The design, fabrication and assembly of an asymptotic timber gridshell

Emil ADIELS*, Cecilie BRANDT-OLSENa, Johanna ISAKSSONb, Isak NÄSLUNDb, Karl-Gunnar OLSSON, Emil POULSENc, Chris J. K. WILLIAMS

* Department of Architecture and Civil Engineering
Chalmers University of Technology
412 58 Gothenburg, Sweden
emil.adiel@chalmers.se

a BIG Engineering
b BuroHappold Engineering
c CORE Studio Thornton Tomasetti

Abstract

This paper describes and discusses the design, fabrication and assembly of an asymptotic gridshell built of plywood laths. The overall question concerns how geometry, structural action, and efficient production can interplay and inform spatial design. The environment is a two-day workshop where architects, engineers and researchers with specialization in structural and digital design cooperate with undergraduate students in a compulsory parametric design and digital fabrication course. The gridshell shape is based on an Enneper surface of threefold rotational symmetry with a boundary baseplate inscribed within a circle of 4.5 m in radius. Utilizing the concept of asymptotic curves, which are surface curves whose osculating plane coincides with the tangent plane of the surface, the structure was built using planar straight laths of plywood made using manually operated drills and saws.

Keywords: Gridshell, differential geometry, parametric design, active bending, structures, timber, education.

1 Introduction

Throughout the history of construction architects and engineers has sought strategies to reduce the cost of production and use of material without limiting our spatial experience or the performance of the design. Schlaich-Bergerman designed numerous roofs with planar quadrilateral panels[1] while Otto, Happold and Liddell connected continuous timber laths into a flat grid that was formed into the gridshell, like a sieve, creating stiffness in Mannheim Germany[2]. For students aspiring to work in the intersection between architecture and engineering these architects and engineers from the history, with knowledge, skills and attitudes intertwined into a creative working environment, are important role models. The question is how this connection between geometry, space and structure can be best explored and taught to serve as inspiration for deeper studies in the respective fields as well as illustrating how mathematics can support virtual and physical explorations of space and form. We investigated this question by building a
gridshell, figure 1, in a three dimensional shape with students from the double degree program in architecture and engineering at Chalmers.

Figure 1: The asymptotic gridshell built during two days in the workshop in December 2018 by the students based on design provided by the workshop leaders, photo: Martin Skarby.

2 Interdisciplinary teaching in an experimental culture

Architecture and Engineering (A&E) is a dual degree program (MArch. and MSc. in Engineering) at Chalmers University of Technology, training students in interdisciplinary design methods combining different areas of knowledge and skills related to both engineering and architecture. The educational program includes mathematics, applied mechanics, building materials, history of architecture, design tools as well as design of architectural projects and experimental workshops integrating mathematics, mechanics and art. These workshops have explored topics such as masonry shells, membranes and weaved structures through both theory and application, figure 2a) - b).

In 2017 the course Parametric Design - Digital Tools (3 ECTS) was initiated with the aim to teach bachelor A&E students in basics of parametric design and digital fabrication. In nine days of teaching roughly six days covers parametric design basics, extraction of production data and digital fabrication methods. Examination are through smaller design exercises and projects. The last two and half days consists of a workshop which aim is to summarize the course by building a temporary structure for a indoor exhibition space. In the 2017 workshop a geodesic gridshell was built and is described in Adiels et al [3], see 2. The well working format, 4.8 out of 5 in the student evaluation, was kept for the 2018 workshop, but a new geometrical concept that links shape and structure to rational and efficient building process was developed by the group of organizers. This group consists of architects and engineers from both academy and industry with specialism in computational design. As the exhibition would take place in the old testing facility at Chalmers the fabrication could be done on site utilizing the institutions wood-workshop, which facilitates tools and machines for processing wood including a CNC for
plywood boards.

Figure 2: a) Building a freeform vault on actively bent gridshell the course “Material Laborations” [4] b) Soap films and membranes was invested a workshop in the course “Virtual Tools in a Material Culture” c) and d) shows the geodesic grid-shell built in the workshop 2017 in this course. [3]

3 Asymptotic lines in gridshell design

The geometrical concept for the workshop and the design of the structure to be built has three main requirements: The underlying theory needs to able to be presented in a comprehensible way based on the student’s prior knowledge. The design must enable a rational production process linked to the design such that it can built in just two days activating all forty students. It should also be transparently implemented in a parametric environment such that the students can learn and modify it for future projects.

