Tolerance Analysis Framework for Cutting Tool Interface Design

Soner Camuz
To my wife Elin and my daughter Ester, for their unconditional love and support throughout the ups and downs of the research project.
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Abstract

Tolerance analysis of cutting tool interface designs is a field with many opportunities for further development of existing methodologies. Cutting tool interface designs require multiple contacting surfaces, allocating the stress generated from the cutting forces, to avoid excessive deformation of the interface. With numerous contact areas, the insert will be overdetermined in the interface. The positioning of the insert in the tool body is dependent on the positioning of the contacting surfaces and of the magnitude and direction of the cutting force. Erroneous or varying positioning of the insert can result in reduced productivity. This research project aims at creating a framework to handle tolerance allocation of cutting tool interface designs. The main issues found within current tolerance analysis methodologies are their inability to incorporate overdetermined surface-to-surface contacts and nonlinear material behaviours.

The backbone of the framework follows a typical empirical research model: set the design space, simulate, build a meta-model, optimize and visualize. The first iteration of the framework relies on current methodologies to gain a holistic view of the research field and to identify areas that need improvements. In the second iteration, a reliability-based optimization routine with a genetic algorithm is used to accommodate the stochastic nature of overdetermined assemblies. The framework in its current state allows the practitioner to set predefined contact zones to define the positioning of the insert in the cutting tool body. The optimization routine finds a nominal set of input parameters that fulfills a predetermined criterion limiting the variation. The proposed framework allows for the practitioner to apply and analyze tolerances in cutting tool interface designs. The conducted research contributes to filling the scientific gaps regarding the positioning of surface-to-surface contacts in assemblies.

An approach to incorporating nonlinear material behaviour in variation simulation of sheet metal parts has been proposed using Taylor’s expansion of the primary variable in a finite element analysis. The approach has shown potential in reducing computational time with limited effect on the accuracy of the simulation. The method has not yet been implemented in the framework and needs further work before being considered for implementation.

Current limitations of the framework involve computationally heavy simulations, which grow exponentially with added input parameters. Further research needs to investigate how computational time can be reduced to increase the applicability of the framework in early design phases.

Keywords: Variation simulation, geometry assurance, cutting tool interface design, reliability-based optimization, tolerance analysis
ACKNOWLEDGEMENT

This study has been carried out at the Department of Industrial and Materials Science at Chalmers University of Technology in Gothenburg and at AB Sandvik Coromant in Sandviken.

My first acknowledgement goes to my academic supervisors Rikard Söderberg and Kristina Wärnfjord for their support and motivational backup throughout the years.

My deepest gratitude goes to the Department of Metal Cutting Technology at AB Sandvik Coromant for their ability to critique and push my ideas further and challenge me intellectually. Special thanks to Jonas Östby for taking his time and helping me by proofreading the thesis.

I would also like to thank Swedish Agency for Innovation Systems (VINNOVA) and AB Sandvik Coromant for their financial support.

Finally, I would like to thank my family, especially my wife Elin and my daughter Ester, for their love, patience and support.
THESIS CONTENT

This thesis consists of an introduction and is based on the following appended papers:

**Paper A**

**Paper B**

**Paper C**

**Additional Paper**

**Distribution of Work**

The appended papers were prepared in collaboration with co-authors. The author of this thesis was responsible for the major progress of work, including taking part in planning the papers, developing the theory, developing and carrying out measurements, the numerical implementations and writing the reports. The co-authors reviewed the papers and contributed with comments and input to the work.
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Summary of thesis

1 INTRODUCTION

This chapter provides an introduction to the conducted research, containing motivation and incentives, aim and scope, research questions and a brief outline.

1.1 Background

As machining operations become increasingly optimized, the effects of deviations in the macro geometry are becoming more apparent for indexable cutting tool interface designs. Small deviations in the macro geometry can have a considerable effects on quality aspects, such as tool life, surface finish and dimensional repeatability. A possible source of variation in the macro geometry for indexable cutting tools is the positioning of the insert in the tool body interface.

The positioning is given by the contacts between the tool body and insert and tends to be overdetermined\(^1\). Overdetermining the positioning of the cutting tool interface causes imprecise positioning, leading to deviations in the cutting load magnitudes or directions. Consequently, the intended productivity of the cutting tool is affected. Factors that will alter the positioning involve cutting force direction/magnitude, the stiffness in the interface, and geometric deviations of the interface caused by manufacturing process variations. Therefore, methodologies for reducing the effects of such variations are necessary to ensure quality aspects such as durability, reliability and performance in cutting tool designs.

The competitiveness of cutting tool manufacturers is steadily increasing. Companies continuously need to improve the quality and productivity, reducing the costs to get an advantage over the competitors. Increasing the quality and productivity of indexable cutting tools involves mainly improving three areas, which are: the machine process, the carbide insert and the tool body. Examples of improvements are:

- Strategies to enhance the machining process involve on-site improvements, where optimal cutting tool paths and parameters are adjusted.
- Improvements on indexable carbide inserts involve enhancing specific properties of the carbide composition as well as various cutting geometries, both macro and micro. Furthermore, increasing the amount of cutting edges per insert reduces the total manufacturing cost.
- Improving the tool body involves finding optimal dynamic and static stiffness, thus improving cutting processes through e.g. increased depth of cut, cutting speed and feed.

In recent years, efforts in increasing the robustness of the cutting point of an insert have been in focus. These efforts have involved both chamfered and serrated interfaces in the tool body. The results have often had the unfortunate, unintended effect of overdetermining the positioning in the interface. With current tolerance analysis and allocation methodologies, it is very difficult to ensure the expected robustness due to the sheer complexity of the interface. Continuously altering the positioning of the insert, during a cutting process, will have a negative impact on

\(^1\)For example, a four-legged chair with different leg heights. As the person sitting on it shifts its centre of gravity, a new set of three legs will be in contact with the ground. Thus causing a wobbling effect as the load changes direction and magnitude.
the cutting performance. Altering the positioning will reduce the expected tool life and generate an inferior surface quality of the machined workpiece. The forces generated from the cutting process will not only affect the positioning, but they will also plastically deform the interface, which can lead to altered macro geometry for future operations with new inserts.

1.2 Scope
The aim of this research project is to create a framework of methodologies to analyze and allocate tolerances for complex overdetermined assemblies with surface-to-surface contacts, which are typically found in cutting tool interface designs. For tolerance analysis and allocation of interface geometries it has been identified that the following elements need to be considered and handled in the methodologies and framework:

- Non-linear material behaviour
- Surface contact with friction
- Overdetermined assemblies
- External effects such as mechanical and thermal loads
- Transient analysis for operations such as drilling and milling

1.3 Related Research
The conducted research combines four fields: tolerance analysis, positioning, overdetermined assemblies and machining. Through literature reviews, it was identified that current methodologies require the incorporation of non-linear material behaviour and surface-to-surface contact formulations for geometry assurance of cutting tool design. This section presents the current state of the research field with relevant articles and defining the gaps.

In (Moos and Vezzetti, 2013) the authors discuss the importance of non-linear material behaviour in resistance spot welding (RSW). They identified that the clamping force by the electrodes deviates and causes plastic deformations which in turn affect the assembly deviations. Adopting, for example, the MIC methodology under these conditions will reduce the accuracy due to non-linearities in the material behaviour. The authors in (Söderberg et al., 2016) also point out the importance of using non-linear material models in variation simulations and the need for incorporating new material models in current methodologies.

