A Simple Method for Robust Vehicular Communication with Multiple Nonideal Antennas

Keerthi Kumar Nagalapur, Erik G. Ström, Fredrik Brännström, Jan Carlsson, and Kristian Karlsson

Abstract—For critical vehicular communication services, such as traffic safety and traffic efficiency, it is advisable to design systems with robustness as the main criteria, possibly at the price of reduced peak performance and efficiency. We describe a simple, low-cost method for combining the output of L nonideal (i.e., nonisotropic) antennas to the input signal to a single-port receiver with the aim to guarantee robustness, i.e., to minimize the probability that K consecutive packets arriving from the worst-case angle-of-arrival are decoded incorrectly. To minimize complexity, the combining network does not estimate or use channel state information (complex channel gains, noise levels, etc.). The combining network consists of $L - 1$ analog phase shifters whose phases are affine functions of time. For a general L and the case when the packet error probability decays exponentially with the received SNR, the optimum slopes of the affine functions can be computed by solving an optimization problem that depends on the antenna far field functions. We provide an analytical solution for the special case of $L = 2$ antennas, which turns out to be independent of the antenna patterns. In an experimental setup consisting of two monopole antennas mounted on the roof of a Volvo XC90, the proposed combining method is shown to give significant performance gains compared to using just one of the antennas.

I. INTRODUCTION

Vehicular traffic safety and traffic efficiency applications demand robust (reliable) communication between vehicles. Many of these applications rely on that vehicles transmits periodic status messages containing current position, speed, heading, etc. These packets are referred to as cooperative awareness messages (CAMs) in Europe and basic safety messages (BSMs) in the US [1], [2]. The time between packets, T, is typically in the order of 100 ms, but can vary due to vehicle dynamics and application requirements. Occasional packet losses are normally not problematic, since the CAMs contain information of physical quantities that vary slowly over the time duration of a few packets. However, if a number of consecutive packets from a vehicle are lost, this might lead to an application failure. It is therefore reasonable to design the communication system to minimize the burst error probability

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(BrEP), i.e., the probability of losing $K > 1$ consecutive packets, where K depends on the application and T. This is in contrast to the more common design goal to minimize the packet error probability (PEP).

A shark fin antenna module located on top of a vehicle's roof is the standard method for housing the antennas used for vehicular communications today. However, conformal/hidden antennas are also being considered for the reasons of safety of the antennas, exterior appearance of the vehicle, and aerodynamics. Radiation patterns of hidden antennas are typically far from isotropic due to the vehicle components that closely surround them. In fact, the antenna patterns might have very low, or even zero gain in certain directions, and packets arriving from an unfavorable angle of arrival (AOA) might be lost due to poor signal-to-noise ratio (SNR). Moreover, since the vehicle positions varies slowly over the time duration of a few consecutive packets, we can expect the AOA of the signal from a certain vehicle to remain approximately the same over this duration, and there is a risk of losing a number of consecutive packets from the same vehicle.

The problems due to nonideal antenna patterns can be remedied by using multiple antennas with complementing radiation patterns. Combining the outputs of the multiple antennas is a well studied topic and methods such as selection combining (SC), equal gain combining (EGC), and maximal ratio combining (MRC) have been investigated thoroughly [3]. These methods either require the knowledge of the instantaneous channel amplitude and phase, or the SNR of the output signal on each antenna. Schemes that do not require the aforementioned information for combining have also been studied. A scheme called random beamforming has been explored in [4], where the antenna pattern is randomized over several time-frequency blocks to achieve omnidirectional coverage on average.

Typically, the combining methods described above require a multiport receiver (RX) to combine the signals digitally. An alternative to this approach is to use an analog combining network (ACN) consisting of analog phase shifters, variable gain amplifiers, and combiners to obtain a single combined signal that requires only a single-port RX [5], [6]. When the antennas and the RX are co-located, it is convenient to use a closed loop system where the information from the RX is used to control the analog combining network. However, from a modularity and implementation complexity point of view, it would be beneficial to devise an ACN that does not require RX feedback or knowledge of the SNR or other channel state

information (CSI).

