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A Method for Simultaneous Creation of an Acoustic Trap and a Quiet Zone

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***Abstract*—An acoustical levitation trap can be created by maximizing the Laplacian of the Gor'kov potential and minimizing the pressure at a given location. Marzo et al. have successfully used Broyden-Fletcher-Goldfarb-Shanno minimization to find the phases to be imposed on the elements of an ultrasonic transducer array to create the required pressure field. We extend this method to create a field with one acoustical trap and a quiet zone at a different location. This can be used to, for example, create an independent second trap at the location of the quiet zone. We show through numerical simulations that it is possible to create such sound fields using ultrasonic arrays, and we demonstrate the advantages of the proposed approach over the direct design of a sound field with more than one trap. Our method can create a quiet zone extending almost two wavelengths without affecting the levitation trap.**

***Index Terms*—Ultrasound, Levitation, BFGS minimization**

I. Introduction

King developed the theory of radiation pressure in 1934 [1], which forms the basis of acoustic levitation. Radiation pressure evolves in high intensity pressure fields due to second order nonlinear interactions between the field incident on an object and the field scattered from the object. This radiation pressure can be integrated over the surface of the object to find the net radiation force. For complex geometries or large objects, one possibility is to solve the scattering problem and integrate numerically [2].

For small spherical objects, it is possible to formulate the radiation force using a scalar energy potential [3]. This so called Gor'kov potential U is a scalar field whose negative gradient describes the radiation force on spherical objects in a sound pressure field [4]. The potential consists of a pressure part and a velocity part as [4]

$$U = \frac{V}{2}\left(f_1\kappa_0|p|^2 - \frac{3}{2}f_2\rho_0|v|^2\right)$$

where

$$f_1 = 1 - \frac{\kappa_\mathrm{p}}{\kappa_0}$$
$$f_2 = 2\frac{\rho_\mathrm{p} - \rho_0}{2\rho_\mathrm{p} + \rho_0},$$

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V is the volume of the spherical object, κ_0 and κ_p are the compressibilities of the medium and the particle, and ρ_0 and ρ_p are the densities of the medium and the object, respectively. Since the particle velocity v is proportional to the gradient of the sound pressure p, it is sufficient to know the pressure field to evaluate the Gor'kov potential.

Acoustic levitation is typically performed with ultrasound and is an emerging technology with applications that include non-contact manipulation of liquids [5], microparticle sorting [6], and pharmaceutical refinement [7].

II. Acoustical Traps

An acoustical levitation trap is a location in space to which the radiation forces are converging. If a small spherical object is placed in the vicinity of a trap, the forces will push the object to the center of the trap. Maximizing the forces converging to a point is identical to minimizing the divergence of the force, or maximizing the Laplacian of the Gor'kov potential. Such traps will be very sensitive to disturbances since the pressure in the center of the trap tends to be very high. A stable levitation trap should therefore have a low pressure in the center [8].

Ultrasonic arrays can be used to create sound fields with the required high intensity. Since practical implementations are often restricted to a single constant amplitude, many formulations only vary the phase of the individual transducer feeds [2], [8], [9], [10]. The optimal relative phase shifts that evoke a sound field with a levitation trap can be found using numerical optimization. A good choice of objective function to create a single acoustical levitation trap at $\vec{r}_0$ is [8]

$$\begin{aligned} O(\varphi_1, \ldots, \varphi_n; \vec{r}_0) &= w_p|p(\vec{r}_0)|^2 \\ &- w_x U_{xx}(\vec{r}_0) - w_y U_{yy}(\vec{r}_0) - w_z U_{zz}(\vec{r}_0), \end{aligned} \tag{1}$$

where φ_i are the phases of individual transducers in the array. Here $U_{qq} = \frac{\partial^2 U}{\partial q^2}$ is the second order partial derivative along the respective Cartesian axes. The Laplacian of the potential was split in separate parts weighted by the weights w_i, which can be used to create different types of traps. Using this objective function Marzo et al. [8] successfully used Broyden-Fletcher-Goldfarb-Shanno (BFGS) minimization [11] to create acoustic traps.