To model how variations propagate through an assembly, different approaches can be taken, each with its advantages and disadvantages (Shen et al., 2005). Point-based approaches, such as the 3-2-1 locating scheme for rigid assemblies (Söderberg and Lindkvist, 1999) and N-2-1 for compliant assemblies (Cai et al., 1996; Söderberg et al., 2006), are most common in sheet metal applications and fixturing of workpieces. Point-based locating schemes require that the positioning between two or more parts are known and unchanged during operation. The large cutting forces will alter the positioning of the cutting, and therefore known locating schemes will provide an unrealistic boundary condition for cutting tools. The authors in (Dahlström and Lindkvist, 2007) present an approach for implementing a contact
search algorithm in the MIC methodology to avoid penetration of contacting surfaces. The implementation shows significance in reducing computational time with a limited loss of accuracy. The contact in cutting tools between the insert and the tool body are surface-to-surface contacts. In (Dantan et al., 2008) a skin model-based approach is used to define a coherent expression of Geometrical Product Specification (GPS) during tolerancing on isolated parts. The authors in (Schleich et al., 2015) extend the skin model approach by incorporating a framework that allows for simulation of assembly and kinematic behaviours. The skin model approach is validated in (Schleich and Wartzack, 2016) where the authors present a quantitative study on tolerance analyzes by comparing a skin model to three well-known methods: tolerance stack up, vector loops and small displacement torsor. The authors highlight the importance of integrating deformation and thermal effects in the skin model approach. In (Garaizar et al., 2016) the authors present a framework for integrating thermal effects in skin models. The skin model is generated through a finite element mesh where geometric deviations, both systematic and random, are added to the nodes. The new mesh is then used to simulate thermal effects using finite element analysis (FEA). However, the framework does not take contact boundary conditions into consideration. There is a high probability that the contacting surfaces will penetrate after added variation. Skin models with integrated effects will require numerous FE simulations in order to get statistical data. It is time-expensive and resource-heavy due to the non-linearity of the boundary conditions. Therefore, the number of simulations must be reduced without losing the accuracy of the results, keeping it possible to collect enough data for statistical assessments.

The authors in (Lorin et al., 2012, 2010) generate a design of experiments where process input parameters, such as mold temperature and cooling time, are varied. Each observation is simulated, and a regression model is created for each node on the surface and used for variation simulations. The approach is independent of the distribution of input parameters, as variation simulations are conducted prior to the FEA simulations. Using regression or meta-models also allows for various distributions of the input parameters in the variation simulations compared to the data collection. The accuracy of the variation simulations is therefore dependent on the coverage of nonlinear behaviours in the design of experiments and the FE-model.

### 1.4 Delimitations

Deformations in the tool body will alter the positioning of the insert, thus changing the macro geometry. Changing the macro geometry will affect the forces generated from the cutting process. In this research, modelled cutting forces are kept constant throughout a simulation. Wear mechanisms on the insert due to the cutting conditions have also been neglected in this thesis. However, future simulation implementations should take consideration to variations in the cutting forces due to the cutting geometry.

### 1.5 Research Questions

From the introduction two research questions have been defined and will be answered in this thesis. They are as follows:
RQ I  

*How can external loads be handled in locating schemes for overdetermined surface-to-surface contact conditions?*

This research question addresses the issue of overdetermined locating schemes in cutting tools. The external thermomechanical loads can alter the positioning of an indexable carbide insert in the interface of a tool body and affect the overall quality of the cutting tool.

RQ II  

*How can nonlinear material behaviour be accounted for in variation simulations?*

Current variation simulation methodologies neglect the effect of plastic deformations, as solving nonlinear material behaviours require an iterative approach, which will increase computational time extensively. This research question addresses this issue and aims to incorporate nonlinear material behaviours in variation simulations.


2 FRAME OF REFERENCE

This chapter will outline the foundation of the theoretical background and frame of reference used in this research project and give an overview of previous research conducted within this research field.

2.1 General Metal Cutting and Classifications

Metal cutting is a manufacturing process that removes excess material as chips from the workpiece. The objective is to get the workpiece to its desired dimensions and surface finish. The cutting edge of a wedge-shaped tool causes the workpiece to plastically deform, forming chips. This process generates large shear strains on the primary shear zone, see Fig. 1. There are two main deformation mechanisms associated with the secondary shear zone. The first mechanism is the chip rubbing against the surface of the rake face, generating heat and shear stresses that exceed the material yield limit. The second mechanism is the material flow over the stagnation point\(^2\) that generates shear stresses. Friction between the flank surface of the insert and the machined surface generates shear stresses in the tertiary zone.

A machining system contains three major subsystems: the machine tool, the tool holder, and the cutting holder. A machine tool is a power-operated machine that cuts or shapes materials such as metal and wood. The most common practice is to distinguish machine tools by the type of machining operation they can perform, which are either rotation symmetric (Fig. 2a) or prismatic (Fig. 2b) (Groover, 1996). Rotation symmetrical machines have a rotating workpiece and a stationary cutting tool, while prismatic machines have a stationary workpiece and a rotating cutting tool. However, there are hybrid machines that can both do rotation symmetric and prismatic machining. This allows for more operations per set-up, which reduces both lead time and variations induced by fixturing the workpiece.

The tool holder is the interface that connects the cutting tool to the machine tool, see Fig. 3. The design of the tool holder is dependent on the application. Tool holders are either made from one solid piece (monolithic) or as a mechanical modular system. Choosing the wrong type for an operation can result in a decrease in quality aspects such as accuracy, repeatability, rigidity and tool life.

\(^2\)The stagnation point is where the material meets the edge and either goes over or under it thus generating intense shear stresses.
The third subsystem is the cutting tool which is either a solid High-Speed Steel (HSS) tool or a cemented carbide tool. HSS tools are preferable for ductile materials and low-speed applications where a sharp cutting edge is required. Solid carbide tools consist essentially of a mix of tungsten and cobalt powder that is compressed in a die to form the tool shape. After that, the tool is sintered\(^3\). The result is a cemented carbide tool that has enhanced wear resistance and can withstand higher temperatures than HSS tools. This makes cemented carbide more suitable for machining tougher materials such as carbon- or stainless steel.

In this research project, indexable cutting tools are in focus. An indexable cutting tool consists of a tool body with a cemented carbide insert that is fixated on the tool body using either a screw, self-clamping mechanism or some other mechanical clamping mechanisms. The term “indexable” refers to the interchangeability of the cutting edge (or insert), as an insert can have more than one cutting edge.

### 2.1.1 Cutting Tool Classification

Cutting tools can be classified in numerous ways. The most common classification is based on the number of cutting edges that are active during an operation: single point, double point and multi-point. Single point cutting tools have only one main cutting edge that is operational during the machining process for turning, boring,

\(^3\text{Sintering is a process that uses heat and/or pressure to fuse masses together without reaching the liquefaction point, which is roughly 1400°C for tungsten/cobalt mix}\)
slotting, etc. Double point tools have two cutting edges that are active during for example drilling. Multi-point cutting tools have more than two main cutting edges that are active during an operation and that work simultaneously to remove material in a single pass during for example milling, broaching, gear hobbing, etc.

2.1.2 Mechanistic Models to Predict Cutting Forces

There are different approaches to model the cutting process and they can be divided into four categories: analytical, experimental, numerical and mechanistic models. This section will briefly outline the different models to predict cutting forces but will mainly focus on mechanistic models.

Analytical models predict the cutting forces based on physical mechanisms during machining. The models are either based on single shear plane theory (Ernst and Merchant, 1941; Lee and Shaffer, 1951; Merchant, 1945a,b), or shear zone theory (Oxley and Hatton, 1963). However, analytical models do not consider high strain rates, temperature gradients and elasto-plastic material behaviours. This results in analytical models not accurately predicting the general case of machining.

Experimental models rely on empirical measurements and focus on collecting data through static and dynamic cutting tests. This information can be used to calculate cutting parameters to e.g. avoid chatter vibrations (Altintas, 2012), which limits the productivity of the machining process.

Numerical models rely mainly on simulations of the cutting process using FEA to predict cutting forces. One of the major challenges within this field is to formulate the material models. Inaccurate material behaviour will give invalid predictions.

Mechanistic models are semi-analytical models that assume that the cutting forces are proportional to the uncut chip area. This means that the models are dependent on the cutting conditions, the cutting geometry and the material properties of the workpiece. These dependencies are referred to as cutting force coefficients or specific cutting forces in the mechanistic models. There are two main approaches to model the mechanistic cutting forces: First-Order Model and Kienzle’s Model. For both approaches, the initial step is to conduct force measurements at different feed rates $f_n$, using specialised cutting tools (Altintas, 2012). Both cutting depth $a_p$ and cutting speed $v_c$ are kept constant. The average forces determine the magnitude of the forces during a stable cut, see Fig. 4.

