THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Optimization Framework for Crushing Plants

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Overview of a conceptual optimization framework with a photo of NCC Industry's Glimmingen plant, Uddevalla, Sweden.

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To my Family

ABSTRACT

Optimization is a decision-making process to utilize available resources efficiently. The use of optimization methods provide opportunities for continuous improvements, increasing competitiveness, trade-off analysis and as a support tool for the decision-making process in industrial applications. One of such industrial applications where optimization methods are needed is coarse comminution and classification processes for aggregates and minerals processing industries. The coarse comminution and classification process, consisting of crushing and screening, is a heavy industrial process characterized by continuous operations. The processes handle large material volumes, are energy intensive, and suffer large variabilities during process operations.

To understand the complexity and to replicate the process performance of the coarse comminution and classification processes, process simulation models have been under development for the past few decades. There are two types of process simulation models: steady-state simulation and dynamic simulation. The steady-state simulation models are based on instantaneous mass balancing while the dynamic simulation models are capable of capturing the process change over time due to non-ideal operating conditions. Both simulation types are capable of capturing the process performance, although the dynamic process simulations have been proven to have a higher fidelity for industrial applications. Both the steady-state and dynamic simulation models lack the capability of optimization methods which can potentially increase the utilization of the developed process simulation models. The optimization capabilities can further increase the functionality of the process simulation models and provide decision-making support.

The thesis presents a modular optimization framework for carrying out process optimization and process improvements in a coarse comminution and classification process using process simulation models. The thesis describes the results of explorative studies carried out for developing the application of optimization methods and key performance indicators for the coarse comminution and classification process. The application of the optimization methods can generate new insights about the process performance with respect to the operating parameters, and non-intuitive results. The application of the key performance indicators can be used to carry out process diagnostics and process improvement activities. As a conclusion, a conceptual framework for carrying out optimization procedure within the coarse comminution and classification process is presented. The development of the optimization system and performance measuring system can be useful for process optimization and process improvements for industrial applications.

Keywords: Modelling, Dynamic Simulations, Comminution, Crushing, Classification, Screening, Minerals Processing, Multi-Disciplinary Optimization (MDO), Multi-Objective Optimization (MOO), Key Performance Indicators (KPIs), Process Optimization, Process Improvement, Industry 4.0

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Kanishk Bhadani

Gothenburg, May 2019

APPENDED PUBLICATIONS

The thesis contains the following papers.

Paper A: Bhadani, K., Asbjörnsson, G., Hulthén, E., Bengtsson, M., Evertsson, C. M., (2017) *State of the art in application of optimization theory in minerals processing*, Presented at the European Symposium on Comminution and Classification, Izmir, Turkey, September 11-14, 2017.

Paper B: Bhadani, K., Asbjörnsson, G., Hulthén, E., Evertsson, C. M., (2018) *Application of multi-disciplinary optimization architectures in mineral processing simulations*. Published in Minerals Engineering (Journal), 2018, Volume 128, pp 27-35.

Paper C: Bhadani, K., Asbjörnsson, G., Hulthén, E., Bengtsson, M., Evertsson, C. M., (2018) *Comparative study of optimization schemes in mineral processing simulations*, Proceedings of XXIX International Mineral Processing Congress, Moscow, Russia, September 17-21, 2018, Volume 1, pp 464-473.

Paper D: Bhadani, K., Asbjörnsson, G., Hulthén, E., Evertsson, C. M., (2019) *Development and implementation of key performance indicators for aggregate production using dynamic simulation*. Submitted to Minerals Engineering (Journal), April, 2019.

WORK DISTRIBUTION

Paper A: Bhadani, Asbjörnsson and Hulthén initiated and conceptualized the idea. Bhadani carried out the literature study and wrote the paper with Asbjörnsson, Hulthén, Bengtsson and Evertsson as active reviewers.

Paper B: Bhadani initiated the idea and carried the development & implementation of the optimization methods. Asbjörnsson provided models for process simulation. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

Paper C: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani carried the development & implementation of the optimization methods. Asbjörnsson provided models for process simulation. Bengtsson provided constructive input for optimization methods. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

Paper D: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani carried the development & implementation of the key performance indicators. Asbjörnsson provided models for process simulation and Hulthén supported with the real-time process data. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

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NOTATIONS

m	Mass
$\dot{m}$	Mass flow
γ	Material properties
x	Vector of design variables
y	Vector of coupling variables or output from sub-process analysis
f	Objective function
c	Vector of design constraints
c^c	Vector of consistency constraints
N	Number of sub-processes
$(\ )_0$	Functions or variables shared between more than one sub-process
$(\ )_i$	Functions or variables applied only to a sub-process *i*
$(\bar{\ })$	Independent copies of variables distributed to other sub-process
$(\ )^0$	Functions or variables at their initial values
$(\ )^*$	Functions or variables at their optimal values
SOO	Single-objective optimization
MOO	Multi-objective optimization
MDO	Multi-disciplinary optimization
GA	Genetic algorithm
IDF	Individual-discipline feasible
MDF	Multi-discipline feasible
KPI	Key performance indicators
CSS	Closed-side setting
SA	Screen aperture
PI	Proportional-integral controller
SPV	Sub-process value
OFAT	One-factor-at-a-time
ISO	International Organization for Standardization
OEE	Overall Equipment Effectiveness

1 INTRODUCTION

This chapter aims to:

> *Introduce the concept of comminution and classification process.*
>
> *Provide an overview of the needs for optimization capabilities in crushing plants.*
>
> *Introduce the area of research and the scope for the development of the optimization framework.*

Crushed rock materials such as crushed stone, gravel, sand, clay are base materials used in infrastructure development for roads, railways and housing constructions. Mineral ores are rock materials which have specific chemical composition containing one or more compound or element. Minerals are used to extract valuable metals such as iron, copper and non-metals used in industrial applications. Both crushed rocks and mineral ores form a strong economic basis for today's societal needs and are associated with mining activities.

Sweden is one of the major mining countries in Europe. The mining activities carried out in Sweden can be broadly classified as Aggregates, Minerals and Mining Industry. Swedish aggregates industry, supplying products such as gravel, sand and crushed rocks for construction purposes, accounted for a total of 86 million tonnes in 2016 (SGU, 2016b). Swedish minerals and mining industry's statistics showed a total of 74.9 million tonnes of mineral ore production (both metals and non-metals) in 2016 (SGU, 2016a). These industries are also an energy-intensive sector consuming a total of around 4 TWh of energy in 2017 (Energimyndigheten, 2018). The aggregates, minerals and mining industries comprise of a common segment of processing operation called comminution and classification process.

A comminution process is defined as the size reduction of particles, while a classification process is defined as the separation of particles based on size, shape, and material properties such as density, chemical affinity (Wills and Finch, 2015, Napier-Munn et al., 1996). The comminution and classification processes handle large volumes of material as shown by the two Geological Survey of Sweden reports (SGU, 2016a, SGU, 2016b) and consume a considerable portion of the total energy expended by these industries. A typical crushing plant consists of size reduction machines (e.g., crushers) with intermediate separation machines (e.g., screens), transportation equipment (e.g., conveyor belts, trucks), and storage (e.g., stockpiles, bins) (Hulthén, 2010). The process is usually divided into multiple stages with different particle size ranges in each. The process is operated by trained operators and is associated with various variabilities due to natural raw materials, wear, etc.

1.1 NEED FOR PROCESS SIMULATION AND OPTIMIZATION

With such a large volume of materials processing in the comminution and classification processes, development of tools and methods for improving utilization and optimization of the resources involved is of great value for today's and future's industrial sustainability goals. At the same time, design and operations of comminution and classification processes are complex and need a broad understanding by the personnel involved, which is developed by training and with experience. The opportunities for increasing recourse utilization compared to today are possible, but support from a decision-making tool is required. Process simulation of the comminution and classification process is one of the cost-effective tools which has been accepted by the industry.

Asbjörnsson (2015) presented a system wide-view on the complexity with the development of process simulation for comminution and classification processes. Figure 1 presents an overview of various factors that can influence plant performance of a comminution and classification process in a crushing plant. The process operation includes changes and variations due to controllable factors such as the setting of single equipment to uncontrollable factors such as wear and segregation. Capturing various phenomena occurring in a physical process into a process simulation requires suitable use of a variety of modelling techniques (Asbjörnsson, 2015).

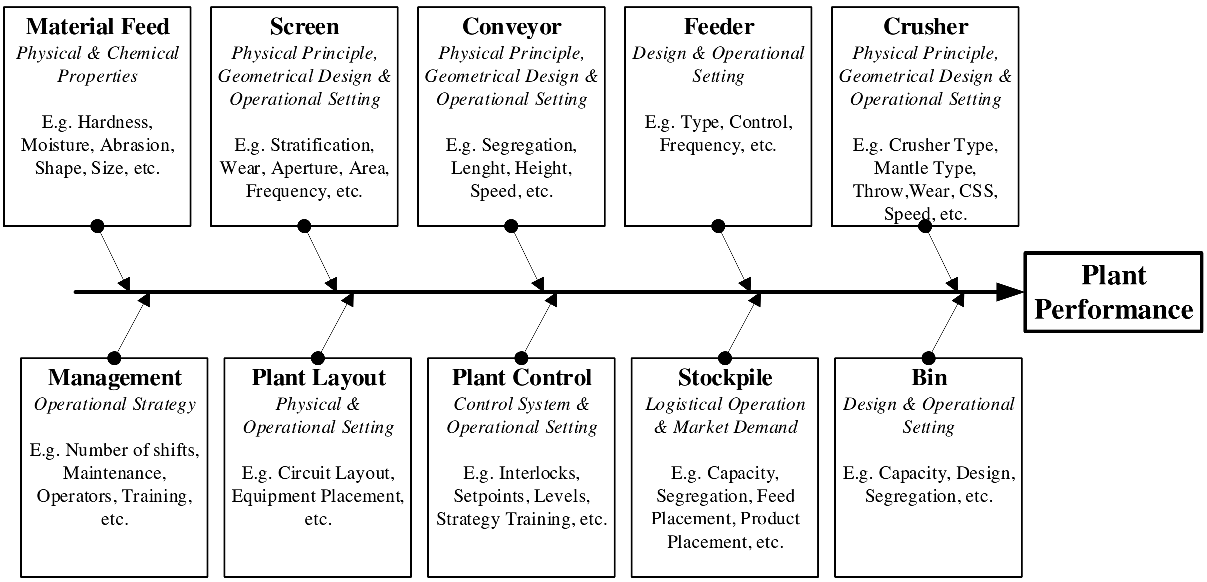


Figure 1. Factors influencing plant performance for a crushing plant (Asbjörnsson, 2015).

Operation of comminution and classification processes needs to consider complex dependencies between various equipment and sub-processes involved. Modelling of those complex relationships has been made possible through the use of dynamic process simulations (Asbjörnsson, 2015). Furthermore, development of the functionality within process simulations, such as optimization capabilities, can enable process improvements and process optimization towards increased plant performance of the comminution and classification process. Extracting higher output with minimal resource consumption also adds competitive value to the industries operating such processes. With the current industrial trends towards digitalization, development in the use of simulation capabilities is a value-adding activity and can potentially lead to new business opportunities.

1.2 Trends Towards Use of Simulation Platforms for Industrial Applications

Development of mathematical models (Evertsson, 2000, Whiten, 1972, Powell and Weerasekara, 2010) and process simulations (King, 2001, Napier-Munn et al., 1996) for comminution and classification process has been going on for the past few decades. Various commercial software packages exist such as *JKSimMet* at JKMRC, *MODSIM* (King, 2001), *Bruno* (Metso Minerals), *IES* (CRC Ore), *Plant Designer* (Sandvik), *HSC Chemistry* (Outotec) and so on. The simulation software is used within the industry for various purposes such as designing new plants, training personnel, diagnostics and improvements.

Most commercial software packages capable of simulating comminution and classification processes are based on the instantaneous mass balance principle of the process, which is also called steady-state process simulation. The recent development in dynamic process simulation for coarse comminution and classification process (Asbjörnsson, 2015) is capable of capturing the discrete and gradual changes happening in the crushing plant due to delays, start-ups, discrete events, wear, etc. which is a closer replication of a physical process. There has been an increasing interest in the use of dynamic process simulation of the comminution and classification process to create a tangible improvement at physical industrial scale (Brown et al., 2016).

Apart from the possible implementation of dynamic simulation capability, the commercial software packages present today are limited in providing features of optimization for the comminution and classification process. Previous researchers have shown the usefulness of the optimization methods (Svedensten, 2007, Huband et al., 2006), but a broader approach in understanding optimization capabilities is needed. Successful industrial implementation cases with a suitable optimization approach are required to drive such development. Optimization capability within process simulation can provide opportunities for further utilization of existing simulation platforms for both the static and the dynamic process simulations and can be a useful support tool for decision-making.

