CHALMERS
UNIVERSITY OF TECHNOLOGY

An additive noise modeling technique for accurate statistical study of residual RF hardware impairments

An Additive Noise Modeling Technique for Accurate Statistical Study of Residual RF Hardware Impairments

Mohammad Hossein Moghaddam
Department of Electrical Engineering
Chalmers University of Technology
Gothenburg, Sweden
mh.moghaddam@chalmers.se

Sina Rezaei Aghdam
Department of Electrical Engineering
Chalmers University of Technology
Gothenburg, Sweden
sinar@chalmers.se

Thomas Eriksson
Department of Electrical Engineering
Chalmers University of Technology
Gothenburg, Sweden
thomase@chalmers.se

***Abstract*—Hardware impairments are the inevitable limiting factors in radio frequency communication systems, and in particular in mm-wave, the impairments can severely affect system performance. In this paper, we propose an additive noise modeling technique for modeling and analyzing the residual hardware impairments, more accurately than previously done in the literature. We analyze the effects of joint residual phase noise and IQI in both transmitter and receiver by using additive noise modeling as a representation method and indicate how other impairments can be described in the same framework. We derive the signal to distortion plus noise ratio (SDNR) for both the joint and the individual effects of impairments and validate the formulations with simulations which also acknowledge the usefulness of the additive noise modeling as a mean for accurate hardware impairments study.**

***Index Terms*—Hardware impairments, residual phase noise, IQI, additive noise modeling, variance of error.**

I. Introduction

According to the ever-increasing demands for achieving higher data rates and higher bandwidth in next generation of wireless networks, the concerns over hardware impairments have got more attention as one of the major limiting factors in communication system designs, and several projects are now contributing to this topic [1]. Relevant hardware impairments are e.g., non-linear distortions in the power amplifier, phase noise (PN), in-phase and quadrature imbalance (IQI), timing jitter, and carrier frequency offset (CFO). To handle the destructive effects of these impairments and compensating the residual effects, several calibrations should be carried out in various parts of the system. For designing and evaluating these compensation algorithms, the system performance needs to be analyzed which can be made by modeling the impairments, and then evaluating the performance in terms of impairment parameters and residual effects. Such calibration/compensation analysis in communication systems, is even more relevant in mm-wave communication due to the more destructive effects of hardware impairments in these higher bands [1].

This work is funded by European union horizon 2020 research and innovation program under the MARIE SKODOWSKA-CURIE agreement No 721732 for Silika project, www.silika-project.eu

Modeling and analysis of hardware impairment effects on radio frequency (RF) communication systems are done in the literature within several different settings. Regarding the statistical hardware modeling, in [2] a statistical model is proposed for residual transmitter RF impairments, that describes the sum of all impairments as an independent and identical distribution (i.i.d) additive Gaussian noise, which is a general interpretation based on central limit theorem. In a similar approach, in [3] a MIMO system which is affected by residual transmit RF impairments is considered, and achievable rates are calculated for such a system. In a more accurate approach, authors in [4] use behavioral modeling of hardware impairments and the Bussgang theorem [5] for separating distortion from signal, and derive an approximate aggregate statistical model for the distortions and apply it to a MIMO-OFDM system. In [6], the joint effect of residual phase noise and IQI is studied focusing on the error vector magnitude (EVM) of single-carrier, linear, and memory-less modulated signals, such as phase-shift keying and quadrature amplitude modulation (QAM). In [7], the authors analyze the impact of joint IQI and PN on the performance of beam-forming OFDM direct-conversion transceivers by quantifying the normalized mean square error (NMSE). In [8] and [9], the effect of hardware impairments are studied for single-carrier scenarios and it is analytically proved that single-carrier transmission is more robust to hardware impairments.

In this paper, we derive an accurate additive noise modeling for hardware impairments which is in general able to provide a term-by-term description for the individual and combined effects of hardware impairments. The proposed method relies on considering accurate behavioral models for different impairments and employing tools such as Taylor expansion to describe the received signal as the sum of a desired signal and a distortion (which is uncorrelated with the desired signal). This framework is signal-agnostic and it can be employed for both single-carrier direct-conversion transceivers or OFDM transceivers. As a case study, we apply this modeling method to a system impaired by residual PN and IQI and demonstrate its usefulness in performance evaluation and identification of the dominant source of distortions.