This objective function could be used simultaneously at different locations to find the required phases for two or more traps. But there is no guarantee that it will converge to traps of similar strength. This occurs, for example, if the random initialization of the algorithm happens to favor one of the traps. The algorithm will modify the transducer phases to reduce the pressure at the second location. But it can cause the Laplacian of the Gor'kov potential still to be low at this location. We present such a case in Sec. IV.

III. METHOD

Instead of optimizing for several traps simultaneously, we propose to create a sound field with an acoustic trap as well as one or more quiet zones. A second sound field can then be superposed that exhibits, for example, a trap where the first sound field exhibits a quiet zone. The specification of the quiet zone can vary depending on the intended use for the other locations. It is desirable to minimize both pressure and pressure gradient if the intention is to use a second location to levitate another object since the Gor'kov potential depends on both.

Following the same approach as in (1), we propose a new objective function as

$$\begin{aligned} O'(\varphi_1, \ldots, \varphi_n; \vec{r}_0, \vec{r}_N) = O(\varphi_1, \ldots, \varphi_n; \vec{r}_0) \\ + w_N \left(|p(\vec{r}_N)|^2 + w_g |\nabla p(\vec{r}_N)|^2 \right) \end{aligned} \tag{2}$$

where w_N controls the relative strength of the trap and the quiet zone, and w_g controls the contribution of the pressure gradient in the quiet zone. This new objective function is minimized using a BFGS optimizer with basinhopping to ensure convergence to a stable minimum [12]. We use the holographic phase signatures combined with a simple focusing element as described by Marzo et al. [8] as the initial starting point for the optimizer, i.e. a set of phases that minimizes the original objective function O.

If a field with a trap at $\vec{r}_0$ and a quiet zone at $\vec{r}_N$ is superposed with a second field with, for example, a trap at $\vec{r}_N$ and a quiet zone at $\vec{r}_0$, then the two fields will not interfere regarding their respective purposes. Assuming that linear superposition is sufficiently valid, it is possible to create both fields simultaneously using a single array. This assumption is commonly used to add contributions from the transducers in ultrasonic arrays [6], [8], [9], [13].

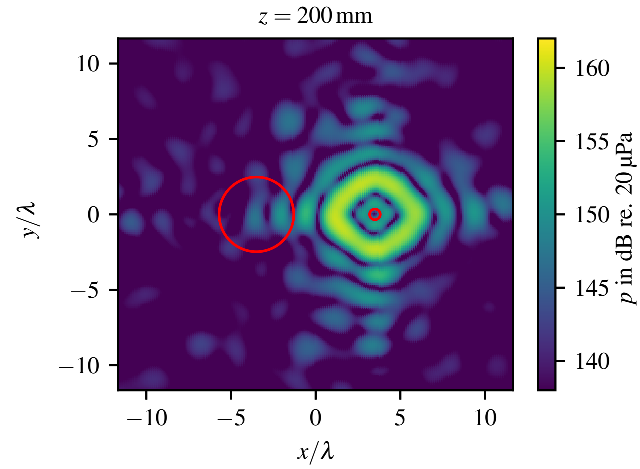
Representing the phase shift for each transducer as a complex amplitude $e^{j\varphi_i}$ the complex amplitudes needed to create the combined field is obtained as

$$C_i = e^{j\varphi_i^{(1)}} + e^{j\varphi_i^{(2)}} \tag{3}$$

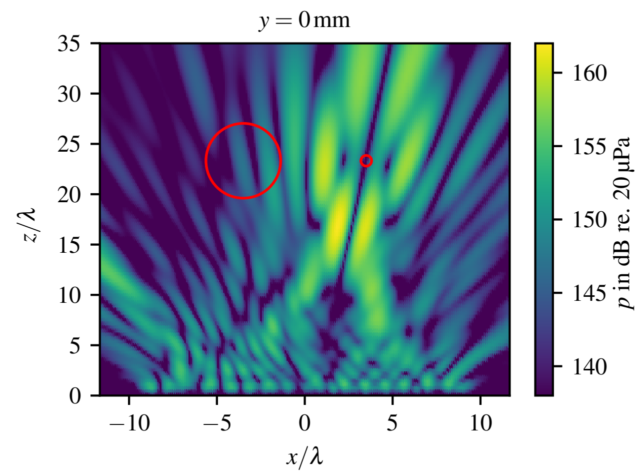
where $\varphi_i^{(1)}$ and $\varphi_i^{(2)}$ are the phase obtained from the two optimizations. It is important to note that this requires control over both phase and amplitude for each individual transducer as $|C_i|$ is not necessarily 1. Since this is not possible on all hardware platforms, this approach limits the choice of hardware.

