CHALMERS
UNIVERSITY OF TECHNOLOGY

Characterizing Swedish school algebra – initial findings from analyses of steering documents, textbooks and teachers' discourses

Characterizing Swedish school algebra – initial findings from analyses of steering documents, textbooks and teachers' discourses

Kirsti Hemmi[1,3], Kajsa Bråting[1], Yvonne Liljekvist[2], Johan Prytz[1], Lars Madej[1], Johanna Pejlare[4], and Kristina Palm Kaplan[1]

[1]Uppsala University, Sweden; kajsa.brating@edu.uu.se, kirsti.hemmi@edu.uu.se, lars.madej@edu.uu.se, johan.prytz@edu.uu.se, kristina.palm-kaplan@edu.uu.se

[2]Karlstad University, Sweden; yvonne.liljekvist@kau.se

[3]Åbo Akademi University, Finland; khemmi@abo.fi

[4]Chalmers University of Technology and the University of Gothenburg; pejlare@chalmers.se

The paper reports the first results of an ongoing research project aiming at characterizing Swedish school algebra (grades 1-9). Both diachronic and synchronic studies are conducted to identify the specific teaching tradition developed in Sweden and different theoretical approaches are applied in the overall project in order to obtain a rich picture of the Swedish case. The results reported here are based on the analyses of mathematics curriculum, textbooks and focus group interviews with teachers in seven schools. The initial results indicate that, since 1980s, algebra is vaguely addressed in the steering documents and the progression of algebraic thinking is elusive in teachers' discourses. Moreover, certain important ideas, such as generalized arithmetic, are largely missing in the curriculum and mathematics textbooks for grades 1-6. We discuss the implications of the initial findings for our project.

Keywords: Algebraic thinking, grades 1-9, teaching tradition, curriculum

Background

Profound knowledge in algebra is important for the understanding of several mathematical areas as well as topics in other disciplines. A multitude of studies among adolescents document students' difficulties with algebra and the serious consequences of these difficulties. Due to its role as a critical gatekeeper and to recent research results that question the earlier school mathematics traditions, many countries including Sweden have revised their curriculum[1] attempting to integrate algebra in school mathematics from the very beginning (Cai, Lew, Morris, Moyer, Fong Ng & Schmittau, 2005; Prytz, 2015). Yet, the international evaluations like The Programme of International Student Assessment (PISA) and The Trends in International Mathematics and Science Study (TIMSS) show that Swedish students' results in algebra have not improved. The overall purpose of the ongoing project is to contribute to the international research field concerning the complex issue of implementing algebra in school mathematics by investigating the Swedish case. More specifically, we attempt to find possible reasons for the failure of raising the quality of algebra teaching by examining how algebra is traditionally treated in the Swedish school curricula and textbooks in Grades 1 through 9 (the diachronic perspective). We also explore the current

[1] With curriculum we refer to the national steering document concerning the contents and goals in mathematics.

situation (the synchronic perspective) by analyzing the treatment of different algebra-related items in the current mathematics curriculum, textbooks and teacher guides (cf. Cai et al., 2005; Hemmi, Lepik & Viholainen, 2013), and investigate how teachers at different school levels relate to these issues and the materials. The focus of the research project is to identify the expected student progression in algebra as interpreted in different *arenas* of the Swedish school system. The school system is regarded as stratified into levels and a basic distinction is made between arenas of formulation and realization (cf. Lindensjö & Lundgren, 2000). Examples of the former are the group of people who decide content in policy documents, for example curricula, but also people producing textbooks. Examples of the latter are the teachers who interpret texts and design and carry out lessons. Our particular interest is also to reveal how more or less tacit traditions in textbook production and teaching practices are related to the intentions of the curriculum in order to find possible mismatches and contribute to the future development at different levels of the educational system. In order to achieve this, the project is built upon three sub projects probing 1) the diachronic perspective on the formulation arenas, 2) the synchronic perspective on the formulation arenas, and 3) the synchronic perspective in the realization arena.

In this paper, we report the first steps of the project taken during the first year and discuss the initial results and their consequences in terms of how to proceed during the following years.