Our proposed method improves on the existing statistical hardware impairment studies, e.g., [2]–[4], in two ways: 1) By taking into account the behavioral models corresponding to different hardware impairments, the proposed modeling method facilitates a more accurate characterization of the individual and combined residual impairments. 2) This model provides a term-by-term description of different components of the distortion which allows for determining the dominant source of errors.

The paper is organized as follows. In Section II, we illustrate additive noise modeling with a case study of joint residual phase noise and IQI. In Section III, we validate the analytic study with simulations. In Section IV, we explain how this framework can be applied on other hardware impairments and finally, the paper is concluded in Section V.

II. Additive Noise Modeling and Analysis

A. Additive Noise modeling of Residual Phase Noise and IQI

In RF communication systems, by assuming non-frequency selective IQI and residual phase noise, the up-converted signal with frequency of f_{T} can be written as [10]:

$$x'(t) = e^{j(2\pi f_{\mathrm{T}} t + \varphi_{\mathrm{T}}(t))}(x(t) + \alpha_{\mathrm{T}} x^*(t)), \tag{1}$$

where α_{T} indicates the impact of IQI in the transmitter, $\varphi_{\mathrm{T}}(t)$ stands for the transmitter phase noise and $x(t)$ is a complex symmetric base-band signal ($x(t)$ and $x^*(t)$ are uncorrelated). At the receiver side (RX), by considering an ideal non-frequency-selective channel with complex channel coefficient A for the base-band received signal, and down-conversion oscillator frequency of f_{R}, we have:

$$y(t) = e^{j(-2\pi f_{\mathrm{R}} t + \varphi_{\mathrm{R}}(t))}(Ax'(t) + w(t) + \alpha_{\mathrm{R}}(Ax'(t) + w(t))^*), \tag{2}$$

where α_{R} indicates the impact of IQI in the receiver, $\varphi_{\mathrm{R}}(t)$ indicates the receiver phase noise and $w(t)$ represents the channel additive noise. Now if we consider perfect frequency offset estimation ($f_{\mathrm{R}} = f_{\mathrm{T}}$), we can rewrite the above formulation as:

$$y(t) = Ae^{j\varphi_{\mathrm{PN}}(t)}x'(t) + w(t) + \alpha_{\mathrm{R}}(A^* e^{-j\varphi_{\mathrm{PN}}(t)}x'(t)^* + w^*(t)). \tag{3}$$

where $\varphi_{\mathrm{PN}}(t)$ is the residual phase noise remained in the system after compensation of phase noise in TX and RX. The equation (3) is a nonlinear function of $\varphi_{\mathrm{PN}}(t)$, but assuming that the residual phase noise is small, we can accurately approximate it with a Taylor expansion[1]:

$$\begin{aligned} y(t) = {} & Ax(t) + A^*\alpha_{\mathrm{T}}{}^*\alpha_{\mathrm{R}}x(t) + A\alpha_{\mathrm{T}}x^*(t) + jA\varphi_{\mathrm{PN}}(t)\alpha_{\mathrm{T}}x^*(t) \\ & + A^*\alpha_{\mathrm{R}}x^*(t) - jA^*\alpha_{\mathrm{R}}\varphi_{\mathrm{PN}}(t)x^*(t) + jA\varphi_{\mathrm{PN}}(t)x(t) \\ & - jA^*\alpha_{\mathrm{T}}{}^*\alpha_{\mathrm{R}}\varphi_{\mathrm{PN}}(t)x(t) + \alpha_{\mathrm{R}}w^*(t) + w(t), \end{aligned} \tag{4}$$

where $x(t)$, $\varphi_{\mathrm{PN}}(t)$ and $w(t)$ are stochastic zero mean and independent random variables. The first two terms contribute to the desired signal $x(t)$, and the rest, uncorrelated with $x(t)$, are considered as distortion terms. Equation (4) clearly indicates the severity of each impairment in the distortion. We can define the error (distortion) term (Δ) and the desired term (Λ) as follows:

$$\begin{aligned} \Delta = {} & jA\varphi_{\mathrm{PN}}(t)x(t) - jA^*\alpha_{\mathrm{T}}{}^*\alpha_{\mathrm{R}}\varphi_{\mathrm{PN}}(t)x(t) + A\alpha_{\mathrm{T}}x^*(t) \\ & + jA\varphi_{\mathrm{PN}}(t)\alpha_{\mathrm{T}}x^*(t) + A^*\alpha_{\mathrm{R}}x^*(t) - jA^*\alpha_{\mathrm{R}}\varphi_{\mathrm{PN}}(t)x^*(t) \\ & + \alpha_{\mathrm{R}}w^*(t) + w(t), \end{aligned} \tag{5}$$

$$\Lambda = Ax(t) + A^*\alpha_{\mathrm{T}}{}^*\alpha_{\mathrm{R}}x(t), \tag{6}$$

where according to the assumptions on $x(t)$, $\varphi_{PN}(t)$, and $w(t)$, the desired term and error term are uncorrelated. The error term (Δ) is zero mean, so we have:

$$\begin{aligned} \sigma_\Delta^2 = {} & |A|^2\sigma_{\varphi_{\mathrm{PN}}}^2\sigma_x^2 + |A|^2|\alpha_{\mathrm{T}}|^2|\alpha_{\mathrm{R}}|^2\sigma_{\varphi_{\mathrm{PN}}}^2\sigma_x^2 + |A|^2|\alpha_{\mathrm{T}}|^2\sigma_x^2 + \sigma_{\mathrm{N}}^2 \\ & + |A|^2|\alpha_{\mathrm{T}}|^2\sigma_{\varphi_{\mathrm{PN}}}^2\sigma_x^2 + |A|^2|\alpha_{\mathrm{R}}|^2\sigma_x^2 + |A|^2|\alpha_{\mathrm{R}}|^2\sigma_{\varphi_{\mathrm{PN}}}^2\sigma_x^2 \\ & + |\alpha_{\mathrm{R}}|^2\sigma_{\mathrm{N}}^2 - 4\Re(A^2\alpha_{\mathrm{T}}\alpha_{\mathrm{R}}{}^*\sigma_{\varphi_{\mathrm{PN}}}^2\sigma_x^2) + 2\Re(A^2\alpha_{\mathrm{T}}\alpha_{\mathrm{R}}{}^*\sigma_x^2). \end{aligned} \tag{7}$$

Similarly, the variance of the desired term can be obtained using (6) as:

$$\sigma_\Lambda^2 = (|A|^2 + |A|^2|\alpha_{\mathrm{T}}|^2|\alpha_{\mathrm{R}}|^2 + 2\Re[A^2\alpha_{\mathrm{T}}\alpha_{\mathrm{R}}{}^*])\sigma_x^2. \tag{8}$$

Equations (7) and (8) provide a clear picture of how different impairments contribute to the desired signal and the distortion. This allows for an accurate characterization of the impact of different hardware impairments (residual phase noise and IQI in this particular example) on the performance of the system. Furthermore, by studying these equations, we gain an immediate insight on how different impairments can amplify each other and more importantly, we can determine the dominant sources of distortion (as we will do in our numerical examples in Section III). To collect the effects of all components together and tracking them more easily we use signal to distortion plus noise ratio (SDNR) as a

[1] This is an accurate approximation for oscillators [11].

performance metric for further discussions, defined as:

$$\text{SDNR} = \sigma_\Lambda^2 / \sigma_\Delta^2. \tag{9}$$

This metric also gives a direct insight to the capacity and can be used in capacity analysis, too, but in this paper we don't analyze the capacity. In the next section we will evaluate the effect of impairments based on analyzing this metric in simulation rounds.

