L∞ algebras for extended geometry

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L_∞ algebras for extended geometry

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Abstract.   Extended geometry provides a unified framework for double geometry, exceptional geometry, etc., *i.e.*, for the geometrisations of the string theory and M-theory dualities. In this talk, we will explain the structure of gauge transformations (generalised diffeomorphisms) in these models. They are generically infinitely reducible, and arise as derived brackets from an underlying Borcherds superalgebra or tensor hierarchy algebra. The infinite reducibility gives rise to an L_∞ structure, the brackets of which have universal expressions in terms of the underlying superalgebra.

The results in this talk, presented by MC at Group32, Prague, July 9-13, 2018, are primarily based on refs. [1, 2].

The motivation for the investigation lies in the dualities appearing in string/M-theory, and the possibility to "geometrise" them. The plan of the talk is to briefly mention how the dualities arise, then display the basics of the general framework of extended geometry, and finally discuss the gauge structure, and the appearance of an L_∞ algebra.

When M-theory is compactified on an n-torus, the U-duality group is $E_{n(n)}(\mathbb{Z})$. The duality mixes momenta and brane windings. If this symmetry is to be geometrised, also diffeomorphisms and tensor gauge transformations need to be unified.

Membranes can wind on 2-cycles, 5-branes on 5-cycles. An example: Take a 6-dimensional torus T^6. There are $\binom{6}{2} = 15$ membrane windings Z_{ij} and $\binom{6}{5} = 6$ 5-brane windings Z_{ijklm}. Collect them together with the momentum p^i into the generalised momentum $P^M = (p^i, Z_{ij}, Z_{ijklm})$. These generalised momenta span a 27-dimensional space, and transform under $E_{6(6)}(\mathbb{Z})$.

The discrete duality group contains the geometric mapping class group. It turns out that it can be geometrised, so that (roughly speaking) the duality group derives from an "extended geometry" like the mapping class group from geometry. Special cases are double geometry (for T-duality) [3–17], and exceptional geometry (for U-duality) [18–37]. Recently, a completely general framework has been formulated [1], where any Kac–Moody group can be used as structure group instead of the continuous version of a duality group, and where any integrable highest

weight representation can be used for the generalised momenta (at least for highest weights with vanishing Dynkin labels for short roots).

The gauge transformations in extended geometry — the generalised diffeomorphisms — unify diffeomorphisms and gauge transformations for tensor fields. Given a Kac–Moody algebra $\mathfrak{g}$ and a lowest weight coordinate representation $R(-\lambda)$ (we use conventions where extended tangent space vectors are in lowest weight modules and cotangent vectors in highest weight modules; $R(-\lambda)$ denotes the lowest weight representation with lowest weight $-\lambda$, for dominant λ), they are expressed in terms of a generalised Lie derivative as $\delta_U \phi = \mathscr{L}_U \phi$, for any field ϕ transforming covariantly, where

$$\mathscr{L}_U V^M = U^N \partial_N V^M + Z_{PQ}{}^{MN} \partial_N U^P V^Q \; . \tag{1}$$

Here the invariant tensor Z has the universal expression [1, 36]

$$\sigma Z = -\eta_{\alpha\beta} t^\alpha \otimes t^\beta + (\lambda, \lambda) - 1 \; , \tag{2}$$

i.e., $Z_{PQ}{}^{MN} = -\eta_{\alpha\beta}(t^\alpha)_P{}^N (t^\beta)_Q{}^M + ((\lambda, \lambda) - 1)\delta_P^N \delta_Q^M$, where $(t^\alpha)_M{}^N$ are representation matrices in $R(-\lambda)$, σ is the permutation operator and $\eta_{\alpha\beta}$ the Killing metric.