Relevant literature

In many countries, algebra has traditionally been postponed until adolescence partly because of former assumptions concerning child cognitive development, and partly because of the parallels made between the learning trajectories of students and the history of mathematics (cf. Carraher, Schliemann & Brizuela, 2006). The dominant view of child cognitive development connected to constructivist learning theories was already challenged by the classroom studies of the Vygotskian based Davydov team showing that Russian children who received instruction in algebraic representation of verbal problems from Grades 1 through 4 performed better than their control peers throughout later school years (Carraher et al., 2006). Also the suggested similarity between a child development and the history of mathematics has been questioned (see for instance Bråting & Pejlare, 2015) and recent studies show that it is possible and even beneficial to start working with algebraic ideas and generalizations in parallel with arithmetic already in early grades (e.g. Cai et al., 2005; Carraher et al., 2006; Blanton, Stephens, Knuth, Murphy Gardiner, Isler & Kim, 2015). Blanton et al. (2015) found that children are capable of engaging successfully with a broad and diverse set of algebraic ideas. The idea of early algebra is to facilitate students' progression towards understanding more formal algebra. Scholars agree that algebraic thinking in early grades should reach beyond arithmetic and computational fluency "to attend the deeper underlying structure of mathematics" (Cai et al., 2005). Kieran (2004), for example, addresses the following adjustments that students need to make in developing an algebraic way of thinking: 1) A focus on relations and not merely on the calculations of numerical answers; 2) A focus on operations as well as their inverses, and on the related idea of doing/undoing; 3) A focus on both representing and solving a problem rather than on merely solving it; 4) A focus on both numbers and letters, rather than on numbers alone; and 5) A refocusing of the meaning of the equal sign. Although these adjustments

are in the domain of arithmetic they represent a shift toward developing fundamental ideas of algebra (cf. Cai et al., 2005).

In Sweden, algebra became a part of all students' schooling after the introduction of the nine-year compulsory school in the 1960s. Moreover, the 1969 policy documents prescribe that algebra should be a part of school mathematics from grade 2 (Prytz, 2015). The algebraic content in upper secondary textbooks has changed from being dominated by algebraic manipulations and expressions to becoming more integrated with other school subjects and thus being more anchored with reality as well as everyday activities (Jakobsson-Åhl, 2006). Besides the directives in the curriculum documents and changes in the textbooks, there have been various attempts to improve algebra teaching in Sweden through in-service training projects for teachers and in teacher education for some decades. However, it is not possible to discern a general positive effect of these efforts on Swedish students' learning in algebra, at least not if we consider the results in the TIMSS evaluations and in FIMS and SIMS that preceded TIMSS. Since 1964 (FIMS) Swedish students have always performed below the international average in algebra.

Häggström (2008) compares algebra tasks used in Chinese and Swedish mathematics textbooks (grade 8) and finds an extensive variation in many relevant aspects in the Chinese textbooks while tasks in the Swedish do not open many dimensions of variation. Concerning the realisation arena, there is some research about how teachers interpret and relate to national mathematics curriculum documents in Sweden in general (cf. Boesen et al., 2014) but studies focusing on certain mathematical areas are largely lacking. An exception is a small case study of Kilhamn (2013) who identifies different approaches to the introduction of variables in grade 6 of two teachers referring to the same piece of national curriculum text and using the same textbooks.

Methods

As mentioned in the introduction we conduct three studies that both separately and related to each other help us to discern important aspects of the issue of implementing algebra in school mathematics. Next we briefly describe them and indicate what is done so far (the focus of this paper).

Study 1: The diachronic perspective on the formulation arenas (1960–2015)

The motivation for having a diachronic perspective is based on the observation that we cannot assume that all actions are based on people's awareness of explicit goals. People might also act according to traditions in a more or less conscious manner. Thus there is a tacit dimension for us to handle. Our point is that if we want to understand people's action today, we also have to consider the possibility that they act according to a tacit tradition. The purpose of the study is to deepen our understanding of the tradition in Swedish school mathematics and the position of algebra within this tradition. The data material consists of steering documents issued by the central school authorities, textbooks, teacher journals and official reports. The category steering documents includes the syllabi and commentary materials.

Thus far, material issued by the central school authorities, i.e. syllabus and commentary material, have been studied. The analysis is focused on how knowledge in mathematics is described by

different types of terminology, mainly expressions for mathematical concepts and expressions for competencies. The results reported in this paper are based on these analyses.

