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A Survey of Voltage Stability Indicators Based on Local Synchronized Phasor Measurements

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Abstract—This paper reviews and evaluates the main types of voltage stability indicators (VSIs) based on local measurements and further provides a background to their development. Due to weaknesses during dynamic conditions, the bus VSIs based on Thévenin's equivalent impedance methods are in general found to be unsuitable for most corrective applications, but may instead be used to estimate local loadability margin to voltage instability. Line VSIs, although requiring some data communication, are in general found to be more robust and may in most cases be used both for predictive and corrective applications. Sensitivity-based VSIs are typically more accurate for detecting voltage instability, but are instead sensitive to measurement noise and are highly nonlinear when the system is close to a voltage collapse, consequently being unsuitable for estimating stability margins. The VSIs based on the local identification of voltage emergency situations (LIVES) concept can take into account the delayed effects from load tap changers, making them suitable for corrective applications and to use in local protection schemes.

Index Terms—Voltage stability index, voltage instability, synchronized phasor measurements, instability detection, emergency control, local measurements

I. Introduction

Voltage instability is a phenomenon that transmission system operators (TSOs) continuously have to take into account during both planning and operation of the power system. An increasing demand of electric power and the driving force of maximizing economic benefits have pushed the operation of the power system closer to the physical limits [1]. In general, the closer a grid can be operated to these limits, the more economic and efficient it will be. However, this will also make the system more vulnerable to contingencies and disturbances. Hence, there exists a balance between a system that is operated efficiently and one that is operated securely.

Another trend in the electrical power system is the increasing amount of inverter-based renewable generation and other power electronic controlled devices (e.g. FACTS and HVDC) that are being integrated into the power system. These appliances have generally different and significantly faster dynamics compared to more conventional equipment (e.g. synchronous generators). This development will thus with high probability increase the need of developing faster and more efficient methods of assessing the system stability [2].

In the last decades, the phasor measurement technology has opened several new perspectives and methods for wide-area monitoring and control of the power system [3]. Several voltage stability indices (VSIs) based on phasor measurements have been proposed in the literature. The phasor-based VSIs may mainly be divided into two categories [4]:

1) *VSIs based on local measurements*: These VSIs are based on few or no input from other measurements and are mainly developed using a maximum power transfer theorem or the existence of solutions for the voltage equations.

2) *VSIs based on observability of whole region*: These methods are generally more accurate than the VSIs based on local measurements. However, as the name indicates, they require full observability of the monitored region and the measurements used in these models should preferably be filtered through a state estimator causing increased computation time and complexity.

This paper will perform an extensive review of the development of VSIs based on *local phasor measurements*. The definition of a local VSI is in this paper defined as a VSI relying on measurements from only two or fewer buses. Although PMUs are becoming more widely deployed, few parts of the power systems fulfill the requirement of full observability. Further, all of the VSIs based on local measurements can be extended as the number of PMUs in a power system increases, allowing TSOs to gradually increase the monitoring system as the number of installed PMUs increase.

Previous studies have examined the development of some VSIs, e.g. as in [5], [6]. However, these reviews are more general in their approach and there is no specific focus on phasor-based VSIs using local measurements. This paper examines more the underlying differences and sensitivities to model simplifications between the VSIs and a specific focus is also spent on evaluating practical applications of the different classes. This field of research is also in development and more recent VSIs are lacking in previous reviews. The paper is aimed to provide researchers a good starting point into the field of phasor-based VSIs, as well as giving TSOs an overview of the potential applications and limits. The paper does not strive to evaluate *all* local VSIs, but rather the most prominent and/or

recently developed ones for each class.

The local VSIs may be divided into two main groups, namely; bus voltage stability indices (bus VSIs) and line voltage stability indices (line VSIs). The paper is then organized as follows. In Section II and Section III, the bus and line VSIs are briefly presented. In Section IV, an evaluation and classification of these VSIs are presented, along with a discussion of potential applications. Finally, concluding remarks are presented in Section V.

Fig. 1. A Two-bus Thévenin Equivalent Circuit

II. Bus Voltage Stability Indices

The bus VSIs are in this paper defined as the VSIs only determining the voltage stability in a single bus and by mainly requiring phasor measurements from that bus in the grid. The bus VSIs are based mainly on 3 approaches; either by using (A) Thévenin's Equivalent (TE) impedance methods, (B) sensitivity-based methods using the systems characteristics in the voltage collapse point, or (C) methods based on the so called local identification of voltage emergency situations (LIVES) method [7].