In many cases (see below), the transformations form an "algebra"

$$[\mathscr{L}_U, \mathscr{L}_V] W = \mathscr{L}_{[\![U,V]\!]} W \; , \tag{3}$$

where the "Courant bracket" is $[\![U, V]\!] = \frac{1}{2}(\mathscr{L}_U V - \mathscr{L}_V U)$, provided that the derivatives fulfil a *section constraint*. The section constraint ensures that fields locally depend only on an n-dimensional subspace of the coordinate space, on which a $GL(n)$ subgroup acts. It reads

$$Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0 \; , \tag{4}$$

where the two derivatives can act on any field (or parameter). The tensor Y is related to Z as $Y = Z + 1$, and can be seen (in eq. (1)) as the deviation of the generalised Lie derivative from a Lie derivative on the extended space. The section constraint means that any two momenta lie in a linear subspace of a minimal orbit of $R(\lambda)$.

The "algebra" is not a Lie algebra. The consequences will be examined below.

The closure of the generalised diffeomorphisms as in eq. (3) takes place only under certain conditions:

○ The algebra $\mathfrak{g}$ is finite-dimensional,

○ $(\lambda, \theta) = 1$, *i.e.*, the highest weight λ is a fundamental weight dual to a simple root with Coxeter label 1.

In other cases, so-called ancillary transformations occur. These are restricted local $\mathfrak{g}$-transformations, that remove degrees of freedom corresponding to mixed tensors (dual graviton, etc.)

The complete list of situations without ancillary transformations is:

(*i*) $\mathfrak{g}_r = A_r$, $\lambda = \Lambda_p$, $p = 1, \ldots, r$ (*p*-form representations);

Figure 1. *Dynkin diagram of $\mathscr{B}(\mathfrak{g})$. The grey node is a fermionic null node. The single line connecting to the Dynkin diagram of $\mathfrak{g}$ can be replaced by multiple lines if λ is not a fundamental weight.*

(ii) $\mathfrak{g}_r = B_r$, $\lambda = \Lambda_1$ (the vector representation);

(iii) $\mathfrak{g}_r = C_r$, $\lambda = \Lambda_r$ (the symplectic-traceless r-form representation);

(iv) $\mathfrak{g}_r = D_r$, $\lambda = \Lambda_1, \Lambda_{r-1}, \Lambda_r$ (the vector and spinor representations);

(v) $\mathfrak{g}_r = E_6$, $\lambda = \Lambda_1, \Lambda_5$ (the fundamental representations);

(vi) $\mathfrak{g}_r = E_7$, $\lambda = \Lambda_1$ (the fundamental representation).

The Jacobi identity does not hold for the bracket $[\![\cdot, \cdot]\!]$. Instead, one typically has

$$[\![U, [\![V, W]\!]]\!] + \text{cycl.} \sim d[\![U, V, W]\!] ,\tag{5}$$

where $[\![U, V, W]\!] \in R_2$ represents reducibility. The derivative d is linear in ∂_M. $R_1, R_2, \ldots$ are the positive levels (level = ghost number) of the Borcherds superalgebra $\mathscr{B}(\mathfrak{g})$ [25, 38] with Dynkin diagram given in Figure 1.

This is the beginning of the L_∞ structure.

What is an L_∞ algebra [39–42]? Consider a full set of ghosts, including ghosts for ghosts, etc. Let $C = C_1 + C_2 + C_3 + \ldots$, where the subscript indicates ghost number. (We will later see this as an element in $\mathscr{B}_+(\mathfrak{g})$, the positive level subspace of $\mathscr{B}(\mathfrak{g})$.) The Batalin–Vilkovisky (BV) action [43], restricted to ghosts, can be expanded as

$$S(C, C^\star) = \sum_{n=1}^\infty \langle C^\star, [\![C^N]\!]\rangle ;\tag{6}$$

where

$$[\![C^n]\!] = [\![\underbrace{C, C, \ldots, C}_{n}]\!]\tag{7}$$

is the n-bracket.