Study 2: The synchronic perspective on the formulation arenas

The second study is synchronic and focuses on the formulation arenas. The aim is to find out and characterize the hypothetical learning trajectory/trajectories (cf. Hemmi et al., 2013) and the typical ways of integrating algebra in the current Swedish compulsory school instruction for grades 1–9 (age 7-12). The main data for this study comprises the current steering document in mathematics and mathematics textbooks with teacher guides.

In an initial study we have identified and classified the algebraic content in the current Swedish curriculum in mathematics and in the two textbook series Matte Direkt and Matte Eldorado for grades 1-6. The two textbook series were chosen on the basis of high popularity (Neuman et. al., 2015), and because they represent different approaches to organization of teaching (Neuman et. al., 2015). Moreover, Eldorado is relatively new at the Swedish textbook market compared to the more established Matte Direkt.

As a starting point for our analysis we used a classification that is based on the analytical framework of Blanton et al. (2015) regarding how algebraic content can be characterized at compulsory school level. They identified the following four main categories (that they call ”big ideas”) in school algebra: 1) Equivalence, expressions, equations & inequalities (EEEI); 2) Generalized arithmetic (GA); 3) Functional thinking (FT); 4) Variable (Var).

Study 3 The synchronic perspective on the realisation arena

The third study is also synchronic and it focuses on the realisation arena. The aim is to find out how teachers talk about algebra progression and texts and tasks produced in the formulation arenas. In the first part of Study 3 we have conducted focus group interviews in seven schools with, in all, 33 certified teachers from grade 1-9 (mean 15.9 years of teaching experience, SD=9.4). The schools were situated in different socio-economic contexts. An interview guide containing 14 open questions steering the conversation into two themes; 1) what is algebra (pre-algebra), and 2) what mathematical tasks are suitable for teaching algebra (at some specific school level). In the second theme we used tasks from Blanton et al. (2015) in order to cover the big ideas to be developed throughout the school years. Moreover, we selected tasks from mathematics textbooks to investigate how the teachers relate to specific aspects, such as informal/formal methods, everyday mathematics/pure mathematics. One project assistant conducted the interviews, and one took notes and collected background data (i.e., a questionnaire). The interviews were audio recorded and transcribed, and the initial thematic analyses were conducted with NVivo software using both a priori categories concerning Blanton’s big ideas and the specific aspects Blanton et al. (2015), and an open approach to capture items that may be invisible in the documents or in previous research.

In the following sections, we will first display the first-year results from these three studies, and thereafter draw some conclusions for further studies. The results can be understood both separately and related to each other, and can help us discern important aspects of algebra teaching and learning in Swedish schools.

Results

The result from the diachronic study concerns how knowledge has been expressed in syllabus. As regards algebra, our preliminary observations indicate that progression has been expressed differently in different topics; especially in the syllabus of 1962 and 1980. Since 1980 progression in arithmetic was expressed more clearly than in algebra. However, from the syllabus of 1994 and onwards, progression in all topics was expressed more vaguely.

Regarding how algebra is addressed in the current Swedish mathematics curriculum for Grades 1-6 the result of our initial study reveals that three of Blanton et. al.'s (2015) categories, namely EEEI, FT and VAR are well-represented in the content of the curriculum. Meanwhile, statements connected to category GA is not represented at all in the Swedish mathematics curriculum for Grades 1-6. Functional thinking (FT) is the most represented category where the dominating items are "proportional reasoning" and "construction of patterns".

The result of our initial study of textbooks for grades 1-6 shows that EEEI is the most represented category in both textbook series, especially in grades 1-3 and in Matte Direkt. However, in both textbook series the EEEI content decreases from grades 1-3 to grades 4-6. The categories FT and VAR are also well-represented in both series, especially in grades 4-6. Apparently the tendency in both textbook series is that FT and VAR increases from grades 1-3 to grades 4-6 while the amount of EEEI decreases from grades 1-3 to grades 4-6. The category GA is the least represented in both textbook series, especially in Matte Direkt. As mentioned above, the category GA is not represented at all in the current mathematics curriculum for grades 1-6 which probably is one reason behind the low representation of GA in the textbooks.

Concerning the teachers' ways of talking about algebra at different school levels, the initial analyses indicate that teachers' considerations about the expected progress of students' algebraic thinking at different grade levels are vague. The next extract illuminates this.

