

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

# Development and evaluation of methods for control of multiple-input multiple-output systems

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# Abstract

In control, the most common type of system is the multiple-input multiple-output (MIMO) system, where the same input may affect multiple outputs, or conversely, the same output is affected by multiple inputs. In this thesis two methods for controlling MIMO systems are examined, namely linear quadratic Gaussian (LQG) control and decentralized control, and some of the difficulties associated with them.

One difficulty when implementing decentralized control is to decide which inputs should control which outputs, that is the input-output pairing problem. There are multiple ways to solve this problem, among them using gramian based measures, which include the Hankel interaction index array, the participation matrix and the  $\Sigma_2$  method. These methods take into account system dynamics as opposed to many other methods which only consider the steady-state system. However, the gramian based methods have issues with input and output scaling. Generally, this is resolved by scaling all inputs and outputs to have equal range. However, in this thesis it is demonstrated how this can result in an incorrect pairing. Furthermore this thesis examines other methods of scaling the gramian based measures, using either row or column sums, or by utilizing the Sinkhorn-Knopp algorithm. This thesis shows that there are considerable benefits to be gained from the alternative scaling of the gramian based measures, especially when using the Sinkhorn-Knopp algorithm. The use of this method also has the advantage that the results are completely independent of the original scaling of the inputs and outputs.

An alternative way to control a MIMO system is to implement an LQG controller, which yields a single control structure for the entire system using a state based controller. It has been proposed that LQG control can be an effective control scheme to be used on networked control systems with wireless channels. These channels have a tendency to be unreliable with package delays and package losses. This licentiate thesis examines how to implement an LQG controller over such unreliable communication channels, and proposes an optimal controller which minimizes the cost function.

When new methods of control system design and analysis are introduced in the control engineering field, it is important to compare the new results with existing methods. Often this requires application of the methods on examples, and for this purpose benchmark processes are introduced. However, in many areas of control engineering research the number of examples are relatively few, in particular when MIMO systems are considered. For a thorough assessment

## ABSTRACT

of a method, however, as large number of relevant models as possible should be used. As a remedy, a framework has been developed for generating linear MIMO models based on predefined system properties, such as model type, size, stability, time constants, delays etc. This MIMO generator, which is presented in this thesis, is demonstrated by using it to evaluate the previously described scaling methods for the gramian based pairing methods.

**Keywords:** Control configuration selection, Decentralized control, Gramian based measures, Input-output scaling, LQG control, Unreliable communication links, Delays, Hold-input, MIMO systems.

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Göteborg, October 2018



# List of publications

This thesis is based on the following four appended papers.

## Paper I

Fredrik Bengtsson, Babak Hassibi, and Torsten Wik. LQG control for systems with random unbounded communication delay. In *Proceeding of the 55th Conference on Decision and Control (CDC)*, pages 1048-1055. IEEE, 2016.

## Paper II

Fredrik Bengtsson and Torsten Wik. LQG control over unreliable communication links. *To be submitted.*

## Paper III

Fredrik Bengtsson and Torsten Wik. A multiple input, multiple output model generator. Technical report, Department of Signals and Systems, Chalmers University of Technology, 2017.

## Paper IV

Fredrik Bengtsson, Torsten Wik, and Elin Svensson. Resolving issues of scaling for gramian based input-output pairing methods. *To be submitted.*





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# **Part I**

## **Introductory chapters**



# Chapter 1

## Introduction

A common issue in industrial processes is that interaction between different parts of the plant gives rise to a multiple-input multiple-output (MIMO) system, where the same input may affect multiple outputs, or conversely, the same output is affected by multiple inputs. Such interactions make MIMO systems are considerably more complex to control than single input single output (SISO) systems [1].

While there are numerous ways to control MIMO systems, the focus here is on two methods, decentralized control and linear quadratic Gaussian (LQG) control, and ways to solve some of the problems associated with these strategies. Moreover this licentiate thesis discusses tools and methods to evaluate the control of MIMO systems.