1.3 RESEARCH AREA OVERVIEW

One of the methods used for increasing competitiveness of an industrial product or process is through the application of optimization methods for a defined problem. According to Papalambros and Wilde (2017), the optimization of a *system* can be defined as choosing the *best* alternative which meets the *original need* within the *available mean*.

For the context of this research, the *system* in focus is a coarse "Comminution and Classification System" which typically consists of multiple crushing processes with intermediate screening processes (crushing plants). The system provides the basic value-adding functionality (size-reduction and separation) to the aggregates production process for producing various aggregate products. For the mineral processing application, the system provides the functionality to reduce the rock particle size to the needs of the next processes such as fine comminution and classification, and concentration depending on the process under consideration.

The *best* alternative can be based on the *objective* of the optimization. In the case of a comminution and classification system, the *objective* can be defined based on the performance of the process operation. The performance of the process can include various indicators consisting of technical, economic, and environmental functions which can give quantitative values of how well the processes are performing. The performance aspects of the comminution and classification system can be collected in a system called "Performance Measuring System".

The term *original need* sets requirements and the term *available means* defines the constraints that can be applied to the process design and operations. The constraints can include technological means, such as size and operating range of equipment, plant layout, to economical means, such as product demand, and operational and energy costs. The means can also include natural constraints like raw material properties and availability.

In order to find the *best* alternative for the defined problem of a comminution and classification system, a wide range of methods and algorithms are available within the research field of optimization, which will be referred as "Optimization System". To summarize the research area using the above descriptions, it can be encompassed into three systems: "Optimization System", "Comminution and Classification System" and "Performance Measuring System" as shown in Figure 2.

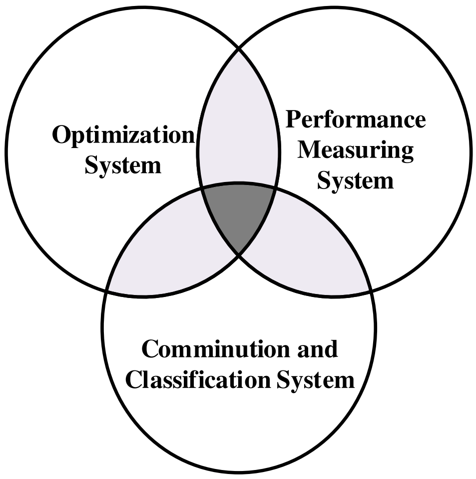


Figure 2. Overview of three research areas involved in the thesis.

1.4 RESEARCH OUTLINE

Research Aim: This research aims to develop a framework for carrying out process optimization and process improvements in a comminution and classification process using process simulation.

Research Objective: The objective of this research is to investigate the application of optimization methods for creating value for the comminution and classification process simulation. Furthermore, the research objective is to create a modular solution which can be transferrable to other similar processes. The results of the research are directed towards increasing the sustainability of the comminution and classification processes.

Research Scope: The research is initiated with the focus on the aggregate processing industry, although the research results are transferable to similar activities in the minerals processing industry. The basic process involved is a coarse comminution and classification process which is presented in Figure 3. In the scope of this research, the coarse comminution and classification equipment is limited to crushers, screens, conveyors, feeders and bins.

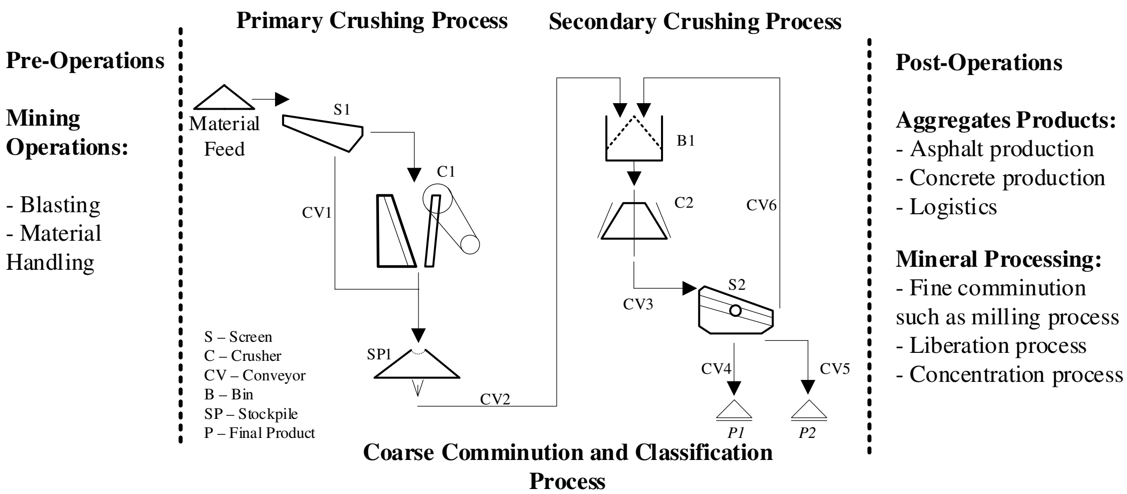


Figure 3. The part of a coarse comminution and classification process considered in this research.

1.5 RESEARCH QUESTIONS

The objective of the thesis can be described by the following research questions:

RQ 1) What optimization approach can be used to implement multi-domain optimization capability in a dynamic comminution and classification process?

RQ 2) How can the process objectives be formulated for process improvements and process optimization in a dynamic comminution and classification process?

RQ 3) How can an optimization system be structured to perform optimization routine for a dynamic comminution and classification process?

1.6 DELIMITATIONS

The research work is initiated with a focus on coarse comminution and classification processes for aggregates and minerals processing industry. The dynamic simulation approach, equipment models and data acquisition system for the coarse comminution and classification process used in this research are based on the previous development by Evertsson (2000), Hulthén (2010) and Asbjörnsson (2015). The development work in this thesis is carried out in MATLAB/Simulink environment. The research work excludes prior processes such as drilling and blasting from mining operations and application-specific subsequent processes such as asphalt and concrete production, fine comminution and classification, liberation, and concentration. The consideration regarding certain physical and chemical properties of the rock material such as ore grade is excluded for this work.

2 FRAME OF REFERENCE

This chapter aims to:

> *Provide an introduction to optimization systems, comminution and classification systems, and performance measuring systems.*
>
> *Describe the basic requirements of an optimization system.*
>
> *Describe the recent research on process simulation and modelling of crushing plants.*

The development within the three research areas: Optimization System, Performance Measuring System, and Comminution and Classification System can be described with hierarchical relations as shown in Figure 4. Each of the systems represents an area of research in itself; however, a brief overview of the three systems in context to the coarse comminution and classification processes is presented in this chapter.

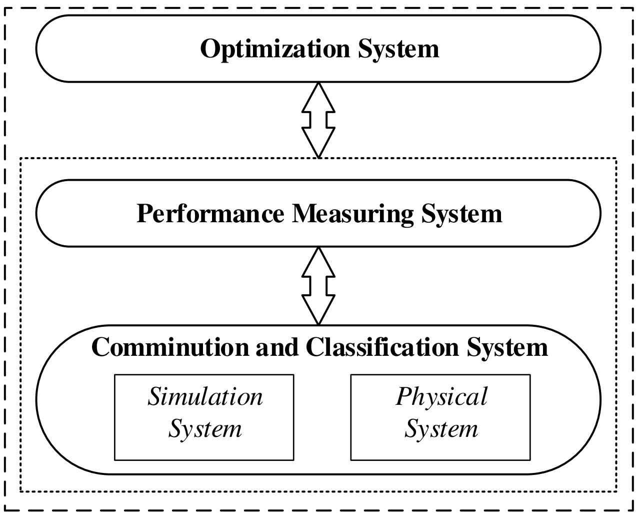


Figure 4. The hierarchical relationship between the three research areas.

2.1 OPTIMIZATION SYSTEM

The research within optimization is a diverse field and has been applied to many areas of engineering such as automotive, aerospace and manufacturing. Researchers within the comminution and classification processes for aggregates and minerals processing have shown various successful examples of optimization applications. These applications are presented at several abstraction levels of operation such as crusher liner design optimization (While et al., 2004), crushing sequence optimization (Lee and Evertsson, 2011, Lee and Evertsson, 2008), crushing plant optimization (Svedensten and Evertsson, 2005, Huband et al., 2005, Huband et al., 2006) and many others (Powell et al., 2009, Carrasco et al., 2017, Farzanegan and Vahidipour, 2009).

Napier-Munn et al. (1996) showed examples of optimization procedures on the crushing and classification process by using experience-based and logical deduction methods. Hulthén (2010) demonstrated the application of real-time optimization to control the operation of a cone crusher to improve productivity. An important aspect to consider with the development of the optimization system is the demarcation between the activities of process improvement (Napier-Munn, 2014), process optimization (Svedensten, 2007, Ding et al., 2017, Huband et al., 2006) and process control (Hulthén, 2010).

The research within the optimization method application for the comminution and classification processes has shown positive trends towards the use of optimization methods and their capabilities, but are limited to the repetition of the results because of the ill-defined optimization problems. Based on the variety of application, a general view on the ingredients of an optimization system is needed to increase the level of understanding and replicability of the optimization problem. The essential purpose of an optimization system as described by Papalambros and Wilde (2017) is for the decision-making process. Several fundamental concepts exist within an optimization system, which is presented in Figure 5 based on interpretation from the book "Principles of Optimal Design" by Papalambros and Wilde (2017). These concepts are briefly addressed in the following section.

- *System Concept and System Function(s)*: A system can be described as a collection of the units which is intended to perform specific functions by taking a set of inputs and producing a set of outputs.

- *Mathematical Models and Mathematical Relationship*: A mathematical model is an approximate representation of physical reality. There are various types of mathematical models which can represent reality, although the degree of quality and generalization of the application can vary (Moeller, 2004). The mathematical models can be of various types such as empirical models (developed based on experimental data), mechanistic models (developed based on the physics of the problem)(Newton, 1687) and so on. The mathematical relationship defines equality and inequality functions that relate the mathematical models' outcome, and indirectly represents a relationship between variables and parameters. These are used to define the optimization problem formulation. It is rather important to know the validity, accuracy, limitations, etc. of the mathematical model in the context of its application to understanding its degree of reliability for its use in an optimization problem.

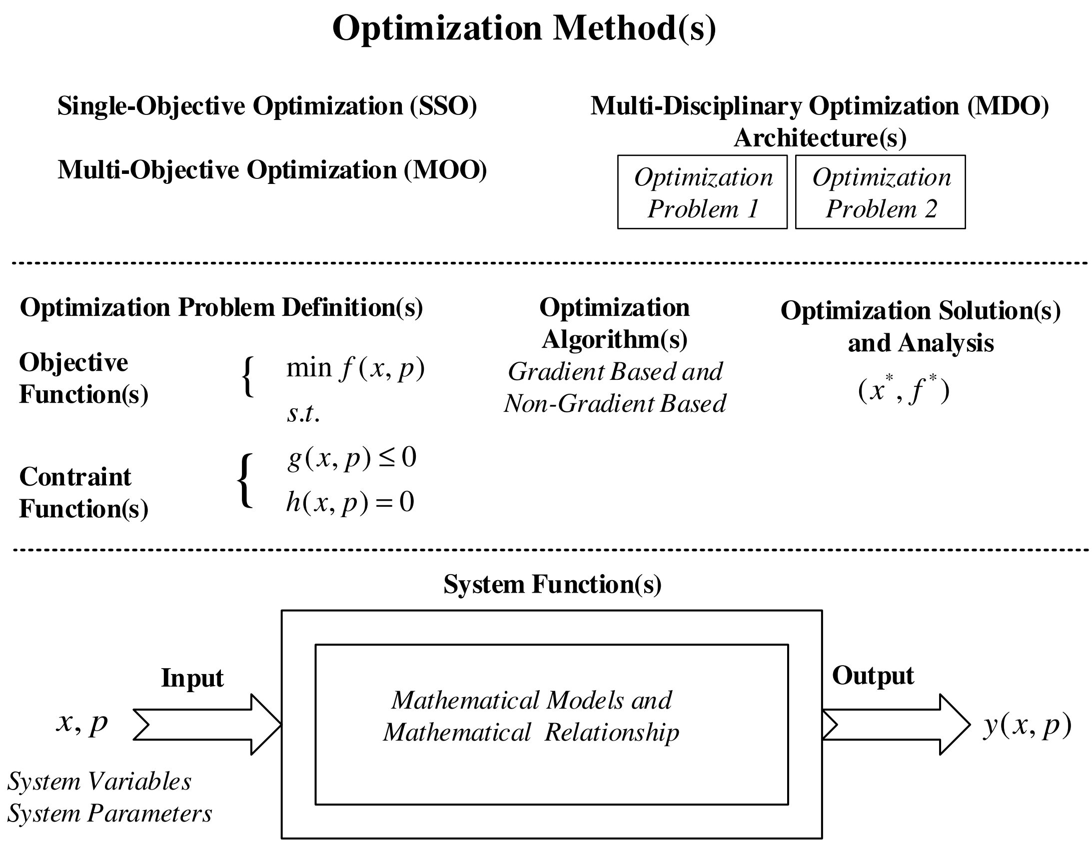


Figure 5. Overview of the optimization system components.

