sigma_{ab}}{d\Omega} g_\varnothing \Delta p^k \Delta p^l d\Omega. \tag{2.5}$$

The angular integrals in eqs. (2.4) and (2.5) are taken over

$$\int d\Omega = \int_{\theta_{\min}}^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi, \tag{2.6}$$

where $\theta_{\min}$ is the minimum scattering angle below which Debye shielding screens out long-range interaction. This shielding effect means that each plasma particle only interacts with particles within a distance of the Debye length λ_{D}; over larger distances, the plasma species will be distributed to ensure macroscopic charge neutrality.

Without the minimum angle $\theta_{\min}$, the integrals in the Fokker–Planck operator (2.3) would diverge, since the Coulomb-interaction cross section scales as $d\sigma_{ab}/d\Omega \propto \sin^{-4}(\theta/2)$ while the lowest-order terms in $\Delta p^k d\Omega$ and $\Delta p^k p^l d\Omega$ are of order $\sim \sin^3(\theta/2) d[\sin(\theta/2)]$ [53]. The collision operator thus acquires terms which are proportional to the *Coulomb logarithm* $\ln\Lambda = \ln(2/\theta_{\min})$, which is typically large in magnetic-fusion plasmas due to the large number of particles within a Debye sphere [54]. Paper C gives an expression for the Coulomb logarithm including its energy-dependence, using an interpolation between the expression obtained in Ref. [15] describing the collision of two thermal particles, and that from Ref. [55] in which the incoming particle is superthermal. The Coulomb logarithm quantifies the dominance of small-angle collisions over large-angle collisions, and thereby determines the validity of the Fokker–Planck operator. Since the Fokker–Planck operator can only model small-angle collisions accurately, it only contains the leading-order terms in $\ln\Lambda$. Unless the order-unity terms (i.e. $\ll \ln\Lambda$) can be neglected, the resulting operator will exhibit unphysical energy transfers between the different species. Such unphysical properties can also appear in the collision operator for partially ionized plasmas that we derive in this thesis, and are addressed in Paper C.

For runaway electrons, the relation between the Fokker–Planck operator and the Boltzmann operator is more than a theoretical curiosity, since both are needed to model the runaway dynamics. In most magnetic-fusion plasmas, small-angle collisions dominate over large-angle collisions in which case the Fokker–Planck operator accurately models the dynamics. Small-angle collisions dominate also in runaway-prone plasmas, but here, large-angle collisions introduce a new runaway mechanism known as the avalanche effect – which causes an exponential growth of the runaway population. The Fokker–Planck and Boltzmann operators must therefore be combined to describe runaway dynamics. Large-angle collisions are also enhanced in partially ionized plasmas, which challenges the validity of the Fokker–Planck operator to model the effect of partial screening. In Paper C, we therefore investigate the validity of

the Fokker–Planck operator by comparison to the Boltzmann operator, and find that the Fokker–Planck operator is adequate also in partially ionized plasmas for typical tokamak parameters.

2.2.1 Collisions with a Maxwellian

To demonstrate the effect of the collisions, we evaluate the collision operator describing the collisions of species a with a background distribution in thermal equilibrium. In a collisionally dominated, relativistic plasma, the equilibrium state is the Maxwell–Jüttner distribution (the *Maxwellian* for short) [52]

$$f_{\mathrm{M}a}(\mathbf{p}) = \frac{n_a}{4\pi m_a^3 c^3 \Theta_a K_2(1/\Theta_a)} \exp\left(-\frac{\gamma}{\Theta_a}\right), \tag{2.7}$$

where $\Theta_a = T_a/(m_a c^2)$ is the temperature[1] of species a normalized to the rest energy and $K_2(1/\Theta_a)$ is the second-order modified Bessel function of the second kind. In the non-relativistic limit $\Theta \ll 1$, the Maxwell–Jüttner distribution simplifies to the Maxwell–Boltzmann distribution, which may be obtained by expanding eq. (2.7) in small Θ, in which case $K_2(1/\Theta) \sim e^{-1/\Theta}\sqrt{\pi\Theta/2}$. The non-relativistic Maxwellian is accordingly given by [54]

$$f_{\mathrm{M}a}(\mathbf{p}) = \frac{n_a}{\pi^{3/2} m_a^3 v_{\mathrm{T}a}^3} \exp\left(-\frac{v^2}{v_{\mathrm{T}a}^2}\right), \tag{2.8}$$

where $v_{\mathrm{T}a} = \sqrt{2T_a/m_a}$ is the thermal speed.

When species b has a Maxwellian distribution, the Fokker–Planck collision operator (2.3) can be parametrized by the three collision frequencies ν_D^{ab}, ν_S^{ab} and $\nu_\parallel^{ab}$ [54]:

$$C_{\mathrm{FP}}^{ab} = \nu_D^{ab}\mathscr{L}\{f_a\} + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\left(\nu_S^{ab} f_a + \frac{1}{2}\nu_\parallel^{ab} p \frac{\partial f_a}{\partial p}\right)\right], \tag{2.9}$$

where the Lorentz scattering operator

$$\mathscr{L} = \frac{1}{2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \tag{2.10}$$

[1] We follow the plasma-physics convention to include a factor of the Boltzmann constant k_{B} in the temperature, which gives units of energy.

causes scattering at constant energy, known as pitch-angle scattering.

