Measurement-Based Modeling of Higher-Order Non-Linearities of the Parametric Loudspeaker

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Measurement-Based Modeling of Higher-Order Non-Linearities of the Parametric Loudspeaker

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Introduction

The so called "Parametric Audio" effect is a way to create highly directional audible sound using ultrasonic carrier waves. It stems from an acoustical phenomena where two high frequency waves at high intensity will interfere and generate intermodulation tones due to non-linear effects in the wave-propagation. Most analytical models apply a second order approximation to predict the level of the audible sound. With a second order approximation it is not possible to compensate distortions in the audible sound caused by higher order non-linear effects. Higher order distortions in the audible sound were observed anecdotally by the authors, raising the question if they as well could be compensated.

This paper describes a set of measurements of the amplitudes of the harmonics of a bifrequency plane wave, and compares the measured results with a simple theoretical model. The measured results indicate that there is significant nonlinear distortion in the electrical and/or mechanical subsystems. Even if a model is developed to predict and compensate for said distortion the model would be tied to a specific amplifier and transducer combination, with very limited real-life applicability.

Theoretical Model

The nonlinear distortion in air of a bifrequency source in a one-dimensional wave guide was treated in the 1970s by Fenlon [1]. The components of primary interest in this work are the difference frequency terms p_{lm}, which can be expressed as

$$p_{lm} = 2p_0 \frac{(-1)^m}{\sigma N_{lm}} J_l(N_{lm} P_a \sigma) J_m(N_{lm} P_b \sigma) \sin(N_{lm} \omega \tau)$$

where p_0 is an arbitrary characteristic sound pressure normalization, $P_a = p_a/p_0$ and $P_b = p_b/p_0$ are the normalized pressure amplitudes of the two sines at the source, $N_{lm} = ln_a - mn_b$, and $\tau = t - x/c_0$ is the retarded time. The fundamental frequency of the combined signal ω and the two angular frequencies of the source, ω_a and ω_b, defines the two coefficients n_a and n_b as $n_a = \omega_a/\omega$ and $n_b = \omega_b/\omega$. Note that n_a and n_b are necessarily integers, otherwise ω is not the fundamental frequency of the combined signal. The normalized distance σ is defined as

$$\sigma = \beta \frac{p_0}{\rho_0 c_0^2} kx$$

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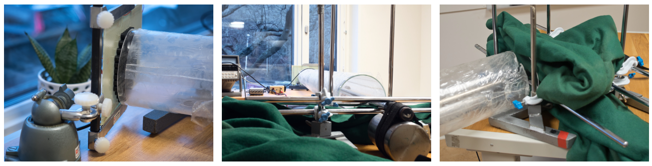

Figure 1: Measurement setup.

where $\beta = 1.2$ is the coefficient of nonlinearity for ideal diatomic gases, ρ_0 and c_0 are the density and speed of sound under linear conditions, $k = \omega/c_0$ is the wavenumber corresponding to the fundamental frequency, and x is the distance from the source. The two indices l and m both range from 1 to ∞ and creates the set of all difference-intermodulation components.

The above expression can be used to determine the amplitudes of the difference-intermodulation tones, given knowledge of the source attributes $(p_a, p_b, \omega_a, \omega_b)$, distance (x) and properties of air (β, ρ_0, c_0). Similar expressions exist for the pure harmonics, and the sum-intermodulation tones. For further details the interested reader is referred to Blackstock and Hamilton [2]. In this work the input frequencies are $\omega_a = 2\pi \cdot 40\,250\,\text{Hz}, \omega_b = 2\pi \cdot 39\,750\,\text{Hz}$ which gives the fundamental frequency $\omega = 2\pi \cdot 250\,\text{Hz}$ and $n_a = 161, n_b = 159$. The source amplitudes p_a, p_b are determined by fitting the model to the measured amplitudes at the two source frequencies.

Measurements

A parametric audio speaker consisting of 120 small ultrasonic transducers was placed at one end of a 2 m long, 12.5 cm diameter tube, see Figure 1. The tube was terminated with absorptive cloth to reduce the reflections due to the impedance change at the end of the tube. The transducers were fed with an amplified signal consisting of two sines at the same amplitude at two frequencies 39 750 Hz and 40 250 Hz. The frequencies were chosen to match the main resonance of the transducers at 40 kHz.