A. Thévenin Equivalent VSIs

The most common approach for the bus VSIs is to use the TE impedance as a measure of the margin to voltage instability. Considering the simple system in Fig. 1, consisting of a TE and a load bus, it can be shown that the maximum transferable power in the system occurs when $|\overline{Z}_{th}| = |\overline{Z}_L|$. This relationship has been used in several papers, e.g. [8]–[10], to develop a tracking algorithm that uses the TE impedance to estimate the proximity to a voltage collapse. The relationship between the TE equivalents may be stated as:

$$\overline{E}_{th} = \overline{V}_L + \overline{Z}_{th} \cdot \overline{I} \tag{1}$$

where $\overline{E}_{th}$ and $\overline{Z}_{th}$ are the TE voltage and impedance, respectively, and $\overline{V}_L$ and $\overline{I}$ the load voltage and current, respectively. Using the relationship in (1), the values of $\overline{Z}_{th}$ can be estimated. The real and imaginary values of $\overline{E}_{th}$ and $\overline{Z}_{th}$ in (1) results in 4 unknowns, requiring measurements to be taken at two or more times to solve for the unknown parameters. The estimation is based on the assumption that the system is in a quasi-steady-state, where the TE impedance and voltage are constant during the time of the measurements.

1) Least-squares TE (LS-TE), Impedance Stability Index (ISI), Total Least Squares TE (TE-TLS): In [8], a least-squares TE method (LS-TE) is introduced, where a larger measurement windows is used to handle measurement noise and the quasi-static TE parameters. The relation between $\overline{E}_{th}$ and $\overline{Z}_{th}$ are then used as the indicator of the proximity to voltage collapse. In [9], the impedance stability index (ISI) is developed by instead using a recursive least-squares algorithm to track these time-varying parameters. The concept is taken further by taking into account and allowing communication of reactive power limits from generators to the local voltage instability predictor relays. In [10], a method based on the
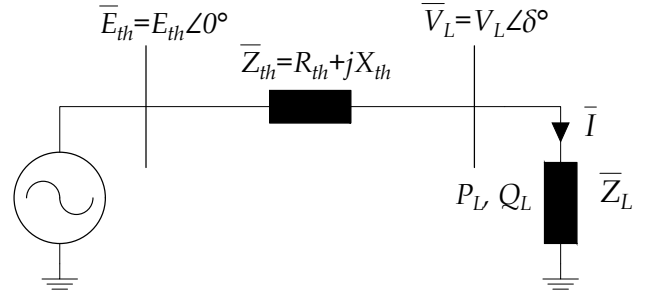
total least squares (TE-TLS) was proposed that proved less sensitive to measurement noise to other compared methods.

2) Adaptive Method (AD): In [11] and [12], the need for significant system variations between two subsequent measurements and a large data window is addressed. The method, denoted as the Adaptive Method (AD) in previous papers, assumes that $X_{th} \gg R_{th}$, causing the complexity of (1) to be reduced from four unknown variables to three. The proposed algorithm further assumes that E_{th} and X_{th} are constant in the brief interval during their identification, which requires a very short sampling time. The adaptive method then introduces an estimation of E_{th}, which allows the TE circuit to be solved directly. From the changes in the X_{th}-value the estimated value of E_{th} is then updated. The speed of the adaptive method is depending on how fast the estimation of the E_{th} is allowed to be, where a balance between a fast estimation and a non-oscillatory estimation in general is desired.

3) Thévenin Equivalent Determination Method (TE-DM): The assumption of a quasi-steady-state system is not always true, and simultaneous changes in the system side and the load side may cause large errors for TE-methods. In [13], this problem, and the impact of measurement errors, are addressed. The paper proposes a VSI, here denoted as the Thévenin equivalent determination method (TE-DM), based on the following developed relationship:

$$E_{th}^2 = V_L^2 + I^2 Z_{th}^2 + 2P_L R + 2Q_L X \tag{2}$$

where P_L and Q_L are the active and reactive power. Using three different measurements and eliminating Z_{th}^2 from the equations allows the equations to be rewritten into:

$$2\Delta PR + 2\Delta QX + \Delta V_L^2 = 0 \tag{3}$$

where

$$\Delta P = det \begin{bmatrix} 1 & 1 & 1 \\ P_{L(1)} & P_{L(2)} & P_{L(3)} \\ I_{(1)}^2 & I_{(2)}^2 & I_{(3)}^2 \end{bmatrix},$$

$$\Delta Q = det \begin{bmatrix} 1 & 1 & 1 \\ Q_{L(1)} & Q_{L(2)} & Q_{L(3)} \\ I_{(1)}^2 & I_{(2)}^2 & I_{(3)}^2 \end{bmatrix},$$

$$\Delta V_L^2 = det \begin{bmatrix} 1 & 1 & 1 \\ V_{(1)}^2 & V_{(2)}^2 & V_{(3)}^2 \\ I_{(1)}^2 & I_{(2)}^2 & I_{(3)}^2 \end{bmatrix}$$

and where the number index in parenthesis is the measurement number. The three separate measurements can then be used to represent and calculate the TE impedance parameters. To compensate for measurement errors and variations in the system side, additional redundant measurements are proposed to be used to reduce the impact of these factors.

B. Sensitivity based bus VSIs

1) S-Difference Criterion (SDC): There are a number of other Bus VSIs based on other approaches than using the TE-theorem. In [14] and [15], a sensitivity-based method denoted as the SDC is presented. The method is based on using two consecutive measurements of the apparent power on the receiving end of a transmission line. The method is based on the fact that at the voltage collapse point, an increase in the apparent power flow will not increase the received power. The SDC is defined as:

$$SDC = \left| 1 + \frac{\Delta \overline{V}_r^{(k+1)} \overline{I}^{(k)}}{\overline{V}_r^{(k)} \Delta \overline{I}^{(k+1)}} \right| \tag{4}$$

where $\overline{V}_r$ and $\overline{I}$ are the measured phasors of the receiving voltage and current for the measurement k and $k+1$. At the point of voltage instability, the SDC equals zero. In [16] and [17], the validity of such local sensitivity indices are proven by introducing a global index, in the paper called the sensitivity-based Thévenin index (STI). The STI, although requiring data from wide area monitoring systems, are proposed to be used as either validating the results, or for predicting the effects of reactive limits from local indices.

2) Real-time Voltage Stability Index (RSVI): In [18], a similar VSI to the SDC is developed, where the relationship between the rate of change of voltage and current magnitudes are used. The RSVI is defined as:

$$RSVI = 1 - \left(\frac{d\left|I_L\right|/dt}{\left|I_L\right|} - \frac{d\left|V_L\right|/dt}{\left|V_L\right|} \right) \tag{5}$$

where $d\left|I_L\right|/dt$ and $d\left|V_L\right|/dt$ are the rate of change of current and voltage magnitudes over a specified period of time (dt). In a stable state, the rate of change of voltage is close to zero, resulting in RSVI values less than 1. Near the point of collapse, the RSVI reaches a value of 1 which indicates an impending voltage collapse.

3) Ambient QV-sensitivity (Γ-VSI): Another sensitivity-based method is presented in [19], where a measure based on the slope of the QV-curve is developed. The VSI is based on the fact that, in the voltage collapse point, the slope of the QV-curve will become infinite. The VSI is based on a positive and a negative index, both calculated by the formula:

$$\Gamma_i = \frac{\Delta Q_i}{\Delta V_i} = \sum_j \frac{\Delta Q_{ij}}{\Delta V_i} \tag{6}$$

where ΔQ_{ij} is the reactive power difference between two measurements for each transmission line connected between two nodes, i and j, and ΔQ_i and ΔV_i represents the incremental change in reactive power and voltage respectively. The data is

split into a positive and a negative subset, which is then used in a weighted mean average to estimate the sensitivities. The methods are further tested in [20], where the sensitivity-based methods are found to be favorable in the sense that that they do not require any model parameters and may be extended to be used in every bus in the grid for higher observability. However, all methods require preprocessing of data as the high sensitivity to noise in the measurements may cause the accuracy of the method to be reduced.