The processes described by three collision frequencies in eq. (2.9) drive the distribution f_a toward a Maxwellian distribution through different mechanisms. The deflection frequency ν_D^{ab} drives the distribution towards isotropy, and will counter-act any beam-like structures which can be generated by, for example, a large electric field. In the energy distribution, the slowing-down frequency ν_S^{ab} describes collisional friction, whereas the parallel momentum diffusion frequency $\nu_\parallel^{ab}$ reduces sharp gradients of the energy distribution. As a consequence of these collision types, a distribution in equilibrium has an isotropic distribution, and the balance between collisional friction and momentum diffusion gives the familiar bell shape of a Maxwellian.

2.2.2 Linearized collision operator

In order to determine the evolution of a distribution function, the collision operator must be evaluated for a non-Maxwellian distribution. The treatment in the previous section is nevertheless valuable, since the collision operator can be linerarized around a Maxwellian if the distribution is close to its thermal equilibrium. Such a nearly-Maxwellian distribution is obtained in highly collisional plasmas, including many tokamak scenarios and runaway events.

The linearized Fokker–Planck collision operator consists of two pieces: the *test-particle* operator $C_{\mathrm{FP,tp}}^{ab}$ and the *field-particle* operator $C_{\mathrm{FP,fp}}^{ab}$. The test-particle operator describes how the perturbation to the Maxwellian is affected by collisions with the Maxwellian background according to eq. (2.9), while the field-particle operator describes the back-reaction from the perturbation on the Maxwellian. As a concrete example, the friction on a runaway population is determined by the test-particle operator whereas the field particle operator modifies the bulk population, so that momentum and energy are conserved.

The linearized electron collision operator

$$C_{\mathrm{FP,lin}}^{\mathrm{e}} = C_{\mathrm{FP,tp}}^{\mathrm{ee}} + C_{\mathrm{FP,tp}}^{\mathrm{ei}} + C_{\mathrm{FP,fp}}^{\mathrm{ee}} \tag{2.11}$$

is composed of the electron-electron and the electron-ion collision operator. Here, the electron-ion collision operator only contains the test-particle operator; in the limit of small electron-to-ion mass ratio, energy

transfers between the two species are negligible, so that

$$C_{\mathrm{FP}}^{\mathrm{ei}} = C_{\mathrm{FP,tp}}^{\mathrm{ei}} = \nu_D^{\mathrm{ei}} \mathscr{L}\{f_e\}. \tag{2.12}$$

Accordingly, the linearized electron collision operator takes the form

$$C_{\mathrm{FP,lin}}^{\mathrm{e}} = \left(\nu_D^{\mathrm{ee}} + \nu_D^{\mathrm{ei}}\right) \mathscr{L}\{f_e\} + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\left(\nu_S^{\mathrm{ee}} f_e + \frac{1}{2}\nu_{\parallel}^{\mathrm{ee}} p \frac{\partial f_e}{\partial p}\right)\right] + C_{\mathrm{FP,fp}}^{\mathrm{ee}}. \tag{2.13}$$

2.2.3 Collisions between electrons and partially ionized atoms

Fast electrons are strongly affected by interaction with partially ionized atoms. Unlike the case of a fully ionized plasma, the electron-ion interaction strength depends on the electron energy due to the energy-dependent screening of the N_e bound electrons around the ion; a low-energy electron will experience a *completely screened* ion with the net ion charge Z_0, whereas an ultra-relativistic electron will approach the limit of *no screening*, where the interaction strength is determined by the full nuclear charge Z. Moreover, the electron experiences an increasing rate of inelastic collisions with the bound electrons as its energy increases. Since the collision frequencies vary approximately quadratically with the ion charge and linearly with electron density, the collision rates for fast electrons are strongly enhanced in the presence of weakly ionized impurities, compared to when the nuclei are completely screened by the bound electrons.[2]

A comprehensive discussion of the fast-electron collision operator in partially ionized plasmas is given in Paper C, including the full derivation of the operator and a discussion of the employed approximations. Therefore, this section is limited to a brief introduction of the main points of the paper.

The collision operator between fast electrons and partially ionized atoms consists of two parts: the operator between electrons and the nuclei and the operator between the free electrons and the bound electrons. In both cases, the target particle can be treated as stationary; ions have a lower velocity than electrons due to the small electron-to-ion mass

[2]Throughout this thesis, we use the limit of complete screening as reference in expressions such as "*enhanced* dissipation rates".

ratio (assuming the electron and ion temperatures are of the same order of magnitude), and the bound electrons are slow since they would not be bound if their energy exceeded the binding energy. Collisions with partially ionized atoms will therefore only affect the test-particle part of the electron collision operator (2.9). Out of the three frequencies ν_D^{ab}, ν_S^{ab} and $\nu_\parallel^{ab}$, $\nu_\parallel^{ab}$ vanishes at superthermal electron momenta and can therefore be described by the completely screened expression, which is valid at thermal energies.