One method to control a MIMO system is to divide it into subsystems of one input and one output and implement SISO controllers for each of the subsystems. This control strategy is called decentralized control and remains widely used in industry [2]. It has several advantages compared to implementing a MIMO controller for the entire system, as it allows the use of relatively easy to design low dimensional controllers. Moreover it is less vulnerable to sensor and actuator failures than more complex control schemes that try to control the entire system with one overarching control scheme. However, a decentralized control scheme leads to the input-output pairing problem: which inputs should be used to control which outputs to best fulfill the control objectives?

Numerous methods have been proposed to find a suitable input-output pairing, many of which are discussed in [3]. The most widely used is the Relative Gain Array (RGA) [4] and modifications of it, such as the dynamic RGA and the Relative Interaction Array (RIA) [5]. Relatively recently a new group of input-output pairing methods have been introduced, namely the gramian based methods. This group includes the  $\Sigma_2$  method [6], the participation matrix (PM) [7] and the Hankel interaction index array (HIIA) [8]. These methods use the controllability and observability gramians to create an interaction matrix which gives a gauge of how much each input affects each output. An attractive property of

these interaction matrices is that they can be used to determine both a decentralized controller structure and a sparse structure (a structure which includes feed-forward or MIMO blocks). Moreover, the gramian based measures take into account system dynamics and not only the steady state properties of the system.

The gramian based methods, however, differ from the RGA and its variants in that they suffer from issues of scaling, in the sense that the results of the methods vary depending on input and output scaling. There is a commonly suggested method to solve this problem, presented in for example [9]. However, in paper IV we demonstrate that this method is insufficient in some situations. We then proceed to propose a new method of scaling, based on the Sinkhorn-Knopp algorithm [10], which removes the problems of scaling dependency.

Another control scheme which can be used to control MIMO systems is LQG control. This is a well established method developed in the 1960s which aims to find the optimal control scheme to minimize a quadratic cost function. While LQG control was relatively quickly adopted for the control of ships and space vehicles, the process industry was generally slow to adopt LQG control [11]. However as industry has become more interested in use of networked control systems to perform remote control of factories [12], control over wireless channels is an issue that has risen into prominence. Here LQG control is one of the proposed methods to carry out control in such situations [13] and in this licentiate thesis how to optimally implement LQG control over unreliable channels is examined.

Wireless communication is prone to issues of package losses and delays, which poses difficulties when implementing control schemes. In Paper I we examine how to optimally implement LQG control in the case where there is such a random unbounded delay between the controller and actuator and in Paper II we expand this to cover general unbounded delays and package losses.

When new methods of design and analysis are introduced in the control engineering field, it is important to compare the new results with those of existing methods, yet it is not always apparent how this can be accomplished in an unbiased and consistent way. To address this, in Paper III we propose a MIMO system generator, which allows for the creation of a large number of random MIMO systems with user defined properties. In Paper IV we demonstrate how the MIMO generator can be used to perform statistical analysis for evaluation of new methods to compare their results with those of existing methods.

The thesis is organized as follows: In Chapter 2 the control configuration problem is presented, and common methods to find an input-output pairing are discussed with special focus on the gramian based pairing measures. In Chapter 3 the difficulties the gramian based pairing measures have with input and output scaling is discussed, along with possible methods to resolve this issue. In Chap-



ter 4 methods to evaluate and compare new methods are discussed. In Chapter 5 LQG control is described and in Chapter 6 control of systems with delay is discussed. In Chapter 7 the papers included in this thesis are summarized and in Chapter 8 possible future work is discussed.

## **Main contributions**

The main contributions in this thesis are as follows:

1. A new method for scaling the gramian based input output pairing methods is proposed, which removes the scaling dependency of the gramian based measures.
2. The construction of a multiple-input multiple-output (MIMO) system generator, which can be used to evaluate and compare control methods.
3. The derivation of optimal LQG control in the case where there are unbounded delays and package losses in the communication channel between the actuator and controller.