> A teacher: Often, I think, that we lack this, what one expects of the students when they leave the 3th grade… and also the other way around, what I can expect when the students come to me in the 4th grade. What have they done? What skills do they have? I can't start somewhere the students have not yet arrived at. And yet, we [the teachers] are often in the same building… what about when the students leave for the 7th grade, and they change the school completely. Then it's even more difficult to know what we teachers can kind of expect from each other.

The excerpt above is representative of the way in which the teachers talk about the lack of consensus concerning what to expect of students in different school grades. Moreover, when they talk about goals they talk in general terms in a tentative manner without specifying what students should actually learn at different levels. We find this interesting as the diachronic studies show that algebra is traditionally vaguely addressed in the Swedish curriculum. Moreover, in line with the results from the analyses of the current steering document and the two textbooks, items connected to EEEI, such as the meaning of the equal sign and informal and formal methods of equation solving dominate the teachers' discourses. The following extract represents a common way of discussing the topic.

A teacher: …one sometimes thinks that a child has no clue of what the equal sign actually means, they think that it results to something, instead of balancing on both sides. If one has understood that it weighs evenly, then one can use that knowledge in almost all mathematics later on.

Understanding of the equal sign and simple equations are stressed as especially important in grades 1-3 but also raised as an important and difficult topic to continue working with in the following grades. Working with patterns sometimes followed by a formulation of a rule (FT) is another item that the teachers in the focus groups raised when discussing algebra at different school levels.

Conclusion

The absence/low occurrence of generalized arithmetic in the curriculum, textbooks and teachers' discourses can be something important to investigate further in search of reasons for the low results at TIMSS and PISA in Sweden, especially considering that the results in algebra is, and has been, the weakest of all mathematical content areas. Generalized arithmetic is stressed as an important part of algebra by several researchers and it can be seen as a bridge between arithmetic and algebraic thinking. The term "generalized arithmetic" has emerged from the part of algebraic thinking that considers the study of structures and relations arising in arithmetic (Kaput, 2008). Previously, generalized arithmetic has been associated with a letter-symbolic algebra, with its equations and unknowns (Kieran et. al., 2016). However, during the years and within the research field of early algebra the term has acquired a much broader sense in that the relations and properties inherent to arithmetical operations are explored and seen by students as being generalizable, without necessarily involving alphanumeric symbols (Kieran et. al., 2016). We believe that a progression in "algebra as generalized arithmetic" throughout compulsory school is necessary in order to improve the algebraic skill of Swedish school students. The high representation of FT may be due to an international trend where "study of change" has been identified as a key area of mathematics in for instance PISA:s framework for school mathematics. This is reflected in the current Swedish curriculum where "Relationship and change" constitutes a separate category within the mathematical content for both compulsory and upper secondary school. Earlier this type of content has been spread out over different content categories.

Methodologically our ongoing project is unique, as we approach the issue from both diachronic and synchronic perspectives, and also investigate how algebra is addressed in the different arenas. Relating the results from the different studies to each other helps us to find explanations for the separate findings in different arenas and we believe that it will help us to increase our understanding both of the specific Swedish tradition and of reasons for why a diversity of mathematics initiatives concerning algebra have not been successful. All the three studies are not yet synchronized, for example only textbooks from grades 1-6 have been analysed so far and the analysis of algebra in the diachronic study has just been started. The initial results are promising and the next step regarding the diachronic study is to consider the status ascribed to algebra by the central school authorities. Another type of future studies concerns textbooks and not at least how and to what extent textbooks have realized the syllabus. A selection of the textbook series will then be made according to their popularity. By using a database regarding textbooks in mathematics published in the period of

1930–2015, constructed for the project, we can do this selection in a reliable way. The results from the diachronic studies can also be related to the variation of Swedish students' results in national and international evaluations and offer us interesting information about the effects of different kinds of steering documents and textbooks on students' learning of algebra.

As to the synchronic textbook studies, we will identify and classify the algebraic content also in the curriculum and textbooks for grades 7-9. We will deepen our knowledge concerning the specific character of the activities/tasks identified in the Swedish materials within the big ideas, as these categories are quite general. We aim to do this by conducting comparative studies with our colleagues in two countries that seem to have different approach to algebra, in order to find more nuanced picture of the Swedish situation. Thereafter the expected student progression within the categories across the grades 1-9 will be investigated in order to understand the hypothetical learning trajectory in current school algebra in Sweden. Finally, concerning the realisation arena, we will gather more data and also deepen the analyses of the categories. We will continuously relate the results from our three studies to each other.