The BV variation of C is

$$(S, C) = \sum_{n=1}^\infty [\![C^n]\!] .\tag{8}$$

In order for it to be nilpotent, $(S, (S, C)) = 0$, the brackets must satisfy the generalised Jacobi identities

$$\sum_{i=0}^{n-1} (i+1)[\![C^i, [\![C^{n-i}]\!]]\!] = 0 .\tag{9}$$

Figure 2. *The Dynkin diagrams of* $\mathfrak{g}_{r+1}$.

Figure 3. *Equivalent Dynkin diagrams of* $\mathscr{B}(\mathfrak{g}_{r+1})$.

The derivative is the 1-bracket.

$$R_1 \underset{d}{\leftarrow} R_2 \underset{d}{\leftarrow} R_3 \underset{d}{\leftarrow} \dots \tag{10}$$

In order to be able to construct all the brackets, one needs to rely on some underlying structure. It would seems like the superalgebra $\mathscr{B}(\mathfrak{g})$ provides it. However, there is no natural way of expressing the generalised Lie derivatives in terms of the (anti-)commutators of $\mathscr{B}(\mathfrak{g})$. For this one needs a further extension, $\mathscr{B}(\mathfrak{g}_{r+1})$. The bosonic extension $\mathfrak{g}_{r+1}$ has the Dynkin diagram in Figure 2 (here, like for $\mathscr{B}(\mathfrak{g})$, the connection of the extending node to the Dynkin diagram of $\mathfrak{g}$ can consist of multiple lines). Two Dynkin diagrams of $\mathscr{B}(\mathfrak{g}_{r+1})$ are given in Figure 3. The leftmost line is always single. The equivalence between the two diagrams, expressing $\mathscr{B}(\mathfrak{g}_{r+1})$ as an extension of $\mathscr{B}(\mathfrak{g})$ or $\mathfrak{g}_{r+1}$, respectively, involves a fermionic Weyl reflection [44].

It was noted in ref. [45] (in that case for the exceptional series, but straightforwardly applied to the general situation) that the generalised diffeomorphisms have a natural expression in terms of the Lie super-brackets of $\mathscr{B}(\mathfrak{g}_{r+1})$. The actual expression is

$$\mathscr{L}_U V = [[U, F^{\flat M}], \partial_M V^{\sharp}] - [[\partial_M U^{\sharp}, F^{\flat M}], V] , \tag{11}$$

The notation needs some explanation. We note that, in the double grading of Table 1, all $\mathfrak{g}$-modules at $p \neq 0$ come in pairs. We use the nilpotent operations $\sharp$ and $\flat$ to raise and lower elements within the pairs. F^M are basis elements for $R(\lambda)$ at $p = -1$, $q = 0$, and their lowered counterpart $F^{\flat M}$ span $R(\lambda)$ at $p = -1$, $q = 0$. The two terms reproduce the two terms in eq. (1).

The doubly extended algebra is needed also in order to harbour so called "ancillary" ghosts, which appear in most cases, including the exceptional series. At some level (ghost number) the derivative fails to be a derivation, and the generalised Lie derivative fails to be covariant. This gives rise to the extra ancillary ghosts. They are naturally encoded in the doubly extended algebra as elements in R_p at $q = 1$. They are restricted with respect to the section constraint in a certain sense, and can be constructed as a derivative together with an element in $\widetilde{R}_{p+1}$. We refer to ref. [2] for details.

