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## Predicting Battery Aging Trajectory via a Migrated Aging Model and Bayesian Monte Carlo Method

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### Abstract

Thanks to the fast development in battery technologies, the lifespan of the lithium-ion batteries increases to more than 3000 cycles. This brings new challenges to reliability related researches because the experimental time becomes overly long. In response, a migrated battery aging model is proposed to predict the battery aging trajectory. The normal-speed aging model is established based on the accelerate aging model through a migration process, whose migration factors are determined through the Bayesian Monte Carlo method and the stratified resampling technique. Experimental results show that the root-mean-square-error of the predicted aging trajectory is limited within 1% when using only 25% of the cyclic aging data for training. The proposed method is suitable for both offline prediction of battery lifespan and online prediction of the remaining useful life.

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*Keywords:* Lithium-ion batteries; Model migration; Bayesian Monte Carlo; Aging trajectory prediction; State-of-health;

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## 1. Introduction

### 1.1. Motivation and challenges

Thanks to the development of battery technologies, lithium-ion batteries become more and more reliable [1], with their lifespan up to 3000 cycles [2]. The long cyclic aging time largely benefits consumers but is a nuisance to researchers or engineers working on battery reliability. For a large variety of battery related products, such as electric vehicles (EV) [3] or uninterruptable power supply (UPS) [4], a warranty for their reliable operation lasting for certain years is compulsory. To do so, the battery lifespan under various load conditions must be ascertained. However, this task is challenging for at least three reasons:

- Lack of effective aging models: Since SONY commercialized the first lithium-ion (Li-ion) battery in 1991, a number of battery types have been commercially available, commonly different in their chemical components. Their aging processes can be influenced by multiple factors such as temperature, current rate, cut-off voltage, pulse current and storage time [5, 6]. Building up an effective physics-based aging model suitable for different Li-ion batteries under various conditions is a significant but very tough task;
- Lack of experimental data: A powerful alternative is to model the aging process using data-driven approaches, which, however, requires a large amount of experimental data;
- Long experimental time: Experiments to generate aging data, especially under normal operating conditions, are costly and time-consuming.

### 1.2. Literature review

There are mainly two approaches to acquire the battery aging behavior quickly, i.e. prediction-based and acceleration-based. Prediction-based approaches leverage partial experimental data to predict the whole aging trajectory, where it is unnecessary to test the battery over its entire lifespan. The prediction can be realized by algorithms [7], such as Extended Kalman filter (EKF) [8], Bayesian prediction [9] and neural network [10]. A common assumption underpinning this kind of methods is that the battery aging process maintains the same over its entire lifespan. Based on this assumption, the historic data is used to predict the future performance. However, the aging trajectory of some Li-ion batteries may contain a turning point, the profiles before and after which are much different (see Fig. 2). This phenomenon implies the underlying assumption of the prediction-based approaches are not always valid.

Accelerate aging associated with stress factors is another approach to shorten the experimental time for battery state of health (SOH) studies. Although aging mechanisms of the normal-speed aging and accelerate aging can be different, attempts have been made to explore these differences on certain battery types, see Refs [11, 12]. However, it should be highlighted that the SOH evolution characteristics appeared under normal conditions can be submerged under accelerate aging tests. Then, prediction deviations are inevitable if evaluating a battery's lifespan based on the results of the accelerate aging test only.

### 1.3. Contributions

The contributions of this paper are twofold: a novel migrated normal-speed aging model is established on the accelerate aging model. Then, the Bayesian Monte Carlo method is employed to predict the aging trajectory of the normal-speed aging.

## 2. Migrated Aging Models

According to the model migration principle [13], if an old process, also known as the base process, has been studied in detail resulting in a based model, then the obtained base model can be used to model a similar new process with fewer data, compared with directly modelling the new process. As a result, a new model requiring a few amounts of data is established. A general diagram of mode migration is shown in Fig. 1-(a).