In the electron-ion collision operator $C_{\mathrm{FP,tp}}^{\mathrm{ei}}$, only ν_D^{ei} is modified by partial screening, since no energy transfer is kinematically allowed between ions and electrons in the limit of small electron-to-ion mass ratio. To calculate the ν_D^{ei}, we evaluate the collision operator (2.3) for stationary target particles. This derivation of the collision operator follows Ref. [53] but the Rutherford cross-section is replaced by the quantum-mechanical cross section for collisions with bound electrons, taken in the Born approximation [56, 57]:

$$\frac{d\sigma_{\mathrm{e}j}}{d\Omega} = \frac{r_0^2}{4\bar{p}^4}\left(\frac{\cos^2(\theta/2)\bar{p}^2+1}{\sin^4(\theta/2)}\right)\left|Z_j - F_j(q)\right|^2 . \tag{2.14}$$

Here, r_0 is the classical electron radius, $\bar{p} = p/(m_e c)$ is the normalized momentum, θ is the deflection angle and Z_j is the charge number for ion species j. The form factor is defined as $F_j(\mathbf{q}) = \int \rho_{\mathrm{e},j}(r)\mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{r}/a_0}\, d\mathbf{r}$, where a_0 is the Bohr radius, and $\mathbf{q} = 2\bar{\mathbf{p}}\sin(\theta/2)/\alpha$ with the fine-structure constant $\alpha \approx 1/137$. The electronic charge density of the ion is denoted $\rho_{\mathrm{e},j}(r)$, and must in general be determined by numerical methods such as density functional theory (DFT); see Paper C. By inspecting the form factor, the limits of complete screening and no screening can be identified. Complete screening is obtained as the exponential approaches unity at low q, which implies that $F \to N_e$ and $|Z-F|^2 \to Z_0^2$. Conversely, the no screening limit is approached at high q since the fast oscillations in the exponential cause the form factor to vanish, which yields $|Z-F|^2 \to Z^2$.

Regarding the electron-electron collision operator $C_{\mathrm{FP,tp}}^{\mathrm{ee}}$, we modify the slowing-down frequency ν_S^{ee} according to the Bethe stopping-power formula [58, 59]. We however neglect the effects in ν_D^{ee}, since there is no analytic expression for the differential cross section for collisions with bound electrons. As discussed in Paper C, this approximation can be motivated by the fact that $\nu_D^{\mathrm{ei}} \gg \nu_D^{\mathrm{ee}}$ if the effects of screening are signif-

icant, and thus the total deflection frequency $\nu_D = \nu_D^{\mathrm{ei}} + \nu_D^{\mathrm{ee}}$ is well approximated even with the completely-screened expression for ν_D^{ee}. Conversely, if the screening effects are insignificant, the completely-screened expression will resemble ν_D^{ee} per definition.

In the next chapter, we will demonstrate the application of this collision operator in the kinetic equation to model runaway electrons.

Chapter 3

Kinetic modeling of runaway electrons

As the kinetic equation is too complicated to solve in its full form, it is usually customized to a particular application using various approximations. Accurate modeling of runaway dynamics requires a relativistic collision operator which accounts for both small-angle and large-angle collisions, since the latter cause the avalanche effect. A further complication comes from the radiation losses that relativistic electrons experience. Meanwhile, the spatial dynamics of runaway electrons is often less important than momentum-space effects, and therefore, the approach in this thesis is to focus on momentum space and ignore spatial effects entirely.

This chapter illuminates the runaway phenomenon by considering the friction force on plasma particles. In order to obtain a kinetic equation for fast electrons, the kinetic equation in the previous chapter is specialized to a spatially homogeneous plasma. We introduce the individual terms in the kinetic equation for fast electrons, and present CODE, which is the numerical tool used here to solve the same equation.

3.1 The runaway phenomenon

The runaway mechanism originates from the non-monotonic property of the friction force, which is determined by ν_S^{ab} introduced in section 2.2.1.

Due to the small electron-to-ion mass ratio, the contribution from ν_S^{ei} can be neglected and the friction on electrons is entirely determined by the electron-electron slowing-down frequency. At non-relativistic bulk temperatures $T_e \ll m_e c^2$, the slowing-down frequency takes the following form[1] [62]

$$\nu_S^{ee} = \frac{n_e e^4 \ln\Lambda}{4\pi m_e \epsilon_0^2} \frac{2G(v/v_{\mathrm{Te}})}{p v_{\mathrm{Te}}^2}, \tag{3.1}$$

where $G(x)$ is the Chandrasekhar function

$$G(x) = \frac{\phi(x) - x\phi'(x)}{2x^2} \to \begin{cases} \frac{2x}{3\sqrt{\pi}}, & x \to 0 \\ \frac{1}{2x^2} & x \to \infty \end{cases}, \tag{3.2}$$