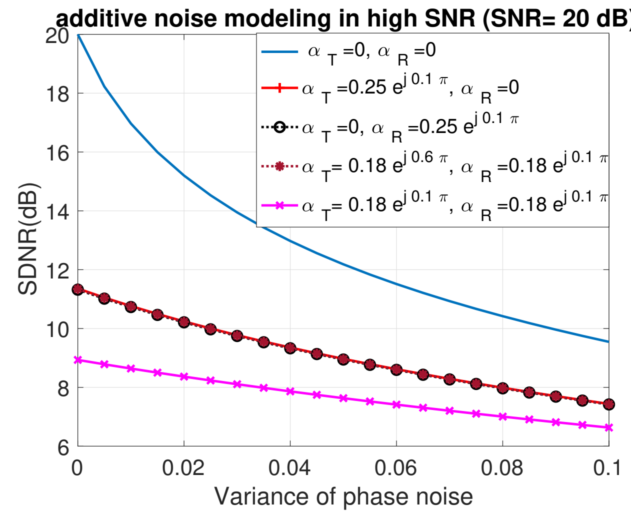
III. Simulation Results and Numerical Analysis

The main message in this paper is the Taylor expansion technique, based on accurate behavioral models, to describe hardware impairments, as discussed earlier. However, in this section we illustrate the power with the proposed technique to understand and qualitatively and quantitatively evaluate the effects of impairments, by case studies.

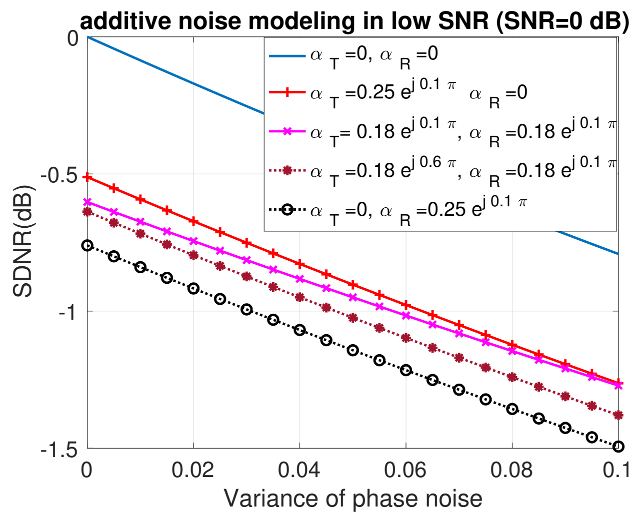
For numerical analysis we have made simulations for several scenarios to accurately indicate the effects of applying each impairment parameter. We study the effect of IQI in terms of α_T and α_R, and the effect of residual phase noise in terms of $\sigma^2_{\varphi_\text{PN}}$. We evaluate the SDNR based on equation (9) as the performance metric in all of the simulations. We define two scenarios, high-SNR with SNR=20 dB and low-SNR with SNR = 0 dB. In all of the simulations, we have applied normalization to the formulations to preserve the total power and to make sure that no energy is added to the system by impairment effects. For a fair comparison, we also keep the total power of IQI constant, so we will consider the following constraint in all of our simulations:

$$|\alpha_\text{T}|^2 + |\alpha_\text{R}|^2 = \gamma^2 \tag{10}$$

where γ is a constant.



(a)