Using this Borcherds superalgebra as an underlying structure, we are able to construct all brackets as derived brackets and check their identities [2].
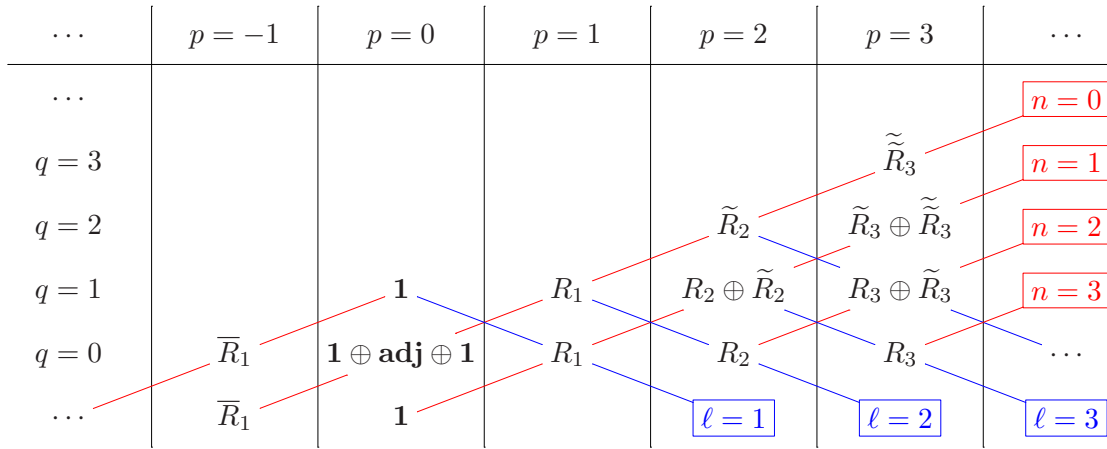
$\cdots$		$p = -1$	$p = 0$	$p = 1$	$p = 2$	$p = 3$	$\cdots$
$\cdots$							$\boxed{n=0}$
$q = 3$						$\widetilde{\widetilde{R}}_3$	$\boxed{n=1}$
$q = 2$					$\widetilde{R}_2$	$\widetilde{R}_3 \oplus \widetilde{\widetilde{R}}_3$	$\boxed{n=2}$
$q = 1$			$\mathbf{1}$	R_1	$R_2 \oplus \widetilde{R}_2$	$R_3 \oplus \widetilde{R}_3$	$\boxed{n=3}$
$q = 0$		$\overline{R}_1$	$\mathbf{1} \oplus \mathbf{adj} \oplus \mathbf{1}$	R_1	R_2	R_3	$\cdots$
$\cdots$		$\overline{R}_1$	$\mathbf{1}$		$\boxed{\ell=1}$	$\boxed{\ell=2}$	$\boxed{\ell=3}$

Table 1. *The general structure of the superalgebra $\mathscr{B}(\mathfrak{g}_{r+1})$. The blue lines are the L_∞-levels, given by $\ell = p+q$. Red lines are the usual levels in the level decomposition of $\mathscr{B}(\mathfrak{g}_{r+1})$, and form $\mathfrak{g}_{r+1}$ modules.*

$$K_{p_0} \xleftarrow{\ d\ } K_{p_0+1} \xleftarrow{\ d\ } K_{p_0+2} \xleftarrow{\ d\ } \cdots$$
$$\Big\downarrow{\flat} \qquad\qquad \Big\downarrow{\flat} \qquad\qquad \Big\downarrow{\flat}$$
$$0 \xleftarrow{\ d\ } C_1 \xleftarrow{\ d\ } \cdots \xleftarrow{\ d\ } C_{p_0-1} \xleftarrow{\ d\ } C_{p_0} \xleftarrow{\ d\ } C_{p_0+1} \xleftarrow{\ d\ } C_{p_0+2} \xleftarrow{\ d\ } \cdots$$

Table 2. *The typical structure of the action of the 1-bracket between the ghost modules, with ancillary ghosts appearing from level $p_0 \geq 1$.*