# Chapter 2

## Control configuration selection

A key property of integrated plants is that they tend to have numerous outputs (controlled variables) and numerous possible inputs (manipulated variables). There are two basic strategies that can be implemented here, either one can treat the control of the entire system as one control problem and design a control scheme for the entire system, using a multiple-input multiple-output control strategy such as for example model predictive control (MPC), or one can divide the system into subsystems and design a separate control scheme for each subsystem. While designing a control scheme for the entire system may yield the best solution in theory, this solution also tends to be complex to implement as it generally requires a good model of the entire system. Furthermore a single actuator or sensor failure may jeopardize the entire control scheme.

Splitting the system into subsystems generally alleviates this problem as each subsystem has a control scheme designed independently of the other subsystems. An extreme case of this is the decentralized control structure, where the system is divided into subsystems of one input and one output. This method is commonly used in industrial processes as it is straightforward to implement using simple PI or PID controllers.

However, to implement a decentralized control structure two problems need to be resolved. Firstly, if there are more inputs than outputs available, decisions have to be made regarding which inputs will not be used (as each output here is controlled only by one input). When this is done one needs to decide which input is to control which output. This is known as the input-output pairing problem and is the focus of this part of the licentiate (it is assumed the decision of which inputs to use has already been made).

## 2.1 Input-output pairing problem

As previously stated, the input-output pairing problem consists of choosing which input should control which output using a decentralized control scheme. While in industry this is still sometimes done using rules of thumb and experience [14], there are pairing methods which give systematic ways to determine the input-output pairings. These pairing methods analyze some properties of the system and from there find a recommended pairing. While these methods often find pairings which allow for good control, there are no guarantees of optimality from any of the methods as there is no definition of what an optimal pairing may be. Moreover, different pairing methods may give different recommended pairings, which presents additional difficulties when determining which pairing to use.

## 2.2 The transfer function matrix

To use most pairing methods the MIMO system is defined using its transfer function matrix (TFM) which describes the interactions between the outputs and inputs of a MIMO system as:

$$Y = G(s)U \quad (2.1)$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1N}(s) \\ g_{21}(s) & g_{22}(s) & & \\ \vdots & & \ddots & \\ g_{N1}(s) & & & g_{NN}(s) \end{bmatrix} \quad (2.2)$$

with  $y_1, \dots, y_N$  being the systems outputs,  $u_1, \dots, u_N$  being the systems inputs and  $G(s)$  is the TFM of the system.

## 2.3 RGA

The most common pairing method is the RGA [4], which determines a pairing by comparing the open loop and closed loop properties of the system. It is calculated from the static gain of the system's TFM as

$$RGA = G(0) \circ G(0)^{-T}$$

with  $-T$  being the inverse transpose of the matrix and  $\circ$  denotes element-wise multiplication. To find a pairing from the RGA matrix one selects a pairing with elements closest to 1, while avoiding negative elements. Explicitly, if the element of row  $i$  and column  $j$  in the RGA is close to 1, then  $u_j$  should be used to control output  $y_i$ .

An important property of the RGA is that it is scaling independent, which means it gives the same results regardless of the scaling of the outputs and inputs. It, however, has a few limitations, one of which is that it only takes into account two way interaction. As a consequence interactions from a triangular TFM would not appear in the RGA. Moreover, the static RGA does not take into account system dynamics including delays. However, it can be expanded with the dynamic RGA which examines a frequency range rather than the zero frequency. The dynamic RGA though is based on the assumption of perfect closed loop control for all frequencies it covers, which is unrealistic for high frequencies [14].

## 2.4 Gramian based measures

Another group of input-output pairing methods which will now be examined and henceforth be referred to as the gramian based measures are the  $\Sigma_2$  method, the participation matrix (PM) and the Hankel interaction index array (HIIA). These methods examine each of the transfer functions of the TFM separately to gauge the impact of each input on each output. Unlike the static RGA they take into account the system's dynamics and not only its steady state properties. The gramian based measures (PM, HIIA and  $\Sigma_2$ ) can be calculated from a system's TFM [6–8]. Given a TFM as described in (2.2) each measure generates an interaction matrix (IM), with

$$[IM]_{ij} = \frac{\|g_{ij}(s)\|}{\sum_{kl} \|g_{kl}(s)\|},$$

using the Hilbert-Schmidt norm, Hankel norm or  $\mathcal{H}_2$ -norm for the PM, HIIA and  $\Sigma_2$ , respectively.