IV. RESULTS

The following simulations assume a planar 16×16 transducer array operating at $40\,\mathrm{kHz}$ with transducers equispaced $10\,\mathrm{mm}$ apart, centered around the coordinate origin in the xy-plane. The transducer directivity was modeled as a circular piston, producing a sound pressure of $6\,\mathrm{Pa}$ at $1\,\mathrm{m}$ distance in the normal direction. We limit the examples to the creation of one trap and one quiet zone in each individual sound field for ease of comparison. The two positions that are considered are $\vec{r}_1 = (-3, 0, 20)$ cm and $\vec{r}_2 = (3, 0, 20)$ cm. These locations are marked in all figures.



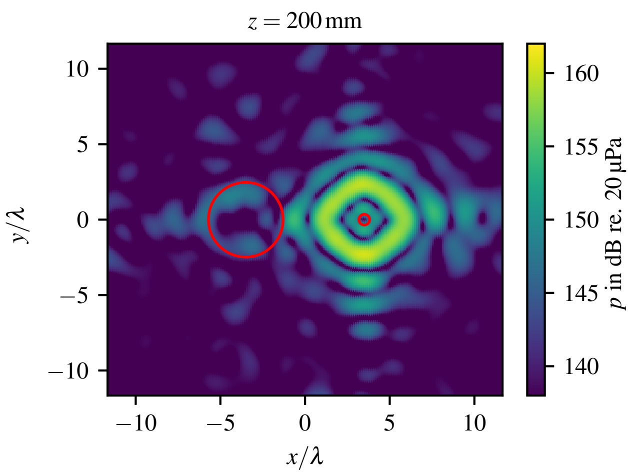
(a) Top view.



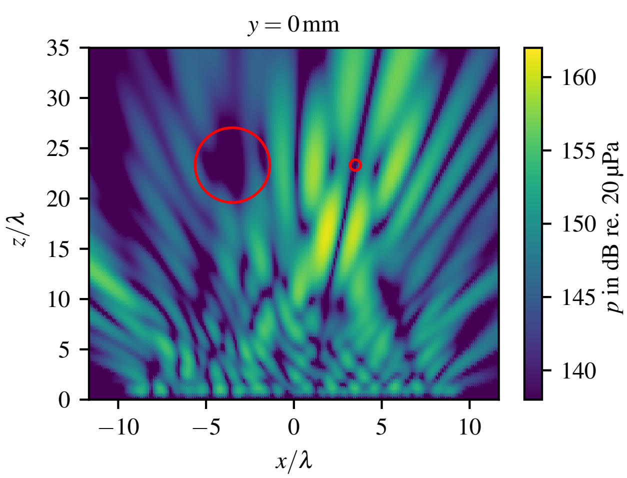
(b) Side view.

Fig. 1. Normalized pressure field created when optimizing phases for an acoustic trap at $\vec{r}_2$ using the original objective function (1), without any constraints on the field at $\vec{r}_1$.

Fig. 1 shows cross-sections of the field created when using the objective function (1) without additional quiet zones. This is the solution from [8]. Several secondary minima are visible outside of the trap at $\vec{r}_2$. This is a consequence of the finite extent of the array as well as of spatial aliasing. The latter arises due to the fact that the transducer spacing is larger than half a wavelength. It is not possible to create precise focus points without creating secondary maxima if the transducers are further apart than half a wavelength [14]. This demonstrates the need for modeling quiet zones in the



(a) Top view.



(b) Side view.

Fig. 2. Normalized pressure field created when optimizing phases for a quiet zone at $\vec{r}_1$ and an acoustic trap at $\vec{r}_2$.

objective function.

