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15th CIRP Conference on Computer Aided Tolerancing – CIRP CAT 2018

Evaluating evolutionary algorithms on spot welding sequence optimization with respect to geometrical variation

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Abstract

Spot welding is the prevalent joining process in the automotive industry. The spot welding sequence has a notable effect on the geometrical variation of the final assembly. Finding the optimal weld sequence for geometrical quality is a fast growing and NP-complete problem. Using exhaustive search for this purpose can be a time-consuming task. In this paper, genetic, particle swarm and ant colony optimization algorithms are applied to three industrial reference cases. The performance of these algorithms for finding the optimal sequence with respect to geometrical variation is compared considering the number of function evaluations.

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Peer-review under responsibility of the Scientific Committee of the 15th CIRP Conference on Computer Aided Tolerancing - CIRP CAT 2018.

Keywords: Joining; Tolerancing; geometrical variation

1. Introduction

In the automotive industry, the body of the car (Body in White) is made using sheet metals. The predominant joining method for these type of assemblies is spot welding. In a typical passenger car body, there are more than 4000 spot-welds. These spots are mainly being welded by industrial robots which are placed in different assembly cells. Different sub-assemblies are being joint together in these cells using weld guns on robot arms. A limited amount of time is available in each of the cells for the welding to be performed. Therefore, a limited amount of spots can be welded by the robots. The spot-welds and the sequence in which they are applied have a significant effect on the assemblies' geometrical variation [1]. The spot-welding sequence is a combinatorial problem which can be categorized as a Hamiltonian graph search problem. Since the number of the possible sequences increases exponentially, with the number of the weld points, finding the global optimal sequence becomes a challenge. This problem can be compared to the traveling salesman problem which is an NP-hard problem in combinatorial optimization. Previously, Genetic Algorithm (GA) has been tested on this problem and shown to have promising results. Although GA has a widespread application in the sequencing and scheduling problems in the industry, there are some shortcomings associated with it. Using GA still, a large number of the solutions need to be evaluated, which can be time-consuming. Moreover, it is not always clear that a global optimum has been reached. There are many competitors to GA, in the category of evolutionary algorithms, for solving the combinatorial problems. These have not yet been tested on the spot-weld sequence optimization problem with respect to geometrical variation. In this work, three different evolutionary optimization algorithms have been successfully applied to the spot-weld sequencing problem. The performances of these algorithms have been evaluated using three industrial reference assemblies.

1.1. Previous research

Different approaches have been tested for finding the optimal welding sequence. Genetic algorithm has been used for finding the optimal sequence and has proven to be an efficient method [2, 3]. Xie and Hsie [3] have used GA to find the optimal spot-weld or clamping sequence minimizing the assembly deformation. They also have taken cycle time into consideration. Segeborn et al. [2] have used GA for finding the

optimal spot welding sequence with respect to geometrical variation and cycle time. Industrial best practice approaches have also been tested for finding the pseudo optimal welding sequence [4]. Neural network approach has also been used for finding optimal continuous welding sequence [5]. Carlson et al. [6] have introduced a systematic search approach for quality and throughput optimization, which exploits the properties of the welding process.

1.2. Non-rigid variation simulation

Sheet metal assemblies are non-rigid and are bent and deformed during assembly. In order to analyze geometrical variation of theses assemblies, caused by over-constrained locating schemes and joining forces, Finite Element Analysis (FEA) calculation is needed to retrieve the deformations. Variation analysis can be performed using the transformation matrices to simulate the propagation of the variation in the assembly [7]. In this work, the CAT tool, RD&T is utilized to calculate the assembly deformations and geometrical variation. This software uses Monte Carlo Simulation (MCS) for variation analysis and method of influence coefficient (MIC) in the FEA solver for calculating the assembly deformations and variation. In order to model the accurate non-rigid behavior of the sheet metal assemblies contact modelling is used to avoid the mating parts to penetrate each other. This is achieved by including a number of contact points in the CAT models. The connection between the MCS and MIC is improved and described by Lindau et al. [8]. The software has the possibility to calculate the deformations after spot welding, taking into account the welding forces.