First-Order Models describe a linear relationship between the normalized cutting force and the uncut chip thickness, such as:

$$\frac{F_q}{b} = K_{qc}h + K_{qe} \quad (2.1)$$

Here, $q$ denotes the cutting force direction, ($t$)angent, ($a$)axial or ($r$)adial. The variable $h = f_n \sin(\kappa)$ is the uncut chip thickness, $b = \frac{a_p \sin(\kappa)}{\sin^{\kappa}}$ and $\kappa$ is the major cutting angle, see Fig. 5. The specific cutting forces $K_{qc}$ and $K_{qe}$ are the only unknown parameters and are determined by the curve fitting equation (2.1) together with force measurements at different feeds ($f_n$), see Fig. 6.

The mechanistic model derived by (Kienzle and Victor, 1957) gives a better prediction for a large variation in chip thickness as the specific cutting forces are dependent on the chip thickness. For lower feed rates Kienzle’s model tends to underpredict the cutting forces, while the linear model overpredicts. Therefore, Kienzle’s model is most suitable for medium to large feed rates. It takes the effects
from strain hardening of the workpiece material, induced in the previous revolution, into consideration. The model is also the most commonly used model to predict cutting forces and cutting energy. Kienzle’s model for turning operations is derived as:

\[ F_q = K_{q1} a_p h^{1 - m_q}, \quad (2.2) \]

here \( K_{q1} \) is the specific cutting force at \( h = 1 \text{mm} \). The tool-workpiece dependent exponent, \( m_q \), describes the behaviour of the cutting force in different materials. The specific cutting force and the dependent exponent are curve-fitted in the same way as for the linear model, see Fig. 6.

2.2 Quality

The concept and the definition of quality will vary depending on whom you may ask and within what discipline they are active in. Walter Shewhart demonstrated in the late twentieth century that quality needs to be distinguished between measurable
and subjective views on quality (Shewhart, 1980). Shewhart highlighted that both views were important but the measurable view was more crucial for the producer. The subjective view is based on the customer experience and his or her point of view. However, they are interlinked, within the automotive industry perceived quality is the most important attribute that defines a successful automotive design (Falk et al., 2017; Stylidis et al., 2018). A measurable quality aspect within the automotive industry is the flush and gap between adjacent parts of the car body. This will trigger a customer’s visual senses and direct their vision towards any inconsistencies of the car body (Wagersten et al., 2014).

A more holistic view on quality was presented in the U.S. by Bryne and Taguchi where they state that: "The quality of a product is the loss imparted by the product to the society from the time the product is shipped" (Taguchi and Bryne, 1986). However, Taguchi’s view on quality was well-established in Japan during the 60’s. What differentiates Taguchi’s view on quality from others is that it involves the effect from the society and how poor quality will have an economic impact on the manufacturer. Taguchi presented together with his view on quality a whole concept and philosophy involving methodologies for increasing and analysing quality (Taguchi, 1986).

Quality could also be divided in two larger groups that consist of both measurable and subjective views on quality, goods and services. The quality concept of goods is presented in Fig. 7 and it can be separated into eight dimensions (Bergman and Klefsjö, 2010):

- **reliability** measures how often problems occur and how severe they are
Quality
Reliability
Performance
Maintainability
Environmental impact
Appearance
Flawlessness
Safety
Durability

Figure 7: Quality of goods

- **performance** is a measure on how well functions and key characteristics are fulfilled

- **maintainability** defines the accessibility, detectability and complexity of a problem

- **environmental impact** is the impact of the product on the environment from manufacturing to end-use

- **appearance** refers to the design and colour choices of the product

- **flawlessness** indicates that the product does not have defects or deficiencies at the time of purchase

- **safety** of the product is that it does not cause harm to persons or damage properties

- **durability** determines that the product can be used, stored and transported without being damaged

In this research project, the quality dimensions are viewed from a metal cutting perspective in the machining industry. The quality dimensions that are emphasized are durability, reliability and performance. The mentioned three dimensions are strongly correlated with each other, meaning the performance will affect the durability of the cutting tool (Denkena and Biermann, 2014). The main effects for each quality dimension for cutting tools are presented in Fig. 8.

2.2.1 Durability

As mentioned before, the investigated quality dimensions within the metal cutting industry are correlated. Setting up a machining process involves determining the
cutting speed, feed rate and cutting depth, which are dependent on the cutting geometry. In an optimized machine process, the durability is typically dependent on insert grade and coating. Meaning, the expected tool life or durability of a cutting tool is dependent on the wear rate and is only controllable by the coating and the grade of the cemented carbide.

In the concept and design phase of new cutting tools there are numerous controlled experiments. The experiments determine the cutting tools expected tool life, performance and the optimum machine process parameters. As a new design is determined, it undergoes numerous on-site experiments to acquire relevant information that was left out during the controlled experiments. Deviations in the cutting tool interface are not considered and critical failures due to deviations in the interface are typically caught once the product is commercially available.

### 2.2.2 Reliability

Reliability is the cutting tool’s ability to reproduce a consistent result, such as surface finish, throughout its tool life. Also, a carbide insert has a lower life expectancy than the tool body. For consistency, switching inserts should not affect the dimensional accuracy, the durability or the performance of the cutting tool.

The main affecting factors on the reliability of cutting tools, in this research project, are interface positioning, tool body material and the clamping mechanism. The tool body material will determine how much the interface plastically deforms, altering the interface positioning for the current insert and future inserts. The stiffness of the clamping mechanism determines the resistance to position changes during a cutting operation. However, too stiff clamping will break the insert.
2.2.3 Performance

The performance of a cutting tool is determined by its capabilities of productively removing metal from the workpiece. The assessment of performance also often involves an estimate of work spent determining optimum cutting conditions to avoid degenerative vibrations, also to increase the metal removal rate.

As mentioned, the main benchmark of the performance quality dimension is the metal removal rate. The metal removal rate is however dependent on the cutting geometry, as it determines the required feed rate and cutting depth for the machining process. The process parameters are set prior to the machining operation and in this research project, the cutting geometry is assumed to change during a cutting operation. Therefore, the cutting geometry is considered to be the main impacting factor on the performance quality dimension.

2.3 Locating Scheme

Locating schemes, or positioning systems, are used to: fixate parts during manufacturing operations, assemble multiple parts and lock parts for inspection. Variations induced by the fixture on the finished product need to be controlled to ensure that the product is within specified tolerances (Söderberg and Lindkvist, 1999).

The most common locating scheme in various industries is the 3−2−1 locating scheme for rigid assemblies or parts (Söderberg and Lindkvist, 1999), see Fig. 9. A rigid part has six degrees of freedom that determine its position and orientation in space. To lock all six degrees of freedom the part needs: three points (A1, A2, A3) to describe a plane and thus locking the $R_x, R_y, T_z$ degrees of freedom, two points (B1, B2) to describe a line and thus locking the $R_z, T_x$ degrees of freedom and one point (C1) to lock the last degree of freedom $T_y$.

![Figure 9: 3-2-1 Locating scheme for rigid parts](image)

An issue with the 3−2−1 positioning system is that it only applies to rigid parts and assemblies. This gives a poor correlation to reality as no parts or materials have infinite stiffnesses. To incorporate flexibility in the positioning system (Cai et al., 1996) presented the $N−2−1$ for compliant assemblies, especially to fixate sheet metal assemblies. In (Söderberg et al., 2006) the authors expanded the positioning systems further by incorporating orthogonal and non-orthogonal systems. Orthogonal positioning systems have all locator directions orthogonal to each other.