The sound pressure was measured along the central axis of the tube in 5 cm steps using a Brüel & Kjær Type 4133 microphone with a known frequency response up to 40 kHz with the protective grid mounted, as it was during the measurements. The microphone is specified for distortions of $< 1\,\%$ at a sound pressure level of 154 dBSPL, approximately 20 dB higher then the measured levels. It is not clear to the authors if this specification is with or without the protective grid and how large effect the grid has on intermodulation distortion. The measurements were done at nine different input amplitudes in

3 dB steps, from a few dB below the maximum rated amplitude of the transducers to 24 dB lower. The amplitudes of the harmonics were determined in the frequency domain, using a flattop window to ensure consistent results [3].

A typical spectrum of the measured sound pressure inside the tube is shown in two frequency ranges in Figure 2. It is clear that the two input tones are the strongest components in the output peaking at 132 dBSPL. The strongest audible component is at 500 Hz. i.e. the fundamental audible tone, and peaks at 107 dBSPL, see Figure 6.

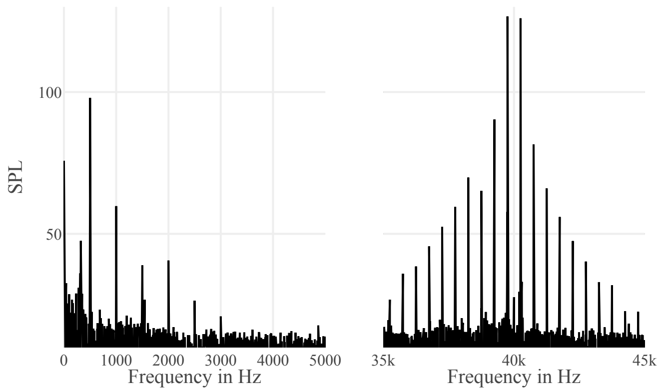

Figure 2: A typical spectrum from a measurement inside the tube.

The spectrum of the measured amplifier signal at the maximum amplitude used can be seen in Figure 3. It is clear that there is some distortion in the amplifier but almost exclusively intermodulation tones in the inaudible range, corresponding to odd order nonlinearities. The strongest distortion component in the amplifier output is 46 dB below the primary output signal.

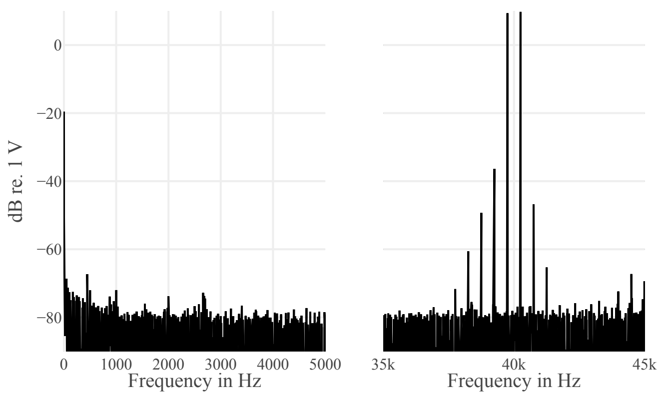

Figure 3: Spectrum of the amplifier output at maximum input amplitude.

Comparison

Some differences between the measured results and the theoretical model is expected, due to differences in source condition and boundaries. One difference is the theoretical assumption of a plane wave. The wave in the tube is generated from an array of transducers, which does not form a perfect plane wave immediately in front of the array. Since the nonlinear distortions in air are a cumulative effect, the difference in spatial structure could in principle be apparent throughout the entire tube. The theoretical model assumes an infinitely propagating plane wave, but the finite extension of the tube might influence the propagation of the wave. Two such effects can be the influence of the termination of the tube, and the influence of the circumference of the tube.

Furthermore the transducers and the amplifier might influence the source pressure by introduce unknown nonlinear distortion. Such distortion will create components in the source pressure at frequencies differing from the intended input frequencies. The propagated wave from this distorted source conditions cannot be predicted using the simple Fenlon solution, but requires a more general treatment.