and $\phi(x) = (2/\sqrt{\pi}) \int_0^x e^{-y^2} dy$ is the error function. The Chandrasekhar function peaks at $x \approx 1$, and then monotonically decreases. Using the superthermal limit of the Chandrasekhar function in eq. (3.2), we can define a relativistic collision time

$$\tau_c = \left(\frac{n_e e^4 \ln\Lambda}{4\pi \epsilon_0^2 m_e^2 c^3} \right)^{-1}, \tag{3.3}$$

such that

$$\nu_S^{ee} \to \frac{m_e c}{p\beta^2} \tau_c^{-1}, \quad v \gg v_{Te}, \tag{3.4}$$

where $\beta = v/c$. Accordingly, the collisional friction force $F = p\nu_S^{ee}$ on fast electrons decreases with increasing momentum:

$$p\nu_S^{ee} \to \frac{m_e c}{\tau_c \beta^2} = \frac{eE_{\mathrm{c}}}{\beta^2}, \quad v \gg v_{Te}, \tag{3.5}$$

Here, we have introduced the critical electric field

$$E_{\mathrm{c}} = \frac{m_e c}{e\tau_c} = \frac{n_e e^3 \ln\Lambda}{4\pi\epsilon_0^2 m_e c^2}, \tag{3.6}$$

which corresponds to the case of force balance between electric-field acceleration and collisional friction for electrons moving parallel to the electric field at speed $v = c$. The critical electric field is therefore the threshold electric field above which the runaway process can occur.

[1] At relativistic bulk temperatures, the slowing-down frequency takes a more complicated form involving non-trivial integral expressions [60, 61]. However, the resulting friction force has the same qualitative behavior as in the non-relativistic bulk temperature limit.

In practice, the threshold field for runaway generation is higher than E_c for several reasons. Electrons moving with an angle θ to the electric field experience a force $\mathbf{F}_\mathrm{E} \cdot \hat{\mathbf{p}} = -eE\cos\theta$ projected in the $\hat{\mathbf{p}} = \mathbf{p}/p$ direction, and thus the electric field may need to significantly exceed E_c in order to balance out the widening of the distribution function caused by pitch-angle scattering. Moreover, the slowing-down force is increased at relativistic electron speeds due to the effect of synchrotron and bremsstrahlung radiation reaction, and partial screening contributes to enhanced collision rates in partially ionized plasmas. The enhancement of the critical electric field due to these three effects is the subject of Paper B, which shows that in the presence of partially ionized impurities, the effective critical field is approximately given by

$$E_\mathrm{c}^\mathrm{eff} \gtrsim E_\mathrm{c}^\mathrm{tot} = \frac{n_e^\mathrm{tot}}{n_e} E_\mathrm{c}, \tag{3.7}$$

where n_e^tot is the total electron density including free and bound electrons. Consequently, bound and free electrons contribute approximately equally to the critical electric field. This result is the combined effect of collisional friction and enhanced pitch-angle scattering, in synergy with radiation reaction losses.

The requirement that the electric field exceeds $E_\mathrm{c}^\mathrm{eff}$ is necessary but not sufficient for runaway generation. If the electric field is so weak that a negligible electron population experiences a net acceleration, the runaway generation will also be negligible. A significantly stronger electric field is therefore necessary to accelerate a substantial runaway tail starting from a Maxwellian distribution. The required electric field strength is set by the *Dreicer field* [63], which approximates the required electric field for *slide-away*, i.e. when the entire electron distribution is accelerated. To estimate the slide-away electric field, consider the maximum of the friction force in eq. (3.5)

$$\max[p\nu_S] = eE_\mathrm{c}\frac{m_e c^2}{T}\max[G(v/v_\mathrm{Te})] \approx 0.2eE_\mathrm{D}, \tag{3.8}$$

so the Dreicer field evaluates to $E_\mathrm{D} = E_\mathrm{c} m_e c^2/T$. If the electric field is close to this slide-away field, the entire electron distribution will be distorted and rapidly deviate from a Maxwellian. In contrast, if the electric field is a few percent of E_D, it will only accelerate the small fraction of particles that are sufficiently fast to experience a positive net force. Due to energy diffusion (described by $\nu_\parallel$ in section 2.2.1),

this region in momentum space will be continuously re-populated, which gives the distribution an energetic tail of runaway electrons [5]. This runaway population will constitute a small, steadily growing fraction of the electron population, while the majority of the distribution will remain close to the Maxwellian distribution. This mechanism of runaway production is known as Dreicer generation.

Even if the electric field is too weak for substantial Dreicer generation of the thermal population, runaway generation may still occur through two other generation mechanisms, provided that $E > E_c$. One of these effects is *hot-tail* generation, which takes place if the electron distribution cools down while it is subject to an electric field [43, 64–67]. Since the superthermal friction force decreases with velocity at subrelativistic speeds, the highly energetic tail of the distribution will be cooled at a slower rate than the thermal population, if the cooling is dominated by collisional processes. When this bulk cooling is accompanied by an electric field, as in a disruption, the hot-tail mechanism can produce a runaway seed population or even convert a large part of the initial current directly [67].