In the machining industry, much research has been conducted in order to optimize the fixture-workpiece design. Early research concluded that fixture-workpiece design

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\[^4\] $R_{x,y,z}$ - Rotation around the $x, y, z$-axis; $T_{x,y,z}$ - Translation in the $x, y, z$-direction
within machining requires high stiffness of the locators to minimize dimensional
errors caused by the machining processes (Shawki and Abdel-Aal, 1965, 1966a,b). Hurta
do and Melkote presented an analytical contact elasticity model in order
to predict the normal and the tangential reaction forces on the locators during
machining for $3 - 2 - 1$ locating schemes (Hurtado and Melkote, 1998). This allows
for the possibility to optimize fixture designs for machining processes to minimize
its effect on dimensional errors caused by the fixturing of the workpiece. Hurtado
and Melkote improved upon this concept by including stiffness optimization based
on the specified tolerance limits of the machined part (Hurtado and Melkote, 2001).

Recent trends of cutting tool design have gone towards more complex features
in the interfaces in order to maintain repeatability in the machining process. This
increases the number of contacting surfaces in the interface, thus over-determining the
positioning of the insert in the tool body interface. Therefore, compliant positioning
systems such as $N - 2 - 1$ cannot be used as they are point-to-point based and are
bound to predefined positions.

2.4 Variation Simulation

Variation simulation is a terminology for tools and methods used to calculate
statistical variation on assembly level. Predicting the variation in the final product
can ensure that the functional, aesthetic and assembly requirements are fulfilled. In
this section, variation simulation methods used within this research are presented.
The common denominator for all variation simulation methods is to simulate how the
tolerances accumulate. Different approaches can be taken to model how tolerances
accumulate through an assembly, each with certain advantages and disadvantages
(Shen et al., 2005). The earliest models to predict the tolerance sum are Worst Case
(WC) and Root Sum Squares (RSS) (Fortini, 1967). The WC model is based on
the assumption that all component dimensions will occur simultaneously at their
lower or upper bound limit. A problem that will occur with WC models is that the
component tolerances will be reduced significantly as the number of components
increases in an assembly, thus increasing the manufacturing cost for each component.
The RSS model is a statistical model that allows for larger component tolerances
compared to the WC model as it accumulates with the root sum squared. The WC
model is described using the following equation:

$$dU = \sum \left( \frac{\partial f}{\partial X_i} \right) T_i \leq T_{asm} \quad (2.3)$$

Here, $dU$ is the predicted assembly variation and $f(X_i)$ is the assembly function
that describes the tolerance sum, such as gap or flush, as a function of the nominal
component dimension $X_i$. $T_i$ is the component tolerances and the tolerance sum
limit is $T_{asm}$. The RSS model is described using the following equation:

$$dU = \left[ \sum \left( \frac{\partial f}{\partial X_i} \right)^2 T_i^2 \right]^{\frac{1}{2}} \leq T_{asm} \quad (2.4)$$

Even though each component is within its dimensional specifications the accu-
rumulated tolerance sum may not be. Also, the interchangeability of components can
be affected by poorly designed component dimension tolerances. A crucial step in tolerance analysis, is to determine the assembly function \( f(X_i) \), which describes how each tolerance specification in a design affects the tolerance sum. The most common practices in tolerance analyses are tolerance chain loops (TCL), see Fig. 10. Relevant linear dimensions that stack in an assembly are represented as vectors for components that mate (Chase et al., 1995; Chase and Parkinson, 1991; GAO et al., 1998). The TCL approach can be used for one-, two- and three-dimensional assemblies where the complexity of building the loops increases with the dimensions. An example for a 1D stack-up can be seen in Fig. 10 where the clearance gap, \( G \), is of interest and can be described using the actual dimensions \( L_1, \ldots, L_4 \):

\[
G = L_1 - L_2 - L_3 - L_4 \tag{2.5}
\]

However, the actual dimensions \( L_1, \ldots, L_4 \) can vary from their nominal values \( \lambda_1, \ldots, \lambda_4 \) in such ways that the constraint on the clearance gap is not satisfied, \( G < 0 \). The main objective is to obtain a clearance gap that is non-negative and not too large. To this extent the gap is quantified as \( G - \gamma \) where \( \gamma \) is the nominal clearance gap. Equation (2.5) can therefore be reformulated as:

\[
G - \gamma = (L_1 - \lambda_1) - (L_2 - \lambda_2) - (L_3 - \lambda_3) - (L_4 - \lambda_4) \tag{2.6}
\]

A more generalised form of (2.6) is given by:

\[
f(X_i) = G - \gamma = \sum_{i=1}^{N} a_i(X_i - \lambda_i) \tag{2.7}
\]

Here \( X_i \) is the measured value of the \( i \)th component in an assembly with \( N \) components. The effect and the direction of the stack-up are given by the coefficient \( a_i \). In the given example the coefficients are \( a_1 = 1, a_2, \ldots, a_4 = -1 \). It should be noted that the assembly function can be any black-box function that describes an input-output relation. For more complex assemblies it may prove to be difficult to apply conventional stack-up methods, and new approaches for complex non-trivial contacting interfaces may be needed.

2.4.1 Non-rigid Variation Simulation of Sheet Metal Assemblies

Variation simulation of deformable, i.e. non-rigid, sheet metal assemblies is a common industrial application of variation simulation. Here the material model is typically
assumed to be within the elastic region of its material properties. The desired output is to calculate or predict the spring-back variation after the assembly of two or more sheet metal plates. The most common approaches for this use Method of Influencing Coefficients (MIC) (Liu and Hu, 1997) or Direct Monte Carlo Simulations (DMCS).

The DMC approach is straightforward and calculates the spring-back variation by using FEA. The first step is to add deviations to the nominal part. The second step is to clamp the parts to its nominal positions in a fixture using FEA thus forming the unwelded assembly. The third step is to weld the parts together and is calculated using FEA. This will also change the overall stiffness of the assembly which will affect the spring-back once the clamps of the welded assembly are released in the fourth step. However, this approach is time-consuming and requires the algorithm to call for an FE-solver twice during one simulation step and numerous iterations are required in order to gather any statistical data of the spring-back variations.

To this extent, (Liu and Hu, 1997) proposed an approach using MIC where a linear relationship is formed between part deviations and the spring-back deviations of the spot welded assembly. The sheet metal assembly and plates are assumed to be within the linear region of the material properties. This gives that the forces required to clamp the unwelded assembly are equal to the forces generated due to the spring-back of the welded assembly.

\[ F_w = F_u = \Leftrightarrow K_w U_w = K_u V_u \]  

(2.8)

Here, \( F_w \) and \( F_u \) are the forces required to clamp the assembly and the forces generated from the spring-back. \( K_w \) is the assembled stiffness matrix of the welded structure and \( K_u \) is the unwelded stiffness matrix of each individual part. Spring-back deviations are given by \( U_w \) and the part deviations by \( V_u \) (Liu and Hu, 1997).

By simple linear algebra, the relationship between spring-back deviations and part deviations can be found as:

\[ U_w = K_w^{-1} K_u V_u = S_{wu} V_u \]  

(2.9)

Here, \( S_{wu} \) is referred to as the sensitivity matrix. This approach will require that FEA is performed to calculate the stiffness matrices. However, since the behaviour of the assembly processes is assumed to be linear, the stiffness matrices only need to be calculated once to form the sensitivity matrix. Then, the only form of variations that can occur are part deviations generated from the forming process and the fixturing of the sheet metal parts. The MIC approach allows for a great reduction in CPU time compared to Direct Monte Carlo simulations (DMC) where the full FEA model is solved at each randomly generated disturbance.