### 2.4.1 Hilbert-Schmidt norm and Hankel norm

The Hilbert-Schmidt norm and Hankel norm both utilize the Hankel singular values (HSV) of the system. These are defined as:

$$\sigma_H^{(i)} = \sqrt{\lambda_i},$$

where  $\lambda_1, \dots, \lambda_n$  are the eigenvalues of  $PQ$ , with  $P$  being the controllability gramian and  $Q$  being the observability gramian. So this is a gauge of the controllability and observability of the system. The Hilbert-Schmidt norm is the sum of the HSV of the system, while the Hankel norm is the maximum HSV.

### 2.4.2 $\mathcal{H}_2$ norm

The  $\mathcal{H}_2$  norm, which is used for the  $\Sigma_2$  method can be written as

$$\|g_{ij}(s)\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |g_{ij}(j\omega)|^2 d\omega}$$

It is proportional to the integral of the squared magnitude of the bode plot, and can be seen as a measure of the energy in the impulse response.

### 2.4.3 Determining the pairing

For the gramian based measures the generated IM is used to determine the pairing. With an interaction matrix of

$$IM = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1N} \\ \vdots & \ddots & \vdots \\ \gamma_{N1} & \cdots & \gamma_{NN} \end{bmatrix} \quad (2.3)$$

the pairing that has the largest total interaction from the IM is preferred. For instance, a diagonal pairing matching  $u_1$  with  $y_1$ ,  $u_2$  with  $y_2$  etc, would have a total interaction of

$$\sum_{i=1}^N \gamma_{ii}$$

while an anti-diagonal pairing would have a total interaction of

$$\sum_{i=1}^N \gamma_{(N+1-i)i}$$

When an initial pairing has been determined, the control structure can be expanded to include feedforward by selecting additional elements from the IM. So if  $\gamma_{1N}$  is large but not included in the original pairing one can still include the interaction by using feedforward on the control of  $y_1$  to compensate for the impact of  $u_N$ .





# Chapter 3

## Scaling the gramian based measures

The gramian based measures are based on various norms. Norms have the property that

$$\|\alpha g_{ij}(s)\| = |\alpha| \|g_{ij}(s)\|,$$

where  $\alpha$  is a scalar constant.

This means that for the gramian based measures input and output scaling will affect the recommended pairing. For example, if one would have a system as in (2.2) which would yield the IM (2.3) and one was to change the scaling of the first input by  $\alpha$ , so the scaled TFM would become:

$$G^*(s) = \begin{bmatrix} \alpha g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ \alpha g_{21}(s) & g_{22}(s) & & \\ \vdots & & \ddots & \\ \alpha g_{n1}(s) & & & g_{nn}(s) \end{bmatrix}, \quad (3.1)$$

which in turn would yield an IM:

$$IM^* \propto \begin{bmatrix} |\alpha| \gamma_{11} & \cdots & \gamma_{1N} \\ \vdots & \ddots & \vdots \\ |\alpha| \gamma_{N1} & \cdots & \gamma_{NN} \end{bmatrix} \quad (3.2)$$

This IM may yield a different recommended pairing than (2.3) depending on  $\alpha$ . Consequently, as different scalings of the system may yield different results, emphasis needs to be placed on how to best scale the system when using one of the gramian based measures to find a pairing. Generally, this is resolved by scaling the inputs and outputs from 0 to 1, setting zero to the lowest value they are likely to reach and 1 to the highest value [9]. However there are a few other methods to scale the system, which will be discussed below.