The measured and simulated amplitudes of the two source pressure tones are compared in Figure 4. Since the source pressure in the theoretical model is tuned using the amplitudes of these tones, the overall amplitude corresponds well. The difference in measured and simulated amplitude can be separated in two types, a negative trend and a local variation. The negative trend is likely due to absorption at the circumference of the tube. The local variation indicates a wave which is not a plane wave, with amplitude variations either along the length of the tube or over the cross-section of the tube. A possible cause of a cross-sectional variation is the discrete distribution of the elements in the array together with reflections from the circumference of the tube. Rapid variations along the length of the tube can be caused by reflections at the end of the tube, causing a partial standing wave in the tube.

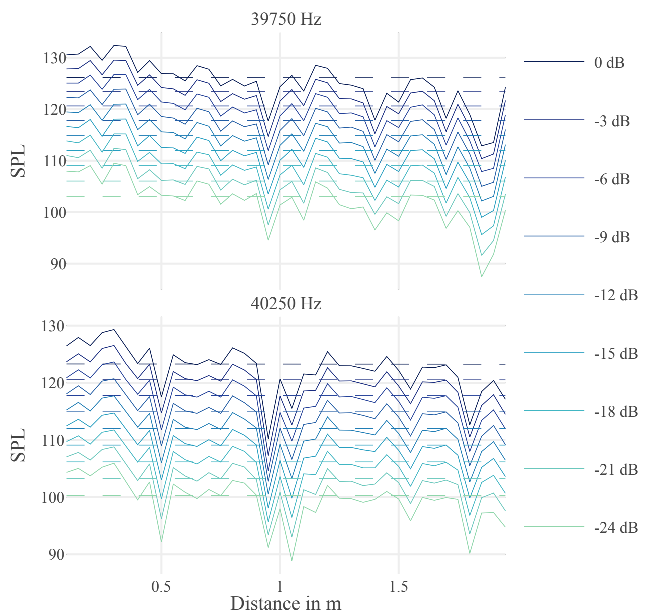

Figure 4: The amplitudes of the main two harmonics for multiple amplitudes and distances. Solid lines indicate measured values while dashed lines indicate simulated.

Figures 5 and 6 show a selection of the inaudible and audible intermodulation tones, respectively. All tones show parts of the same variations over space as the two source pressure tones, see Figure 4. More interesting are the other differences from the theoretical results only apparent for the intermodulation tones. The measured tones show much higher levels than what is expected from the theoretical model. Furthermore the theory predicts the amplitude of the distortion components to increase over space, with no components present at all very close to the source. The measured amplitudes are present from the very start, and show no tendencies to increase as the wave propagates. It is unclear if these tones are present to the high levels measured here, or to what extent the measurement procedure introduced nonlinear distortion in the output signal from the microphone. If the microphone is regarded as sufficiently linear, the measured levels indicate that much of the audible components is not due to the nonlinear behavior of the propagating wave, but is present in the source pressure at the start of the tube. For these components to be present in the source pressure, the combined distortion in the amplifier and transducers has to be the cause of the tones.

Note that the noise floor in the measured amplitudes is around 15 dBSPL, see Figure 2. For the 500 Hz tone all measurements are fully above the noise floor. For the 1000 Hz tone the 6 highest amplitudes are above the noise floor at all distances, and the next two are above the noise floor close to the array. For the 1500 Hz tone the 3 highest amplitudes are above the noise floor at all distances, and the next two are above the noise floor close to the array. For the inaudible tones all measured amplitudes shown here are fully above the noise floor.

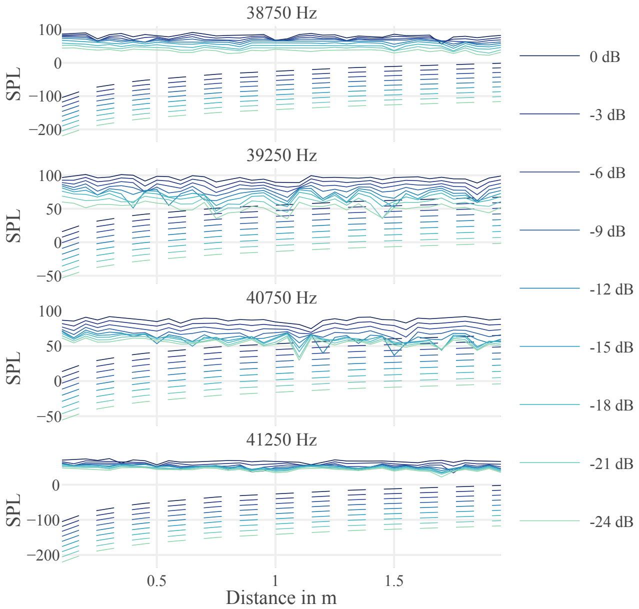

Figure 5: The amplitudes of a selections of the inaudible intermodulation tones for multiple amplitudes and distances. Solid lines indicate measured values while dashed lines indicate simulated.