The MIC methodology within variation simulations laid the basis for continued development of the methodology and the field of variation simulations. Robustness evaluation and locating schemes for variation simulations was presented by creating a variation simulation software RD&T (Söderberg and Lindkvist, 1999). An approach of implementing a contact search algorithm in the MIC methodology to avoid penetration of contacting surfaces was presented by (Dahlström and Lindkvist, 2007). The implementation shows great significance in reducing computational time with limited loss in accuracy.
2.4.2 Meta-Model

A meta-model is a model of a model and is not bound to any specific type of field. Typically within engineering, it is a simplified model of a complex physical behaviour. In this section, the meta-modelling process that has been used within this thesis project is presented. The conducted research is based on finite element simulations, where meta-models are used to simulate variations of the controllable parameters in order to reduce the simulation time for variation simulations. The meta-model describes the relationship between geometric deviations and stress magnitudes of each individual node on the tool-body interface. The problem definition defined earlier states that the interface is over-determined, meaning that the contact locations will vary depending on the input. As a result, the stress magnitudes for the nodes will exponentially decrease or increase depending on the geometric deviation. The issue is resolved by forcing the response in each node to be linear by taking the logarithm of the response. This gives the general function that is used to model the impact of geometric deviations on contact location in the interface of cutting tools.

\[
\ln(Y^{(s)}_i) = \beta_0^{(s)} + \sum_{j=1}^{r} \beta_j^{(s)} X_{ij}^{(s)} + \epsilon_i^{(s)}, \quad \text{for } i \leq r
\]  

(2.10)

The nodal response, \( Y^{(s)} \in \mathbb{R}^{r \times n^{(s)}} \), is, as mentioned, logarithmic where \( r \) is the number of design points or observations and \( n^{(s)} \) is the number of nodes for the surface \( s \). The matrix \( X \in \mathbb{R}^{r \times (1+m)} \) is a matrix containing all terms of a polynomial with an arbitrary\(^5\) order where \( m \) is the number of independent variables, and a column of ones to give the \( \beta_0 \) terms for each observation. The matrix \( \beta^{(s)} \in \mathbb{R}^{(1+m) \times n^{(s)}} \) is a matrix of the coefficients in the meta-model and \( \epsilon^{(s)} \in \mathbb{R}^{r \times n^{(s)}} \), defined by \( \epsilon^{(s)} = Y^{(s)} - \tilde{Y}^{(s)} \), is the matrix of residuals between the true response and the predicted response (Draper and Smith, 1998). The model is then fitted by using least squares and finding the minimum vertical distance between the data points and the polynomial line.

2.5 Design Optimization

Design optimizations can be divided into two sub-groups, deterministic or probabilistic, referring to the constraints of the problem definition. A deterministic approach does not take consideration to any production or manufacturing uncertainties that exist, (Arora, 1989; Haftka and Gürdal, 1992). This means that the most probable point (MPP) is most likely at a peak or a valley depending on the problem definition. A general deterministic constrained minimization problem with an objective function \( f(x) \) can be formulated as:

\[
\min_x f(x) \quad \text{subjected to } \begin{cases} g(x) = c \\ h(x) \geq d \end{cases}
\]  

(2.11)

Here, \( g(x) \) is called an equality constraint, \( h(x) \) is called an inequality constraint and the constants \( c \) and \( d \) are arbitrary deterministic values or limits. By finding

\(^5\)The order of the polynomial is determined for each case study separately
the MPP at a peak or valley in a sensitive system, any uncertainties in the input variables $x$ can have a significant impact on the response $f(x)$ and all industrial applications have uncertainties.

Probabilistic optimization methods can be further separated into two groups, robust design optimization (RDO) and reliability based design optimization (RBDO). Robust design optimization aims at finding a local optimum that is insensitive to noise. This is most commonly done by adding variations to the input variables $x$ for a found deterministic optimum. The second probabilistic approach, RBDO, uses probabilistic constraints instead of deterministic constraints to take account for any uncertainties on $x$ by finding an optimum design at a sufficient distance (Chandu and Grandhi, 1995; Enevoldsen, 1994; Enevoldsen and Sørensen, 1994; Grandhi and Wang, 1998; Yu et al., 1998) from the deterministic optimum, see Fig 11.

![Figure 11: Deterministic optimum $x_{det}$, probabilistic optimum $x_{rbdo}$](image)

The general RBDO problem definition can be written as:

$$\begin{align*}
\min_{x} & \quad f(x) \\
\text{subjected to} & \quad P_f[h(x)] \geq 0 \\
& \quad x^l \leq x \leq x^u
\end{align*}$$  \quad (2.12)

Here, $P_f[h(x)]$ is the reliability constraint and can be formulated as:

$$P_f[h(x)] = P_{allow} - p_f$$  \quad (2.13)

Here, $p_f$ is failure limit of the system and $P_{allow}$ is the allowable probability of failure and is estimated using various approaches such as the first order reliability method, which is used in this research project for its capabilities and robustness of approximating the reliability.
2.5.1 First Order Reliability Method (FORM)

FORM is a method to predict the reliability of a system by approximating the probability integration of the joint probability density function. The definition of reliable in FORM is that the probability of the performance function $g(X)$ being greater than zero (Du, 2005),

$$P\{g(X) > 0\}$$

where $X = (X_1, X_2, \ldots, X_n)$ are the normally distributed random variables. It can also be seen as the stable region, while $P\{g(X) < 0\}$ is the unstable region, or failure region. The performance function, $g(X)$, is a black-box model which can be built using various kinds of data. These models often involve higher dimensions, which may mean that a direct evaluation of the probability integration of failure

$$p_f = P\{g(X) < 0\} = \int_{g(X)<0} f_x(x)dx$$  \hspace{1cm} (2.14)

can prove very difficult to solve. Here $f_x(x)$ is the joint probability density function of $X$. Using FORM or other approximation methods, the probability integration can be approximated with good coherence. The derivation of FORM is divided into two basic steps (Du, 2005):

1. Simplify the integrand

2. Approximate the integration boundary

By simplifying the integrand the random variables in the original space, $X$-space are transformed, using the Rosenblatt transformation (Rosenblatt, 1952), to the $U$-space. The $U$-space is a standard normal space with a mean of 0 and a standard deviation of 1.

$$U = \Phi^{-1}[F_x(X)] = \Phi^{-1}\left[\Phi\frac{X - \mu}{\sigma_{std}}\right] = \frac{X - \mu}{\sigma_{std}}$$  \hspace{1cm} (2.15)

By transforming to the $U$-space, the contours of the integrand become concentric circles without any loss of accuracy. This provides a probability integration that is less complicated to solve than in the original, $X$-space.

The joint probability density function (pdf) in the $U$-space is the product of each individual pdf of the normal standard distribution, due to the fact that the random variables are independent. The probability integration of failure in the transformed $U$-space becomes

$$p_f = \int \cdots \int_{g(u_i)<0} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u_i^2\right) du_i, \ i \in n.$$  \hspace{1cm} (2.16)

To simplify the integration boundary further, the performance function for the integration boundary, $g(U) = 0$, is approximated using first-order Taylor expansion.

$$g(U) \approx u^* + \nabla g(u^*)(U - u^*)^T$$  \hspace{1cm} (2.17)
This allows that the following optimization problem can be formulated:

$$\min_{\mathbf{u}} ||\mathbf{u}||$$

subject to $g(\mathbf{u}) = 0$ \hspace{1cm} (2.18)

By solving the optimization formulation given in (2.18), one will find the Most Probable Point (MPP), $u^*$. The MPP describes the minimal Euclidean distance from a starting point to the limit state $g(\mathbf{U}) = 0$. The reliability or the probability of failure are given in each iteration $i$ as:

$$\Phi(-\beta_i) = \Phi \left( - \left[ \beta_{i-1} + \frac{\nabla g}{||\nabla g||} \right] \right)$$ \hspace{1cm} (2.19)

Here $\Phi$ is the normal cumulative density function. A FORM based approach will only find the closest point to the linearized limit-state function based on its starting origin point, design point, in the standard space. However, more than one design point may exist that satisfies the limit-state function and other constraints. To handle the existence of multiple design points a "bulge" or a restricted area is created around a found solution, and integrated it into the limit-state function as (Kiureghian and Dakessian, 1998):

$$g_{m-1}(\mathbf{u}) = g_{m-2}(\mathbf{u}) + B_{m-1}(\mathbf{u}) = g(\mathbf{u}) + \sum_{i=1}^{m-1} B_i(\mathbf{u})$$ \hspace{1cm} (2.20)

Here $B_i$ is the "bulge" of the $i$-th design point. This continues until all the $m$ design points are found.
3 RESEARCH APPROACH

This chapter briefly outlines different approaches for conducting quantitative research, mainly describing Mitroff's quantitative research cycle which was used within this research project.