In this paper, we will present a general framework for designing an ACN to combine the outputs of L antennas to a signal that is fed to a single-port receiver. The ACN is optimized to minimize the BrEP from the worst-case AOA without requiring RX feedback or channel state information. Interestingly enough, it turns out that the design does not require knowledge of the individual antenna patterns. For the special case of $L = 2$, we present close-form results for the ACN design and compare the proposed method with standard combining methods for a setup with two monopole antennas placed on roof of a Volvo XC90. The proposed combining method gives significant performance gain compared to using just one of the antennas. However, SC, EGC, and MRC perform even better, but at the price of higher receiver complexity.

II. System Model

Consider $L \geq 2$ antennas located on a vehicle at the same height over ground. For simplicity, we assume that the antennas are vertically polarized and that the incident electrical field is also vertically polarized and arriving in the azimuth plane. We can then characterize the lth antenna with its far-field function $g_l(\phi)$, $l = 1, 2, \ldots, L$, where ϕ is the azimuth angle. The far-field function is normalized such that $|g_l(\phi)|^2$ represents the relative directive gain of the lth antenna with respect to an isotropic antenna.

We assume that a single multipath component is impinging on the antenna configuration and that the delay spread is negligible[1]. This is a reasonable model for highway environments, which typically have a dominating line-of-sight component or a few strong scatterers. Hence, relatively few multipath components contribute to the received power. Considering the $l = 0$ antenna as the reference, the complex channel gain at the lth antenna can be written as [7, Eqn. 8]

$$h_l(t) = a(t)g_l(\phi)e^{-J\Omega_l},$$

where $a(t)$ is the complex gain to antenna 0 and Ω_l is the relative phase difference experienced by lth antenna with respect to the reference antenna (which depends on the physical placement of the antennas and the angle-of-arrival). Note that ϕ and Ω_l can be considered to be time-invariant as long as changes in vehicle positions and propagation environment can be neglected. In fact, we assume that this will be the case for the time it takes to transmit K packets, i.e., for KT seconds, where T is the time between consecutive packets.

The signal at the output of the lth antenna is given by

$$r_l(t) = s(t)h_l(t) + n_l(t), \qquad (1)$$

where $s(t)$ is the transmitted signal and $n_l(t)$ for $l = 0, 1, \ldots, L-1$ are independent identically distributed complex additive white Gaussian noise (AWGN) processes with power $\mathbb{E}\{|n_l(t)|^2\} = P_n$ over the bandwidth of the signal $s(t)$. We restrict the ACN to consist of analog phase shifters and an

[1]The delay spread assumption can be relaxed without invalidating the derivation—we simply interpret $s(t)$ in (1) as the convolution of the transmitted signal and the channel impulse response to the reference antenna.
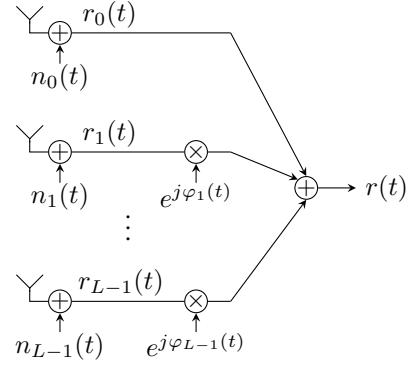

Fig. 1. The analog combining network with L antennas.

adder as seen in Fig. 1. The outputs of the $L-1$ antennas are phase rotated and added to the output of the reference antenna. The output of the combiner $r(t)$ is given by

$$r(t) = \sum_{l=0}^{L-1} r_l(t)e^{J\varphi_l(t)},$$

where $\varphi_l(t)$ is the time-varying phase shift applied to the lth antenna output and $\varphi_0(t) = 0$. For simplicity, we let

$$\varphi_l(t) = \alpha_l t + \beta_l, \qquad l = 0, 1, \ldots, L-1.$$

Hence, α_l is the phase slope and β_l is the phase offset of the lth phase shifter, and $\alpha_0 = \beta_0 = 0$ (since $\varphi_0(t) = 0$). The output of the ACN is