### 3.1 Row or column scaling

Each column in the IM corresponds to the interactions from one input, while each row corresponds to the interactions affecting one output. One way to scale the system prior to pairing is to divide the elements in each column of the IM by the corresponding column sum. This was presented in [15] for the  $\Sigma_2$  method and ensures that when conducting the pairing algorithm, equal importance is given to each input. In the new IM the scaled elements would become:

$$[IM_c]_{ij} = \frac{[IM]_{ij}}{\sum_{k=1}^N [IM]_{kj}},$$

where  $IM_c$  is an interaction matrix with normalized columns. If one instead wishes to ensure that equal importance is given to each output, one could instead chose to normalize the rows, which gives an interaction measure defined by

$$[IM_r]_{ij} = \frac{[IM]_{ij}}{\sum_{k=1}^N [IM]_{ik}}.$$

### 3.2 Sinkhorn-Knopp algorithm

By scaling the columns or rows one can guarantee that equal importance is given to either each input or each output when determining the pairing. However, if one wishes to have both the columns and rows scaled, the Sinkhorn-Knopp algorithm can be used. This algorithm combines row and column scaling by alternating between normalizing the rows and normalizing the columns. In cases where the matrix can be made to have positive elements on the diagonal (as is always the case with gramian based measures) this algorithm is guaranteed to converge to a matrix that will have both rows and columns normalized [10]. While the Sinkhorn-Knopp algorithm can be implemented by simply alternating between dividing the elements in each column of the IM by the corresponding column sum and dividing the elements in each row by the corresponding row sum, it can also be implemented as described in [16]:

$$\begin{aligned} r_0 &= e \\ c_{k+1} &= \mathcal{D}(IM^T r_k)^{-1} e \\ r_{k+1} &= \mathcal{D}(IM c_{k+1})^{-1} e, \end{aligned}$$

where  $e$  is a vector of ones, and  $\mathcal{D}(x)$  turns a vector into a diagonal matrix by creating a matrix with the elements of the vector on its diagonal and zeros in all remaining positions. The scaled IM then becomes:

$$IM_{SK} = \mathcal{D}(r)IM\mathcal{D}(c).$$

To calculate how far the solution is from being perfectly scaled (that is having both column and row sums equal one), one can use the following formula [16]:

$$err_k = \|c_k \circ \mathcal{D}(c_{k+1})^{-1} - e\|_1,$$

where  $\circ$  denotes element-wise multiplication. This can be used as a stopping criterion.

Scaling the IMs with the Sinkhorn-Knopp algorithm has the additional benefit of removing the impact of input and output scaling on the IMs. Using the Sinkhorn-Knopp algorithm to scale the system will yield the same IM, regardless of what the original scaling of the system was.

In Paper IV, we compare the Sinkhorn-Knopp scaling with alternative scalings on a large number of randomly generated MIMO systems and find that it performs significantly better.



# Chapter 4

## Evaluation of control methods

Whenever a new method, or a change to an existing method is proposed, it needs to be evaluated to determine if it offers a significant improvement. Moreover, in cases where there are numerous competing methods to solve the same problem (such as the input-output pairing problem) there is a need of a way to compare the methods and determine for which types of system each method is preferable.

### 4.1 Generating system models for evaluation

When analyzing new methods for control system design, it is common to demonstrate their benefits on one or a few example systems. While this is a useful way to demonstrate a new method, it does not easily allow for general conclusions of the strengths and limitations of the new method. To do this it would be beneficial to implement the method on a large number of systems with varying properties. For single-input single-output system a large batch of process models have been collected for such evaluations [17], but there is no similar batch for MIMO systems.

In Paper III we present a MIMO model generator which allows for the generation of a large number of MIMO systems to enable comprehensive testing on MIMO systems. The MIMO model generator generates TFMs with predefined properties such as system size, stability, time constants, delays etc. It is implemented in MATLAB and the code is freely available [18].

### 4.2 Determining a cost

To compare different methods there needs to be a method to evaluate how well the control performs on a given system. A well established method to assess the performance of control systems is to evaluate its response to reference steps and

to various types of disturbances by integrating the squared deviation from the reference, i.e.

$$c = \int_0^T (R(t) - Y(t))^2 dt$$

where  $c$  is the derived cost, and  $R(t)$  is a vector containing the reference signals. Typical disturbances one may test would be step disturbances on the inputs and high frequency noise on the outputs. This cost can be expanded to include a cost on the control inputs, for example

$$c = \int_0^T (R(t) - Y(t))Q_1(R(t) - Y(t)) + U(t)Q_2U(t)dt$$

where  $Q_1$  and  $Q_2$  are user defined matrices, used to weight the different parts of the cost.