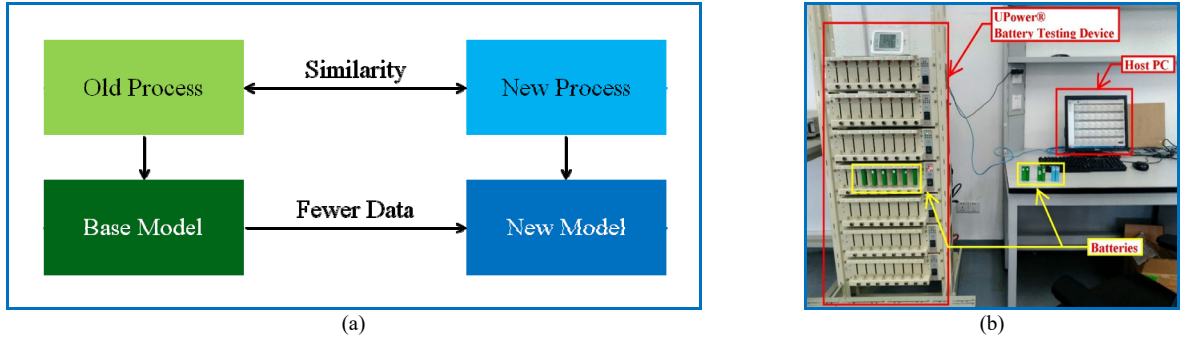


Fig. 1. (a): Illustration of model migration method; (b): Experimental platform.

Because obtaining aging data with stress factors is much faster, the accelerate aging process is selected for building the base model, while the normal-speed aging process is regarded as the “new process”.

Before the development of both models, the SOH is defined to evaluate the degree of battery aging [14]:

$$SOH(k) = C_n(k) / C_{n0} \tag{1}$$

where  $C_n(k)$  is the actual battery capacity at time  $k$  (usually represented in cycle) under the manufacturer specified temperature (usually 25°C), while  $C_{n0}$  is the fresh cell capacity under this temperature.

Then, the accelerate aging model describing the relationship between SOH and time can be formulated in the following exponential form [15]:

$$SOH_{acc}(k) = f_{acc}(k) = a \cdot e^{b \cdot k} + c \cdot e^{d \cdot k} \tag{2}$$

where the subscript *acc* means that the model is describing the accelerating aging process.  $a, b, c$  and  $d$  are model parameters which can be estimated using nonlinear least-square method. Since the accelerate aging model is selected as the base model, the above-mentioned factors are assumed to be offline determined accurately.

Based on the key idea of model migration, we can develop a migration between the normal-speed aging model and accelerate aging model:

$$SOH_{ns}(k) = f_{ns}(k) = x_1 \cdot f_{acc}(x_2 \cdot k + x_3) + x_4 \tag{3}$$

Where the subscript *ns* means that the model is describing the normal-speed aging process.  $\bar{X} = [x_1, x_2, x_3, x_4]$  is the factor for model migration between the accelerate aging model and normal-speed aging model. They represent the re-arrangement of the amplitude, time scale, time shift and bias, respectively.

### 3. Bayesian Monte Carlo prediction for battery aging trajectory prediction

#### 3.1. Bayesian Monte Carlo method

To open-loop predict the aging trajectory of the normal-speed aging, we need to accurately estimate the vector  $\bar{X}$ . In this paper, we assume that the vector  $\bar{X}$  and the error of the migrated model are subject to the Gaussian distribution:

$$\bar{X}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} = \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \\ x_4(k-1) \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \overline{X(k-1)} + \bar{\Omega}; \tag{4}$$

$$\begin{aligned} \omega_1 &\sim N(0, \sigma_1) \\ \omega_2 &\sim N(0, \sigma_2) \\ \omega_3 &\sim N(0, \sigma_3) \\ \omega_4 &\sim N(0, \sigma_4) \end{aligned}$$

$$SOH_{ns}(k) = f_{ns}(k) + v = x_1(k) \cdot f_{acc}(x_2(k) \cdot k + x_3(k)) + x_4(k) + v; \quad v \in N(0, \sigma_5) \tag{5}$$

The goal is to estimate the probability distribution:  $P(\bar{X}(k) | SOH(1:k))$  given the historical SOH measurements:  $SOH(1:k) = [SOH(1), SOH(2), \dots, SOH(k)]$ . In the Bayesian framework [16], this can be implemented using two steps: one-step prediction and state update. At cycle  $k-1$ , we have  $P(\bar{X}(k-1) | SOH(1:k-1))$ , and a one-step prediction is given by:

$$P(\overline{X}(k) | SOH(1:k-1)) = \int P(\overline{X}(k) | \overline{X}(k-1)) \cdot P(\overline{X}(k-1) | SOH(1:k-1)) \cdot d\overline{X}(k-1) \tag{6}$$