$$r(t) = s(t)a(t)\underbrace{\sum_{l=0}^{L-1} g_l(\phi)e^{-J(\Omega_l - \alpha_l t - \beta_l)}}_{g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, t)} + \sum_{l=0}^{L-1} \tilde{n}_l(t),$$

where $g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, t)$ is the effective time-varying far-field function, $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_{L-1}]^{\mathsf{T}}$, $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_{L-1}]^{\mathsf{T}}$, and $\tilde{n}_l(t) = n_l(t)e^{J\phi_l(t)}$ has the same distribution as $n_l(t)$ since $n_l(t)$ is circularly symmetric.

When the phase shift, $\varphi_l(t)$, over a packet duration is negligible, $g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, t)$ remains approximately constant over the duration of a packet, and the effective far-field function during the kth packet can be approximated to be $g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, kT)$. Consequently, the average SNR of the kth packet is given by

$$\bar{\gamma}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, k) = \frac{\mathbb{E}\left\{|a(t)s(t)|^2\right\}|g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, kT)|^2}{\mathbb{E}\left\{\left|\sum_{l=0}^{L-1} \tilde{n}_l(t)\right|^2\right\}}$$

$$= \frac{P_r}{LP_n}|g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, kT)|^2,$$

where P_r is the received signal power from an ideal isotropic antenna.

The packet error probability $P_e(\bar{\gamma})$ is a function of the average SNR that depends on modulation, coding, packet

length, and channel characteristics. For simplicity, we model the PEP function as

$$P_{\text{e}}(\bar{\gamma}) = a \exp(-b\bar{\gamma}), \qquad (2)$$

where $a, b > 0$ are constants. As mentioned earlier, we intend to minimize the probability of having a burst of K consecutive packet errors. Assuming that packet error events are statistical independent[2], the BrEP is

$$P_{\text{B}}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, K) = \prod_{k=0}^{K-1} P_{\text{e}}(\bar{\gamma}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, k))$$
$$= \prod_{k=0}^{K-1} a e^{-b\bar{\gamma}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, k)}.$$

Since we want to determine the optimum $\boldsymbol{\alpha}$ that minimizes the BrEP for the worst-case AOA $\phi \in [0, 2\pi)$ and worst-case initial offset $\beta_l \in [0, 2\pi)$, the optimum $\boldsymbol{\alpha}$ is found as

$$\boldsymbol{\alpha}^{\star} = \arg\inf_{\{\alpha_l\} \in \mathbb{R}} \sup_{\phi, \{\beta_l\} \in [0, 2\pi)} P_{\text{B}}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, K)$$
$$= \arg\inf_{\{\alpha_l\} \in \mathbb{R}} \sup_{\phi, \{\beta_l\} \in [0, 2\pi)} \ln P_{\text{B}}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, K)$$
$$= \arg\sup_{\{\alpha_l\} \in \mathbb{R}} \inf_{\phi, \{\beta_l\} \in [0, 2\pi)} \sum_{k=0}^{K-1} \bar{\gamma}(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, k)$$
$$= \arg\sup_{\{\alpha_l\} \in \mathbb{R}} \inf_{\phi, \{\beta_l\} \in [0, 2\pi)} \sum_{k=0}^{K-1} \frac{P_{\text{r}}}{LP_{\text{n}}} |g(\phi, \boldsymbol{\alpha}, \boldsymbol{\beta}, kT)|^2$$
$$= \arg\sup_{\{\alpha_l\} \in \mathbb{R}} \inf_{\phi, \{\beta_l\} \in [0, 2\pi)} \sum_{k=0}^{K-1} \left| \sum_{l=0}^{L-1} g_l(\phi) e^{-\jmath(\Omega_l - \alpha_l kT - \beta_l)} \right|^2 .$$