The concrete expression for all brackets read:

$$[\![C]\!] = dC \,,$$

$$[\![K]\!] = dK + K^\flat \,,$$

$$[\![C^n]\!] = k_n \Big((\mathrm{ad}\, C)^{n-2}(\mathscr{L}_C C + X_C C) + \sum_{i=0}^{n-3} (\mathrm{ad}\, C)^i R_C (\mathrm{ad}\, C)^{n-i-3} \mathscr{L}_C C \Big) \qquad (12)$$

$$[\![C^{n-1}, K]\!] = \frac{k_n}{n} \Big((\mathrm{ad}\, C)^{n-2} \mathscr{L}_C K + \sum_{i=0}^{n-3} (\mathrm{ad}\, C)^i \mathrm{ad}\, K (\mathrm{ad}\, C)^{n-i-3} \mathscr{L}_C C \Big) \,,$$

where the coefficients have the universal model-independent expression in terms of Bernoulli numbers

$$k_{n+1} = \frac{2^n B_n^+}{n!} \,, \quad n \geq 1 \,. \qquad (13)$$

All non-vanishing brackets except the 1-bracket contain at least one level 1 ghost C_1. No brackets contain more than one ancillary ghost. In the expressions for the brackets, $\mathscr{L}_C = \mathscr{L}_{C_1}$, *i.e.*, only

the ghost number 1 part of C enters the generalised Lie derivative. X and R are ancillary contributions, and we refer to ref. [2] for their exact expressions.

The L_∞ algebra for double geometry was constructed in refs. [41, 46, 47]. Then, there are no ancillary ghosts, and the algebra stops at ghost number 2 and a 3-bracket. This is because the corresponding Borcherds superalgebra is finite-dimensional.

The Borcherds superalgebra is unable to handle situations where ancillary transformations appear in the commutator of two generalised Lie derivatives. The presence of ancillary ghost at ghost number n relies on the occurrence of a $\mathfrak{g}$-module $\widetilde{R}_p$ in $\mathscr{B}(\mathfrak{g}_{r+1})$ (see Table 1). The Borcherds superalgebra never contains $\widetilde{R}_1$, which would be the signal of ancillary ghosts with ghost number 1, *i.e.*, of ancillary transformations Σ in the commutator of two generalised Lie derivatives,

$$[\mathscr{L}_U, \mathscr{L}_V]W = \mathscr{L}_{[\![U,V]\!]}W + \Sigma_{U,V}W \tag{14}$$

The ancillary parameter Σ is a parameter for a restricted (with respect to the section constraint) local $\mathfrak{g}$-transformation, see refs. [34–36, 48]. Then one needs a *tensor hierarchy algebra* [49, 50], a generalisation of the Cartan-type superalgebras $W(n)$ and $S(n)$ in Kac's classification [51]. Tensor hierarchy algebras are non-contragredient superalgebras, and therefore *a priori* not defined by standard Chevalley–Serre relations from a Dynkin diagram. In ref. [50] we presented a set of generators and relations for the (finite-dimensional) tensor hierarchy algebras $W(r+1) = W(A_r)$ and $S(r+1) = S(A_r)$, based on the same Dynkin diagram as that of $\mathscr{B}(A_r)$. The straightforward generalisation of these relations seems to provide a good definition of $W(\mathfrak{g})$ and $S(\mathfrak{g})$ in general (see the talk by JP at the present meeting [52]). The superalgebra used for extended geometry with structure algebra $\mathfrak{g}$ is $S(\mathfrak{g}_{r+1})$. These algebras agree with the Borcherds superalgebras at positive levels, and turn out to "know" when ancillary transformations appear: they contain a module $\widetilde{R}_1$ exactly in these cases (*i.e.*, when $\mathfrak{g}$ is infinite-dimensional and when $\mathfrak{g}$ is finite-dimensional and $(\lambda, \theta) > 1$). In the series of exceptional duality symmetries, this development is necessary starting from E_8. It also seems promising for incorporating dynamical fields (vielbein, torsion,...) in the present framework. Work is in progress concerning the rôle of tensor hierarchy algebras in extended geometry [53, 54], and we believe that they will encode the information needed for the gauge structure and dynamics also in cases with infinite-dimensional structure groups, such as E_9 [36, 37], E_{10} and maybe E_{11} [55].

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