At cycle  $k$ , with a new observation  $SOH(k)$ , the posterior distribution of  $\overline{X}(k)$  can be calculated based on Bayes' rule:

$$P(\overline{X}(k) | SOH(1:k)) = \frac{P(\overline{X}(k) | SOH(1:k-1)) \cdot P(SOH(k) | \overline{X}(k))}{\int P(\overline{X}(k) | SOH(1:k-1)) \cdot P(SOH(k) | \overline{X}(k)) \cdot d\overline{X}(k)} \tag{7}$$

It should be pointed out that the high-dimensional integrals are very difficult to calculate, and Monte Carlo (MC) method is introduced to represent the probability density function by a set of random samples:

$$P(\overline{X}(k) | SOH(1:k)) \approx \sum_{i=1}^{N_s} w^i(k) \cdot \delta(\overline{X}(k) - \overline{X}^i(k)) \tag{8}$$

where  $\overline{X}^i(k)$  is a set of random independent samples drawn from  $P(\overline{X}(k) | SOH(1:k))$ ,  $w^i(k)$  is the Bayesian importance weight associated with each  $\overline{X}^i(k)$ ,  $\delta(\cdot)$  is the Dirac delta function, and  $N_s$  is the size of the random samples. In general cases,  $P(\overline{X}(k) | SOH(1:k))$  is usually unknown, and in response, one can sample  $\overline{X}^i(k)$  from an arbitrarily distribution (called the importance function). If this importance function is selected as  $P(\overline{X}^i(k) | \overline{X}^i(k-1))$ , we can have the following recursive updating law to update the weight [17]:

$$w^j(k) = w^j(k-1) \cdot P(SOH(k) | \overline{X}^j(k)) = w^j(k-1) \cdot \frac{1}{\sqrt{2\pi}\sigma_s} \cdot \exp\left(-\frac{(SOH(k) - f_{ns}^i(k))^2}{2\sigma_s^2}\right) \tag{9}$$

where  $f_{ns}^i(k)$  is used to calculate the estimated SOH with migration factor  $\overline{X}^i(k)$ , and the weight should be normalized by:

$$w^j(k) = w^j(k) \cdot \left[ \sum_{j=1}^{N_s} w^j(k) \right]^{-1} \tag{10}$$

When the number of samples  $N_s \rightarrow \infty$ , Eq. (8) converges to its true posterior density.

Now, the h-step ahead prediction of each trajectory can be calculated by  $f_{ns}^i(k+h)$ , and the mathematical expectation can be calculated by:

$$E[SOH_{ns}(k+h)] = \sum_{j=1}^{N_s} w^j(k) \cdot f_{ns}^j(k+h) \tag{11}$$

### 3.2. Stratified Resampling

Particle degradation is a main problem of the BMC based methods, in other words, at the end of a BMC simulation, only one or two particles can have a significant weight, while the weights of the other particles are close to zero. Resampling is believed to be an important approach to solve the particle degradation problem and many different forms of resampling method is proposed. The basic idea of resampling is to resample the particles with the higher weight, while throw away those with lower weight. Amongst the different types of resampling methods [18], stratified resampling method is a relative simple and efficient method with complexity  $O(N_s)$ . There are two steps for the stratified resampling:

1. Generate  $N_s$  random number:  $\widetilde{u}_k, k \in \{1, 2, \dots, N_s\}$

$$\widetilde{u}_k = \frac{(k-1) + u_k}{N_s}, u_k \in U[0,1); k = 1, 2, \dots, N_s \tag{12}$$

2. The sample  $\overline{X}^i(k)$  and the corresponding weight  $w^i(k)$  should be copied  $n_i$  times, where

$$n_i = \text{countif} : \widetilde{u}_k \in \left( \sum_{j=1}^{i-1} w^j(k), \sum_{j=1}^i w^j(k) \right] \tag{13}$$

where “countif:  $M$ ” is a function that returns the number of elements that satisfy the condition  $M$ .