### 4.3 Comparison of costs

While this cost works well to evaluate different controllers on one single system, for a thorough comparison of control methods one would need to evaluate more than one system. However the cost are not immediately comparable between different systems, as the systems may be of different scale and of varying difficulty to control. To allow comparison between different systems the costs can be normalized for each control configuration on the system using the following equation to produce a score for each configuration:

$$S = \frac{c_{min}}{c},$$

where  $S$  is the score of the configuration,  $c$  is the configuration's cost, and  $c_{min}$  is the lowest cost of all configurations for the system. This ensures that each configuration has a score from 0 to 1 for each system, which allows comparisons to be made for the result on different systems.

### 4.4 Controller Tuning

When performing evaluations and comparisons in cases where controller design is not the focus of the evaluation, for instance in cases of input output pairing, controllers needs to be implemented in a generalized and consistent way that yields reasonable results without favoring one method over the other. There are numerous methods to design PID controllers automatically, some of which are discussed in [19]. The method that will be discussed here is the lambda method. This is one of the most common methods for commercial auto-tuners [20], so it is

a reasonable method to be used for comparison purposes (as it is a method fairly likely to be applied when the control system is implemented in the industry). The lambda method [21] is a two step procedure where the first step is to approximate the transfer function by a first order system with dead time, i.e

$$G^*(s) = \frac{K}{1 + Ts} e^{-Ls}.$$

To derive a PI controller

$$C(s) = K_p \left( 1 + \frac{1}{T_i s} \right)$$

where the controller parameters are derived from  $G^*(s)$  according to

$$\begin{aligned} K_p &= \frac{1}{K} \frac{T}{L + \lambda} \\ T_i &= T \end{aligned}$$

and  $\lambda$  is the targeted time constant of the closed loop system. Every step in implementing this control scheme can be done automatically, and hence does not require any user input which may add bias to the results.





# Chapter 5

## LQG control

LQG control is a well established controller design method applicable to MIMO systems. It is based on control of linear systems, defined as

$$\begin{aligned}x(t+1) &= Ax(t) + Bu(t) + w(t) \\y(t+1) &= Cx(t) + e(t),\end{aligned}$$

were  $w(t)$  and  $e(t)$  are Gaussian model noise and measurement noise respectively. LQG control yields a full multi-variable controller where all the inputs are used to control all the states. It is based on finding the control input that minimizes a cost function similar to what is described in Chapter 4.2, namely

$$J_N = \min_u E \left[ \sum_{i=0}^N u_i^T Q u_i + \sum_{i=0}^N x_i^T R x_i + x_{N+1}^T S_{N+1} x_{N+1} \right] \quad (5.1)$$

where  $Q$  is a positive definite symmetric matrix and  $R$  and  $S_{N+1}$  are positive semi-definite symmetric matrices.

To derive the optimal solution dynamic programming is used. This means that first the  $u_N$  that minimizes the cost function (5.1) is found, then for the remaining cost a control signal  $u_{N-1}$  that minimizes this cost is found. After this,  $u_{N-2}$  is found to minimize the now remaining cost. This is repeated until all  $u_k$  have been found. It is possible to show [22] that the optimal control signal is

$$\begin{aligned}u_i &= -L_i x_i \\L_i &= (B^T S_{i+1} B + Q)^{-1} B^T S_{i+1} A \\S_i &= A^T \left( S_{i+1} - S_{i+1} B (B^T S_{i+1} B + Q)^{-1} B_i^T S_{i+1} \right) A + R\end{aligned}$$

In this case, as  $N$  is a finite number this is the solution to what is called the finite horizon problem. If  $N \rightarrow \infty$  this becomes what is known as the infinite horizon problem which has the solution:

$$u_i = -Lx_i$$

$$L = (B^T S B + Q)^{-1} B^T S A,$$

where  $S$  is found by solving

$$R + A^T S A - S - A^T S B (Q + B^T S B)^{-1} B^T S A = 0.$$

It can also be shown that for sufficiently large  $N$  the finite horizon solution  $L_i$  will tend towards the infinite horizon solution  $L$ .