Several different weights w_N and w_g for the strength of the quiet zone and the gradient in (2) were evaluated. The pressure gradient is approximately of order k, the wavenumber, larger than the pressure. In order to have a balance between minimizing pressure and minimizing pressure gradient at the quiet zone, $w_g \sim 1/k^2$ was found to be appropriate. It was found that it is necessary to prioritize the trap and not the quiet zone using $w_N \sim 0.01$.

Comparing Fig. 1 with 2 shows that the traps caused by the original objective function (1) and the proposed one (2) have a similar shape and pressure amplitude, which suggests that they are similarly strong. Comparing Fig. 1b and Fig. 2b clearly demonstrates that a quiet zone is apparent around $\vec{r}_1$ when using the proposed objective function (2).

Fig. 3 shows the mean pressure at a given distance from the center of the desired quiet zone, both for the simulation with the quiet zone based on (2) and for the same location in the simulation using the original objective function (1). The zone

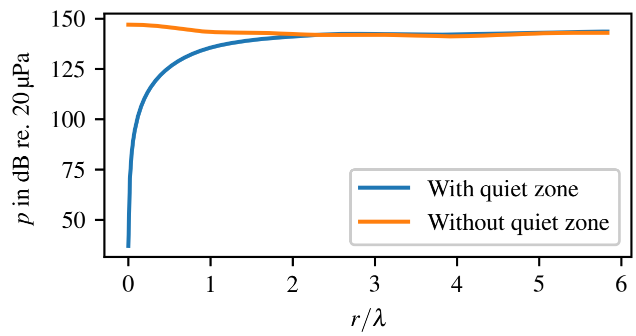


Fig. 3. The mean pressure on spherical shells of radius r around the center of the quiet zone, with and without the additional constraint in the objective function.

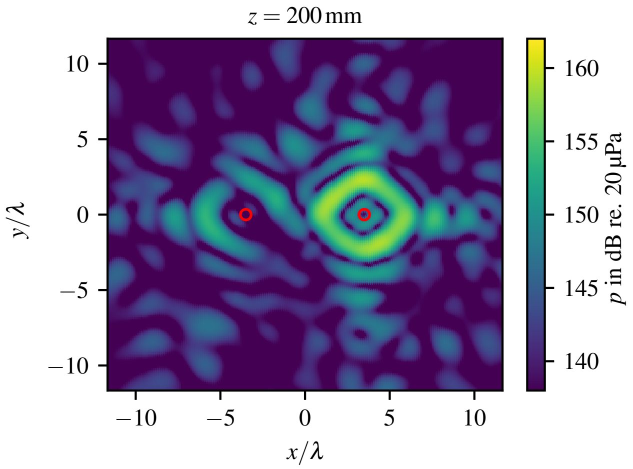
with reduced pressure extends almost two wavelengths from the specified location, and the pressure at the center almost vanishes completely (blue curve).

To highlight the advantage of the proposed method, we compare two ways for creating two simultaneous traps, one at $\vec{r}_1$ and one at $\vec{r}_2$: i) We superpose the original solution (1) for a trap at $\vec{r}_1$ with the original solution for a trap at $\vec{r}_2$, and ii) we superpose the proposed solution (2) for a trap at $\vec{r}_1$ and a quiet zone at $\vec{r}_2$ with the proposed solution for the locations $\vec{r}_1$ and $\vec{r}_2$ of the trap and the quiet zone swapped. The superposition is performed using (3). Note that in case i), a trap is superposed with whatever residual field occurs at the trap's location due to the formation of the trap at the other location and vice versa. In case ii), a trap superposes with the quiet zone of the sound field creating the other trap.