The third runaway generation mechanism is the avalanche mechanism [23, 43, 68–70]. This effect resembles snow avalanches not only by its rapid growth, but also by the need for a trigger, such as the first snowball. In the runaway case, this first snowball is a seed runaway population generated through either Dreicer generation, the hot-tail mechanism or – in future tokamak reactors – tritium decay or radiation from the first wall [71]. In the presence of a super-critical electric field, a seed population generated by any of these sources will grow exponentially in time through the avalanche effect. This avalanche is created when a runaway electron collides with a thermal electron and transfers enough momentum that both electrons run away. Large-angle collisions thereby provide a shortcut in momentum-space compared to the combination of collisional diffusion and electric field acceleration that gives Dreicer generation. In other words, if the runaway population is sufficiently large, this mechanism will dominate over Dreicer and hot-tail generation, despite the rarity of large-angle collisions in plasmas mentioned in the previous chapter. The small-angle Fokker–Planck operator must therefore be complemented by the large-angle Boltzmann operator in order to accurately describe runaway generation.

To summarize, there are three distinct electric-field regions with different fast-electron behavior:

(i) $E < E_c$: runaway decay,

(ii) $E_c < E \lesssim 0.2E_D$: runaway generation can occur through the Dreicer, hot-tail and/or avalanche mechanisms,

(iii) $E \gtrsim 0.2E_D$: slide-away.

Of these, the electric field is typically in range (ii) during runaway scenarios, and consequently this is the most important region to model from a runaway perspective. In this range [as well as range (i)], a linear collision operator is adequate as long as the runaway population is trace.

It is beneficial for modeling purposes, and for physical insight, to separate the three generation mechanisms and compare analytical predictions to numerical results. Specifically, analytical steady-state growth rates due to both the Dreicer [4, 63, 72, 73] and avalanche [23] mechanisms have been successfully benchmarked against kinetic simulations [37, 38, 70]. However, the analytical formulas neglect several effects including partial screening and spatial effects. Moreover, it is unknown how well the steady-state growth rates match the full numerical solution in dynamic scenarios which have time-dependent background parameters [45]. Compared to Dreicer and avalanche generation, modeling of the hot-tail mechanism is at an earlier stage; the complexity of the problem challenges analytical modeling, and analytical results [66] only have limited predictive capabilities [67]. Consequently, several aspects of the runaway generation rates remain unclear.

3.2 Kinetic equation for runaway electrons

In order to obtain a realistic yet simple kinetic equation for the runaway problem, we introduce new terms compared to the standard formulation of the kinetic equation (2.1), but also neglect certain aspects of the problem. The following section argues for such specialization of the kinetic equation in four aspects: adding the radiation reaction, accounting for large-angle collisions, using a linear collision operator and neglecting spatial effects. This approach has also been applied in the numerical

tool CODE, which has been used for numerical modeling in the present thesis and is described in the following section.

To model runaway electrons, the force in eq. (2.1) must include radiation losses. Synchrotron radiation is emitted as particles gyrate around magnetic field lines, and bremsstrahlung emission is a result of inelastic collisions with ions (not to be confused with inelastic collisions with bound electrons in a partially ionized plasma). Synchrotron radiation reaction can be modeled as a continuous momentum-dependent force [74–76], whereas bremsstrahlung is dominated by large-angle collisions and generally requires a Boltzmann operator in order to capture the effect on the runaway distribution [77]. As discussed in Paper B, both bremsstrahlung and synchrotron radiation reaction losses are stronger in partially ionized plasmas. Bremsstrahlung, which is fundamentally a collisional process, is enhanced due to the effect of screening and can be modeled with a form factor similarly to the Fokker–Planck operator in section 2.2.3. The enhancement of synchrotron radiation reaction is an indirect result; the synchrotron force remains unchanged but the increased pitch-angle scattering rate resulting from partial screening leads to stronger synchrotron emission.

As previously mentioned, a collision operator for runaway electrons must include both a small-angle Fokker–Planck operator and an avalanche operator, which describes the effect of large-angle collisions. Since the energy transfer required to produce a runaway electron typically far exceeds the binding energy, the analytic structure of the avalanche source is largely unaffected by collisions with partially ionized impurities. The only difference is that the multiplying density should include the free as well as the bound electrons, since they have equal probability of becoming runaway electrons.

Many runaway scenarios have a trace runaway population, since even a small fraction of ultrarelativistic particles are enough to carry the full initial plasma current. Full current conversion sets the upper limit of the runaway current due to induction, and thereby restricts the maximum runaway density, if the average runaway speed is close to the speed of light. Therefore, a linear collision operator – which accounts for the effect of screening – can well describe many aspects of runaway dynamics. Two interesting exceptions where a linear operator may be inadequate are if the distribution is strongly distorted during a thermal quench, or if the electric fields are comparable to the slide-away field introduced in

the previous section. Such scenarios can not be modeled by the linear collision operator employed in CODE, but require a numerical tool with a non-linear collision operator, such as NORSE [39].