#### 4. Experimental results

In this paper, a set of experiments are carried out using the equipment listed in Table 1 and Fig. 2-(b).

Table 1. Experimental Platform.

Item	Device/Method	Specifications
Battery Testing System	UPower® Battery Testing System	5% Accuracy, Room Temperature
Battery	SONY® US18650VTC6	Rated Capacity: 3Ah
Normal-Speed Aging	CC-CCCV @ 1C Rate	Cut-off Conditions: 4.2V, 2.75V, 0.05C
Accelerate Aging	CC-CCCV @ 1C Rate	Cut-off Conditions: 4.4V, 2.75V, 0.05C

Table 2. Experimental results.

Experiment	Model	Parameters				Overall RMSE
1	$f_{con}()$	$a=0.02437$	$b=-0.1171$	$c=0.9803$	$d=-0.0009077$	8.16%
	$f_{ns}()$	$x_1=0.9390$	$x_2=0.6473$	$x_3=0.3494$	$x_4=0.0338$	0.70%
2	$f_{con}()$	$a=0.1788$	$b=-0.00778$	$c=0.8147$	$d=9.105e-06$	1.78%
	$f_{ns}()$	$x_1=0.9563$	$x_2=0.8655$	$x_3=0.4499$	$x_4=0.0333$	0.67%

The results are provided in Fig. 2 and Table 2, where  $f_{con}()$  represents the result of the conventional direct fitting approach. It can be seen that compared with the direct fitting of the aging model in (2), the proposed migrated model shows better accuracy. The accuracy of the predicted SOH trajectory is higher than 1% even if the training data is only 25% of the entire lifespan. The proposed method basically extracts the information of “time scale” from the normal-speed aging process, and extracts the SOH changing tendency from the accelerate aging process. This combination provides a better result than using only the data of normal-speed aging process, especially when the experimental time is so limited that the SOH changing trend cannot be properly extracted from the normal-speed experimental data. It can be observed from Fig. 2 –(b) that the direct fitting approach can also provide a good prediction result when enough data is provided, however, this requires a longer experimental time. From the deriving of the proposed method, it can be seen that the proposed method can also be directly used for online remaining useful life (RUL) prediction. The similarity between the base model and the new process determines the efficiency of the proposed method.

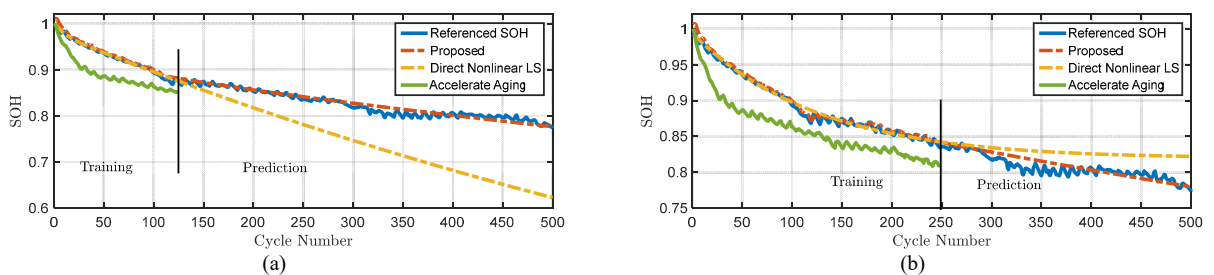


Fig. 2. Experimental results. (a): Experiment 1; (b): Experiment 2.

#### 5. Conclusions

In this paper, a migrated model is proposed to predict the aging trajectory of lithium-ion batteries. The accelerate aging is migrated for the normal-speed aging process through a linear transformation. The migration factors are determined through the Bayesian Monte Carlo method and the stratified resampling technique. The RMSE of the predicted aging trajectory limited to 1% with only 25% of the cyclic aging data. The proposed method is suitable for predicting the remaining useful life in real time as well as offline predicting the battery lifespan before designing the products such as EVs or UPSs as mentioned in the introduction.

Future research interests exist in quantifying the similarity between the accelerate and normal-speed aging processes, e.g., through incremental capacity analysis (ICA) method, and in comparing the effects of different accelerate aging operations.

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