- *System Variables and System Parameters*: System variable (x) represent the set of variables that can be altered in the mathematical model while the system parameter (p) are set to a specific value for a particular mathematical model.

- *Optimization Problem Definition(s)*: The basic optimization problem in negative null form is defined in Figure 5, where the objective is to minimize a function $f(x, p)$ for a given inequality constraint $g(x, p)$ and equality constraint $h(x, p)$. The mathematical model and relationships are used to define various functions in the optimization problem definition.

- *Objective Function(s)*: An objective function $f(x, p)$ is a representation of the goal of the optimization problem. The objective function is either described as minimization form or maximization form.

- *Constraint Function(s)*: A constraint provides a set of requirements which the optimization solution set needs to fulfil. The constraints are of two types: equality constraint $h(x, p)$ and inequality constraint $g(x, p)$.

- *Optimization Algorithm(s)*: The optimization algorithm presents the numerical scheme of solving an optimization problem. It represents a set of processes carried out to find the optimal solution(s) to the optimization problem(s). These can be broadly classified as a gradient-based algorithm (e.g., interior point method, sequential quadratic programming, etc.) and non-gradient based algorithm (e.g., genetic algorithm, etc.). There exist other multitude variants of the optimization algorithms which can be referenced under other categories of optimization algorithms (Arora, 2015, Uryasev and Pardalos, 2013, Carson and Maria, 1997).

- *Optimization solution(s) and Analysis*: The optimization solution consists of a set of optimal solution point(s) for the variables called *optimizer* (x^*) and the value(s) of the objective function at those optimal solution points are called *optimum* (f^*). The optimization solution(s) need to be analysed for finding the feasibility and boundedness of the solution space. The term feasibility means the solution set is meeting the requirements (constraints) defined in the optimization problem while the term boundedness means the optimizer set is within the defined limits of upper and lower bounds of the defined variable. The solution set also needs to be reviewed whether it is a local optimization or a global optimization result.

- *Optimization Method*(*s*): An optimization method represents the approach towards solving the optimization problem based on the requirements of the problem. For this research, the optimization method is broadly classified into three categories: *Single-Objective Optimization* (*SOO*), *Multi-Objective Optimization* (*MOO*) and *Multi-Disciplinary Optimization* (*MDO*) *architecture.* An *SOO* approach is usually applicable for a simplified problem where the objective is to find optimum for a single function. A *MOO* approach is applicable to a problem where there is more than one objective function for the problem. An *MDO* architecture approach is applicable where the optimization problem represents different disciplines or large system scope. Depending on the choice of the optimization method, the optimization problem definition and optimization algorithm are selected.

- *Multi-Objective Optimization* (*MOO*): A *MOO* represents the simultaneous optimization of multiple objective functions involved in a given problem. For a *MOO* problem, the solution set can be used to plot trade-off curves (Pareto Optimality) between various objective functions. A *MOO* problem can be solved using various approaches such as weighted-sum approach, constraint-based approach, and use of a heuristic algorithm, for example, a genetic algorithm (Belegundu and Chandrupatla, 2011).

- *Multi-Disciplinary Optimization* (*MDO*) *Architecture*: The *MDO* architecture provides a strategic algorithm to handle multiple optimization problems belonging to various disciplines within a system to reach a global solution(s). It represents the mathematical schemes for organising and coordinating such a set of optimization problems. The *MDO* architecture can be broadly classified into two categories: monolithic architecture and distributed architecture depending on the hierarchical level of the problem definition. (Martins and Lambe, 2013)

2.2 COMMINUTION AND CLASSIFICATION SYSTEM

The essential function of a comminution and classification system is to reduce the particle size of rock (crushing process) and separate the particle sizes (classification process). The system represents the *crushing plant* for the aggregates and minerals processing industries. The comminution and classification system for this research is divided into two sub-systems: the physical system and the simulation system.

The physical system basically represents the physical entities such as crushers, screens, conveyors, material which are involved in the physical process. The simulation system represents the virtual representation of the physical systems using mathematical models and computer simulations. The simulation system contains process simulation and equipment models for the crushing and classification processes.

2.2.1 Process Simulation

Numerous research has been conducted over the past 40 years for numerical simulation of crushing plants (Lynch, 1977, Napier-Munn and Lynch, 1992, Whiten, 1972, King, 2001). There are commercial pieces of software available which performs steady-state crushing plant simulation such as JKSimMet at JKMRC, MODSIM (King, 2001), Bruno (Metso Minerals), IES (CRC Ore), AggFlow (BedRock Software) and NIAflow. The steady-state process simulation is typically based on the instantaneous mass balance of the process as shown in Eq. 2.1.

$$\dot{m}_{in} = \dot{m}_{1,out} + \dot{m}_{2,out} \tag{2.1}$$

The steady-state process simulations have been found useful for plant design, optimization and comparison of different circuit configurations (Mular et al., 2002, Csöke et al., 1996, Powell et al., 2014). Asbjörnsson (2015) demonstrated that the steady-state process simulations for crushing plants are limited to predict operational perspectives such as changes in the process over time and non-ideal operating conditions. The initial work for the dynamic process modelling for comminution and classification process was carried out by Whiten (1984) who introduced the idea of transition from the steady-state to the dynamic-state model to include the effect of material delays during physical processing. To further develop dynamics of the process simulation, Liu and Spencer (2004) showed the application of PID (Proportional-Integral-Derivative) controller in the grinding circuit, and Sbárbaro and del Villar (2010) demonstrated the application of model-based control system in comminution and classification processes. Asbjörnsson (2015) showcased the capability to capture discrete and gradual changes happening in the crushing plant due to delays, start-ups, discrete events, wear, etc. in the dynamic process simulation which is a closer representation of the physical process. In dynamic process simulation developed by Asbjörnsson (2015), each equipment model includes the derivative for mass m and properties γ of the material with respect to time as given in Eq. 2.2 and 2.3.

$$\frac{dm(t)}{dt} = (\dot{m}_{i,in}(t) - \dot{m}_{j,out}(t)) \tag{2.2}$$

$$\frac{d\gamma_i(t)}{dt} = \frac{\dot{m}_{i,in}(t)}{m(t)}(\gamma_{i,in}(t) - \gamma_i(t)) \tag{2.3}$$

2.2.2 Equipment Models

In order to generate mathematical models of each equipment type in the coarse comminution and classification process simulation, there has been an advancement for describing, explaining and modelling the fundamental relationships of particle breakage and particle separation. The particle breakage typically deals with equipment like cone crushers, mills, etc. while the particle separation is related to equipment like mechanical vibratory screens, etc.

The earlier work within the comminution equipment modelling was aimed to explain the fundamental relationship between energy and size reduction of the particles (Kick, 1885, Bond, 1952, Rittinger, 1867) which are termed as classical theories of comminution (Asbjörnsson, 2015, Jankovic et al., 2010). A general relationship between energy and particle size is presented in Eq. 2.4 (Walker et al., 1937, Hukki, 1961), where E is the net specific energy; x is the characteristic dimension of the product; n is the exponent, and it is dependent on the characteristic dimension of the particle; C is a constant related to the material.

$$dE = -C\frac{dx}{x^n} \tag{2.4}$$

The population balance model, introduced by Epstein (1947), is one of the commonly used models to represent the particle size reduction in comminution equipment such as cone crushers (Whiten, 1972), high pressure grinding rolls (Dundar et al., 2013), grinding mills (Valery Jnr and Morrell, 1995), etc. The population balance model is characterised as a probability-based model and is dependent on a large empirical dataset generated by testing of different materials (Asbjörnsson, 2015). The cone crusher, which is one of the primary size reduction equipment in the coarse comminution process, can be also be modelled with high fidelity using a mechanistic approach as developed by Evertsson (2000). The mechanistic model relies on the geometry of the crusher chamber and the sequence of operations of the crushing process within the equipment which can give higher predictability of reality (Asbjörnsson et al., 2016).

The classification process, consisting of the screens in a crushing plant, can be modelled using a simple phenomenological model given by an efficiency curve. The efficiency curves can be modelled using a probabilistic function (Reid, 1971), an exponential sum expression (Whiten, 1972), or a polynomial function (Hatch and Mular, 1979). Other models for vibratory screens include an analytical model (Soldinger, 2002) which provides higher fidelity in the simulations. Equipment such as bins can be modelled by using a perfect mix principle, first-in-first-out (FIFO) principle and a mechanistic model principle (Asbjörnsson et al., 2012). Conveyors in the process act as a material delay unit and can be modelled as a state-space model (Asbjörnsson et al., 2013).

2.2.3 Operation and Control

The physical operation of a comminution and classification process is built with multiple layers of the control system application (Tatjewski, 2007). Various equipment involved in a crushing plant needs to be coordinated and controlled during the operation to produce the desired plant performance. The process and the equipment involved in the crushing plant suffers variability due to many factors such as variability in material feed, wear in crushers and screens (Asbjörnsson, 2015, Lindqvist, 2005). To manage these variabilities, various types of control systems are in place to stably operate the crushing plant. These include simple controls such as electrical control (on/off), simple interlock to advance process control such as model predictive control. For individual equipment, the control systems are installed to maintain and regulate the operation based on the equipment purpose. For a process, the control systems can be installed to stabilize or regulate the process operation.

The purpose of the control systems depends on the application, and it is broadly classified into two categories: regulatory control and supervisory control (Asbjörnsson, 2015). The purpose of a regulatory controller is to maintain a stable process operation by manipulating certain variables within the real-time process (Åström and Hägglund, 1995). Example of a regulatory controller is a proportional-integral (PI) controller. The supervisory controller is usually applied to achieve process optimization based on the target for real-time plant performance (Korbicz and Kościelny, 2010). Examples of supervisory control are model predictive control (Johansson and Evertsson, 2018) and real-time optimization using a Finite State Machine (FMS) algorithm (Hulthén and Evertsson, 2011).

2.3 PERFORMANCE MEASURING SYSTEM

A performance measuring system can be defined as the collection of functions which comprises of various decision-making indicators defined towards performance monitoring of a coarse comminution and classification system. These decision-making indicators can be used for two purposes: process improvements and process optimization.

Early research within the area of minerals processing circuits has highlighted the use of numerous objective functions for optimization which was driven by the need of technical and economic measurements of the processes (Buskies, 1997, Meloy, 1983a, Meloy, 1983b). Recent researchers followed a similar approach to demonstrate optimization objective functions such as production and operation cost (Bengtsson et al., 2015, Bengtsson et al., 2009, Svedensten and Evertsson, 2005), profit and quality (Bengtsson et al., 2017), material throughput rate (Hulthén and Evertsson, 2011, Asbjörnsson et al., 2016, Muller et al., 2010), technical parameters (Farzanegan and Vahidipour, 2009), yield and energy (Lee and Evertsson, 2008), net present value (Huband et al., 2006), yield and cost (Huband et al., 2005), grade engineering (Carrasco et al., 2017, Carrasco et al., 2016), crusher geometrical design (Lee and Evertsson, 2011, While et al., 2004) and so on.

From the operational management research point of view, there are specific indicators which have also been applied to minerals processing such as overall equipment effectiveness, availability, performance, effectiveness and so on (Powell et al., 2012, Powell et al., 2011, Kullh and Älmegran, 2013). The ISO 22400 standard states a set of key performance indicators for managing manufacturing operations (ISO, 2014a, ISO, 2017, ISO, 2014b, ISO, 2018). Other important aspects which have been under recent development are the environmental indicators for the aggregate industry (Asbjörnsson et al., 2017). The performance measurement system is also related to the development of cost-effective data acquisition techniques such as mass flow measurement on conveyors based on head pulley power draw (Hulthén and Evertsson, 2006). These operation related indicators are useful for process improvements, but can also be potentially applied for process optimization.