Asymptotic lines are special curves on a surface that has zero normal curvature. Extruding these curves in the normal direction these developable surfaces can be unrolled both straight and planar without distortion. This concept can be utilized in gridshell design to be built with straight planar laths. Asymptotic lines can be found at the top of a torus or physically along the free edges of a prestressed membrane, as in figure 2b. Asymptotic lines are restricted to anticlastic surfaces and surfaces of zero Gaussian curvature. For the former there exist two asymptotic directions in each point while in the latter only one. This is evident looking at Mohr’s circle of curvature[5], figure 3. Knowing the magnitude and direction of the principal curvatures one can find the asymptotic directions using Mohr’s circle or Euler’s theorem[6], knowing normal curvature to be zero.

\[
\kappa_n = k_1 \cos(\phi)^2 + k_2 \sin(\phi)^2
\]  

\[
\phi \pm \arctan\left(\frac{-k_1}{k_2}\right)
\]

The principal directions bisect the angle between the asymptotic directions, but not necessarily the other way around. Only on minimal surfaces the two principal curvatures are equal and opposite, therefore the asymptotic directions are found at a 45° angle to the principal directions.

Inspired by the work of Schling et al[7], a series of scale model prototypes where designed and constructed, all of them following the asymptotic curves. Specifically, three mechanical properties and features were studied: The curvature and bending capacity related to the choice of cross section. The built-in structural mechanism allowing to reshape the structure from a flat shape to a three-dimensional shape. The by active bending built in strain energy creating
Figure 3: Mohr’s circle of curvature where the axis represents the normal curvature $\kappa_n$ and surface torsion $\tau_g$. The blue circle represents a minimal surface where $\kappa_1$ and $\kappa_2$ are of equal magnitude with opposite sign, therefore the asymptotic directions are given with $\phi = \pi/4$ and form right angles. For the gray case $\phi \neq \pi/4$ and therefore the asymptotic directions are not orthogonal to each other.

Figure 4: Small physical model of Enneper surface. It was built using planar straight 1mm plywood laths, slotted together. It can be made flat but the inbuilt energy in combination with the structure being a mechanism makes it transform into an average shape between the purely geometrical and flat state[9].

4 Prepatory studies

The Enneper surface was chosen due to that it is an anticlastic shape which reminds of shell form which also creates a defined interior space. The openings also create a potential exterior space connected to the structure. The symmetry would also ensure a reduced number of unique elements which speeds up the production process and lessening the probability of errors.

Going further the final dimensions of the grid and the structure would be set to fit the exhibition, creating a space. Also, further analysis of the individual laths and the behaviour during erection and the in-plane. A time-efficient fabrication procedure must also be ensured along with generation of necessary drawings and production data.

The laths were chosen to be 100 mm wide and made from six mm thick birch plywood boards. To create the slots a method was designed using a drill and regular chop saw set to cut. By pre-calibrating the saw this method would not require much prior experience in carpentry. When...
evaluating the bending behaviour of these laths it became clear that the slots would attract a lot of stresses causing them break unexpectedly. The limiting curvature was therefore hard to predict and the results varied from decent to bad[10]. Therefore, a test-rig was made where the curvature was plotted, and the laths tested[11]. It was observed that by weakening the lath, i.e. making more cuts, the stress distribution increasing the curvature limit. The test-rig was essential during the design process as well as during the workshop evaluating laths in areas with high curvature.

One of the unknowns was how the structure, being a mechanism with little in-plane stiffness, would perform and behave. The unknowns in the analysis was the quality of the manually made elements and the actual kinematic behaviour in the nodes. The structure was modelled FEM-models were made using different conditions in the nodes as well with and without active bending accounted. The results were compared to the small-scale model to forward with the design. During these exercises, with the presumptions that it would be indoors and temporary, the structure seemed to have enough stiffness in the plane to not cause a collapse, even though it was still believed to be quite flexible. Therefore, no bracing was designed since it would complicate the production.

To simulate the erection the small-scale model was compared to a simulation of continuous spline elements using the dynamic relaxation method[12]. At this stage it was interesting to know both the amount of force necessary and the best position to avoid failure or collapse. Since the numerical method was using rods the physical model seemed more accurate and it was easier orchestrate different possible scenarios. The worst case seemed to be when flat since then the laths would have maximum curvature and no twist, meaning the laths will be relieved some of its stresses while being erected.