1.3. Scope of the paper

The problem of welding sequence optimization has been studied before, using various methods. For spot-welding sequence optimization, the analysis of evolutionary algorithms has been limited to GA. The applicability and efficiency of other evolutionary algorithms are remained ambiguous in this application. In this work, two other suitable evolutionary algorithms are chosen and their performances are compared to the GA algorithm. This will give a broader perspective on the applicability of other methods on spot-welding sequence optimization with respect to geometrical variation. The structure of the paper is as follows:
In the first section, an introduction to the problem and previous research are presented. A brief introduction to the utilized method is also given. Built upon this information, the rest of the paper is structured as follows. Section 2 introduces the three chosen evolutionary algorithms and implementation of them on the welding sequence optimization for geometrical variation. In section 3, the reference cases and experiment setup are described. In Section 4, the results retrieved from the reference cases are presented and discussed. In section 5, the conclusions are drawn based on the results.

2. Evolutionary Algorithms on Spot-Weld Sequencing

Evolutionary Algorithms (EA) are those algorithms that are inspired by biological evolution processes. The main characteristics of such a process are fitness and reproduction. There are a number of algorithms available and developed in this arena. Genetic algorithms (GA), Ant Colony Optimization (ACO), and Particle Swarm Optimization (PSO) are of this kind. These algorithms are widely applied to the combinatorial type of problems. Spot welding sequence optimization with respect to geometrical variation can be categorized as one of the combinatorial problems. The formulation of this problem is close to the Travelling Salesman Problem (TSP). There are several evaluations on the performance of the EAs on the TSP problem [9]. ACO has abroad applications for this type of problem [10]. PSO is also one of the major competitors of the GAs, especially for the continuous type of problems. Discrete PSO has also been applied to the TSP problem, and have been shown to be efficient [11]. The ACO and PSO algorithms are chosen to be evaluated and the performance to be compared with the GA algorithm on this basis, for the spot welding sequence optimization.

2.1.1. Genetic algorithm

Genetic algorithms are one of the stochastic search methods within the evolutionary algorithms category. Natural selection processes inspire these algorithms. In GAs a set of design alternatives, also referred to as population, in a specified generation is reproduced, crossed over among themselves with the bias towards the fit member to create a new set of alternatives [12]. The three main operators in GAs are reproduction, crossover, and mutation. The reproduction operator copies an old string into the new population based on the strings fitness/cost through function evaluation. Crossover allows the exchange of the characteristics of a string among themselves. Mutation is another operator which allows a number of the members in a binary string, with a specified location, switch 1s to 0s, vice versa [12]. The algorithms steps on spot welding sequence problem are shown in Fig.1.
In order to apply GAs to the sequencing problems, some modifications need to be applied to crossover and mutation operators. This is to overcome the problem of repetitive numbers being proposed to the different permutations. These modifications can be performed through different methods, of
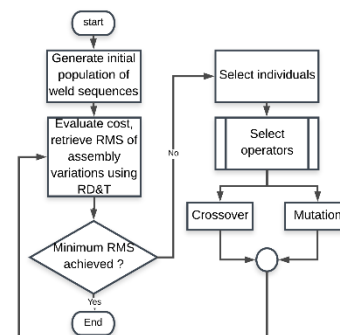

Fig. 1 GA principle on spot welding sequence

which random key encoding and GASP are widespread within the sequencing type of problems [13]. In this work, random key encoding approach has been applied to the GAs to solve the spot-weld sequence optimization problem. Like other evolutionary algorithms, GAs involve a set of parameters that can affect the efficiency of the search. Selection pressure, mutation percentage, crossover percentage, population size and number of generations are parameters that can influence the quality of the results and efficiency of the search.