A spatially independent model of runaway dynamics can correspond to, for example, a spatially homogeneous atmospheric plasma, or the motion of particles close to the magnetic axis of a tokamak. To understand the latter, we note that the rapid motion around the toroidally symmetric torus tends to quickly smooth any toroidal asymmetries. Moreover, the equation can be *gyro-averaged* over the helical gyro-motion around the magnetic field lines, and *orbit-averaged* over the poloidal angle. The latter is the angle encircling the torus from its outboard to its inboard side and back, in which particles with sufficiently large magnetic moment $\mu_\mathrm{B} = p_\perp^2/(2mB)$ are *magnetically trapped* and make a "bouncing" motion. These averaging processes leave a three-dimensional system in the variables $\{p_\parallel, p_\perp, r\}$, which can be solved by numerical tools such as LUKE [40–42] and CQL3D [43, 44] (parallel and perpendicular are defined here with respect to the local magnetic field direction). In the limit of small radial variation over each gyration or poloidal orbit, the spatial dependence is limited to the radial variation of the background parameters. This can be weak if we furthermore assume that the runaway electrons are located at a large aspect ratio, i.e. their distance to the magnetic axis is small in relation to the major radius of the tokamak. In this sense, the spatially homogeneous kinetic equation can be regarded as the large-aspect-ratio limit of the orbit- and gyro-averaged kinetic equation. Such an equation undoubtedly neglects certain aspects of runaway dynamics; for example, the particles trapped in the magnetic well will be prevented from experiencing runaway acceleration. Nevertheless, the simplified spatially independent equation may still capture the dominating generation and loss mechanisms for runaway electrons and therefore offers the opportunity to study these mechanisms in detail.

With the considerations above, we obtain an equation which describes the essential effects of runaway dynamics. We introduce the cosine of the pitch angle $\xi = \cos\theta = \mathbf{p}\cdot\mathbf{B}/(pB)$ and let E be the component of the electric field antiparallel to the magnetic field $\mathbf{B}$ (so that electrons

are accelerated in the positive ξ direction).[2] After a transformation to the $\{p, \xi\}$ coordinate system, we arrive at the kinetic equation as it is solved in CODE:

$$\frac{\partial f_e}{\partial t} + \underbrace{eE\left(\xi\frac{\partial f_e}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f_e}{\partial \xi}\right)}_{\text{electric field}} = \underbrace{C_{\mathrm{FP}} + C_{\mathrm{ava}}}_{\text{collisions}} + \underbrace{C_{\mathrm{br}} - \nabla_k\left(F^k_{\mathrm{syn}} f_e\right)}_{\text{radiation reaction}}. \tag{3.9}$$

The form of the linear Fokker–Planck collision operator C_{FP} was given in eq. (2.13), where the effect of screening is accounted for as described in section 2.2.3, which was detailed in Papers A and C. The Lorentz scattering operator can be simplified in our gyro-averaged system:

$$\mathscr{L} = \frac{1}{2}\frac{\partial}{\partial \xi}\left(1-\xi^2\right)\frac{\partial}{\partial \xi}. \tag{3.10}$$

Avalanche losses are described by C_{ava}, and radiation losses are modeled by C_{br} (the bremsstrahlung collision operator) and $\mathbf{F}_{\mathrm{syn}}$ (the synchrotron radiation reaction force). Additionally, eq. (3.9) can be supplemented by terms describing sources of energy and momentum to the system (such as terms to account for the spatial dynamics).

3.3 Numerical solution of the kinetic equation with CODE

While certain aspects of runaway dynamics can be understood directly from the kinetic equation and by analytic equations, quantitative results typically rely on numerical calculations. This thesis is no exception: In Papers A-C the numerical tool CODE (COllisional Distribution of Electrons) [37, 38] has an important role.

CODE solves the kinetic equation in a uniform cylinder as given in eq. (3.9). It calculates the time-evolving electron distribution function f_e under the influence of electric-field acceleration, collisions and radiation reaction, as well as an externally set background temperature and

[2] In a tokamak, particles are accelerated in the direction parallel to the *magnetic* field, since the motion caused by the orthogonal electric field will cancel by design of the tokamak. In a magnetized plasma, E thus refers to the component of the electric field that is antiparallel to the magnetic field. In a non-magnetized atmospheric plasma, ξ is instead defined by the angle to the electric field; $\xi = -\mathbf{p}\cdot\mathbf{E}/(pE)$.

plasma composition. To model prescribed density and temperature evolution, particle and heat sources are added to the kinetic equation (3.9). By using a linearized collision operator, CODE can only handle a trace runaway population which may, however, give a non-trace contribution to the plasma current. Since modeling of non-trace runaway currents requires a self-consistent electric field evolution, the electric field in CODE can either be set externally or determined self-consistently by a zero-dimensional inductive model.