3.1 Qualitative and Quantitative Research

Scientific research is distinguished between two sub-groups, qualitative and quantitative research. Qualitative research approaches focus particularly on discovering underlying meanings and interrelating phenomena and entities, without involving mathematical modelling. Quantitative research methodologies use statistics, mathematical or computational techniques on empirical observations, to form theories or draw conclusions. Even though there is a clear distinction between the two research methodologies, they are not inseparable. For example, case research can often combine both qualitative and quantitative methods in its research design. A method in this sense refers to the technique of data collection and analysis rather than how data is interpreted and presented. Meredith et al. proposed a generic framework for the classification of a research method (Meredith et al., 1989). The framework is not intended to guide a researcher to choose what method to use, e.g. case study or action research, but to visualize the paradigmatic influence upon different methods (Karlsson, 2009).

3.2 Methods and Methodologies in Quantitative Research

Methods, as mentioned, are the tools or techniques that we use in order to collect data for the research. A methodology is how we conduct our research. Quantitative model-based research is quantified according to (Meredith et al., 1989) as a rational knowledge generation method. This is based on the assumption that it is possible to construct objective models that describe operational processes. Relationships between the variables are described as causal, which indicates that a change of $a$ in a variable $x$ will lead to a change of $f(a)$ in another variable $y$. For causal and quantitative relationships it is possible to predict future states of the modelled process rather than being bound to the observations made. This requires all claims that are made within the modelled process to be unambiguous and verifiable. Quantitative modelling can therefore be categorized into two classes: axiomatic quantitative modelling research and empirical quantitative modelling research. Axiomatic research is primarily driven by an idealized model (Karlsson, 2009). Idealized models will tend to simplify the problem to such an extent that relevant information could be lost. Therefore, the main research methodology used in this research project is an empirical quantitative modelling approach with computer simulations, due to the complexity of the investigated problem. For empirical research, the main issue of the practitioner is to make certain that there is coherence between a model and observations from reality or simulations. Empirical research can be both descriptive and prescriptive. Descriptive empirical research mainly aims at creating a model that sufficiently describes the causal relationship. Prescriptive empirical research tends to create policies, strategies and actions to improve the processes. In this research project, a prescriptive empirical research approach is conducted to create a framework with a set of tools to handle tolerance analysis and allocation of cutting tool interface designs.
3.2.1 Mitroff’s Model

Mitroff and Sagasti presented a research methodology for studying science from a holistic or systems point of view in (Mitroff et al., 1974; Sagasti and Mitroff, 1973) because anything less will fail to pick up certain aspects of science’s most essential characteristics (Mitroff et al., 1974). This is one of the earliest contributions to the field of quantitative research methodologies. The model is referred to as the Mitroff’s Cycle, see Fig.12. The model consists of four phases (I) conceptualization, (II) modelling, (III) model solving and (IV) implementation.

![Mitroff’s Cycle](image)

Figure 12: Mitroff’s Cycle (Mitroff et al., 1974)

The conceptualization phase consists of the researcher building a conceptual model of the system of interest. This usually specifies which variables should be addressed and the aim and scope of the model. Previous studies and literature reviews are often used to build upon. In the modelling phase the quantitative model is built which defines the causal relationships between the independent variables. In the next phase, the model is solved and finalized by implementing its results in the implementation phase. However, Mitroff et al. state that a research cycle can begin and end in any of the four phases if the practitioner is aware of the conclusions that can be made based on the results of the research. Mitroff et al. also discuss the shortcuts, (F) narrow feedback and (V) validation, which practitioners can use and which are often applied in research projects. This tends to lead to less desirable research designs. The authors also adress the II-III-(F) cycle that many practitioners following the cycle tend to mistake the model solving phase for implementation of the model. Also, practitioners following the I-(F)-IV cycle tend to misinterpret conceptualization for modelling. The Mitroff’s cycle is an essential tool to identify methodological paths that certain work follow in order to relate the validity of the claims that were made.

Axiomatic research can, as empirical, be both descriptive and prescriptive. For axiomatic descriptive (AD) research, the central part of the cycle is the modelling.
The practitioner most often takes a conceptual model from literature and creates a scientific model of it. Typically in axiomatic descriptive research, the practitioner does not move to the model solving phase thus giving a I-II-(V) cycle. However, for axiomatic prescriptive (AP) research, the practitioner enters the model solving phase, which will lead to the practitioner taking the narrow feedback shortcut. The results of the model are then feedback to the conceptual model, which can be confused with implementation. This is often mistaken for implementation and most often claims are made in that sense (Karlsson, 2009).

The typical empiric descriptive (ED) research practitioner tends to follow a I-II-(V) and is someone who is over-concerned with the validation of the model (Mitroff et al., 1974). For example, the practitioner is trying to overfit the model with respect to reality. This typically leads to a noisy model that only describes the observations made. Empiric prescriptive (EP) research usually follows the complete cycle, I-II-III-IV, and in many cases, empiric prescriptive research is based on earlier published research from the axiomatic descriptive research approach (Karlsson, 2009).

### 3.3 Verification and Validation

The definition of verification and validation varies and is dependent on the subject. In this research, the objective is to create a functional framework that employs multiple simulation models to analyze tolerance in cutting tool seat designs.

From a manufacturing perspective, (Boehm, 1979) states that verification defines how the model corresponds to its specifications and defines validation as to how well the model describes its intended purpose. The following questions are stated by (Boehm, 1979) to clarify the definitions further. **Validation:** Are we building the right product? **Verification:** Are we building the product right?

Verification and validation of simulation models are slightly different in definition. The authors in (Sargent, 2013) state that model verification is to ensure that the programmed model and its implementation are correct. Model validation ensures that a model holds a satisfactory range of accuracy for its intended field of application (Sargent, 2013). When developing a model it has to be for a specific purpose or application. The validity of the model is therefore determined depending on its purpose. If a model has to answer multiple questions, then the validity needs to be determined for each question (Sargent, 2013).

(Sargent, 1981) presents a simplified version of the model development process (MDP) in Fig. 13, based on standards set by (SCS Technical Committee on Model Credibility, 1979). The MDP involves three main phases:

- **Problem entity** is the phenomena to be modelled.
- **Conceptual model** is the mathematical/logical/graphical representation of the problem entity and is developed through analysis and modelling.
- **Computerized model** is the conceptual model implemented on a computer.

For each step in the MDP, it is possible to relate model verification and validation, dashed lines in Fig. 13. Conceptual model validation establishes that the assumptions and hypotheses underlying the conceptual model are correct and reasonable for the intended purpose of the model. The conceptual model validation process typically involves ensuring that assumptions on, for example, linearity and variable reductions
are correct. Computerized model verification is to ensure that the implementation and programming of the conceptual model are correct. An example of computerized model verification can be ensuring that adding variation to the mesh in an FEA will not cause surface penetrations between contacting surfaces. Operational validation refers to the process of establishing that the response of the model is within a satisfactory range of accuracy for the intended purpose of the model. Operational validation can be performed for both observable system and non-observable systems. The creator of the model needs to decide what approach is required, subjective or objective, for operational validation of the system, see Tab. 1 (Sargent, 1981, 2013).

<table>
<thead>
<tr>
<th>Decision approach</th>
<th>Observable system</th>
<th>Non-observable system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective</td>
<td>- Comparison using graphical displays</td>
<td>- Explore model behaviour</td>
</tr>
<tr>
<td></td>
<td>- Explore model behaviour</td>
<td>- Comparison to other models</td>
</tr>
<tr>
<td>Objective</td>
<td>- Comparison using statistical tests and procedures</td>
<td>- Comparison to other models using statistical tests</td>
</tr>
</tbody>
</table>

Data validity ensures that any data, such as material models, are sufficient and correct. Data validity is typically not considered due to the fact that it is usually difficult, time-consuming and costly to obtain appropriate, accurate and sufficient data (Sargent, 2013). Verification and validation of the framework presented in this research mainly focus on the outer circle of Fig. 13, conceptual model validation and computerized model verification. Operational model validation is neglected as the system is non-observable, and therefore it is not likely to obtain a high confidence in the model. The overall validity of the results is dependent on each method used within the framework. As the research, in its current state, is concentrating on method development the validation of the models is subjective. Furthermore, each appended paper state under what circumstances the models are valid and their limitations.
Figure 13: Simplified version of the model development process (Sargent, 2013)
4 RESULTS

This chapter will present a summary of the results in the appended papers.