2.1.2. Particle swarm optimization

PSO is a fairly simple optimization algorithm which is inspired by the social behavior (intelligence) of swarms. This algorithm was first introduced for optimization of continuous nonlinear functions. Just like GAs, the algorithm is initialized by generating a random initial population of solutions. These solutions are referred to as particles. Random velocities are assigned to each particle and they maneuver in the problem space. The particles are assigned with global and personal memories, which lead them towards the best cost/fitness evaluated in the whole population. The algorithms main steps applied to the spot welding sequence problem are shown in Fig.2. The PSO algorithm also contains a set of parameters that can affect the performance of the algorithm. These parameters are often referred to as exploration and exploitation parameters. Exploration is the ability of the algorithm to search the problem space for identifying the optimum candidate.

Exploitation is the ability to search around the good candidate in order to find the global optimum. Several studies have been made on fine-tuning these parameters [14]. For making PSO compatible with discrete combinatorial problems some modifications are required. Introduction of a mutation operator is necessary and in this work, this is done through the application of random key encoding. The same approach as for GA has been used in this algorithm.

2.1.3. Ant colony optimization

ACO is one of the meta-heuristic optimization algorithms which is inspired by the behavior of the ants. This behavior entails finding the shortest path from the nest to the food. This is achieved through the medium of pheromones, used for communication between the individuals for sharing the path information. ACO is a rather different algorithm compared to the other EA algorithms since it constructs a new set of solutions, also referred to as colonies, in each generation. The
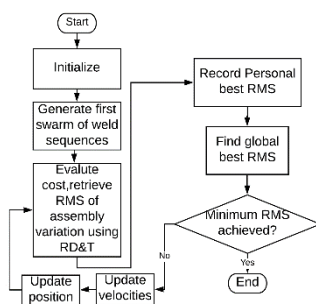
Fig. 2 PSO principle on spot welding sequence

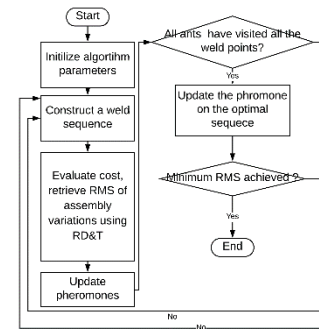constructed solution contains the information of the previous

Fig. 3 ACO principle on spot welding sequence

solutions. Furthermore, an approach to add or remove (evaporation) pheromone on the paths is applied. A local search for exploring the neighboring areas in the solution space is also applied in this algorithm [15]. A number of parameters are affecting the efficiency of this algorithm just like other EAs. Initial pheromone rate, pheromone exponential rate, heuristic exponential rate, evaporation rate together with population size are among these parameters. The algorithms main steps applied to the spot welding sequence problem are shown in Fig.3.

3. Reference Case Evaluations

In this section, the setup required for applying the algorithms on the reference cases is introduced. Three chosen industrial reference cases are also introduced.
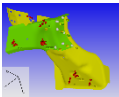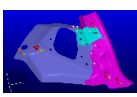
3.1. Reference cases

Three Reference Cases (referred to as RC1-3 for simplicity) from vehicle industry have been chosen for optimizing the spot welding sequences. These are sheet metal assemblies from the vehicles' body. In Table 1, description of each RC is presented. All these RCs are pre-processed (meshed) and modeled for variation analysis in RD&T.

In RC1 the locating scheme consists of 6 points. No extra supports were used in the measurement fixture. RC2-3, however, have an extra support. The mating surfaces in RC1 are quite small compared to the overall geometry. In RC1, the CAT model includes 159 contact points between the ingoing parts in the sub-assembly. The same information for RC2-3 are presented in Table 1. The spot welds in RC2 are on the small formed flanges. The directions in which these spot welds lock the geometry are presented in Table 1, for RC1-3.