C. LIVES concept

1) LIVES and the New LIVES Indicator (LIVES & NLI): In [7], [21], a method called local identification of voltage emergency situations (LIVES) is introduced and tested. The LIVES stability condition is based on monitoring the change in the secondary voltage after at tap decrease on the primary side ($\Delta r < 0$) of a load tap changing (LTC) transformer, which simplified may be stated as:

$$\frac{\Delta V_2}{\Delta r} < 0 \tag{7}$$

where ΔV_2 is the change in the secondary voltage. Thus, if a tap decrease leads to a negative change in ΔV_2, this indicates an unstable condition. Further, the criterion indirectly takes into account the effect of other taps acting in the system as it can observe the net effect of various LTCs over a cycle of tap operations. In [22], this concept is developed further by monitoring the stability condition of (7), solely from the transformer bus, by assuming that primary voltage and current measurements are available. The decreasing tap change is measured indirectly as an conductance increase seen from the primary side, whilst the secondary voltage is indirectly monitored as an increase of consumed active power, P. The new index, denoted as the New LIVES Index (NLI) is formulated as:

$$NLI = \frac{\Delta P}{\Delta G_1} > 0 \tag{8}$$

where

$$G_1 = Re\{\overline{I}_1/\overline{V}_1\}$$

Simulations shows promise during several different grid conditions and topologies, allowing early indication of impending voltage collapses. The method is further tested in [23], where the method is extended and applied for distance relays of transmission lines feeding weak areas.

III. LINE VOLTAGE STABILITY INDICES

The line VSIs are based on phasor measurements being available from both sides of a two-port transmission line and are mainly based on using one or a combination of three different approaches: (A) maximum power transfer theorem, (B) existence of solutions to the voltage equation, and (C) sensitivity-based line VSIs.
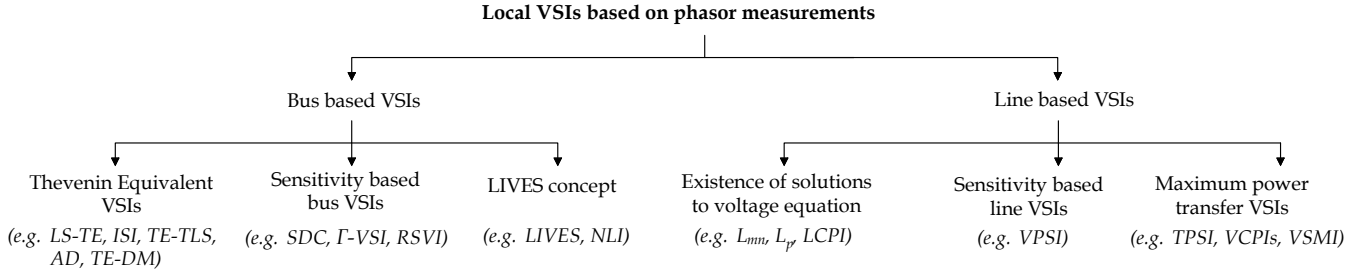
Fig. 2. Classification and some examples of local VSIs based on phasor measurements

A. Maximum power transfer VSIs

Over the years, several line VSIs based on the concept of maximal transferable power have been developed. These are similar to the TE-based methods for the bus VSIs, with the difference being that phasor measurements are required in each end on of a transmission line.

1) Transmission Path Stability Index (TPSI), Voltage Collapse Proximity Indicators (VCPIs), Voltage Stability Margin Index (VSMI): One of the first presented suitable for PMU applications, was the transmission path stability index (TPSI) in [24]. In the TPSI, the maximum power transfer occurs when the voltage drop equals the load-side voltage, according to:

$$TPSI = \frac{V_s}{2} - (V_s - V_r \cos \delta) \qquad (9)$$

where V_s and V_r indicates the sending and receiving end voltage, and δ is the angle difference between the two nodes. This measure is similar to the equal impedance theorem, although it only uses the voltage measurements on each side of a transmission line. Other line VSIs based on similar concepts are the voltage collapse proximity indicators (VCPIs) in [1], where four so called VCPIs are developed, based on the maximum transferable power and the maximum possible line losses that may occur over a transmission line. This is further examined in other papers such as in [25], where similar VSIs based on the same principle are proposed. A simple index, called the voltage stability margin index (VSMI), presented in [26], uses the angle differences between two buses. The VSMI, although showing promise, was found to have limited accuracy for transmission lines with high Q/P ratios.