From the literature, it can be seen that the scope of optimization objective function has been varied depending on the application and there is not a clear consensus on how these process objectives can be classified. The fundamental measurements associated with the coarse comminution and classification processes can be formulated as:

- Physical and chemical properties of raw material
- Form characteristics such as particle size and shape (e.g. particle size distribution)
- Material flow rate indicating processing capacity
- Energy consumption for performing the size reduction and separation processes

The importance of these measurements can change depending on which process they are applied to. For example, in minerals processing, the raw materials' (mineral ore) chemical composition is of importance as it indicates how much metal can be recovered in later stages. At the same time, the ore-type and techno-economic feasibility decide what kind of equipment and processes are applied to which the capacity and the energy indicators become important. For aggregates processing, the particle size distribution and processing capacity determine which size range of products are produced and their respective product quantities.

3 RESEARCH APPROACH

This chapter aims to:

> *Introduce the research methodology used in this thesis.*
> *Introduce optimization methods.*

This research was carried out at the Chalmers Rock Processing Systems (CRPS) research group, which is a part of the Machine Element Group, Division of Product Development at the Department of Industrial and Materials Science at Chalmers University of Technology. The research group is active within the field of comminution and classification processes involving crushing and screening equipment for almost three decades.

3.1 RESEARCH METHODOLOGY

The research approach used in this research has been inspired and adopted based on the characteristic of the problem-based approach used at CRPS. Evertsson (2000) initially described the problem-based approach as a systematic search for new knowledge focusing on the problem. Asbjörnsson (2015) applied the systems thinking approach together with the problem-based approach to integrate the wide scope of system development. The scope of the current work is towards the development of a wide-system of optimization for coarse comminution and classification processes, which involves multiple sub-system studies connected to multiple problems. The research methodology used for this research is presented in Figure 6 and is based on the methodology from Asbjörnsson (2015). The modifications are made to address the multidisciplinary nature of this research work which involves complex aspects of different fields such as engineering, technical solutions, management, and their interactions. The thesis work is organised based on the theories of general system theory, where a complex system can be visualised as a combination of various sub-systems, and their interactions (Skyttner, 2005, Von Bertalanffy, 1950).

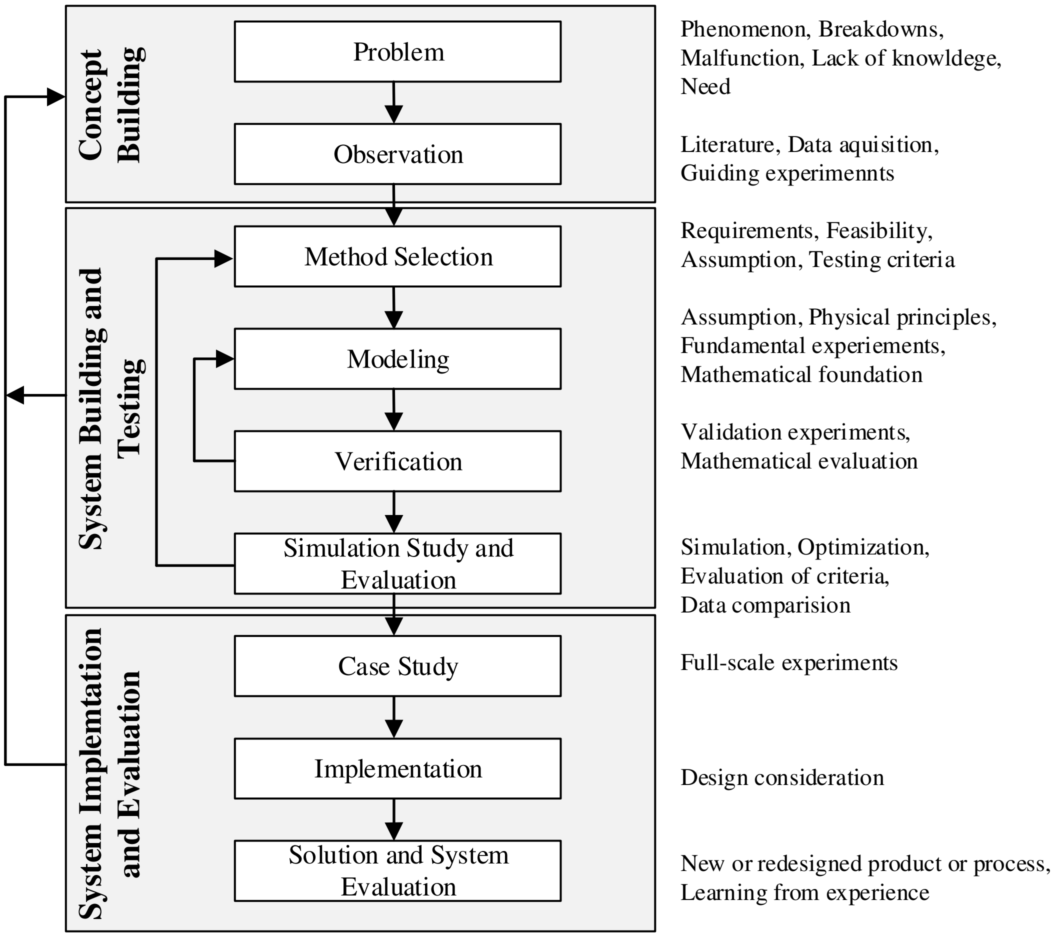


Figure 6. The applied problem-oriented research model for the development of an optimization framework (Based on Evertsson (2000) and Asbjörnsson (2015)).

The work was initiated by identifying the possible knowledge gaps and industrial relevance with the development of the optimization system. These resulted in a set of problems which were undertaken individually for literature study, process and equipment understanding, data acquisition through interviews and knowledge exchange, etc. This was aimed to further clearly define a distinct scope of individual study and to have a modular perspective in the main system development.

When a clear problem for each study was defined, the system building and testing activities were carried out to develop and evaluate each sub-system. The development work carried out in this stage was iterative in nature. The evaluation for each sub-system is carried out based on the criteria of relevance for the problem at hand. Svedensten (2007) and Hulthén (2010) highlighted that early implementation is important, for cost and fidelity reasons; the implementation at this stage is relying on the simulation results and partial implementation. The iterative sub-system development leads to a new insight into the main system itself.

Once various sub-systems involved in the development gain a certain confidence level, a full-scale experimental implementation is possible by defining case studies. The results of the implementation are evaluated which leads to a new or redesigned product or process. The learnings from the research process can be reiterated to further develop higher fidelity of the system. CRPS works in close collaboration with Swedish aggregates, and international minerals and mining industries where the need for this research originated, and the research outputs are relevant to the industrial demands and their challenges.

3.2 RESEARCH EVALUATION

Validity and reliability are two central recurring concepts for evaluating research quality. According to Bryman and Bell (2007), validity relates to the integrity of the research conclusion while reliability concerns with the repeatability of the results. The validity can be represented as internal validity, which relates to the particular claims made based on the research, and external validity, which relates to the particular claims of the study that can be generalized in another research context (Bryman and Bell, 2007). Pedersen et al. (2000) described the concept of research validity into two categories: structural validity and performance validity which can be applied both for theoretical and empirical perspective. The structural validity is based on the qualitative process wherein the system is built on sufficient background information to demonstrate the application of result, while the performance validation is a quantitative process which determines the accuracy of the result for its application (Pedersen et al., 2000). Myrtveit et al. (2005) described reliability as an extent to which the experimental results are consistent on repeated trials indicating the reliability of the measuring procedure. In comparative simulation studies, it is recommended to use the same underlying parameters for a particular simulation model under study to increase the reliability of the results (Myrtveit et al., 2005). These different aspects of the validity and reliability of the research will be discussed in Chapter 5 - Discussion & Conclusions.

3.3 OPTIMIZATION METHODS

In order to solve the optimization problems, two optimization methods have been applied in this thesis. The two methods are Multi-Disciplinary Optimization (MDO) Architecture and Multi-Objective Optimization (MOO). The optimization methods follow a set of notations for defining the optimization problem in the form of MDO architecture and MOO which is shown in Table 1 (Martins and Lambe, 2013).

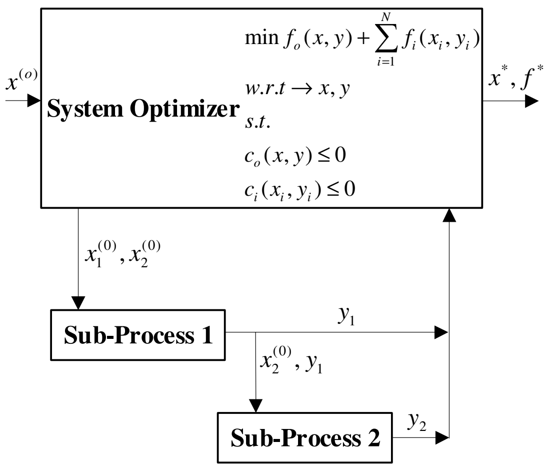
Table 1. Notation for defining an optimization problem.

Symbol	Definition
x	Vector of design variables
y	Vector of coupling variables or output from sub-process analysis
f	Objective function
c	Vector of design constraints
c^c	Vector of consistency constraints
N	Number of sub-processes
$(\ )_0$	Functions or variables shared between more than one sub-process
$(\ )_i$	Functions or variables applied only to a sub-process i
$(\bar{\ })$	Independent copies of variables distributed to other sub-process
$(\ )^0$	Functions or variables at their initial values
$(\ )^*$	Functions or variables at their optimal values

3.3.1 Multi-Disciplinary Optimization (MDO) Architecture

The MDO architecture is a representation of organising, coordinating and solving a set of optimization problems defined for a cross-disciplinary problem (Martins and Lambe, 2013). Two simple MDO architectures have been used in this work: Multi-Discipline Feasible (MDF) and Individual-Discipline Feasible (IDF). The optimization problem formulation and the algorithms of the two architectures are shown in Figure 7 and 8 which are based on the work by Martins and Lambe (2013).

The MDF architecture presented in Figure 7 is monolithic in nature as it contains a single level of the optimization problem. The optimization problem is solved by sequentially evaluating each sub-process involved in the system. The objective function consists of two sets of functions, i.e., function (f_o) which is shared between the sub-processes and function (f_i) representing individual sub-process i. Similarly, the problem definition contains two sets of constraints, i.e., constraint (c_o) which is shared between the sub-processes and constraint (c_i) representing individual sub-process i.



Algorithm:

Input- Design variable x

Output- Coupling variable y^*, optimised variable x^*, objective function f^*

0: Initiate MDF loop

Repeat

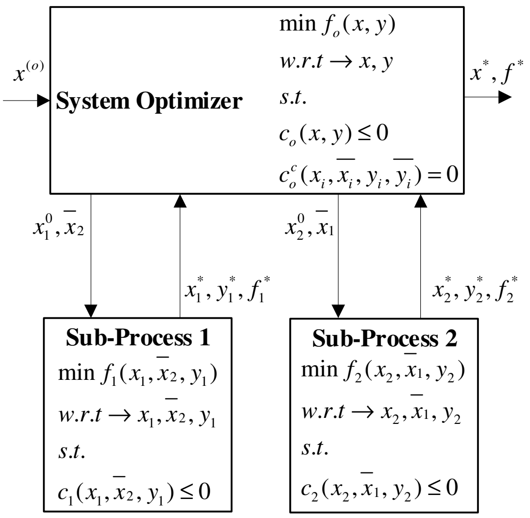
1: Evaluate sub-process 1 and update y_1

2: Evaluate sub-process 2 and update y_2

Until →MDF converged

Figure 7. Optimization problem formulation and algorithm for the monolithic MDF architecture.

The IDF architecture, presented in Figure 8, is a distributed architecture which contains two levels of optimization problems. The system optimization problem is iteratively solved by solving the individual sub-process optimization problem in parallel. The system optimization problem contains objective function (f_o) and constraint (c_o), which are shared between sub-processes, and an additional consistency constraint (c^c) is introduced to maintain the consistencies of the design variables. Each sub-process optimization problem consists of the objective function (f_i) and constraint (c_i) belonging to the particular sub-process i. The individual sub-process optimization receives independent copies of the design variables ($\overline{x}$) belonging to the other sub-process through the system optimizer. The sub-process optimizer delivers a local optimal value for design variable (x_i^*) and function value (f_i^*) to the system optimizer.