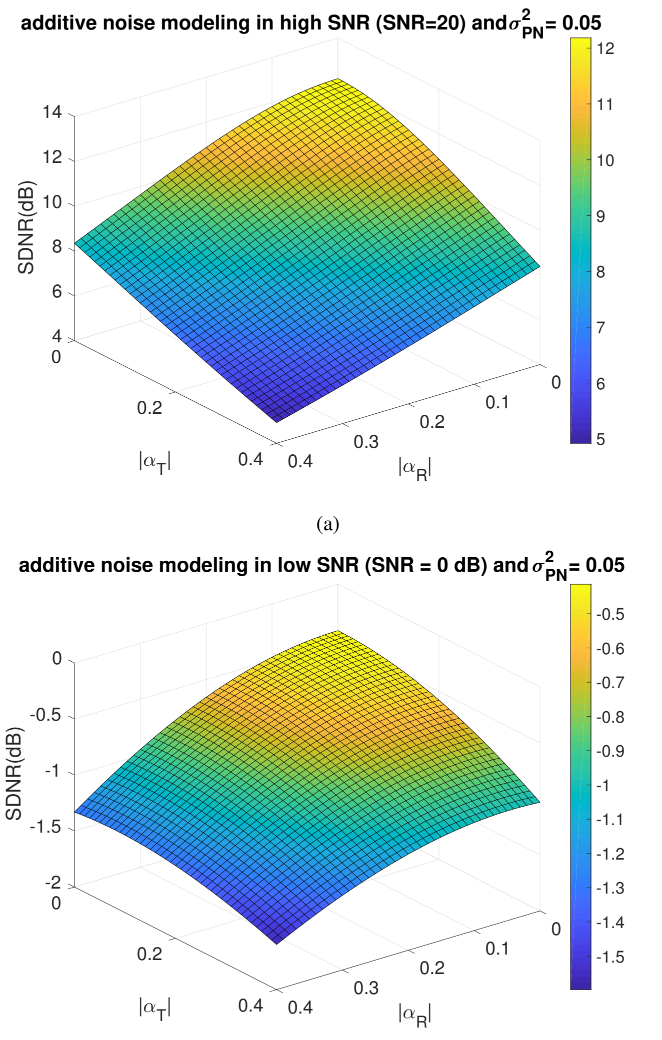


(b)

Fig. 1: SDNR vs. variance of phase noise in: a) high and b) low SNR regimes

In Fig.1.a and b, the SDNR is represented versus variance of residual phase noise with different settings of IQI for both high-SNR (Fig.1 (a)) and low-SNR (Fig. 1 (b)). Comparing the cases when we have IQI in TX (red curves marked with plus) or at the RX (black curves marked with circle), we can see that in the high-SNR regime both curves are completely on top of each other, but in the low-SNR regime the RX IQI is more serious. This is due to the term $(\alpha_\text{R}^2\sigma_\text{N}^2)$ in equation (7), which is the contribution of additive white Gaussian noise. For the case of having IQI in both TX and RX (with the sum IQI power the same as in the one-sided cases), we have considered two different scenarios when the phase difference (η) between α_T and α_R is 0 (magenta curve marked with cross) or $\pi/2$ (brown curve marked with asterisk) we explain more on this parameter on Fig. 4. In the low-SNR regime, we can see that both of them have better performance in comparison to the case of having IQI only in the RX. This is due to the fact that in equation (4) and more precisely on the numerator and the denominator of SDNR (given in (7) and (8)), the effect of α_T and α_R appears on both the desired signal and the distortion signal as you can see the portions on each equation. As seen in Fig. 1, for both $\eta = 0$ and $\eta = \pi/2$, the performance is similar in high-SNR regime which is due to dominant role of signal power. In the low-SNR regime, for $\eta = 0$, we see higher SDNR by increasing the variance of residual phase noise. This is due to the term $(-4\Re(A^2\alpha_\text{T}\alpha_\text{R}^*\sigma^2_{\text{PN}}\sigma_x^2))$ in equation (8). This term is proportional to $\cos(\eta)$ which is maximized for $\eta = 0$ ($\cos(\eta) = 1$) and then by increasing the variance of residual phase noise, the denominator in equation (10) subsequently decreases and the SDNR become higher.