CODE is equipped with linear operators for both small-angle (Fokker–Planck) and large-angle (Boltzmann) collisions. The Fokker–Planck operator includes a test-particle collision operator based on that given in Ref. [78], which was constructed from an asymptotic matching between the non-relativistic collision operator for the thermal population [54] and the relativistic superthermal operator [62]. In accordance with the assumption of a non-relativistic bulk population, the Fokker–Planck operator also contains a non-relativistic field-particle operator [37]. The possibility to model collisions with partially ionized impurities was added to the Fokker–Planck test-particle operator in conjunction with Paper A. CODE also contains several avalanche source term options. In its most advanced form, the implemented avalanche operator is fully conservative, which means that the runaway-generating field-particle term (see section 2.2.2) is combined with a test-particle term, which describes how the runaway-electrons are deflected by large-angle collisions [70]. A Boltzmann operator is also employed for bremsstrahlung radiation reaction [77], while synchrotron radiation reaction is modeled as a continuous force [37].

As for its numerical implementation, CODE is a continuum code. Since the pitch-angle dependence of the Fokker–Planck collision operator is diagonal in a Legendre polynomial basis [54], the kinetic equation is discretized in N_ξ Legendre modes. This is combined with a fourth-order finite difference scheme in the p variable, with N_p non-uniformly spaced grid points. The kinetic equation is thus represented on a $N_pN_\xi \times N_pN_\xi$ grid. The Fokker–Planck collision operator, the electric field terms and synchrotron radiation together form a sparse matrix and are treated implicitly in time, but the Boltzmann integral operators for avalanche and bremsstrahlung radiation reaction constitute dense matrices and are consequently treated explicitly for computational efficiency.

The computational power needed for a CODE simulation heavily depends on the problem. On one hand, a simple Dreicer growth rate scenario requires a fraction of a second on a laptop. On the other hand, a self-consistent electric-field simulation with an advanced avalanche operator including both bremsstrahlung and synchrotron radiation losses may require hundreds of CPU hours using more than 100 GB of RAM.

The present thesis contains the results of both light and computationally demanding CODE calculations. Along with an outline of analytical results on the collision frequencies and the effective critial elecric field, the results of these simulations will be summarized in the final chapter.

Chapter 4

Summary and outlook

Accurate fast-electron modeling is essential for the understanding and control of runaway electrons. Consequently, it is an active area in tokamak fusion research, where runaway electrons have the potential to do severe damage. As impurity injection is the most promising mitigation method in this context, it is crucial to be able to model the runaway-impurity interaction in a cold, post-disruption plasma, where the impurities are only partially ionized. Therefore, the effect of screening of nuclei by bound electrons must be accounted for.

This thesis focuses on deriving a collision operator for a partially ionized plasma, and investigates several aspects of runaway-electron dynamics in such plasmas. The present chapter summarizes the main findings of the appended papers, followed by a few possible directions in which the present work can be extended.

4.1 Summary of papers

In the papers attached, we have systematically investigated the effect of screening on the momentum-space dynamics of fast electrons. By combining analytical derivations and numerical simulations using the tool CODE, we have built up a qualitative as well as quantitative understanding of partial screening.

In Paper A, we initially presented the partially ionized collision operator, which includes a quantum-mechanical description of both elastic

collisions with the partially screened nuclei, and of inelastic collisions with the bound electrons. The former process requires knowledge of the electronic charge density, which we obtained from density functional theory calculations. From the numerical charge density, we constructed an analytical model of the elastic scattering rate by generalizing a previously used formula for the cross section [12]. By matching the scattering rates with bound electrons to the low-energy limit, we obtained a self-contained collision operator. This operator can therefore be straightforwardly implemented in numerical tools, as has been done recently [79] in the particle-orbit-following tool KORC [47].

Our analytical results indicated that a detailed model of the fast-electron dynamics requires the usage of the partially screened collision operator. We found that neither complete screening (i.e. treating the ion as a particle with the net charge) nor no screening (treating the bound electrons and the nucleus independently) give good approximations of the collision frequencies; complete screening gives a significant underestimation, whereas no screening provides a considerable overestimation of the collision frequencies. This picture was supported by kinetic simulations of runaway-electron dynamics in a partially ionized plasma. These were obtained using CODE, upgraded to include the partially ionized collision operator described, and were also presented in Paper A. We investigated the evolution of a runaway beam in a constant electric field and found screening gave significant changes to the shape of the runaway distribution. Enhanced scattering rates also led to a more rapid runaway current decay in a constant electric field compared to previous models.