There are 7 weld points included in RC1-3. This will make the number of possible sequences in each assembly equal to 5040. This approach is chosen mainly for two reasons. Firstly, in each assembly cell in a BIW assembly shop, there is a time limitation, which does not allow the sequencing procedure to perform more than a limited number of welds. Secondly, it has been shown that the first weld points that are welded in each assembly cell have the most effect on the geometry outcome. These points are also referred to as geometry spot welds (geo-spots). The geo-spots are usually a limited number of points in each assembly. The rest of the weld points are considered to have less effect on the final geometry and therefore the

sequence of welding them have minimal effect on the geometry outcome [16].

Table 1 Reference cases

Reference Cases (RC)	1	2	3
Figures			
No. of parts	2	3	3
Locating scheme+supports	6	6+1	6+1
No. of contact points	159	62	194
Welds included in sequence	7	7	7+4*
Welds locking the parts in direction	6(X)-1(Z)	2(X)-2(Y)-3(XYZ)	3(X)-3(X)-1(Z)

* Out of the 11 weld points, 7 were included in sequence analysis and 4 welded simultaneously

3.2. Experiment setup

In the following sections, the approach for applying the EA algorithms and screening their corresponding parameters are described.

3.2.1. Implementation of the EA Algorithms

For analyzing the non-rigid behavior of the assemblies after welding, the three cases were modeled in the CAT-tool RD&T [17]. This model is used to evaluate the cost function of the optimization algorithms, where the cost for a sequence is defined as the total geometrical variation in the assembly. The number of times that an EA calls the cost function is referred to as Number of Function Evaluation (NFE). This means that for every time the algorithm calls the cost function, non-rigid variation simulation in the tool RD&T, the NFE counter accumulate one instance. The cost function for a sequence is defined as the Root Mean Square (RMS) of the variation in all nodes (from the meshed model) of the assembly. For each node j, $j=1,...,k$ the magnitude of the variation is calculated as the sum of the variations in the X, Y and Z directions [18]:
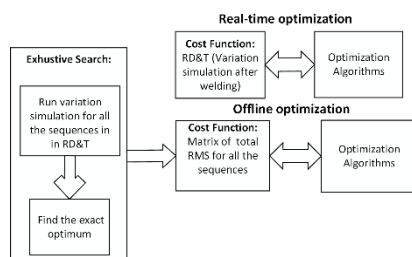
$$S_j^2 = S_{jx}^2 + S_{jy}^2 + S_{jz}^2 \qquad (1)$$

Fig. 4 Overview of the experiment approach

The RMS value is then defined as:

$$S_{RMS} = \sqrt{\frac{S_1^2 + ... + S_k^2}{k}} \qquad (2)$$

Each sequence of seven spot welds requires approximately 44 seconds for cost function evaluation. This is done using a regular working station with CPU i7-2.7 GHz and 16 GB RAM. Having a coupled simulation tool to the algorithms for retrieving the RMS value of each generated solution would be time-consuming (real-time optimization). This means that the algorithms should stand by for 44 seconds until the cost is evaluated. Therefore, for evaluating and comparing the algorithms in spot welding sequence applications, knowing the actual optimum is of importance, due to time limitations. For this, an exhaustive search was performed. All the possible 5040 sequence combinations, for all the three cases, were simulated. The total RMS of the geometrical variation over all nodes after welding was retrieved and saved for each sequence.

The optimum sequences for minimizing the total RMS of geometrical variation were identified, by a simple search on each column of the retrieved results. In RC3, only the deformation data was available, therefore the analysis for RC3 only considers the geometrical deformation and not variation. These data then are used in the cost functions of the algorithms. The algorithms will run much faster thorough using the exhaustive search results since the FEA calculations do not need to run at the same time (Offline optimization). An overview of this approach is shown in Fig.4.

The main bottleneck in spot-welding sequence analysis is the simulation time needed for evaluating each sequence. This is why that the NFE is chosen as a comparison parameter. The algorithms were then successfully applied on the RCs using MATLAB, based on the approach discussed in 2.1.1-2.1.3. For evaluating the algorithms' performance on each case, each algorithm has run 100 times, where the ending condition was reaching a desired value (optimum value, known from the exhaustive search). The NFE for reaching the exact optimum RMS is then considered for comparison. These results are presented in Section 4.

