Resolving issues of scaling for gramian based input-output pairing methods

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A common issue in many industries is that interaction between different parts of the plant gives rise to a multiple input multiple output (MIMO) system, where the same input may affect multiple outputs, or conversely, the same output is affected by multiple inputs. This is the core of the input-output pairing problem; which inputs should be used to control which outputs. While one often solves this by matching one input to one output by a decentralized configuration, at times it can be necessary to add additional feed-forward between the inputs, or even implementing a full MIMO controller for parts of the system.

There are numerous proposed input-output pairing methods, many of which are discussed in [6]. The most prominent one is probably still the Relative Gain Array (RGA), and modifications of it, such as the dynamic RGA and the Relative Interaction Array (RIA)[8]. Relatively recently a new group of input-output pairing methods have been introduced, namely the gramian based methods. This group includes the Σ_2 method [2], the participation matrix (PM) [3] and the Hankel interaction index array (HIIA) [7]. These methods use the controllability and observability gramians to create an interaction matrix which gives a gauge of how much each input affects each output. An attractive property of these interaction matrices is that they can be used to determine both a decentralized controller structure and a sparse structure (a structure which includes feed-forward or MIMO blocks). Moreover, the gramian based measures take into account system dynamics and not only the steady state properties.

The gramian based methods however differ from the RGA and its variants in that they suffer from issues of scaling in the sense that the results of the methods vary depending on input and output scaling. While some methods are suggested to solve this problem in for example [4], we will show by an example of a heat exchanger network how these methods are in some situations insufficient. A method to scale the Σ_2 interaction matrix by normalizing either its columns or rows was presented in [1] and here we will examine this method in more detail and also apply it to the PM and the HIIA. Furthermore, we will examine a new scaling scheme which uses the Sinkhorn-Knopp algorithm [5] to normalize both the rows and columns of the interaction matrix.

To demonstrate the benefit of the new scaling schemes we have developed a MIMO-generator to generate a large number of systems. The scaling method for the gramian, with either columns or rows normalized, or both using the Sinkhorn-Knopp algorithm was then compared to the gramian based methods without any additional scaling on each of the generated systems. This is done by determining the control configuration of each method and evaluating their responses to reference changes and load disturbances. It is shown that using the Sinkhorn-Knopp algorithm to scale the interaction matrices generally gives the control configuration which handled both references and disturbances the best.

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