Algorithm:

Input- Design variable x

Output- Optimized variable x^*, objective function f^*

0: Initiate system optimizer iteration

Repeat

1: Compute sub-process objective and constraints

For each sub-process i, **do**

1.0 Initiate sub-process optimization

Repeat

1.1 Evaluate sub-process i

1.2 Compute sub-process i objective and constraints

1.3 Compute new design point for sub-process (i+1)

Until 1.3 → Optimization i has converged

End for

2. Compute new system design points

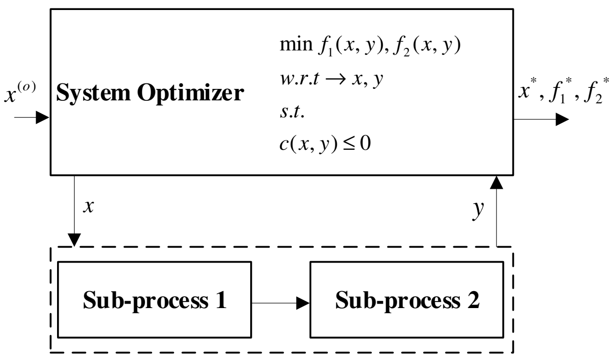
Until 2 → System optimization has converged

Figure 8. Optimization problem formulation and algorithm for the distributed IDF architecture.

3.3.2 Multi-Objective Optimization (MOO)

The MOO method represents a synchronised optimization of multiple objective functions involved in a given problem. The central concept for using MOO is to generate trade-off curves (Pareto Optimality) between various objective functions. The MOO problem can be solved using a various approach such as weighted-sum approach, constraint-based approach, and use of a heuristic algorithm, for example, a genetic algorithm (Belegundu and Chandrupatla, 2011).

A general form for defining a MOO problem using genetic algorithm is shown in Figure 9 (Kramer, 2017, Kalyanmoy, 2001). A genetic algorithm is a heuristic based algorithm and is developed based on inspiration from a natural evolution process. The system optimizer parses the design variables (x) to the process simulation. The simulation returns the output variable (y) to the system optimizer, and this process is repeated until the convergence criteria are achieved. The optimization problem contains multiple objective functions (f_1, f_2), also referred to as fitness functions and a set of constraints (c). The choice of the objective functions and problem formulations are critical in generating the relevant results using this approach.



Algorithm:

Input- Design variable x

Output- Pareto front for multiple-objective functions (f_1^*, f_2^*) and optimized variable set (x^*)

0: Initiate population

Repeat

Repeat

0.1: Crossover

0.2: Mutation

0.3: Fitness computation

Until → Population complete

1: Selection of parental population

Until → Termination condition

Figure 9. Optimization problem formulation and algorithm for the MOO problem using Genetic Algorithm.

4 RESULTS

This chapter aims to:

> *Describe the development of the optimization system.*

> *Present the development in performance indicators.*

> *Present a system-wide view on optimization framework.*

During the iterative development work for this research, various optimization methods and key performance indicators (KPIs) for coarse comminution and classification processes have been defined, explored, developed and implemented. A conceptual framework for performing the optimization routine for industrial use in a crushing plant has been developed. The results from the above-mentioned development work are briefly presented in this section.

4.1 DEVELOPMENT OF OPTIMIZATION SYSTEM

The optimization system, in general, is aimed at exploring non-intuitive solutions for a defined problem towards designing, operating and controlling a coarse comminution and classification process. Paper A, B and C present explorative studies that have been carried out to understand and implement optimization methods. The process of the current development work is carried out from a top-down approach to the problem and the steps involved are presented in Figure 10.

4.1.1 Defining Scope of Optimization Application

One of the main challenges of starting an optimization procedure is to define the scope of the optimization application. Specific important questions need to be answered to understand the scope of the optimization application in coarse comminution and classification processes such as:

- What is the focus area (boundary) of the optimization application?
- What is the context of the optimization application?
- At what abstraction level does the optimization need to be performed?

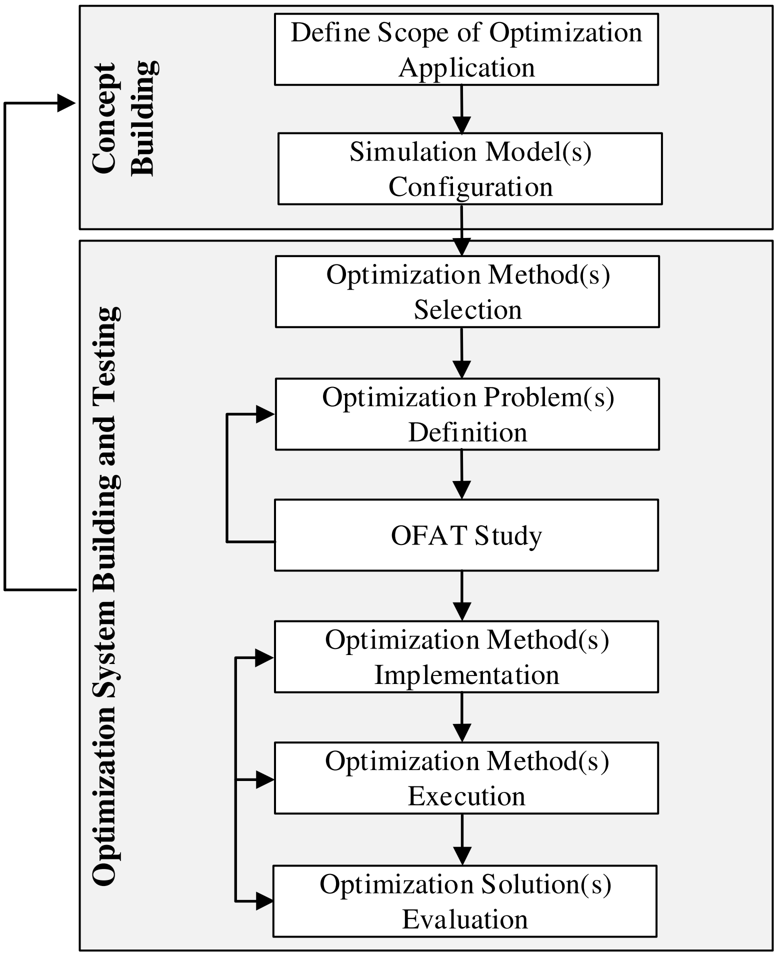


Figure 10. Overview of the development of the optimization system.

The implication of these questions leads to decisions for choosing the appropriate abstraction level of mathematical models that can be used to carry out optimization. Based on the literature review in Paper A, a general classification scheme is established to define the scope of the optimization for coarse comminution and classification processes. Table 2 represents the classification scheme in two dimensions: *State of Application Area Units* and *State of Development Stage*.

Table 2. Classification scheme to define the scope of optimization application (Paper A).

	State of Development Stage →		
State of Application Area Unit ↓	**Design**	**Operations**	**Control**
Equipment			
Sub-Process		Paper B	
Main Process		Paper C	

The state of application area units represents the abstraction-level based on the hierarchical position of physical entities in the crushing plant. This is categorized as: *Equipment, Sub-Process* and *Main Process*. The equipment represents an individual physical unit in the processing operation such as crusher, screen, and conveyor which can perform one or more functionalities. The sub-process represents a collection of equipment performing a specific functionality for the main process of the physical plant. An example of a sub-process can be a primary crushing process with a function to reduce material size under a certain size. The main-process represents a collection of sub-processes to reach the overall goal of the physical plant operations which can be the production of desired aggregate products.

The state of development stage represents the purpose of the optimization and is divided into three categories: *Design Stage, Operation Stage,* and *Control Stage*. These are briefly described below:

Design Stage: Deals with the optimization application towards developing and designing a completely new process or equipment. This also includes re-configuration of an existing design concept for a process or equipment. The validity of the optimization results is dependent on the accuracy of the mathematical models used for the process and equipment optimization. The possibility for the verification of the optimization results is limited.

Operation Stage: Deals with the optimization application towards understanding and finding operational settings of an existing process or equipment based on the user requirements. The validity of the optimization results is dependent on the type of mathematical models used for the process and equipment optimization. The optimization results can be implemented into real-time operations and the results can be verified by collecting and comparing them with operational data.

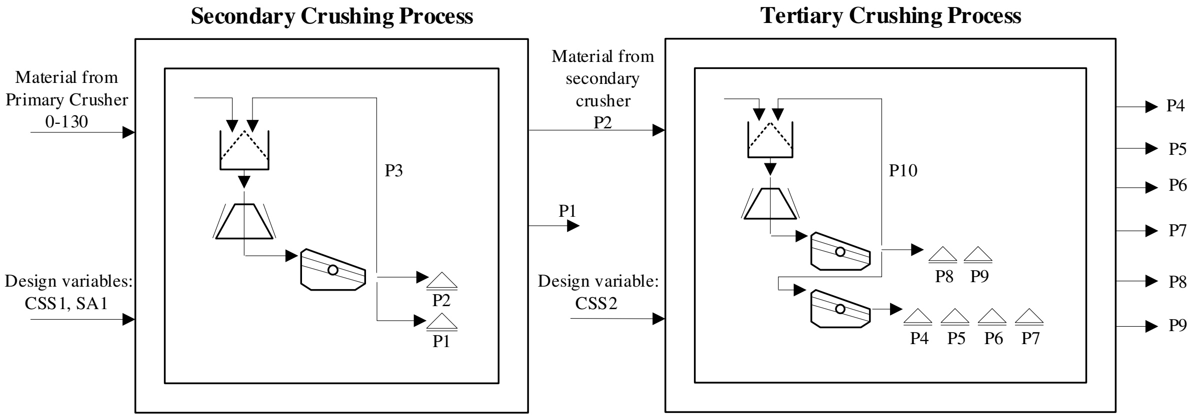
Control Stage: Deals with the optimization application towards regulatory control and supervisory control of the process and equipment under real-time operation. The application is related to stabilising or regulating an existing process or equipment towards maintaining their nominal performance. The usefulness of the optimization application can be observed from the real-time plant performance.

For paper B and C, the application of the optimization methods was carried out to find suitable operating parameters for a fixed crushing plant layout and are categorized under *Operation Stage* (See Table 2).

4.1.2 Simulation Model Configuration

One of the primary uses of the classification scheme presented in the previous section is to communicate the scope of optimization application to the research community and the industry. Based on the scope, one can configure the process simulation model which can be used as an underlying mathematical model for running an optimization method. For both Paper B and C, a dynamic simulation model for a coarse comminution and classification process (crushing plant) developed by Asbjörnsson (2015) is used. The advantage with the dynamic simulation is that it provides a closer replication of the actual physical operation compared to the steady-state simulation models, although, the optimization methods can be run on both types of the process simulation models. However, the dynamic simulation requires more computational power and knowledge compared to the steady-state simulation for its configuration and operation.

In Paper B, the scope of the optimization application was to investigate the operating parameters for an existing layout of an aggregate production plant as shown in Figure 11. The purpose was to find a balance between the goal of the individual sub-processes and the overall process goal. In paper C, the purpose of the optimization application was to study the trade-off and balancing point between the two sub-processes involved in the operation of a crushing plant for a minerals processing application.



Note: P_i represents a set of aggregate products based on the plant configuration. The value of screen apertures for two screens in the tertiary crushing process are fixed.

Figure 11. Crushing plant for aggregates production consisting of two sub-processes (*Paper B*).

4.1.3 Optimization Method Selection

Based on the scope and the purpose of the optimization application, a multitude of optimization methods can be applied. Explorative studies of two different optimization methods have been carried out: Multi-Disciplinary Optimization (MDO) Architecture and Multi-Objective Optimization (MOO). In paper B, implementation of two MDO architectures: Multi-Discipline Feasible (MDF) and Individual-Discipline Feasible (IDF) was carried out. In paper C, an additional implementation of the Multi-Objective Optimization (MOO) using Genetic Algorithm (GA) was carried out to demonstrate the Pareto-front for the multiple objectives present in the crushing plant.

The choice of the optimization method at this stage is based on exploring the functionality of each method for the creation of useful decision-making results. Both the optimization methods are applicable for multi-domain optimization problems which typically exist in a crushing plant. The design and operation of a crushing plant deal with multiple sub-process (e.g., primary crushing process, secondary crushing process, etc.) interactions and dependencies. In multi-domain optimization terms, each one of the sub-process can equivalently represent one domain. A crushing plant can be characterized by a long chain of hierarchical dependencies between the sub-processes as the material flows from one sub-process to the subsequent sub-process. For example, a change in *CSS* in the primary crushing process influences the characteristics of the material produced in this process, which in turn influences the performance of the subsequent secondary crushing process. The sub-processes are loosely coupled with input design variables of the other sub-processes but are strongly coupled with the material output of one sub-process to another. (Paper B and C)

4.1.4 Optimization Problem Definition

Optimization problem definition enlists the goal in the form of the objective(s) function and the requirements in the form of constraint(s) function. The formulation also presents the design variable(s) which is varied during the optimization method execution. The objective functions which have been applied for optimization are throughput rate (Paper B and C), sub-process value (SPV) (Paper B), and power consumption (Paper C). The SPV function is a simple abstract function representing the technical-economic performance of the crushing plant. The SPV function is based on approximate cost, selling price, and throughput rate of the aggregate product (Paper B).