3.2.2. Screening of the EA parameters

In section 2.1.1-3, it was mentioned that the EA's parameter setting can significantly affect the performance of the algorithms. For identifying a suitable set of parameters for each RC, a screening process was conducted. This process is initiated by running multiple tests on the EA parameters to retrieve the appropriate range of each parameter on each RC. This means that the parameters of each algorithm were changed on each test and the algorithms were run with 100 iterations and the corresponding NFE was monitored. The initial starting values of the parameters were based on the recommendations in the literature.

After this monitoring process, a suitable range for each parameter was identified. Latter, 20 experiments were conducted by changing the parameters of each algorithm for

each RC, within the identified ranges. The best combination of the parameters which resulted in the lowest NFEs, out of the 20 experiments, were then chosen for implementation of the algorithms. These parameter values can be found in Table 2. During the screening process, it was observed that by using the larger population size the NFEs increases to a high extent. Using the population 50 in the PSO algorithm in RC1 require at least 200 to 300 NFEs. Therefore, for reaching the fewer function evaluations, all the algorithms required smaller population size for reaching the exact optimum. The optimum is known form the exhaustive search results. Table 2 shows that using the smaller population sizes the optimum can still be found using 7 to 82 function evaluations for all the RCs.

Table 2 Parameter values

GA	Selection Pressure	Mutation Percentage	Crossover percentage	Population size	Observed NFE
Case 1	8	0.9	0.5	2	30
Case 2	8	0.9	0.5	2	82
Case 3	7	0.8	0.6	2	34
PSO	Inertia Weight	Personal learning coefficient	Global learning coefficient	Population size	Observed NFE
Case 1	0.5	0.6	0.4	2	42
Case 2	1	0.2	0.6	2	7
Case 3	1	2	2	3	10
ACO	Selection Pressure	Mutation Percentage	Crossover percentage	Evaporation rate	Population size	Obs. NFE
Case 1	1.6851	1	1	0.05	2	24
Case 2	1.6851	1	1	0.05	2	24
Case 3	1.6851	1.2	1.5	0.15	2	16

4. Results

The results of the evaluations are presented in Table 3. This table shows the mean and range of the NFEs required for finding the minimum value. This also shows the number of solutions (sequences) that have been searched to retrieve the minimum value. The mean and the range of the minimum 6*s* RMS of the geometrical variation (geometrical deviation for case 3) after spot welding (in millimeters) also have been shown as a decision support to the evaluating criteria. The other evaluation criterion is the number of the times that the actual optimum was found by the algorithm during the 100 trials. This can help to evaluate the ability of the algorithm to find the global optimum. The global optimum values are known, through the exhaustive search results. In Table 4 the number of the occurrence of the global optimum in 100 trials is shown.

For reference cases 1 and 2, where geometrical variation was taken into account, ACO resulted in the lowest mean value in the required NFEs.

For RC2, ACO has higher mean in the RMS values. This can also be evaluated with the results in table 4, where ACO has the lowest ability to find the global optimum. This also applies to RC1 for the ACO algorithm. PSO and GA performed quite similar, with approximately 12 NFEs difference, in RC2.

In RC1, GA performed better compared to PSO. This is due to the occurrence of the 1274 NFEs once in 100 trials. If this number was disregarded, the required NFEs will reduce to 178 and range to 480 for the PSO algorithm. In RC3, where geometrical deviation was considered and the range in the problem space was smaller compared to RC1 and RC2, GA resulted in lower NFEs.