B. VSIs based on existence of solutions to voltage equation

The methods based on the existence of solutions to the voltage equation are mainly based on different formulations of the classical power-voltage relationship with negligible line resistance. This relationship may be stated as [27]:

$$V_r = \sqrt{\frac{V_s^2}{2} - QX \pm \sqrt{\frac{V_s^4}{4} - X^2 P^2 - X V_s^2 Q}} \qquad (10)$$

where P and Q is the active and reactive power respectively, and R and X the line resistance and reactance. It can be shown that the maximum power transfer occurs when the value of the inner square root in (10) is zero.

1) L_p, & L_{mn} & Line Collapse Proximity Indicator (LCPI): In [28] and [29], two popular indices called L_p and L_{mn} are presented, using either the expression for the active or the reactive power and reformulating with respect to solvable values for the discriminant of the voltage equation.

In most of the line VSIs, the shunt susceptance is neglected, which naturally leads to a more restrictive assessment of the proximity to the voltage instability point. This is addressed in [30], where a VSI based on the classical ABCD-matrix of a π-modeled transmission line is defined according to:

$$LCPI = \frac{4A \cos \alpha (P_L B \cos \beta + Q_L B \sin \beta)}{(V_s \cos \delta)^2} \qquad (11)$$

where A and B are the transmission line parameters from the ABCD-matrix, and α and β are the respective phase angles of the A and B components. A large amount of other line VSIs based on the similar concept are also presented in other papers.

C. Sensitivity-based line VSIs

1) Voltage-Power Sensitivity Index (VPSI): In [31], the sensitivity of the voltage-to-power characteristics at the voltage instability region is used to form a VSI. The VSI, in this paper denoted as the VPSI, is based on the existence of solutions to the voltage equation and is based on the fact that $dV/dP \to \infty$ at the point of a voltage collapse. The VPSI is then defined as:

$$VPSI = \frac{V_L}{\sqrt{2V_s^2 + 2(P_r R + Q_r X)}} \qquad (12)$$

When the system is close to the voltage collapse point, the index of VPSI approaches 1. Although showing effectiveness in simulations, the practical aspects of the VPSI are affected by it being highly non-linear when close to the collapse point. Other events, such as generator capability limits being met, may also affect the accuracy of the VSI.

IV. CLASSIFICATION AND EVALUATION OF VSIs

A. Classification and attributes

The classification developed in this paper is presented in Fig. 2 and the general attributes are presented in Table I. The inherent local feature of the TE-VSIs is one of the main advantage of that type of indicators, with in principle no requirements of communication from other buses. However, several studies, such as in [32], have shown the weakness

Type	Subcategory	Index	Attributes & applications
Bus VSIs	Thévenin Equivalent VSIs	LS-TE [8] ISI [9] TE-TLS [10] AD [11] TE-DM [13]	Low requirement on data communication; Provides information on loadability margins; In general unsuitable for corrective applications
	Sensitivity-based bus VSIs	SDC [14] Γ-VSI [19] RSVI [18]	Sensitive to measurement noise; Nonlinear indicator in collapse point; Suitable mostly for corrective applications
	Methods based on the LIVES concept	LIVES [7] NLI [22]	Suitable for corrective applications; Fast local assessment of voltage stability; Used either for weak areas and/or buses with for LTCs
Line VSIs	Existence of solutions to voltage equation	L_{mn} [28] L_p [29] LCPI [30]	Some data communication requirements; Predictive and corrective applications possible; Affected by line parameters errors
	Maximum power transfer	TPSI [24] VCPIs [1] VSMI [26]	Similar to the VSIs based on existence of solutions to voltage equation
	Sensitivity-based line VSIs	VPSI [31]	Similar to sensitivity-based bus VSIs

of the TE-methods when modeling meshed power systems during nonlinear and dynamic conditions. The fact that the TE parameters are estimated over a time window which has to be wide enough to result in sufficient change in the operating conditions, whilst at the same time narrow enough to assume the quasi-steady state of the system, may also significantly reduce the speed and/or accuracy of these VSIs. Since the line VSIs use measurements from both sides of a transmission line, these are less sensitive to changes in, for instance, system topology. Additionally, they do not have the same requirement for a filtering window as the TE-VSIs.