In Paper B, the partially screened collision operator was applied to the special problem of the critical electric field. The critical electric field is not only important fundamentally since it is the threshold field above which the runaway generation mechanism occurs, but also because it sets the decay rate of a runaway beam in an inductive device, such as a tokamak. The latter is particularly suitable for experimental validation; it is one of few quantities that are relatively straightforward to both predict from kinetic theory and diagnose experimentally. The critical electric field is therefore a useful indicator by which to evaluate the current status of momentum-space runaway modeling. In Paper B, we employed the approximation of fast pitch-angle dynamics [7, 80] to calculate the critical electric field. This resulted in an analytical formula for the effective critical field. This formula was demonstrated to agree within a

few percent with numerical results from CODE, which was extended to include partial screening in the bremsstrahlung operator and the avalanche source as described in section 3.2. The effective critical field was seen to be significantly enhanced compared to previous estimations, and even exceeded the value obtained by replacing the electron density in the conventional formula by the total electron density (including bound and free electrons). We also found that bremsstrahlung and synchrotron losses individually elevate the effective critical electric field by tens of percent. Additionally, we used CODE to verify the prediction that the induced electric field is close to the effective critical electric field during runaway in high-current inductive devices.

While Paper A introduced the collision operator and presented the initial studies, the main purpose of Paper C was to present the full derivation of the collision operator and investigate several subtle issues that arose. Specifically, we compared the Fokker–Planck collision operator with the more advanced Boltzmann operator. We found that the runaway distributions produced with our partially-screened collision operator and the full Boltzmann operator had negligible differences in all key runaway parameters such as runaway current and density, although the synchrotron spectrum was somewhat different in shape at large electric fields. Consequently, our generalized collision operator is adequate for most purposes of runaway-electron studies in tokamaks.

The partially screened collision operator contains a parameter which can be interpreted as a length scale related to the ion radius. While Paper A presented these constants for a few ionization states, Paper C calculated the constants for a wide range of ion species and compared them to previous models. We found that a previously used simplified model without any free parameters [12] gave acceptable accuracy for use in the collision operator. This will facilitate future modeling as it reduces the need for computationally heavy density functional theory simulations.

Finally, Paper C investigated the steady-state avalanche growth rate in the presence of partially ionized impurities. Unlike the results in Paper A and Paper B, where the increased collisional rates enhanced the runaway dissipation rates and decreased the near-critical growth rate, it was found that the steady-state growth rate at high electric fields ($E \gtrsim 30E_c^{\mathrm{eff}}$) was enhanced by partially screened nuclei. This is

because the bound electrons lead to a stronger avalanche source which dominates over the increased collision rates.

In summary, we found that an accurate treatment of partial screening can have large and rather non-trivial effects on runaway-electron momentum-space dynamics. These results take us one step forward in understanding how runaway electrons can be affected by material injection. However, a full understanding requires these momentum-space effects be combined with spatial effects, as well as a time evolution of the impurity ionization states in the background plasma [13, 67, 81]. Possible steps toward such a complete model are discussed in the following section.

4.2 Outlook

Now that screening effects have been included in the kinetic equation, the runaway model in CODE includes the most important pure momentum-space effects, which do not depend explicitly on the variations of the background plasma. Having explored the main momentum-space features of runaway interaction with partially ionized ions, the natural continuation of this work is to combine screening and other momentum-space effects, recently characterized and studied with CODE [37, 38, 70, 77, 82], with further effects that are crucial for predictive modeling of runaway dynamics in tokamaks.

Current computational resources do not allow a simultaneous treatment of the full kinetic problem together with the spatial dependence and the evolution of the electromagnetic fields, implying that several approximate approaches can usefully offer complementary perspectives to understand runaway-electron dynamics. One possible approach is reduced kinetic modeling, where the momentum-space runaway dynamics can be replaced by approximate formulas for the steady-state runaway growth rates describing the hot-tail, Dreicer and avalanche mechanisms as a function of background parameters. This approach has been extensively used previously [5, 46, 71, 81, 83, 84], but accurate models of, for example, the effect of partial screening and an energy-dependent Coulomb logarithm have not been included in these growth rate formulas. Furthermore, the applicability of the steady-state formulas in dynamic scenarios has not been thoroughly studied. In scenarios where

the steady-state growth rates are accurate, reduced kinetic modeling would be useful to improve the understanding of runaway dynamics. A systematic investigation of the applicability of steady-state growth rates, and determining formulas for these in the presence of different ion species, would therefore be a highly relevant usage of CODE.

Although many aspects of runaway dynamics can be studied with reduced kinetic modeling as described above, certain effects require a simultaneous treatment of momentum and real space. One example where geometrical effects are important for the momentum dynamics is the effect of magnetic trapping, which heavily influences runaway electrons that are not located close to the magnetic axis of the tokamak [29]. The combined effect of screening and trapping could be investigated by for instance incorporating the partially screened collision operator into LUKE [40–42]. Another effect producing an interplay between spatial and momentum-space effects is turbulent transport, which depends on runaway energy [85]. A transport model has recently been implemented in both CODE and LUKE, but it has not yet been investigated in the presence of partially ionized impurities and would therefore be a suitable topic for future research.

Several possible routes for future research could undoubtedly be added to the list above. Nevertheless, any of the suggested topics would build upon the results in this thesis and offer valuable insights into runaway dynamics. In the longer run, it may contribute to improved runaway mitigation schemes which would be one milestone toward operational fusion power plants.

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