Table 3 NFEs and RMS values in 100 trials

RC 1	Optimum RMS* after welding from exhaustive search: 1.8752 mm			
	Min NFE	*Max NFE*	*Range NFE*	*Mean NFE*
GA	10	398	388	147.98
PSO	22	1274	1252	190.58
ACO	**2**	**198**	**196**	**96.03**
	Min RMS	*Max RMS*	*Range RMS*	*Mean RMS*
GA	1.8752	1.9443	0.0691	1.8928
PSO	1.8752	1.9430	0.0678	1.8937
ACO	1.8752	1.9310	0.0558	1.8902
RC 2	Optimum RMS after welding from exhaustive search: 1.0303mm			
	Min NFE	*Max NFE*	*Range NFE*	*Mean NFE*
GA	9	390	381	212.39
PSO	22	492	470	199.92
ACO	**2**	**200**	**198**	**99.31**
	Min RMS	*Max RMS*	*Range RMS*	*Mean RMS*
GA	1.0303	1.0478	0.0175	1.0333
PSO	1.0303	1.0428	0.0125	1.0311
ACO	1.0303	1.4050	0.3747	1.0511
RC 3	Optimum RMS after welding from exhaustive search: 0.6554mm			
	Min NFE	*Max NFE*	*Range NFE*	*Mean NFE*
GA	**6**	**334**	**328**	**60.89**
PSO	10	465	455	113.82
ACO	3	261	258	89.73
	Min RMS	*Max RMS*	*Range RMS*	*Mean RMS*
GA	0.6554	0.6673	0.0119	0.6574
PSO	0.6554	0.6591	0.0037	0.655463
ACO	0.6554	0.6591	0.0037	0.655492

*All the RMS values are in millimeters

In this RC, GA algorithm resulted in a higher mean for the RMS value. Again, this can be evaluated with the results in Table 4, where GA has the lower ability to find the global optimum compared to the other two. In the ability to find the global optimum, PSO algorithm has performed better in all the RCs. This also has to be mentioned that in RC1 the range of the problem space is higher, 0.61 *mm*, than the RC2 and RC3 where the ranges are 0.2773 and 0.0571 *mm* respectively.

In summary, it can be stated that the ACO performed better in RC1 and RC2 for the NFEs with a satisfying mean value in RMS. While, in RC3, GA performed better with an acceptable difference in the mean RMS from the optimum. The PSO algorithm has a strong ability to find the global optimum in all the three RCs.

Within the field of geometry assurance, there are also similar processes to spot welding sequence. As an example, the effect of the clamping sequences on the final geometry can be seen as the same problem. Therefore, the EAs used in this work can be applied to the clamping problem as well. Other types of joining

sequences, like riveting and clip fasteners can also be analyzed using the applied EA algorithms.

Table 4 Occurrence of the global optimum in each case in 100 trial

Reference Case 1		Reference Case 2			Reference Case 3	
GA	40%	GA	67%		GA	68%
PSO	**65%**	**PSO**	**91%**		**PSO**	**99%**
ACO	11%	ACO	2%		ACO	93%

5. Conclusion

Three evolutionary algorithms have been successfully applied on three reference cases. The purpose was to evaluate the performance of these algorithms and compare them to each other. The comparison has been made for the spot welding sequence optimization with respect to geometrical variation. Previously, only GA algorithms have been implemented on this problem. Through this evaluation, it was realized that other evolutionary algorithms like ACO and PSO can be used as an alternative to GA and retrieve satisfactory or even better results, depending on the assembly complexity.

Improvements are still required in this area to minimize the amount of time required for solving this problem. At the current state, all the three evaluated, and perhaps other stand-alone EAs will perform somewhat the same for NFEs. It can be concluded that the three evaluated algorithms are highly dependent on the quality of the initial position of the randomly generated population. This might reduce the efficiency of the EAs where the range in the problem space is considerably high. The bottleneck of this problem lies in the required simulation time needed to evaluate each sequence. This, in another word, means that the NFEs need to be kept to the minimum.

Working on simplifications of these calculations for spot welding sequence optimization is one approach that can be further analyzed. Other evaluations can be made on the ability of these algorithms to run parallel, meaning evaluating different solutions at the same time. One other improvement worth analyzing is the introduction of a biased initial position to the EAs. Industrial best practice approaches could be one alternative for these initial positions.