Let $\psi_l = \mod(\Omega_l - \beta_l - \angle g_l(\phi), 2\pi)$, where $\mod(u, v)$ is the remainder after dividing u by v. Now, varying β_l in the interval $[0, 2\pi)$ for fixed Ω_l and $\angle g_l(\phi)$ implies that ψ_l takes on all values in $[0, 2\pi)$. Hence, the optimization problem can be rewritten as

$$\boldsymbol{\alpha}^{\star} = \arg\sup_{\{\alpha_l\} \in \mathbb{R}} \inf_{\phi, \{\psi_l\} \in [0, 2\pi)} \underbrace{\sum_{k=0}^{K-1} \left| \sum_{l=0}^{L-1} |g_l(\phi)| e^{-\jmath(\psi_l - \alpha_l kT)} \right|^2}_{J(\phi, \boldsymbol{\alpha}, \boldsymbol{\psi}, K)} . \qquad (3)$$

For $L = 2$, a closed-form solution to (3) is given by Theorem 1 below. Due to space constraints, we postpone a treatment of the general case, $L > 2$, to an upcoming journal paper.

Theorem 1. Let $J(\phi, \boldsymbol{\alpha}, \boldsymbol{\psi}, K)$ be defined as in (3). Then, for an arbitrary ϕ and for $K \geq L = 2$,

$$J^{\star}(\phi) \triangleq \sup_{\alpha_1} \inf_{\psi_1} J(\phi, \alpha_1, \psi_1, K) = K(|g_0(\phi)|^2 + |g_1(\phi)|^2)$$

Moreover, the optimum is obtained for

$$\alpha_1 = \alpha_1^{\star} \triangleq \frac{2\pi}{KT}.$$

[2]Error events are independent if noise is independent from packet to packet, which is a standard assumption, and if any small-scale fading is also independent from packet to packet, i.e., when the coherence time is small compared to T, which typically is the case for highway mobility.

Proof: See Appendix. ∎

We note, somewhat surprising, that $\alpha_1^{\star}$ does not depend on the antenna patterns. Moreover, as shown in the appendix, $J(\phi, \alpha_1^{\star}, \psi_1) = K(|g_0(\phi)|^2 + |g_1(\phi)|^2)$, which is independent of ψ_1 and, therefore, independent of β_1. Hence, for any initial offset β_1, the worst-case AOA ϕ, i.e., the AOA that results in the highest BrEP is given by

$$\phi^{\star} = \arg\min_{\phi \in [0, 2\pi)} |g_0(\phi)|^2 + |g_1(\phi)|^2 .$$

As a consequence, when the proposed combining scheme is used, the antennas should be designed and oriented such that $|g_0(\phi^{\star})|^2 + |g_1(\phi^{\star})|^2$ is maximized.

III. COMPARISON WITH STANDARD SCHEMES

In this section, the performance of the proposed combining scheme is compared with a few standard combining schemes. In the case when the PEP function is exponential as in (2), minimizing the BrEP is equivalent to maximizing the sum of the average SNRs of the K packets. Therefore, the sum of SNRs is used as a performance criterion to compare the performance of the combining schemes.

1) Single antenna: the sum of average SNRs at the output of the lth antenna is given by

$$\rho_l(\phi) = \sum_{k=0}^{K-1} \bar{\gamma}_l(\phi, k) = \frac{KP_{\text{r}}}{P_{\text{n}}} |g_l(\phi)|^2 .$$

For an ideal isotropic antenna, the sum of average SNRs is given by $\rho_{\text{ISO}}(\phi) = KP_{\text{r}}/P_{\text{n}}$.

2) MRC: this scheme requires L RF-chains, L analog to digital converters (ADCs), and a multi-port receiver that estimates the complex-valued channel gains and performs combining digitally. The sum of average SNRs is given by

$$\rho_{\text{MRC}}(\phi) = \sum_{k=0}^{K-1} \bar{\gamma}_{\text{MRC}}(\phi, k) = \frac{KP_{\text{r}}}{P_{\text{n}}} \left(\sum_{l=0}^{L-1} |g_l(\phi)|^2 \right) .$$

3) EGC: this scheme requires L RF-chains, L ADCs, and a multi-port receiver that estimates the channel phases and performs combining digitally. The sum of average SNRs is given by

$$\rho_{\text{EGC}}(\phi) = \sum_{k=0}^{K-1} \bar{\gamma}_{\text{EGC}}(\phi, k) = \frac{KP_{\text{r}}}{LP_{\text{n}}} \left(\sum_{l=0}^{L-1} |g_l(\phi)| \right)^2 .$$