4.1 Summary of Appended Papers

4.1.1 Paper A: Tolerance Analysis of Surface-to-Surface Contacts Using Finite Element Analysis

In this paper, an approach to analyze tolerances of surface-to-surface contacts subjected to external loads such that elastic and plastic deformations occur in the contact zones was suggested. In the case study a Corocut QD parting tool was used. The effect of the tool body geometry on the stress distributions in the interface was studied using a parametric CAD model. After simplifications it was assumed that 10 parameters, defining the contacting surfaces in the tool body, would affect the positioning of the insert in the tip seat, see Fig. 14. Both the translational\textsuperscript{6} and rotational\textsuperscript{7} degrees of freedom were assigned with uniformly distributed values within ±0.01 mm and ±1 deg.

A meta-model was built for each individual node in the contacting surface between the insert and the cutting tool body. Creating a meta-model of the contacting surface allowed for visualization and optimization of the contact stress distribution, see Fig. 15, for any distribution of the input parameters within the simulated parameter space.

Paper A also incorporates a simplified cutting force model and considers how the contact variation affects the cutting forces as a result of variations in the rake angle.

\textsuperscript{6}Subscript TX, TY or TZ
\textsuperscript{7}Subscript RX, RY or RZ

Figure 14: Contact surface degree of freedom
Paper A concludes that it is possible to analyze the effect of tolerances on the contact variation and to analyze the impact of contact variation on cutting forces as a result of rake angle variations.

### 4.1.2 Paper B: Reliability based design optimization of surface-to-surface contact for cutting tool interface designs

In this paper, a methodology for reliability based design optimization of overdetermined surface-to-surface contacts for cutting tools is presented. The reliability based design optimization uses a genetic algorithm with an implemented first order reliability method (FORM) approach to approximate the reliability of the performance functions. The performance functions are based on the percentage of contact in the preferred contact zones (PCZ) and can be retrieved through sensitivity analyses. The PCZ in this paper are chosen such that the leverage load acting on the insert due to the positioning of the insert on the tool body is minimized in order to avoid breaking the inserts. The methodology is presented through calculations on assemblies, containing two individual parts for different surface geometries found within the field of metal cutting tools. One part is defined as flexible (grey), see Fig. 16, with a linear material model which will represent the tool body. The other part is seen as rigid (white) which represents a cemented carbide insert. The flexible body rests on a frictionless surface. This allows translation in x,y-directions while prohibiting translation in the z-directions. A distributed load is acting on the rigid body that will compress the flexible body.

![Figure 16: Illustration of the FE models used in the case studies](image)

The results of Paper B is presented in four case studies. The first case study shows the validity of using a FORM based approach to calculate reliability using numerical
data. The second case study presents the validity of the calculated reliability by comparing the results to direct Monte Carlo simulations. The third case study expands the complexity of the interface further with a two-dimensional serrated surface which can be described using four separately interfering surfaces. The fourth case study uses a three-dimensional serrated surface which can be described using eight separately interfering surfaces. The geometries of case studies three and four are chosen to resemble the interfaces of modern cutting tools out in the market today. The contact variation optimization algorithm is applied for case studies II-IV.

Paper B concludes that a FORM based approach on predicting the reliability of design variables with respect to a performance function can be used to define contact zones where contact is preferred. The FORM based approach on numerical data reduces computational time of the reliability with limited loss on the accuracy.

4.1.3 Paper C: Non-Linear Material Model in Part Variation Simulations of Sheet Metals

In Paper C an adaptation of the MIC for non-linear material models is presented and is referred to as the non-linear MIC method (NLMIC). The NLMIC approach incorporates an elasto-plastic material model with isotropic hardening through a first order Taylor expansion of the primary variable around a nominal load. The derivative of the primary variable is identified as the Newton step and can be retrieved from the FE formulation. An elasto-plastic material model with isotropic hardening was used for demonstration purposes. For highly non-linear material models, it is expected that the error will increase as the distance from the nominal load increases. The presented case studies show that it is possible to incorporate plastic strains for single and multiple loads in variation simulations with limited effect on accuracy.

In the first case, the same load deviation vector was applied for both the proposed method and FEA with 1,000 generated numbers with a normal distribution \( u_y \sim N(6.15, 0.2) \). The primary variables for the nominal prescribed displacement can be seen in Fig. 17 and the \( L^2 \) normalization of the residual between FEA and NLMIC. Results are presented in Fig. 18. This indicates that the correlation between the simulated and the approximated solution are coherent.

\[ \text{Figure 17: Nominal prescribed displacement } u_y | \Gamma_3 = 6.15 \, [\text{mm}] \]

---

\(^8\)Normal distribution of a variable \( x, x \sim N(\mu, \sigma) \), where \( \mu \) is the mean of the variable \( x \) and \( \sigma \) is the standard deviation
The second case is intended to show the validity of the superposition principle assumption. The quarter symmetric plate is subjected to a uniformly distributed prescribed displacement $u_x \sim U(5, 0.2)$ on $\Gamma_2$ and $u_y \sim U(6.15, 0.2)$ on $\Gamma_3$. A FE simulation is conducted with the mean prescribed displacements, where the components in the energy functional are obtained for the NLMIC. The assumption of superposition requires that the affected degrees of freedom due to the prescribed displacement are decoupled, for each load case. Once the components of the matrices are decoupled, the load deviation vector is applied and the primary variables can be calculated. A 2-level full factorial test space was created to validate the NLMIC for multiple boundary conditions. The primary variables, with nominal prescribed displacement applied to the boundaries, are presented in Fig. 19 and the $L^2$-normalization of the residuals is presented in Fig. 20.

Paper C concludes that for small variations in a prescribed displacement it is
possible to use Taylor’s expansion to linearize the effect of plasticity on a sheet metal part. Paper C also concludes that the principle of superposition is valid as the model is linearized making it possible to apply the approach for assemblies in future research.
5 DISCUSSION

In this chapter, the answering of the research questions and the relevance of the used research methodology are discussed. The contribution this work makes to new knowledge is also considered.

5.1 Answering the Research Questions

The research questions will be answered one question at a time.

RQ I  \textit{How can external loads be handled in locating schemes for overdetermined surface-to-surface contact conditions?}

This question is addressed in Paper A and Paper B where methods to
1. simulate overdetermined surface-to-surface contact assemblies with mechanical loads,
2. detect critical areas, w.r.t geometric variations, on the contacting surface, and
3. define and optimize an overdetermined locating schemes for surface-to-surface contact designs using a first order reliability based approach

were suggested. Those three methods together form a framework to handle positioning and tolerance analysis of surface-to-surface contact conditions with external loads.

RQ II \textit{How can non-linear material behaviour be accounted for in variational simulations?}

An approach is suggested in Paper B that incorporates non-linear material behaviour in the MIC methodology and was given the name NLMIC. The NLMIC shows great potential and applicability to take in to account material hardening effects and plastic strains with greatly reduced simulation times, compared to direct Monte Carlo simulations with a FE-solver.