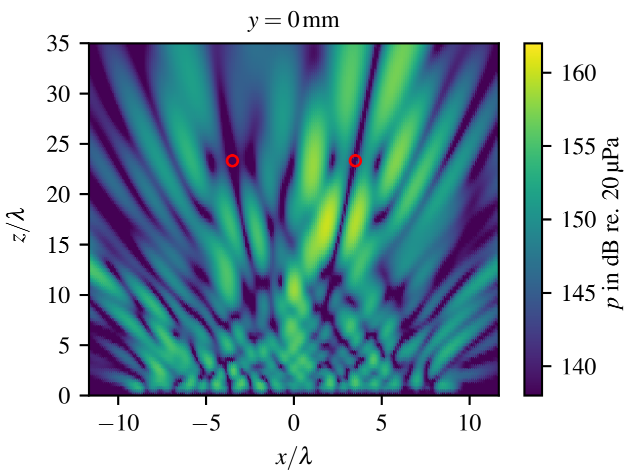
It is not straightforward to measure the actual strength of a trap. As an indicator for this strength, we compare the superposed objective functions (1) and (2), respectively. The two unmodified objective functions evaluated at the locations $\vec{r}_1$ and $\vec{r}_2$ in case i), i.e., when superposing the fields without quiet zones, are $O(\vec{r}_1) \approx 5600$ and $O(\vec{r}_2) \approx 3300$, respectively, whereas in case ii), i.e., when superposing the fields with quiet zones, the objective functions are both in the order of $O(\vec{r}_{1,2}) \approx -1 \cdot 10^{-5}$. The proposed approach provides an improvement of several orders of magnitude.

Finally, we compare the original solution (1) optimizing for two simultaneous traps in Fig. 4 with the above described superposition of the proposed solution (2) with a trap at $\vec{r}_1$ and a quiet zone $\vec{r}_2$ as well as the proposed solution with the two locations swapped in Fig. 5.

As can be seen in Fig. 5, the proposed solution creates two simultaneous traps, which both have very similar strength, which in turn is similar to the stregnth of the original trap from Fig. 1. Optimizing for two simultaneous traps using the original approach causes one strong and one weaker trap. Note that the initial phases for this last example were randomized, and the weights for the two traps were equal. The reported outcome was by far the most likely one that occurred in our experiments.

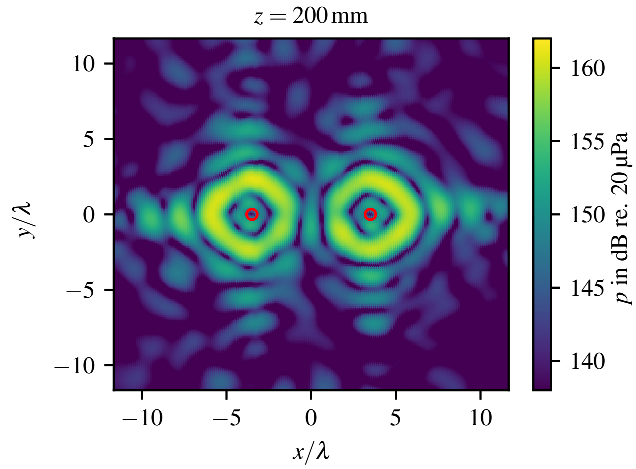


(a) Top view.

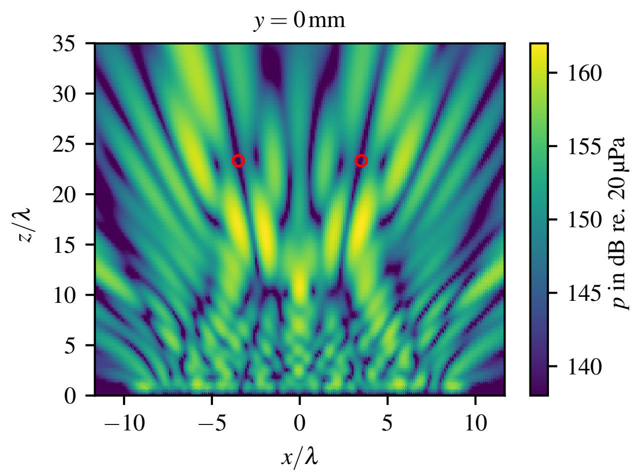


(b) Side view.

Fig. 4. Normalized pressure field created when optimizing phases for two levitation traps simultaneously. It is apparent that the solution favors the trap at $\vec{r}_2$ so that the second trap at $\vec{r}_1$ is hardly visible in the plot.



(a) Top view.



(b) Side view.

Fig. 5. Normalized pressure field created when combining two fields with quiet zones.