4) SC: this scheme requires L RF-chains and circuitry to measure the SNRs on each antenna and to select an antenna. The sum of average SNRs is given by

$$\rho_{\text{SC}}(\phi) = \sum_{k=0}^{K-1} \bar{\gamma}_{\text{SC}}(\phi, k) = \frac{KP_{\text{r}}}{P_{\text{n}}} \max_l |g_l(\phi)|^2 .$$

5) ACN: the proposed scheme requires analog phase shifters on $L-1$ branches operating independently and a combiner. For $L = 2$ and $\alpha_1 = \alpha_1^{\star}$, the sum of average SNRs is

$$\rho_{\text{ACN}}(\phi) = \frac{KP_{\text{r}}}{2P_{\text{n}}} (|g_0(\phi)|^2 + |g_1(\phi)|^2) .$$
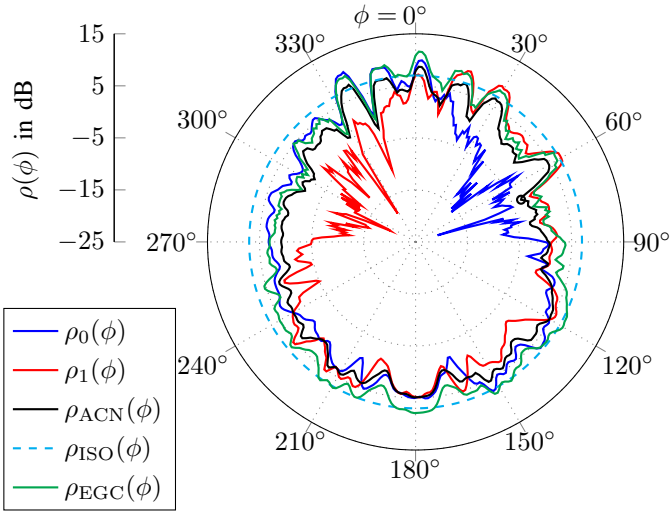
Fig. 2. Antenna placement on the roof of a Volvo XC90.

Fig. 4. BrEP as a function of AOA ϕ for the individual antennas and the combined output when $K = 5$, $\alpha_1 = \alpha_1^\star = 2\pi/(KT)$.

IV. NUMERICAL RESULTS

In this section, the performance of the ACN is studied by using the measured antenna far-field functions of two monopole antennas placed $0.8\,\text{m}$ from each other on the roof of a Volvo XC90, see Fig. 2. The sum SNRs $\rho_0(\phi)$ and $\rho_1(\phi)$ for $P_\text{r}/P_\text{n} = 1$ and $K = 5$ are shown in Fig. 3. As seen in the figure, both antennas exhibit very low $\rho(\phi)$ at certain, but different, AOAs. Clearly, if only one of the two antennas is used, the packets arriving in the AOAs of low $\rho(\phi)$ will have high BrEP. However, by combining the output of the antennas with the proposed ACN, the BrEP is reduced compared to using the antenna with the lowest gain (since $\rho_\text{ACN}(\phi) \geq \min_l \rho_l(\phi)$). The sum of average SNRs in the case of a single isotropic antenna and in the case of the measured antennas combined using EGC are also shown in the figure. The plots corresponding to MRC and SC have been omitted. However, they are related to the plots in the figure as $\rho_\text{MRC}(\phi) = 2\rho_\text{ACN}(\phi)$ and $\rho_\text{SC}(\phi) = \max\{\rho_0(\phi), \rho_1(\phi)\}$. As expected, the more advanced combining methods perform better than the proposed ACN.