An example of a bi-level optimization problem formulation for the distributed IDF architecture is shown in Eq. 4.1, 4.2 and 4.3 (Paper B). The objective function of the system optimizer is to maximize the sum of throughput rate of the desired aggregate products for the crushing plant shown in Figure 11. The objective function of each individual sub-process is to maximize the SPV. The optimization problem also highlights the design variables which in this case are the operational settings of the crushers and screen. The design variables are $CSS1$ and $CSS2$ which represent the closed-side setting of the crushers in the secondary and tertiary crushing process respectively, and $SA1$ which represents the top-deck screen aperture setting of the secondary crushing process (see Figure 11). The optimization formulation also presents the upper and lower limits for each design variables.

System Optimization

$$\begin{aligned}
&\max \sum_{j=1,\, j\neq 2,3,10}^{10} P_j(x, \bar{x}, y) \\
&w.r.t \rightarrow x, \bar{x}, y \\
&x = \{(CSS1, SA1)_1, (CSS2)_2\} \\
&\bar{x} = \{(\overline{CSS}1, \overline{SA}1)_1, (\overline{CSS}2)_2\} \\
&y = \{(P1, P2, P3)_1, (P4, \ldots, P10)_2\} \\
&s.t.: \left\| x - \bar{x} \right\| = 0 \\
&x_{lb} = \{20, 50, 10\}, x_{ub} = \{55, 65, 30\} \\
&\bar{x}_{lb} = \{20, 50, 10\}, \bar{x}_{ub} = \{55, 65, 30\}
\end{aligned} \tag{4.1}$$

Secondary crushing process optimization

$$\begin{aligned}
&\max \sum_{j=1}^{3} P_j V_j(x_1, \bar{x}_2, y_1) \\
&w.r.t \rightarrow x_1, \bar{x}_2, y_1 \\
&x_1 = \{(CSS1, SA1)_1\}, \bar{x}_2 = \{(\overline{CSS}2)_2\} \\
&y_1 = \{(P1, P2, P3)_1\} \\
&s.t.: (F_{80} / P_{80})_2 - 4 \leq 0 \\
&x_{lb} = \{20, 50\}, x_{ub} = \{55, 65\}
\end{aligned} \tag{4.2}$$

Tertiary crushing process optimization

$$\begin{aligned}
&\max \sum_{j=4}^{10} P_j V_j(x_2, \bar{x}_1, y_2) \\
&w.r.t \rightarrow x_2, \bar{x}_1, y_2 \\
&x_2 = \{(CSS2)_2\}, \bar{x}_1 = \{(\overline{CSS1}, \overline{SA1})_1\} \\
&y_2 = \{(P4, ..., P10)_2\} \\
&s.t.: (F_{80} / P_{80})_2 - 4 \leq 0 \\
&x_{lb} = \{10\}, x_{ub} = \{30\}
\end{aligned} \tag{4.3}$$

The optimization problem formulation is dependent on the choice and purpose of the optimization method. In the case of the distributed IDF problem formulation presented above, each optimization problem is decoupled and is represented by a simple objective function (Paper B and C). The optimization problem formulation for monolithic MDF can be found in Paper B and MOO using GA can be found in Paper C which represents a comprehensive optimization problem definitions compared to the IDF problem formulation. The optimization problem formulation in a standard format is useful for replication of the results.

4.1.5 OFAT (One-Factor-at-a-Time) Study

The purpose of performing an OFAT study is to investigate the behaviour of the objective function with respect to the design variable selected in the optimization problem. This is useful to further gain insight into the optimization problem formulation and understand the potential improvements possible with the optimization. OFAT studies are carried out in both Paper B and C. Figure 12 shows an example of the OFAT study for the SPV function applied in the crushing plant (see Figure 11). The crushing plant has complex relationships and conducting OFAT study is useful for understanding the mathematical response of the objective functions with respect to the important design variables.

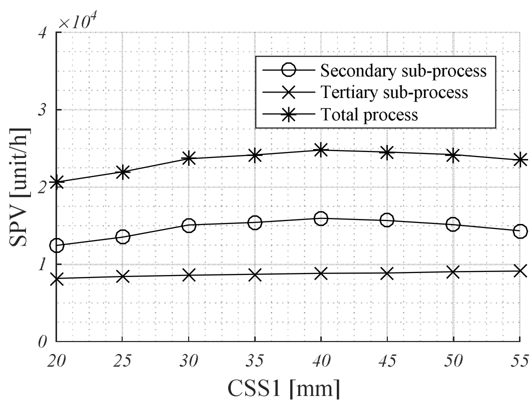


Figure 12. Example of SPV for varying CSS1 (For SA1 = 65 mm, CSS2 = 15 mm) (Paper B).

4.1.6 Optimization Method Implementation

The implementation of the optimization method deals with the creation of a coordinating algorithm to pose the optimization problem formulated. The implementation also includes coupling of the optimization problem with the configured model. The implementation at this stage is carried out in MATLAB/Simulink environment. An example of the implementation framework for the problem posed in the set of Eq. 4.1, 4.2 and 4.3 using distributed IDF architecture is shown in Figure 13 (Paper B). Similarly, the implementation framework for monolithic MDF can be found in Paper B and for MOO using GA can be found in Paper C.

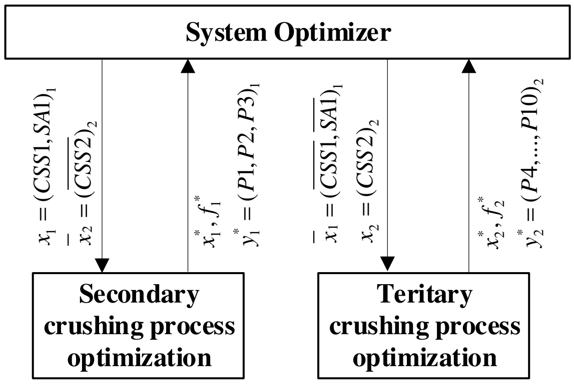


Figure 13. Example of distributed IDF formulation for the two-stage crushing process (*Paper B*).

4.1.7 Optimization Method Execution

The execution of the optimization method involves multiple micro-level decision entities for running an optimization method and algorithm. For instance, to run an optimization problem using MDO and MOO, a number of settings need to be decided such as:

- Choice of solver algorithm, e.g. sequential quadratic programming, genetic algorithm,
- Setting of the tolerance criteria in the functional evaluation,
- Setting of the convergence criteria for the optimization,
- Setting of the maximum number of iterations,
- Defining weights in the objective functions,
- Initial start points for design variable(s) in the algorithm and so on.

The recommendation of these settings is developed by the experience of using the methods. The choice of these settings influences the computation time and quality for the optimization method. For example, in paper B, the MDO algorithms were found to be sensitive to the initial start point of the algorithm. This indicates that the method lacks robustness in the application and it is recommended to test the MDO algorithm at different start points. In paper C, the results from a genetic algorithm are dependent on the initial definition of population size, the number of generations, etc. The settings made at this stage affect the results of the optimization and an iterative process of investigation is required to gain insights.

4.1.8 Optimization Solution Evaluation

The solution obtained from the optimization method needs to be evaluated for two categories: convergence analysis and physical relevance. The convergence analysis deals with the understanding of the iterations required for the optimization algorithm, constraint activity and the behaviour of solution point(s) (optima and optimizers). The number of iterations reflects the computation time, while the satisfaction of the constraint function reflects the feasibility of the solution point(s). The solution point(s) reflect the boundedness of the solution based on the upper and lower limits of variables defined in the optimization problem formulation. The solution points(s) also needs to be checked for local optimization and global optimization which can be reflected together with the understanding developed from OFAT study. The physical relevance with the optimization results is to check whether the solution is feasible to the practical operation in the physical plant. This can be discussed with the experienced personnel in the management and operation of such processes of the plant. The practical implementation of optimization results can be carried out after this stage.

An example of the convergence analysis for the optimization problem posed in Section 4.4 is shown in Figure 14 (Paper B). Figure 14 (a) illustrates the number of iterations required for the convergence of the algorithm where constraints are satisfied, while Figure 14 (b) demonstrates the corresponding design variables behaviour for the respective iterations. The design variables and their duplicate copies reached convergence in six iterations. The design variables reached the final values as $CSS1$ = 44.60 mm, $SA1$ = 65 mm, and $CSS2$ = 10 mm. It can be seen that the two variables $CSS2$ and $SA1$ are hitting the boundary value limits defined in the problem, while the $CSS1$ is converging to an interior optimum value.

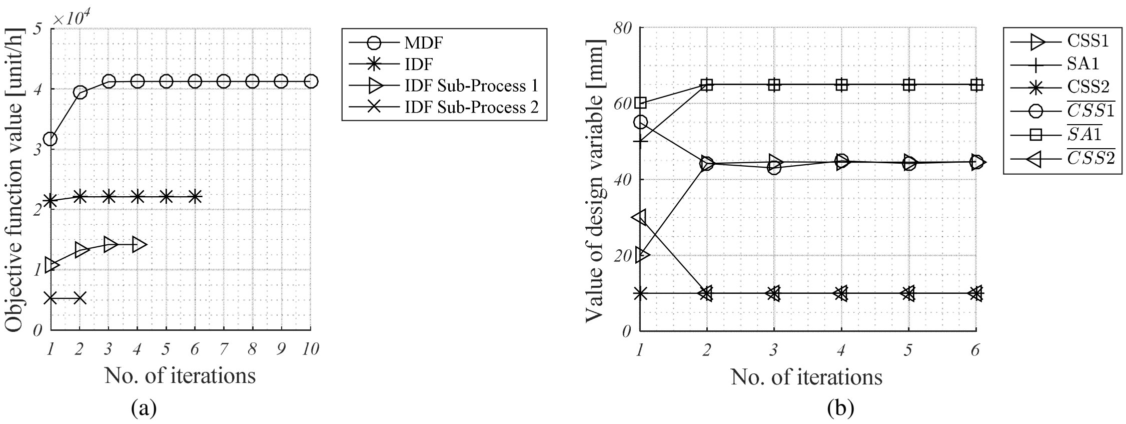


Figure 14. Example of a convergence study for distributed IDF formulation (Paper B).

An example of the Pareto-front result for the MOO problem using GA for two objectives is shown in Figure 15 (Paper C). The purpose of the optimization application was to explore the solution space to find a trade-off between the maximization of the production of the fine materials and minimization of the power draw by the crushers. The details of the crushing plant layout and optimization problem formulation can be found in Paper C, while a brief essence of the results is presented here. The MOO method using GA generates a wide range of solutions and the choice of the solution is based on the reasoning of the solution space. The user can select the most competitive solution depending on the judgement considering the real plant as the results obtained is dependent on the underlying models used in the simulation.

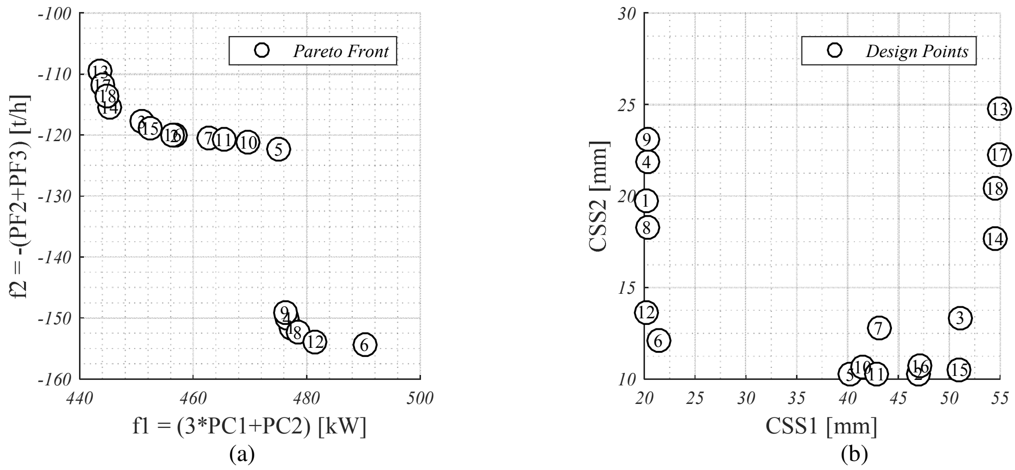


Figure 15. Example of Pareto-front and solution points of the MOO problems using GA (Paper C).

A general comparison between the MDO using distributed IDF method and MOO method using GA is presented in Table 3 (Paper C).

Table 3. Comparison of the two optimization method (Paper C).

Criteria	MDO using distributed IDF	MOO using GA
Problem Formulation	Can be decoupled into two or more levels of optimization problems.	Weighted sum approach to formulate comprehensive optimization objectives.
Result Type	Balanced solution between system optimization and sub-process optimization.	Pareto-front highlighting the spectrum of solutions. The choice is based on the reasoning of the solution space.
Computation Time	Low. Dependent on the initial start point of the algorithm.	High. Dependent on algorithm settings such as population size and generation.