The sensitivity-based VSIs, both for the bus and the line based, are favorable as they do not require any model parameters. However, they have a drawback of being highly nonlinear when the system is close to the voltage collapse point. The sensitivity-based VSIs are also highly sensitive to measurement noise, which requires some filtering algorithm either on the measurement values or on the signal of the VSI. The VSIs based on the LIVES concept, requires similarly as previous VSIs a filter to reduce noise and short term transients. These method are developed mainly to be applied to either buses with LTCs, or as in the case of the NLI, any transmission bus feeding a weak area.

B. Potential Applications

The characteristics of the VSIs are of high importance to what kind of practical applications they would be used for. In general, they may either be used for (i) preventive applications, or (ii) for emergency/corrective applications [4]. Preventive applications include the possibility of estimating the local loadability margin, which can be be used by system operators to take preventive actions against voltage instability. Corrective applications include to in real-time detect and warn system operators of voltage instability, as well as initiate local system protection schemes (SPS) that, for instance, can give signals to relays for undervoltage load shedding.

1) Preventive applications: The TE-VSIs and for the line VSIs, with the exception of the sensitivity-based VSIs, are mostly suitable for preventive applications. Due to the discussed weaknesses of the TE-VSIs during dynamic conditions, these may in general be unsuitable for corrective and emergency purposes. However, in more stable conditions, the difficulties of estimating the TE parameters, such as the need of using a large time window for filtering, will be reduced. Thus, the TE-VSIs may instead be used to, in near real-time, allow system operators to determine the loadability margin for that specific bus. Such estimations will allow the system operators to track the margin in between the conventional, slower, voltage stability assessments. Most of the line VSIs, being able to both estimate the distance to a voltage collapse, and being more robust during dynamic conditions, allows them to in a larger extent be used for both types of applications.

2) Corrective applications: For all categories of the sensitivity-based VSIs, the indicator is mainly useful corrective applications, as those indicators in general are highly nonlinear closer to the collapse point. For corrective applications, speed and accuracy of the assessment is fundamental. However, the inability of most local VSIs to take into account the impact of overexcitations limiters (OELs) and/or the delayed effects of LTC transformers, will cause slower assessments during emergency conditions. This has led recent papers to in a larger extent use so called coupled single-port Thévenin equivalent model (e.g. in [33]), that in a larger extent can take into account the effects of, for instance, OELs. Such methods, although seemingly effective, do require more communication infrastructure and the simplicity of the VSIs based on local measurements are thus lost. For the VSIs based on the LIVES concept, the dynamics of the LTC transformers are being taken into account, allowing them to perform quick identification of impending voltage collapses, and thus being highly suitable for corrective applications and for local SPS.

3) Practical experience: Even though PMUs for a quite long time have been deployed into the power system in several countries, practical applications of local VSIs are uncommon and is to a large extent still considered as a "future" application [3]. This notion is confirmed when examining technical reports, where very little practical experience from local VSIs are reported. Although the technology and the methods have been developed for several years, most TSOs seem reluctant of implementing these methods practically. The rather limited practical use of these indicators are, according to the authors of this paper, mainly

due to fact that the robustness of the VSIs still to some extent is undetermined. As blackouts and other major failures are connected with extremely high costs, the robustness of the VSIs are of highest concern to the TSO. Thus, from the view of a TSO, it is more important that a VSI is robust and accurate than having a fast computation time. Furthermore, the overall lack of practical experience may itself deter TSOs to use such methods. Thus, even more research and field testing of the developed VSIs are required.

V. CONCLUSION

This paper presents a review of the development of VSIs based on local phasor measurements, and further attempts to classify the VSIs based on their attributes and applications. The TE-VSIs, simple in their design but somewhat inaccurate during dynamic conditions, are mainly proposed to be used to in near real-time monitor the local loadability margin to voltage instability. Line VSIs are in general found to be more robust, whereas the fully local feature is somewhat lost. For the sensitivity-based VSIs, the main drawback is the high sensitivity to measurement noise and the nonlinearity of the indicators. These type of VSIs are thus mainly proposed to be used in corrective assessments, allowing warnings to in real-time be communicated to the system operators. The methods based on the LIVES are able to take into account some of the delayed effects from e.g. LTCs, making them suitable for corrective applications and local SPS. The practical implementations of the local VSIs are limited and more accurate estimations of the robustness and accuracy of the methods are required for a more widespread use.

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