## 5.1 State observers

LQG control is a state based control scheme, that is that the control input is calculated based on the states of the system. However, this assumes that the state is known. This is not always the case as many of the states are not measured, and the measurements which are available are subject to measurement noise. This creates the need to estimate the states using what is called an observer, which combines measurements and model based estimates to derive an estimate of the states. The most common observers using state space models can be written on the innovation form [22]

$$\hat{x}(t+1|t) = A\hat{x}(t|t-1) + Bu(t) + K(t)[y(t) - C\hat{x}(t|t-1)]$$

where  $\hat{x}(t+1|t)$  is the estimate of  $x(t+1)$  at time  $t$ .  $K(t)$  is the observer constant specified by the user. The optimal way to determine  $K(t)$  is to use a Kalman filter, which calculates  $K(t)$  as:

$$K(t) = AP(t)C^T (CP(t)C^T + R_2)^{-1}$$

$$P(t+1) = AP(t)A^T + R_1 - K(t)(CP(t)C^T + R_2)^{-1}K(t)$$

where  $R_1$  is the variance of the model noise  $w(t)$  and  $R_2$  is the variance of the measurement noise  $e(t)$ .

In state feedback LQG control these estimated states are used to derive the control signal. In particular the separation principle states that the estimation problem and the control problem can be solved as two independent problems [22]

# Chapter 6

## Control over unreliable channels

With the increasing use of wireless communication in networked control systems, the issue of control over unreliable channels has risen into prominence [12]. As controllers, sensors and actuators are often positioned on different locations, it can be difficult or expensive to create reliable communication links between the components. Therefore, the question of control over lossy networks is one of increasing importance. Some methods to optimize control algorithms over lossy channels are discussed in [23–25].

Depending on how data is sent and decoded, a communication channel could also be subject to delays. For instance, if tree codes are used to characterize the submission of the data from the controller to the actuators, and from the sensors to the controller, as discussed in [26, 27], a lossy channel can be turned into a channel with a random delay. This delay is not bounded, but it follows a probability function that depends on the reliability of the channel. The problem that is examined in Paper I is this case, where the system is subject to an unbounded delay. In Paper II this problem is expanded to cover a system subject to both package losses and delays.

### 6.1 Unreliable communication links

When there is a risk of package loss or package delays in a system there are no guarantees that the signal sent from the controller at time  $t$  is applied as this time. Hence there is a need to distinguish between the signal the actuator applies and the signal the controller sends. Thus the system can be described as

$$x_{k+1} = Ax_k + Bu_{ak} + w_k, \quad (6.1)$$

where  $u_{ak}$  is the control signal applied by the actuator at time  $k$ . However  $u_{ak}$  is not necessarily the control signal our controller sends at time  $k$ , which is denoted

as  $u_{ck}$ . Consequently if LQG control was to be implemented in cases with risks of package loss or package delays, the optimization problem would be posed as

$$J_N = \min_{u_c} E \left[ \sum_{i=0}^N u_{ai}^T Q u_{ai} + \sum_{i=0}^N x_i^T R x_i + x_{N+1}^T S_{N+1} x_{N+1} \right] \quad (6.2)$$

## 6.2 Hold input or zero input

When there is an unreliable channel between the controller and the actuator this means that the latest signal sent might not yet have arrived. When this occurs there are two basic strategies the actuator can adopt [28, 29]. One is to set the input to zero if the latest control signal sent is delayed or lost, which is known as zero-input. The other alternative is to continue to apply the latest input received until a more recent one arrives, this is known as hold-input.

## 6.3 TCP or UDP like case

There are two basic types of unreliable communication links. In one the sender does not know if the sent package has arrived, which is known as the UDP-like case. In the other one there is a system of acknowledgment that ensures that the sender knows if the sent package has arrived. This is referred to as the TCP-like case. There are two principal differences when designing a controller for the UDP-like case and the TCP-like case.