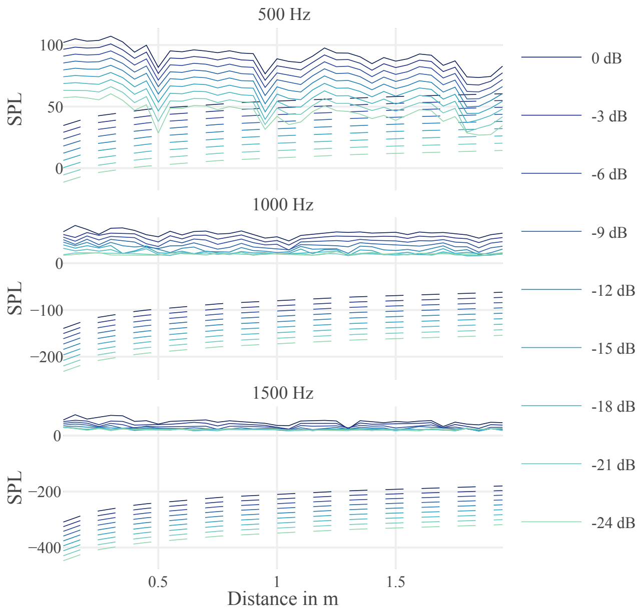

Figure 6: The amplitudes of a selections of the audible intermodulation tones for multiple amplitudes and distances. Solid lines indicate measured values while dashed lines indicate simulated.

Conclusions

There is a considerable difference between the prediction from a theoretical solution to the bifrequency source condition in a one dimensional waveguide and our measured results. Due to the large difference in both magnitude of the distortions but also the spatial behavior it is very likely that the difference is explained by additional distortion in the electrical and mechanical systems, i.e. the amplifier and the transducers. These distortions earlier in the signal path cannot be taken into account using the simple equations used here, but a generalization to an N-frequency source pressure exists [4], which could be used to predict the sound pressure in a waveguide if the source conditions could be determined precisely enough.

For the amplifier it is simple enough to measure the voltage output and use that as a new N-frequency input signal. This could in part explain the higher measured levels in the audible tones, as a result of intermodulation in air between the intended tones and the inaudible intermodulation tones caused by the amplifier. The distortions which occur in the transducers are more difficult to measure, requiring precise mechanical measurements of a small surface with a high enough dynamic range to detect the distortion components. A mechanical distortion in the transducers could be responsible for the difference in spatial structure between the theoretical and measured amplitudes. Any even order nonlinearity in the transducers combined with the odd order nonlinearities in the amplifier would create source pressure components in the audible range, which then exist directly in front of the array. Purely electrical distortions cannot be the cause of the audible components of this magnitude close to the array, since the transducers barely radiate anything at those frequencies even if the full power of the amplifier is given at 500 Hz.

We have shown that measurement based methods to reduce higher-order nonlinearities of a parametric loudspeaker is complicated due to distortions to the signal occurring in the electrical and mechanical systems. Modeling these systems to reduce the distortions might be possible but any passive model will be limited to a particular combination of amplifier and transducer array. A method capable of reducing the distortions of a general system will likely include active monitoring of the distortion components close to the array, either sensing the acoustical sound or the mechanical vibration of the transducers.

References

[1] F. H. Fenlon, "An Extension of the Bessel-Fubini Series for a Multiple-Frequency CW Acoustic Source of Finite Amplitude," J Acoust Soc Am, vol. 51, no. 1, pp. 284–289, 1972.

[2] Blackstock, D. T., et al.: "Nonlinear Acoustics", Acoustical Society of America, 2008, p.79.

[3] D'Antona, G., Ferrero, A.: "Digital Signal Processing for Measurement Systems", Springer Media, 2006, p. 70.

[4] F. H. Fenlon, "Derivation of the multiple frequency Bessel-Fubini series via Fourier analysis of the preshock time waveform," J Acoust Soc Am, vol. 53, no. 6, pp. 1752–1754, Jun. 1973.