5.2 Scientific Contribution

Contributions to the scientific community can be considered in light of the difficulties the cutting tool industries are facing regarding tolerance stack-up analysis of interface designs. In Section 1.2 the requirements on a tolerance analysis framework for cutting tool interface designs were outlined. Through extensive literature review it was found that two of the points were lacking in scientific publications, (1) non-linear material models in variation simulations and (2) surface-to-surface contact positioning. The contribution to the scientific community is summarized as:

- New knowledge and a method for variation simulation of sheet metal parts with nonlinear material models
- A method to optimize position surface-to-surface contacts
- Increased knowledge of tolerance analysis in cutting tool interface designs
5.3 Industrial Contribution

Difficulties the cutting tool industries are facing as a result of this research is summarized as:

- A framework with methods, necessary for tolerance analysis of complex surface-to-surface contacts with external loads, typically found in cutting tool interface designs

- Increased knowledge of tolerance analysis in cutting tool interface designs

5.4 Applied Research Approach

The aim of this research project is to create a framework for engineers and researchers to analyze and allocate geometric tolerances for assemblies with overdetermined surface-to-surface contacts with external loads acting upon them, such as tip seat geometries. Currently, there are no methodologies to handle the defined problem. Therefore, the first iteration starts in phase one of Mitroff’s cycle with the problem situation:

*Lacking a methodology to analyze tolerances that alter the positioning of the insert in the interface.*

The conceptualization in phase two outline the set of tools, based on current methods, required to form the framework. The required steps, in a subsequent order, are:

- Create a design of experiments (DOE)
- Run FE-simulations
- Build a meta-model over the results
- Find an optimum set of input variables
- Visualize the results

Defining the type of DOE to use and boundary conditions in the FEA are defined in phase two, modelling. Phase three involves solving each subsequent step defined in phase one and two. This forms the first iteration of the framework necessary to address the problem situation, Fig. 21.
Finite Element Analysis
Matlab script
Generate CAD
for
ith observation
Start
Build mesh
FE-solver
Extract
surface
nodes
i = i + 1, for i ≤ r
yes
no
yes
i = r
Satisfied?
Adjust
tolerances
End
Visualize results
Direct Monte
Carlo Sim-
ulation
Build meta-
model
Interpolate
mesh
Figure 21: First iteration of the framework

With the defined problem situation a holistic view over the current state of this research field is given. Upon completing the first cycle, the reality of the problem situation has changed. Moreover, it is possible to formulate new realities and problem situations by analysing the feedback provided by the previous cycle. Based on Paper A, two problem situations could be defined; (1) The process of finding an optimal set of design variables is not robust and (2) the FEA is too time-consuming. The new problem situations outline the problem definitions of paper B and paper C where Mitroff’s cycle initiate the second iteration of the framework, see Fig. 22.

The first defined problem situation after Paper A, "the process of finding an optimal set of design variables is not robust", was addressed in Paper B where a first-order reliability based method was developed to handle surface-to-surface contact positioning of cutting tool interface designs. Paper B follows a complete Mitroff cycle, I-II-III-IV, where implementation of the algorithm into the framework is the
last phase. After implementation, the framework is updated according to Fig. 23.

The second problem definition, "FEA is too time-consuming", was partially addressed in Paper C and followed a typical prescriptive axiomatic research cycle, I-II-III-(F). Common pitfalls in prescriptive axiomatic research, defined by (Meredith et al., 1989; Mitroff et al., 1974; Sagasti and Mitroff, 1973), include a danger to get stuck in a continuous loop of constantly improving the conceptual model as not enough knowledge exists about the goal. Therefore, the results of the model solving phase are validated with related research, which is discussed further in section 5.4.1. The feedback path gave additional problem situations that are necessary to take into consideration. The problem situations are defined as sub-problems and involve (2.1) expanding the NLMIC to multiple sheet metal parts and (2.2) verification and validation of the NLMIC method. This research has not been implemented in the framework as it is not in its current state appropriate for surface-to-surface contact positioning.

Figure 23: Second iteration of the framework, after implementation of Paper B
5.4.1 Verification and Validation of the Results

This section discusses the verification and validation of the results and how it affects the validity of the framework.

The results in Paper A and Paper B have undergone multiple procedures to collect necessary data for the final results. The first step was to generate a sufficient test space using LHS. For linear regression, it is recommended to use \( 15 - 20 \) observations per variable (Green, 1991). In Paper A, approximately 30 observations per variable were assumed necessary for constructing the simulation model.

The second step is to build the mesh, add boundary conditions, run the FE simulations and export the nodal responses in the contacting surfaces. Building the mesh for each observation can result in that the node positions are adjusted. Constructing the meta-model requires that the nodes in the mesh do not alter its position. Therefore, the third step is to interpolate the nodal responses in observation to a nominal mesh. The interpolation error is calculated using \( L^2 \) normalization and since linear element shape functions were used the interpolation error was negligible.

In the fourth step, a meta-model is built for each node on the contacting surfaces. The verification of meta-models is quantified using the \( R^2 \)-value. A genetic algorithm is utilised to remove irrelevant predictors to avoid overfitting the meta-model to ensure that subsequent predictions do not have random variations (Hawkins, 2003).

To conclude the data collection in Paper A and Paper B, the number of observation per variable for an acceptable meta-model matches the literature. The interpolation error is negligible as the simulations use linear element shape functions. The verification of the meta-model relies on the \( R^2 \)-value and on reducing irrelevant predictors. Paper A and Paper B are both non-observable systems and will rely on a subjective validation of the model behaviour in the FE simulation.

In Paper B the validity of the results is divided into four case studies. The first case study shows a negligible loss in accuracy when numerical data is used compared to analytical data. This allows for more complex models to be analyzed. The reliability of the found optimum was validated in the second case study by comparing the results to DMC simulations of the performance function. The third and fourth case study expand the complexity of the interface further and show the effectiveness of the presented approach with regards to restricting the contact variations to the PCZ.

The methodology and results in Paper C are validated using an objective decision approach. The implementation of the conceptual model is validated with known research of a deterministic case and is extended to a non-deterministic case.

5.5 Limitations

The current state of the framework relies on FEA to simulate the assembly process and the effect of cutting forces on the interface. The FE simulations are computationally heavy and the computational time increases exponentially with added controllable parameters to simulate, which limits the simulation model. Future implementations need to handle the stated issue and limit the number of simulations necessary to increase the number of parameters to study.

A situation can occur in which a few data points in a dataset can have a disproportionate effect on the slope. Such data could be what are typically referred to as outliers. However, in an overdetermined system, the reason for an outlier can, for example, be a new set of contact points. Deterministic simulation models
do not generate outliers. The issue that arises with a meta-model is that it will only describe the general slope of the dataset and neglect any major effects of an outlier. Neglecting the outliers will result in that the meta-model does not capture the complete behaviour of the overdetermined system. Therefore, a new approach to constructing the meta-models is necessary, taking outliers into consideration without overfitting the meta-model.
6 CONCLUSION AND FUTURE WORK

In this chapter, the results are summarized and future work is outlined.

RQ I How can external loads be handled in locating schemes for overdetermined surface-to-surface contact conditions?

The aim of the first research question is to address the overall issues concerning tolerance analysis of cutting tool interface designs. A framework that takes advantage of the link between CAD and FEA software to conduct parametric studies of the geometry variations in the cutting tool interface is suggested. The generated data is post-processed and meta-models are created to conduct variation simulations within the parameter space. A FORM based approach is implemented to find a reliable set of input parameters that confine contact variations to pre-defined zones.

RQ II How can non-linear material behaviour be accounted for in variational simulations?

The goal of the second research question is to address a common issue within variation simulation, and that is non-linear material behaviour. The framework, in its current state, has only been tested on linear material models to reduce computational time. However, the methods within the framework do not rely on linear material models. Outside of the framework, a method for linearizing material model behaviour was developed. The method uses Taylor’s expansion around a nominal prescribed displacement on the primary variable. For small variations of the prescribed displacement, it was found that the method is accurate and fast compared to FEA.

6.1 Future work

Future work within this topic should focus on reducing the time needed for FEA simulations, increasing the accuracy of the results, including sensitivity analysis and validation and verifying the framework. Based on the research method and feedback from the second iteration more specific topics can be formed and are as followed:

- NLMIC for assemblies using multiple sheet metal parts.
- Error analysis of the NLMIC with respect to the material model.
- An adaptive design of experiments that refines areas around "outliers".
- Cluster analysis of datasets to define multiple meta-models at specific intervals.
References