4.2 PERFORMANCE INDICATORS

The performance measuring system consists of a set of process and equipment performance indicators which can be used to carry out process improvements and process optimization for a crushing plant. The performance improvement is iterative in nature and can be carried out based on the process diagnostics with the support of the performance indicators. The process optimization can be carried out by using the performance indicator as an objective function in the optimization system.

In Paper D, a set of key performance indicators (KPIs) has been developed based on the ISO 22400 Standards (ISO, 2014a, ISO, 2017, ISO, 2014b, ISO, 2018) as shown in Table 4. These KPIs are the initial development which can be used both towards process improvement and process optimization. The KPIs are useful for operators and plant managers of a crushing and screening process to make decisions and can be viewed as a support tool for the decision making process. The KPIs were calculated based on the dynamic process simulation output and in parallel real-time measured data of the process. The real-time data for the physical crushing plant was retrieved from the data-acquisition system consisting of a cloud-based solution. The real-time data contained measurements of mass flow rate from conveyors and various process and equipment set points.

Table 4. List of KPIs developed for the measurement of performance in aggregate production (Paper D).

Measurement Basis	KPI	Unit
Planned and Real-Time	Equipment Utilization	%
	Equipment Availability	%
	Process Availability	%
Logistical Quantities and Quality	Throughput Rate	tph
	Equipment Effectiveness	%
	Process Effectiveness	%
	Yield of Product	%
	Quality Ratio	%
Power Consumption	Specific power	kW/tph
	Direct Power Effectiveness	%
Overall Performance	Overall Equipment Effectiveness Index	%
	Overall Process Effectiveness Index	%

To demonstrate the implementation of the KPIs, a three-stage crushing plant for aggregate production was used, see Figure 16. The physical plant was equipped with an online data-acquisition system. The mass flow during the process operation was measured by a power-based belt scale developed based on the relationship between power draw and mass flow (Hulthén and Evertsson, 2006). The various mass flow measurement points are indicated with red marking in Figure 16.

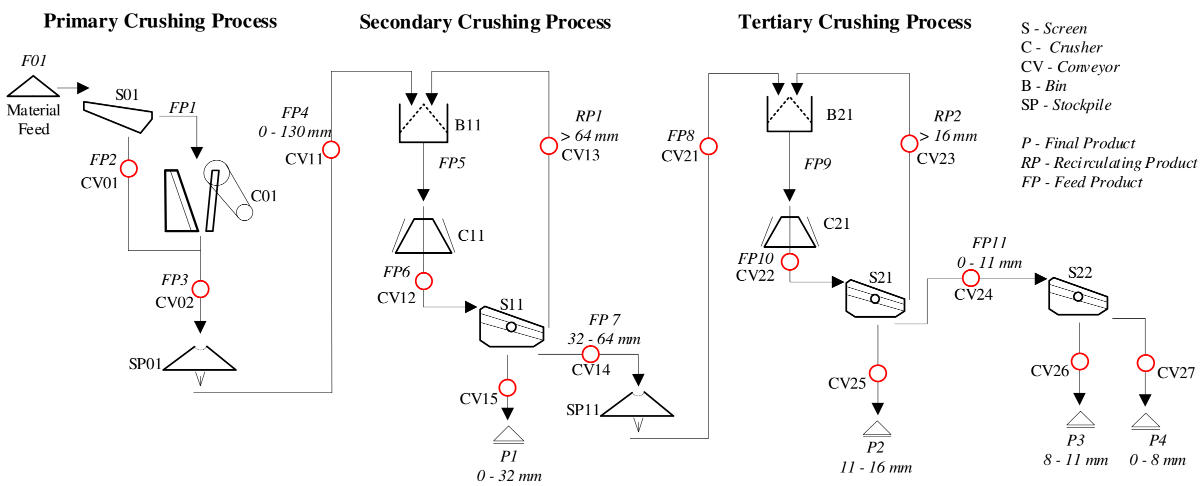


Figure 16. Crushing plant for aggregates production consisting of three sub-processes (Paper D).

An example of the KPI calculation for two crushers (*C11* and *C21*) in the aggregate production plant is shown in Figure 17 and 18. It can be noticed that the KPIs have the capability to demonstrate the start-up and shutdown sequences of the crusher operation and also, highlight performance losses happening during the operations. The KPI value drop due to unscheduled stops occurring in the crushing operation can also be seen in crusher *C11* at around the 3rd hour of the operation while in crusher *C21* at around the 8th hour of the operation. The dynamic process simulation predicts the average KPIs over a shift close to the real-time data-based KPIs as it can be seen for both the crushers, although there exists a difference at the hourly calculations. The KPIs can be applied to different time intervals (e.g. 30 min to 30 days) depending on the requirement (Paper D).

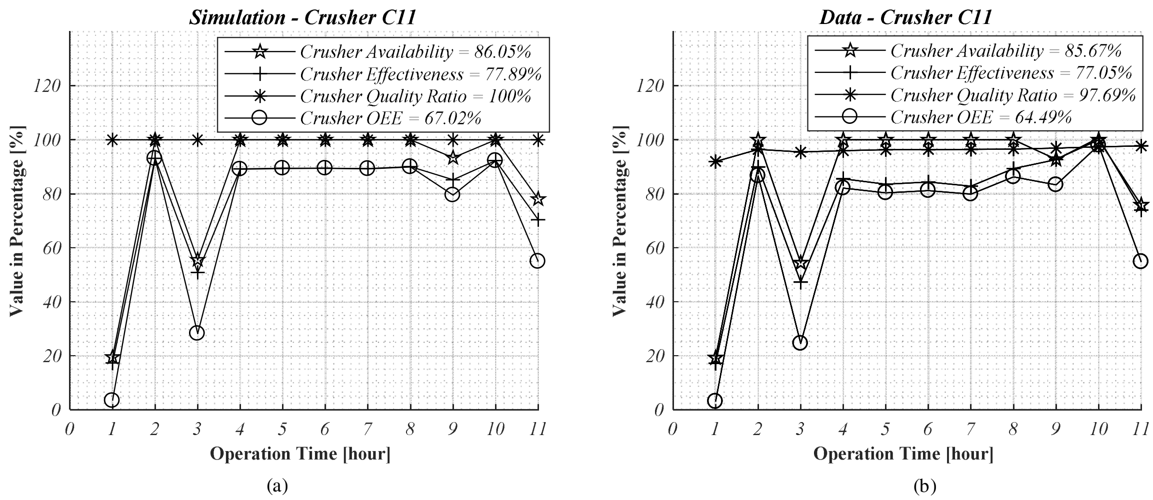


Figure 17. Crusher C11 performance based on process (a) simulation and (b) real-time data (Paper D).

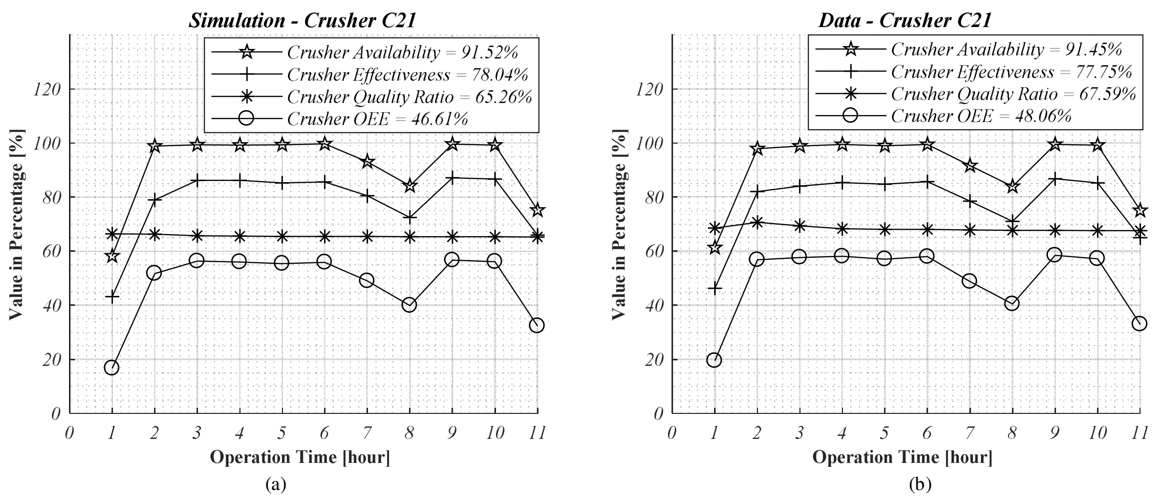


Figure 18. Crusher C21 performance based on process (a) simulation and (b) real-time data (Paper D).

A novel approach to represent an error propagation within the implementation of the KPIs for both physical and simulation system is presented in Figure 19 (Paper D). The choice of the type of equipment and process models directs the degree of fidelity of the simulation results. It is of interest to estimate the accuracy in each stage of development.

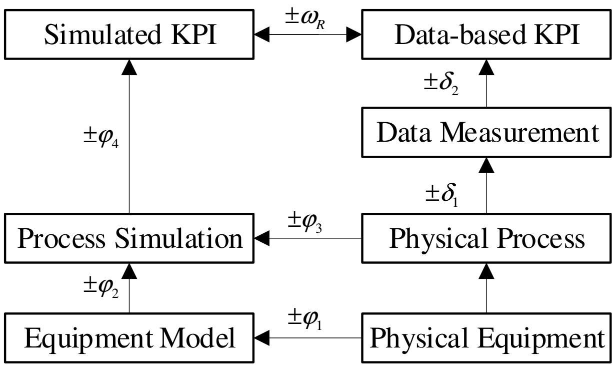


Figure 19. Error propagation in the calculation of KPI from the simulated process and real-time data (*Paper D*).

The degree of trustworthiness of the simulation results is dependent on how well the underlying mathematical models are developed and their resulting accuracy. In order to demonstrate the validity of the underlying simulation model with respect to the real-time process data, a comparison of the mass flow rates between the two was performed, see Figure 20.

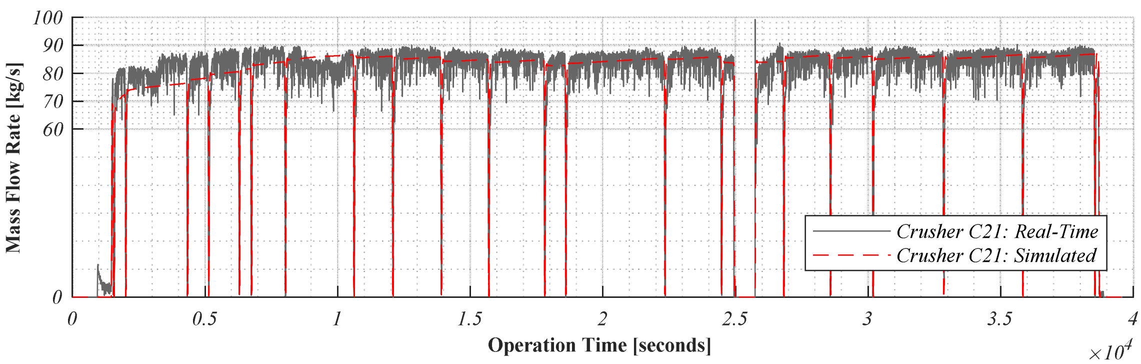


Figure 20. Comparison of the simulated and real-time mass flow rate for the crusher C21 (*Paper D*).

For the physical process data measurement, there are dependencies on the technological aspects of the data-acquisition system. The measurement of all the process output is sometimes economically or technically not feasible where the simulated process could be of potential use. Complimentary use of simulation models and real-time data are needed to make useful decisions. This is expected to increase the digitalization of the crushing and screening processes. It was shown that the KPIs results from the dynamic simulation capture the real-time effects to a degree with sufficient reliability where the decision-making process can happen. Comparison between two or more similar entities in a crushing plant can be carried out using the KPIs as a measure and further improvement opportunities can be identified.

4.3 CONCEPTUAL OPTIMIZATION FRAMEWORK

A conceptual framework for performing the optimization routine for industrial use is presented in Figure 21. The framework represents a general overview of the various sub-systems involved, and their interaction using a simple communication model. The conceptual framework has been developed over iterative studies carried out during the period of this thesis. The optimization system is hierarchically built on the development of the dynamic simulation platform and performance measuring system. The dynamic simulation platform and the data-acquisition system are based on the previous development by Evertsson (2000), Hulthén (2010) and Asbjörnsson (2015). The studies conducted in Paper A, B and C were aimed at exploring and defining the *Optimization System* while the study conducted in Paper D was directed towards *Performance Measuring System.*

The conceptual framework highlights a system-wide view of the optimization process for an industrial application. There are multiple pre-requisites for an industry operating a coarse comminution and classification process to be able to apply and utilize this framework. These pre-requisites are the availability of the configured process simulation model and the installed data acquisition system. The additional advantage would be to have a cloud-based solution for data acquisition system. The framework also inherits multiple sources of error (see Figure 19) and one needs to critically review each subsystem for the validity and reliability of the entire system.