The first pertains to state estimation. If there is a random delay between the sensors and the controller, this means that at time  $n$  the controller may not have access to measurements from time  $n$ . An algorithm is then needed to estimate the states (as they may not have arrived).

$$\hat{x}_{N|t=T} = \begin{cases} x_N & D(x_N) \leq T - N \\ A\hat{x}_{N-1|t=T} + B\hat{u}_{aN-1} & D(x_N) > T - N \end{cases}$$

where  $\hat{x}_{N|t=T}$  is the estimate of the state  $x_N$  at time  $T$  and  $D$  is the delay.  $D(x_N) \leq T - N$  then means that the delay of the measurement  $x_N$  sent from the sensors is less than or equal to  $T - N$ , i.e  $x_N$  has arrived at time  $T$ .  $\hat{u}_{aN}$  is an estimate of the control signal applied by the actuator at time  $N$ . In the TCP-like case the control signal applied by the actuator is known as there are acknowledgments that inform the controller when a control signals arrives at an actuator. This means that  $\hat{u}_{aN-1} = u_{aN-1}$  and the control signal will not impact the quality of the estimation. However, for the UDP-like case  $\hat{u}$  must be estimated, which leads to a more complex problem as the choice of control signal will impact the optimal estimation and therefore one cannot treat the control problem as a problem separate from

the estimation problem. However the focus here is the TCP-like case where the acknowledgments lead to the separation principle holding and thus the control problem can be solved separately from the estimation problem [23].

Another difference is that knowledge of which control signal that have arrived is useful information when calculating subsequent control signals. In Paper I we do not utilize this knowledge, but in Paper II we utilize this knowledge to improve the results.



# Chapter 7

## Summary of papers

### Paper I

Fredrik Bengtsson, Babak Hassibi, and Torsten Wik. LQG control for systems with random unbounded communication delay. In *Proceeding of the 55th Conference on Decision and Control (CDC)*, pages 1048-1055. IEEE, 2016.

Here we examine LQG control when there is a random unbounded delay between the controller and the actuator. We derive an optimal controller for the TCP-like case and evaluate it in simulations of an example system.

### Paper II

Fredrik Bengtsson and Torsten Wik. LQG control over unreliable communication links. *To be submitted*.

This article expands on the work done in Paper I, to present a solution for a more general type of unreliable communication channel with any type of random delay as well as package losses. Moreover, we properly utilize the knowledge of which control signals have arrived to derive a solution which yields a lower cost than in Paper I.

### Paper III

Fredrik Bengtsson and Torsten Wik. A multiple input, multiple output model generator. Technical report, Department of Signals and Systems, Chalmers University of Technology, 2017.

In this technical report a MIMO model generator is presented. This generator allows the user to generate a large number of linear MIMO systems with prede-

finer properties such as system size, stability, time constants, delays etc. These systems can be used to evaluate various control methods and tuning.

### **Paper IV**

Fredrik Bengtsson, Torsten Wik, and Elin Svensson. Resolving issues of scaling for gramian based input-output pairing methods. To be submitted.

In this article we examine ways to resolve the issues of input and output scaling that the gramian based input output pairing methods have. We propose a new method based on the Sinkhorn-Knopp algorithm which removes the scaling dependence of the gramian based measures. We then test this along with other scaling methods on a large number of systems using the MIMO generator described in Paper III. We find that using the Sinkhorn-Knopp algorithm to scale the systems yields significant improvements compared to the other scaling methods.



# Chapter 8

## Future Work

There are multiple areas which can still be expanded on. For example, the gramian based measures can be used to design decentralized system with feedforward control. While this is examined in Paper IV, further work needs to be done to determine how to best design a system with feedforward using a scaled IM.

When it comes to evaluation of methods, further work is needed in the area of automatic controller tuning. The automatic controller tuning should ensure that comparison of methods for control configuration selection is as little dependent on the tuning of individual control loops as possible. However it is not always clear what the best method to use is, and if each controller should be found with the other outputs in open loop, or if one should successively close the loops with each implemented controller.

In the area of LQG control for lossy channels there is still considerable work which needs to be done. For example the UDP-like case, where there are no acknowledgments between the sensor and actuator, needs to be examined. Moreover, there is a need for further work in the area of implementing an optimal Kalman filter for cases with an unreliable channel between the sensor and controller.



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