We aim to apply Bernstein's (2000) theories on classification and framing in the entire study, to understand our results from a broader perspective. Drawing on Bernstein, the analysis focuses on how boundaries related to classification and frames are, and have been, created and maintained at different levels, in our case different arenas. By identifying differences in the creation and maintenance of these boundaries, both within arenas and between arenas, we achieve a better understanding of the implementation problem regarding algebra in the Swedish educational system. We can also provide an explanation, at least to some extent of the Swedish results in international comparisons.

Acknowledgment

The research project that this paper is based on is financed by The Swedish Research Council.

References

Bernstein, B. (2000). *Pedagogy, symbolic control and identity. Theory, research, critique.* NewYork, NY: Rowman & Littlefield. Revised edition.

Blanton, M., Stephens, A., Knuth, E., Murphy Gardiner, A., Isler, I., & Kim, J.-S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education, 46*(1), 39–87. doi: 10.5951/jresematheduc.46.1.0039

Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmgren, B. (2014). Developing mathematical competence: From the intended to the enacted curriculum. *Journal of Mathematical Behavior, 33*, 72–87. doi: 10.1016/j.jmathb.2013.10.001

Bråting, K., & Pejlare, J. (2015). On the relations between historical epistemology and students' conceptual developments in mathematics. *Educational Studies in Mathematics, 89*(2), 251-265. doi:10.1007/s10649-015-9600-8

Cai, J., Lew, H., Morris, A., Moyer, J., Fong Ng, S., & Schmittau, J. (2005). The development of students' algebraic thinking in earlier grades: A cross-cultural comparative perspective. *ZDM – The International Journal on Mathematics Education, 37*(1), 5-15. doi:10.1007/bf02655892

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education, 37*(2), 87-115.

Hemmi, K., Lepik, M., & Viholainen, A. (2013). Analysing proof-related competences in Estonian, Finnish and Swedish mathematics curricula – towards a framework of developmental proof. *Journal of Curriculum Studies, 45*(3), 354-378. doi:10.1080/00220272.2012.754055

Häggström, J. (2008). *Teaching systems of linear equations in Sweden and China: What is made possible to learn?* (Göteborg Studies in Educational Sciences, 262). Doctoral thesis, Göteborg: Acta Universitatis Gothoburgensis.

Jakobsson-Åhl, T. (2006). *Algebra in upper secondary mathematics: A study of textbooks used in the years 1960–2000 in Sweden*. Licentiate thesis, 2006:33. Department of Mathematics, Luleå University of Technology.

Kaput, J.J. (2008). What is algebra? What is algebraic reasoning? In J.J. Kaput, D.W. Carraher & M.L. Blanton (Eds), *Algebra in the early grades* (pp. 5-17). New York, NY: Lawrence Earlbaum Associates.

Kieran, C., Pang, J., Schifter, D., & Ng., S.F. (2016). *Early algebra. Research into its nature, its learning, its teaching. ICME – 13 Topical Surveys.* SpringerOpen.

Kieran, C. (2004). Algebraic thinking in the early grades: what is it? *The Mathematics Teacher, 8*(1), 139-151.

Kilhamn, C. (2013). Hidden differences in teachers' approach to algebra-a comparative case study of two lessons. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *The Proceedings of the Eighth Conference of the European Society for Research in Mathematics Education – CERME-8*, Middle East Technical University, Ankara.

Lindensjö, B., & Lundgren, U.P. (2000). Utbildningsreformer och politisk styrning. Stockholm, Sweden: HLS förlag.

Neuman (Lindvall), J., Hemmi, K., Ryve, A., & Wiberg, M. (2015). Mathematics textbooks' impact on classroom instruction: examining the view of 278 Swedish teachers' views. In Silfverberg, H., Kärki, T., & Hannula, M.S. (Eds.) (2015). *Nordic research in mathematics education – Proceedings of NORMA14, Turku, June 3-6, 2014. Studies in Subject Didactics 10.* Turku: The Finnish Research Association for Subject Didactics.

Prytz, J. (2015), Swedish mathematics curricula, 1850-2014. An overview. In Bjarnadóttir, K., Furinghetti, F., Prytz, J. & Schubring, G. (Eds.). *Dig where you stand 3: Proceedings of the Third International Conference on the History of Mathematics Education*. Uppsala, Sweden: Uppsala University.