Fig. 4 shows the BrEP as a function of AOA for the individual antennas and the ACN. The exponential PEP function $P_\text{e}(\bar{\gamma}) = \exp(-\bar{\gamma}/5)$ is considered and $P_\text{r}/P_\text{n} = 10\,\text{dB}$ is used. It is seen that the BrEP in the case of the individual antennas is very close to 1 for the AOAs that have very low $\rho_l(\phi)$ (see Fig. 3). The BrEP for the AOAs corresponding to low gains in one of the two antennas is reduced by the ACN. The BrEP for certain AOAs when using the ACN is higher in comparison to one of the individual antennas, this is expected as the ACN operates without the knowledge of branch SNRs and the complex-valued channel gains. The figure also shows the BrEP in the case of a single isotropic antenna and in the case of the measured antennas combined using EGC, the BrEP in these cases is in agreement with their $\rho(\phi)$ in Fig. 3.

Fig. 3. $\rho(\phi)$ of the two monopoles mounted on the roof of a vehicle and the ACN. The monopoles exhibit nonisotropic power gains. $P_\text{r}/P_\text{n} = 1$ and $K = 5$.

We note that the sum of average SNRs for MRC and ACN for $L = 2$ are related as $\rho_\text{MRC}(\phi) = 2\rho_\text{ACN}(\phi)$.

The MRC scheme outperforms EGC, SC, and ACN for any far-field functions $g_l(\phi)$. The relative performance of SC and EGC for an AOA ϕ depends on the far-field functions $g_l(\phi)$. The sum of average SNRs of MRC, EGC, and SC schemes is higher compared to the ACN scheme, implying lower BrEP. However, these gains come at a cost of additional hardware and/or signal processing as mentioned above.
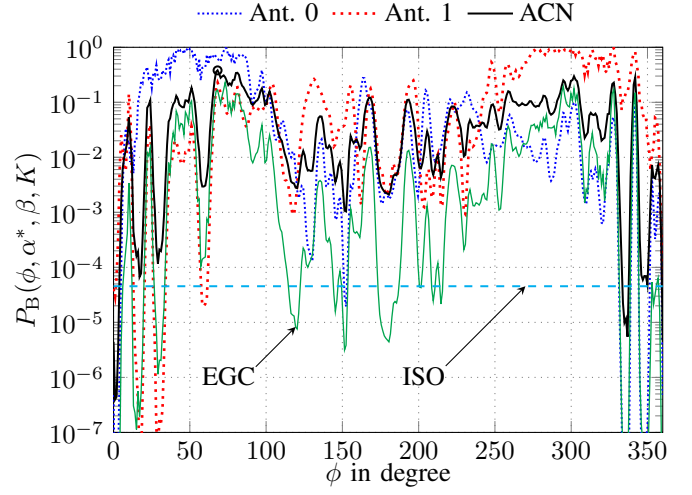
V. Conclusions

In this paper, we have proposed to use a simple analog combining network consisting of $L-1$ phase shifters to combine the outputs of L nonisotropic antennas to the input of a single-port receiver with the goal to minimize the burst-error probability, i.e., the probability of K consecutive packet errors, for the worst-case angle of arrival. The combining network is defined by the offset and slope of the phase shifters and does not require knowledge of the channel state (SNR, fading statistics, etc.). For a general L and when the PEP decays exponentially with the received SNR, the optimum phase slopes can be found by solving the optimization problem (3). For $L=2$, the combining network contains a single phase shifter, and the optimum phase slope is proven to be $\alpha_1^\star = 2\pi/(KT)$, where T is the time between consecutive packets. Somewhat surprising, this result is valid for all initial phase offsets and antenna patterns and will improve the BrEP for all AOAs compared to using the worst antenna. Hence, we cannot further improve performance by optimizing the initial phase offset β_1. However, the BrEP for AOA ϕ is minimized by maximizing $|g_0(\phi)|^2 + |g_1(\phi)|^2$.

The proposed scheme was evaluated using the measured far-field functions of two monopole antennas placed on the roof of a Volvo XC90. It was shown that the proposed scheme gives significant performance gains by combining the antenna outputs compared to just using one of the antennas. The standard MRC, EGC, and SC combining schemes perform even better, but will also require channel state information and more complex receiver circuitry.