To generate the final geometry of the grid a custom script for constructing asymptotic lines was implemented in the parametric environment using Euler’s theorem, equation 1. The grid layout was adjusted using a heuristic approach. This model would be used to construct all the production drawings. The necessary joints were placed in areas with low curvature. Since the geometry would need to be flat initially slots requires to be slightly bigger than in the erected state. By generating a flat configuration of the geometry digitally each slot in the lath could be individually designed based on the nodal angles in the flat state. To avoid to many drills three different slots sizes were chosen. The entire structure was built from nine laths that was repeated six times and were built from thirty elements. In plan the structure took the shape roughly like a six sided polygon with three sides of five meter and three of four meter. The
height in the middle was designed such that one could stand upright in the centre.

![Image](image.png)

Figure 6: a) The asymptotic gridshell design based on a Enneper surface of 3-fold symmetry. b) The thirty drawings needed to produce all the laths.

5 Workshop

The workshop started with two lectures to prepare the students. The first covering the concept with the underlying theory from differential geometry. In the second lecture the design process was described along with a tutorial through the parametric model[13], which generates the geometry and all production drawings associated. The lectures ended with describing the different workstations for fabricating the laths as well as the base plate to anchor the structure. Each student was then responsible to signing up before the production process would start.

![Image](image.png)

Figure 7: Fabrication of the laths using paper templates a) drilling holes for the slots and the joints. A paper template was printed aid this process b) The slots were finished using a pre-calibrated chop saw, cutting halfway through the width of the lath c) The laths were assembled into full length using two m6 bolts and nuts.

The first day of construction aimed to fabricate all structural elements. The laths were cut up in the correct width in the wood-workshop and transported into exhibition hall for final processing, figure 7a)-c). In the big hall four groups of four people were assigned to process them, each equipped with two drills and a chop saw, which was pre-calibrated for making the slots. Two groups shared a separate chop saw for cutting the lengths. In order to make the process run smoothly paper templates printed out from the parametric model. When all the elements were drilled and cut the rest of the students assembled them into their full length. The base plates were milled using the CNC in the wood-workshop and consisted of two layers of 18 mm birch plywood which were glued together.
The second day started by connecting the baseplates together into a frame and assembling all the laths into a flat grid, see figure 8 and [14]. A computer model visualizing the structure flat was a great during this process. Strings were tied around the nodes to avoid separation during erection. When assembled the flat grid was lifted onto trestles so that it could pushed into along the edges into shape, as in figure 5. The structure was pushed using only manual power to be able to push it slowly and with care, due to the brittle nature of the laths. When pushed into the correct position the laths were fixed in milled cavities in the base plates. Three cables going was connected through a flying steel ring in middle to reduce the displacement in the openings. The workshop ended with a seminar summarizing the workshop and lectures from the invited guests. In the evening structure was used for a book release party with students from the entire Architecture and Engineering program.

![Figure 8: a) connecting the laths into a flat grid shown in b). c) - d) the structure was erected by pushing the structure with manual force, photo: Brobäck, Ivarsson & Skarby. [14]](image)

### 6 Discussion

![Figure 9: The mechanism in the system was apparent and triggered by motions in the tangent plane. While in motion the rigid parts system could be read, three triangles along the boundaries of the baseplates, coloured blue in this picture. More investigations could be done to find strategies the most optimal places to lock the system using a minimum amount of bracing.](image)
The structure was built on time, it did not collapse and there were no major incidents during the production meaning that the preparations were enough. The reception from the students was overall very positive and one felt they had taken ownership of the structure during these two days. In the course evaluation the course received an average rating of 4.8 out 5 and the workshop was mentioned, both this year and prior year, as a very good segment and exercise.

The built-in mechanism in the structure is necessary for the erection but disadvantageous in its 3-dimensional form. An optional seminar for the students was arranged to discuss how these mechanisms effect the in-plane stiffness and if there are simple strategies increase to lock these mechanisms, without bracing the entire system. The mechanism was evident and could be activated through motions in the tangent plane. While in motion one could read three triangles, coloured blue in figure 9, which was stiff and rigid due the fixed boundary along the baseplates. Adding three cables to the system, going cross each opening from each base plate, the stiffness the tangent plan was noticeably increased. This could be something to investigate further for future workshops in which the structure utilizes a mechanism for the erection.

References