V. Conclusions

We presented a method to create sound pressure fields for acoustical levitation that is more versatile than previous methods. Our method is capable of levitating a particle in one place while minimizing the sound field in a different location. It is therefore possible to define multiple locations for different purposes without interference. We showed the example of creating two independent levitation traps here. It is also possible to use the proposed method to create simultaneous ultrasonic haptic feedback or parametric sound [13], [15].

The method is straightforward to extend to more control points by adding additional quiet zones to the objective function so that more sound fields serving other purposes can be superposed. The price for this is that each new field that is superposed reduces the energy that is available to each of the fields. The hardware used for acoustic levitation is typically driven close to or at maximum amplitude. Superposing two transducer feeds can double the amplitude of the feed of a given transducer. It is straightforward to weight the fields differently then adding the complex amplitudes in order to prioritize one over the other.

REFERENCES

[1] L. V. King, "On the Acoustic Radiation Pressure on Spheres," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 147, no. 861, pp. 212–240, Nov. 1934.

[2] S. Inoue, S. Mogami, T. Ichiyama, A. Noda, Y. Makino, and H. Shinoda, "Acoustic Macroscopic Rigid Body Levitation by Responsive Boundary Hologram," Aug. 2017.

[3] L. P. Gorkov, "On the Forces Acting on a Small Particle in an Acoustical Field in an Ideal Fluid," *Soviet Physics Doklady*, vol. 6, p. 773, Mar. 1962.

[4] H. Bruus, "Acoustofluidics 7: The acoustic radiation force on small particles," *Lab on a Chip*, vol. 12, no. 6, pp. 1014–1021, 2012.

[5] E. H. Trinh, "Compact acoustic levitation device for studies in fluid dynamics and material science in the laboratory and microgravity," *Review of Scientific Instruments*, vol. 56, no. 11, pp. 2059–2065, 1985.

[6] B. W. Drinkwater, "Dynamic-field devices for the ultrasonic manipulation of microparticles," *Lab on a Chip*, vol. 16, no. 13, pp. 2360–2375, 2016.

[7] C. J. Benmore and J. K. R. Weber, "Amorphization of molecular liquids of pharmaceutical drugs by acoustic levitation," *Phys. Rev. X*, vol. 1, p. 011004, Aug 2011. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevX.1.011004

[8] A. Marzo, S. A. Seah, B. W. Drinkwater, D. R. Sahoo, B. Long, and S. Subramanian, "Holographic acoustic elements for manipulation of levitated objects," *Nature Communications*, vol. 6, 2015.

[9] C. R. P. Courtney, B. W. Drinkwater, C. E. M. Demore, S. Cochran, A. Grinenko, and P. D. Wilcox, "Dexterous manipulation of microparticles using bessel-function acoustic pressure fields," *Applied Physics Letters*, vol. 102, no. 12, p. 123508, 2013.

[10] T. Hoshi, Y. Ochiai, and J. Rekimoto, "Three-dimensional noncontact manipulation by opposite ultrasonic phased arrays," *Japanese Journal of Applied Physics*, vol. 53, no. 7S, p. 07KE07, Jun. 2014.

[11] D. C. Liu and J. Nocedal, "On the limited memory BFGS method for large scale optimization," *Mathematical Programming*, vol. 45, no. 1-3, pp. 503–528, 1989.

[12] D. J. Wales and J. P. K. Doye, "Global Optimization by Basin-Hopping and the Lowest Energy Structures of Lennard-Jones Clusters Containing up to 110 Atoms," *The Journal of Physical Chemistry A*, vol. 101, no. 28, pp. 5111–5116, 1997.

[13] Y. Ochiai, T. Hoshi, and I. Suzuki, "Holographic Whisper," in *the 2017 CHI Conference*. New York, New York, USA: ACM Press, 2017, pp. 4314–4325.

[14] J. Ahrens, *Analytic Methods of Sound Field Synthesis*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2012.

[15] T. Carter, S. A. Seah, B. Long, B. Drinkwater, and S. Subramanian, "UltraHaptics," in *the 26th annual ACM symposium*. New York, New York, USA: ACM Press, 2013, pp. 505–514.