Appendix

To set up the proof for Theorem 1, we start by proving two lemmas. Define the function $f : \mathbb{R}^2 \to \mathbb{R}$ as

$$f(x,y) \triangleq \sum_{k=0}^{K-1} \cos(y - k2x), \qquad (4)$$

where $K > 1$ is a positive integer. It can be shown that

$$f(x,y) = \begin{cases} K\cos(y), & x \in \mathcal{X} \\ \dfrac{\sin(Kx)}{\sin(x)}\cos(y-(K-1)x), & x \notin \mathcal{X} \end{cases} \qquad (5)$$

where

$$\mathcal{X} \triangleq \{q\pi : q \in \mathbb{Z}\}. \qquad (6)$$

Lemma 1. Let f and $\mathcal{X}$ be as defined in (4) and (6), respectively. Then,

$$f(x,y) = 0, \qquad x \in \mathcal{X}^\star, y \in \mathbb{R},$$

where

$$\mathcal{X}^\star \triangleq \{q\pi/K : q \in \mathbb{Z}\} \setminus \mathcal{X}. \qquad (7)$$

Proof: If $x \in \mathcal{X}^\star$ then $x \notin \mathcal{X}$, then (5) implies that

$$f(x,y) = \frac{\sin(Kx)}{\sin(x)}\cos(y-(K-1)x), \qquad x \in \mathcal{X}^\star.$$

The lemma follows since $\sin(Kx)/\sin(x) = 0$ iff $x \in \mathcal{X}^\star$. ∎

Lemma 2. Let f and $\mathcal{X}^\star$ be as defined in (4) and (7), respectively. Then,

$$\inf_{y \in \mathbb{R}} f(x,y) \le 0$$

where equality is achieved iff $x \in \mathcal{X}^\star$.

Proof: From Lemma 1, we know that $f(x,y) = 0$ for $x \in \mathcal{X}^\star$ and all $y \in \mathbb{R}$. To show the lemma, it is therefore sufficient to show that $\inf_y f(x,y) < 0$ for $x \notin \mathcal{X}^\star$. We split up the condition $x \notin \mathcal{X}^\star$ into two cases, (a) $x \in \mathcal{X}$ and (b) $x \in \mathcal{X}' \triangleq \{\mathbb{R} \bigcap \mathcal{X} \bigcap \mathcal{X}^\star\}$. For case (a): $f(x,y) = K\cos(y)$ and $\inf_y f(x,y) = -K < 0$. For case (b):

$$f(x,y) = \underbrace{\frac{\sin(Kx)}{\sin(x)}}_{\ne 0, \forall x \in \mathcal{X}'} \cos(y - (K-1)x).$$

Clearly, there exist a y such that $\cos(y - (K-1)x) = -\operatorname{sgn}[\sin(Kx)/\sin(x)]$, which implies that $\inf_y f(x,y) = -|\sin(Kx)/\sin(x)| < 0$. Hence, the lemma follows. ∎

We are now ready to prove Theorem 1.

Proof: It is easily shown that the objective in (3) for $L=2$ can be written as

$$J(\phi, \alpha_1, \psi_s, K) = K(|g_0(\phi)|^2 + |g_1(\phi)|^2) + 2|g_0(\phi)||g_1(\phi)|f(x,y),$$

where $x = \alpha_1 T/2$ and $y = \psi_0 - \psi_1$. From Lemma 2, we have that $\sup_x \inf_y f(x,y) = 0$. Hence,

$$\begin{aligned} J^\star(\phi) &= \sup_x \inf_y \big\{ K(|g_0(\phi)|^2 + |g_1(\phi)|^2) \\ &\qquad + 2|g_0(\phi)||g_1(\phi)|f(x,y) \big\} \\ &= K(|g_0(\phi)|^2 + |g_1(\phi)|^2). \end{aligned}$$

Moreover, from Lemma 2, we know that the supremum is achieved for $x \in \mathcal{X}^\star$, i.e., for $\alpha_1 \in \{2x/T : x \in \mathcal{X}^\star\}$. The smallest nonzero optimum phase slope is therefore $\alpha_1^\star = 2\pi/(KT)$, and the theorem follows. ∎

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