Acknowledgements

References

[1] Dahlström S, Söderberg R. Analysis of Final Geometry Due to Welding Process Effects in Sheet Metal Assemblies. Fourth International Symposium on Tools and Methods for Competetive Engineering; Wuhan, Hubei, China. 2002.

[2] Segeborn J, Carlson JS, Wärmefjord K, Söderberg R. Evaluating Genetic Algorithms on Welding Sequence Optimization With Respect to Dimensional Variation and Cycle Time. Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering. Volume 5: 37th Design Automation Conference, Parts A and B. 2011. p. 697-704.

[3] Xie LS, Hsieh C. Clamping and welding sequence optimisation for minimising cycle time and assembly deformation. International Journal of Materials and Product Technology. 2002;17.p. 389-99.

[4] Wärmefjord K, Söderberg R, Lindkvist L. Strategies for Optimization of Spot Welding Sequence With Respect to Geometrical Variation in Sheet Metal Assemblies. Volume 3: Design and Manufacturing, Parts A and B. 2010. p. 569-77.

[5] Fukuda S, Yoshikawa K. Determination of welding sequence: a neural net approach. Engineering Analysis with Boundary Elements. 1990;7.p. 78-82.

[6] Carlson JS, Spensieri D, Wärmefjord K, Segeborn J, Söderberg R. Minimizing Dimensional Variation and Robot Traveling Time in Welding Stations. Procedia CIRP; 2014/01/01/. 2014. p. 77-82.

[7] Söderberg R, Wärmefjord K, Carlson JS, Lindkvist L. Toward a Digital Twin for real-time geometry assurance in individualized production. CIRP Annals. 2017;66.p. 137-40.

[8] Lindau B, Lorin S, Lindkvist L, Söderberg R. Efficient Contact Modeling in Nonrigid Variation Simulation. Journal of Computing and Information Science in Engineering. 2016;16.p. 011002--7.

[9] Liao Y-F, Yau D-H, Chen C-L. Evolutionary algorithm to traveling salesman problems. Computers & Mathematics with Applications. 2012;64.p. 788-97.

[10] Dorigo M, Gambardella LM. Ant colony system: a cooperative learning approach to the traveling salesman problem. IEEE Transactions on Evolutionary Computation. 1997;1.p. 53-66.

[11] Wen-hang Z, Jun Z, Wei-neng C. A novel discrete particle swarm optimization to solve traveling salesman problem. 2007 IEEE Congress on Evolutionary Computation; 2007 25-28 Sept. 2007.

[12] Huang M-W, Hsieh CC, Arora JS. A genetic algorithm for sequencing type problems in engineering design. International Journal for Numerical Methods in Engineering. 1997;40.p. 3105-15.

[13] Bean JC. Genetic Algorithms and Random Keys for Sequencing and Optimization. ORSA Journal on Computing. 1994;6.p. 154-60.

[14] Clerc M, Kennedy J. The particle swarm - explosion, stability, and convergence in a multidimensional complex space. IEEE Transactions on Evolutionary Computation. 2002;6.p. 58-73.

[15] Dorigo M, Caro GD. Ant colony optimization: a new meta-heuristic. Proceedings of the 1999 Congress on Evolutionary Computation-CEC99 (Cat No 99TH8406); 1999 1999.

[16] Wärmefjord K, Söderberg R, Lindkvist L. Variation Simulation of Spot Welding Sequence for Sheet Metal Assemblies. Proceedings of NordDesign2010 International Conference on Methods and Tools for Product and Production Development; Gothenburg, Sweden. 2010.

[17] RD&T Technology. 2017. Improving Decision Making by Simulating and Visualizing the Effect of Geometrical Variation. [ONLINE] Available at: http://rdnt.se/index.html. [Accessed 17 October 2017].

[18] Wärmefjord K, Söderberg R, Lindkvist L. Simulation of the effect of geometrical variation on assembly and holding forces. International Journal of Product Development. 2013;18.p. 88-108.