A process improvement work can be carried out based on experience and understanding of the process. The data acquisition system together with the real-time performance calculations will help in the decision-making process and also looking into the results of the decisions. The process improvement work is iterative in nature and can be carried out by the operators and plant managers.

A process optimization work can be carried out by utilizing the optimization system using the underlying simulation system and performance measurement system. For operations purpose, the user, such as operator or plant manager, can set the goal of the optimization using either the historical performance or new requirements to find recommended operational settings. It could also be used as an explorative tool to get recommended settings for new requirements. For design purpose, the optimization can be used to explore the performance boundaries of a new process layout.

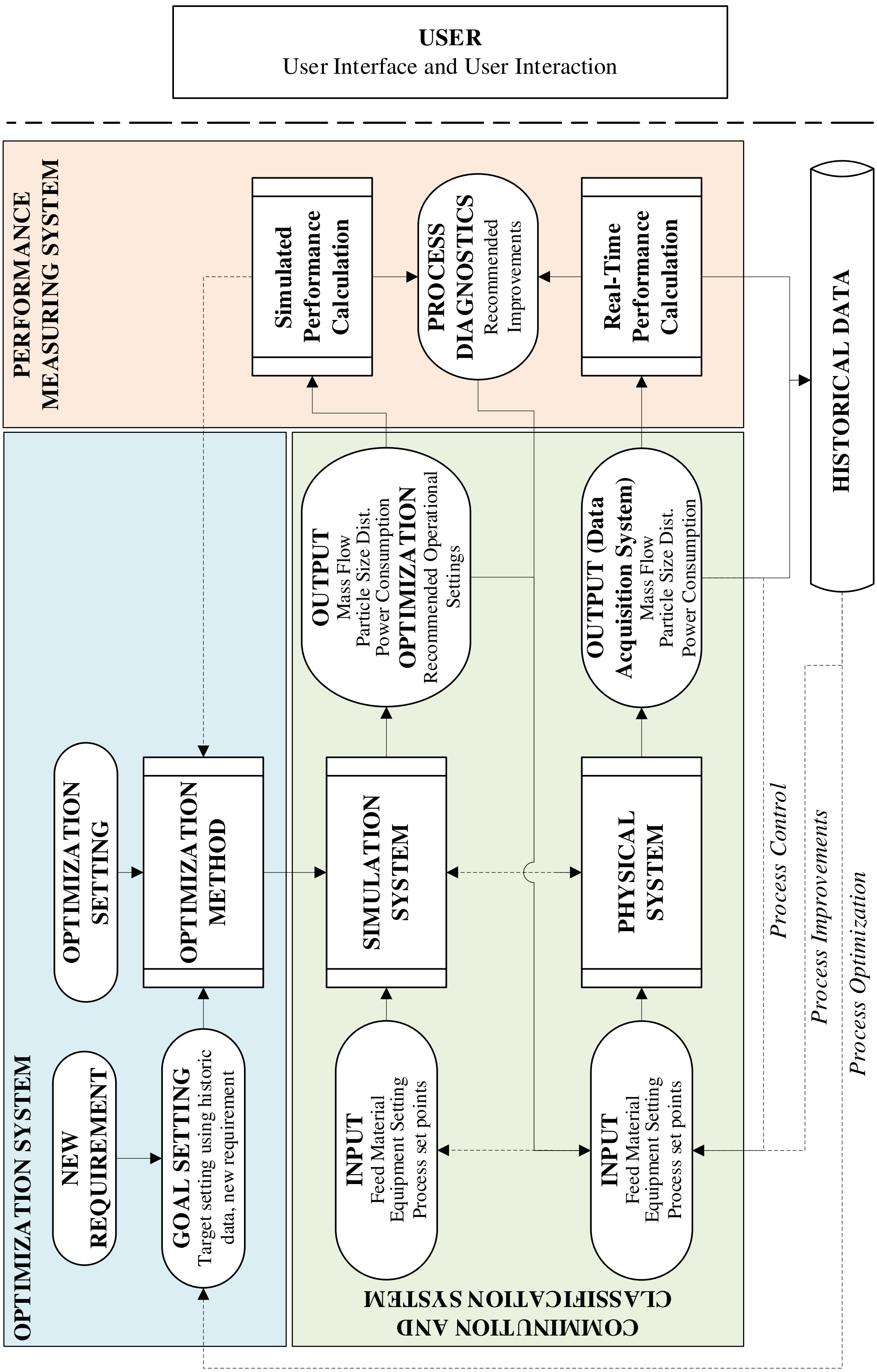


Figure 21. Conceptual optimization framework.

5 DISCUSSION & CONCLUSIONS

This chapter aims to:

- *Present and discuss the most important conclusion drawn in this thesis.*
- *Answer the research questions stated in Chapter 1.*
- *Discuss the validity of the research and future work.*

The aim of the thesis was to develop a framework for executing process optimization and process improvements in a coarse comminution and classification process using dynamic process simulations. During the iterative development work, application of two optimization methods (MDO and MOO) were defined and explored. The application of the optimization methods can generate new insights about the process performance with respect to the operating parameters and non-intuitive results. A set of key performance indicators for coarse comminution and classification processes were developed and implemented. The application of the key performance indications can be used to carry out process diagnostics and process improvement activities. The conceptual framework (see Figure 21) shows a communication model between the different systems involved in process improvements and process optimizations. Table 5 presents the contribution of the appended papers towards answering the research questions which will be discussed in the following section.

Table 5. The contribution of the appended papers towards the research questions.

Research Questions	Paper A	Paper B	Paper C	Paper D
RQ1	●	●	●	
RQ2	●	●	●	●
RQ3	●	●	●	●

5.1 ANSWERS TO RESEARCH QUESTIONS

The following answers are given to the research question stated in this thesis:

RQ 1: What optimization approach can be used to implement multi-domain optimization capability in a dynamic comminution and classification process?

Firstly, to explore the opportunities with the optimization implementation, a classification scheme to define the scope of the optimization application for the coarse comminution and classification process was developed in Paper A (see Table 2, Section 4.1.1). The classification scheme is useful for communication of the optimization applications to the research community and the industry. Thereafter, an overview of the procedure to conduct an optimization exploration for a coarse comminution and classification process using process simulation is presented based on the studies of Paper B and C (see Figure 10, Section 4.1). Depending on the scope and purpose of the optimization application, two optimization methods: Multidisciplinary Optimization (MDO) Architecture and Multi-Objective Optimization (MOO) were explored.

In paper B, two MDO architectures approach: multidiscipline feasible (MDF) and individual discipline feasible (IDF) were studied in context to a crushing plant consisting of two sub-processes. Both approaches produced reasonable results for a given operation case, although the MDF approach was found to be computationally cheaper than the IDF approach. The IDF approach was found to be better at handling larger systems compared to the MDF approach, as it is modular in the application.

In Paper C, a comparative study between the MDO and MOO methods was carried out. It was demonstrated that the optimization problem for MDO using distributed IDF approach can be decoupled into two or more levels using simple objective functions. The MOO using GA used comprehensive optimization problem formulation and is useful for exploring the solution space. The problem formulation of the different optimization methods are not completely equivalent, and one needs to be aware of this as the results of each method are based on the optimization problem formulation.

The MDO methods used in the studies were aimed to find a balance point between the various objectives of the coarse comminution and classification process while the MOO using GA method was aimed to explore the spectrum of the solution space using Pareto-front. The purpose of the optimization application directs the choice of methods. It is recommended to use more than one method at the beginning of these studies as it generates a comparison of whether the results are triangulating at similar solution points. This can further generate confidence in the optimization results. It is also recommended to check the convergence graphs for the optimization results as it further gives insights on the optimization problem formulation.

RQ 2: How can the process objectives be formulated for process improvements and process optimization in a dynamic comminution and classification process?

A process objective can be defined as a mathematical function which indicates the process performance of interest. Based on the ISO 22400 standards, a set of key performance indicators (KPIs) have been developed and implemented for a coarse comminution and classification process in Paper D. In case of process improvements, the KPIs can be used to perform diagnostics on the equipment or the process to find opportunities for incremental improvements in the considered performance criteria. The investigation of the KPIs can also highlight the dependencies between the equipment and can be potentially used for finding bottlenecks in the process.

In the case of process optimization, certain process indicators such technical function (e.g., throughput rate, power consumption) and techno-economic function (e.g., sub-process value) have been applied in Paper B and C. These functions represent the main goal for the optimization problem formulation and careful consideration is needed to use different objectives. Use of the simulation platform provides an opportunity to explore in a cost-effective way.

RQ 3: How can an optimization system be structured to perform optimization routine for a dynamic comminution and classification process?

A conceptual model for the optimization framework is presented highlighting a general overview of the use of various systems involved in performing process improvements and process optimizations (see Figure 21). Partial studies have been performed to the development work such as *Optimization System* using Paper A, B and C and *Performance Measuring System* using Paper D. The framework relies on the previous development of comminution and classification equipment model and process simulation, (Evertsson, 2000, Asbjörnsson, 2015) and data acquisition system (Hulthén, 2010). It is also highlighted that the framework inherits multiple sources of error and it is recommended to critically review the results of each sub-system against the sub-system testing criteria. The testing criteria for the optimization system are to review the convergence analysis, boundedness, feasibility, etc. For the performance measuring system, a validation of the process model results against the physical process data is an indicator for the fidelity of the system. The complete implementation of the conceptual framework towards the use for an industrial application is yet to be carried out, while partial studies have been performed at this stage.

Overall, the studies carried out in this thesis have elevated the current knowledge and understanding of the optimization methods and performance indicators for coarse comminution and classification processes. The studies have also highlighted the benefits of utilizing dynamic process simulation platform for process improvements and process optimization towards industrial benefits.

5.2 RESEARCH VALIDATION

The research conducted in this thesis is explorative and partially confirmative in nature. Multiple findings and insights about the *optimization system* and *performance measuring system* have been developed. The research within the *optimization system* is built on the theoretical understanding of the concepts from Papalambros and Wilde (2017) and previous research on optimization within comminution and classification processes (see Section 2.1). Each of the simulation study within the *optimization system* in Paper B and C was evaluated against certain testing criteria such as convergence analysis and comparison of the results by different methods. Similarly, the development of the KPIs (Paper D) is based on the ISO 22400 Standard and previous research within comminution and classification processes. In this particular research (Paper D), the results were also evaluated against the real-time data which adds additional verification (testing criteria) for the simulation implementation. These arguments support the internal validity (Bryman and Bell, 2007) of the results obtained from the studies conducted for this thesis.

The optimization methods applied in the research (Paper B and C), and the KPIs theory developed in Paper D have the external validity (Bryman and Bell, 2007) as they can be transferred to the other simulation studies within the coarse comminution and classification processes. The use of previous theoretical knowledge also supports the structural validity of the results as proposed by Pedersen et al. (2000), although the specific results and values obtained in the results were site-specific, and therefore have weak performance validity. Further use of the developed methods in a new simulation case and implementation of the results for the industrial case will add to the performance validation of the research. The comparison of different optimization methods in each simulation study was carried out at equivalent parameter settings of the underlying process simulation, thus allowing fair reliability of the comparative results obtained in Paper B and C (Myrtveit et al., 2005).

5.3 FUTURE WORK

The work presented in this licentiate thesis has mainly focused on the explorative study of various optimization methods and process improvement opportunities. Further development of the optimization system, performance measuring system and implementation of the framework using an industrial case study is planned for the future research. The aim with this is to further develop higher fidelity of the methods and provide tangible proof of the potential benefits of the use of process simulations for process improvements and process optimization.

The studies related to the robustness and reliability of optimization methods with respect to other optimization problem were not carried out at this stage and is under consideration for the future work. It was also observed in paper B and C that the optimization results using the MDO architecture methods are not robust as the solutions were dependent on the start point of the algorithm, which needs further investigations. Other potential MDO architectures and optimization methods can also be investigated in future work. The exploration of the developed KPIs for optimization problem formulation can be carried out to study a coarse comminution and classification process. The KPIs can be applied to the optimization methods to further develop insights on the process performance. The objective functions developed at this stage are directed towards the aggregates production, and the transferability of these to minerals processing in mining will be studied in future work.

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