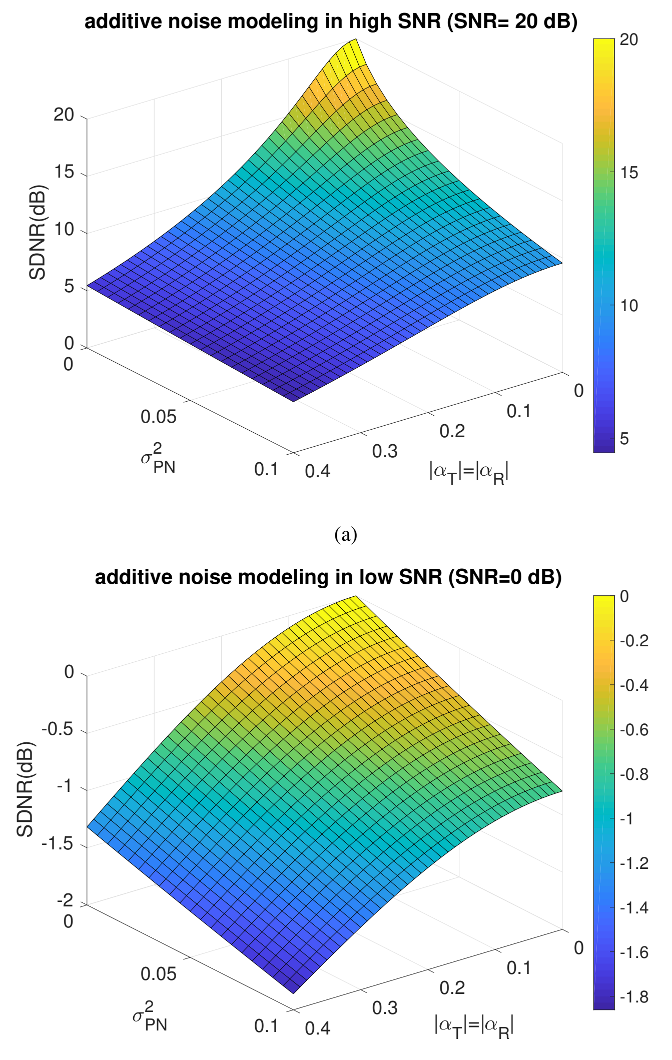
In Fig.2, the SDNR is plotted versus $|\alpha_\text{T}|$ and $|\alpha_\text{R}|$ for both high-SNR and low-SNR regimes with the variance of residual phase noise 0.001. In the low-SNR regime, the effect of $|\alpha_\text{R}|$ is more severe than the effect of $|\alpha_\text{T}|$, but in the high-SNR regime, where the signal power is dominant, the effect of both are quite similar. In Fig. 3, the SDNR is plotted versus the IQI and the phase noise. We see that, when the magnitude of the IQI components are low, the effect of the phase noise is more severe, but when the IQI is high, the effect of phase noise



(a)

(b)

Fig. 2: SDNR vs. IQI in: a) high and b) low SNR regimes both with $\sigma_{PN}^2 = 0.001$



(a)

(b)

Fig. 3: SDNR vs. IQI and Variance of Phase Noise in: a) high and b) low SNR regimes

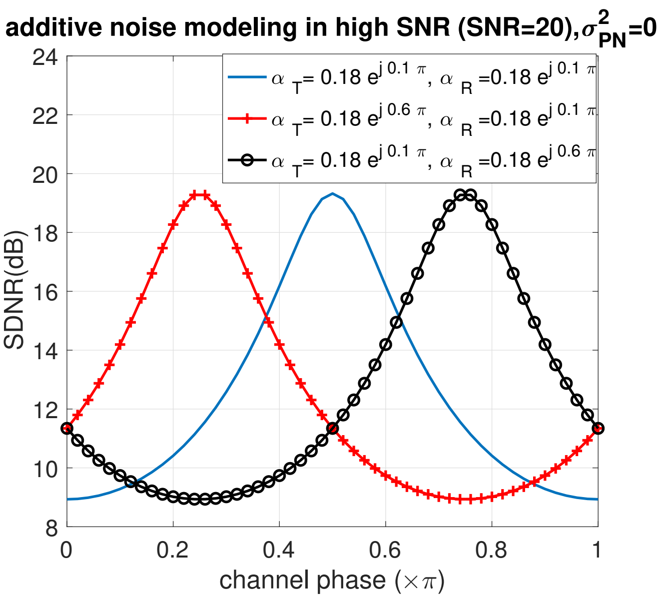
is not that much severe. We can also see a little threshold effect for the IQI, such that the system even at high SNR can tolerate a small amount of IQI with small effect, but for phase noise there is no threshold effect. These two figures give us a clear sense of the dominant impairment in different scenarios of high-SNR and low-SNR.

In Fig. 4, we study the effect of the channel coefficient A. The channel coefficient is not part of the hardware impairments, but since it has an important role on the SDNR metric as in equations (7)-(9), it deserves a separate study. In Fig. 4.a, the SDNR is depicted versus channel phase in high-SNR regime for different values of η ($\eta = -\pi/2,\ 0,\ \pi/2$). Interestingly, as we see in the figure, the SDNR varies with the channel phase and relative IQI phase values. In Fig. 4.b, the SDNR versus the channel phase is depicted for different variances of residual phase noise, and we see that the SDNR decreases by increasing the variance of residual phase noise.

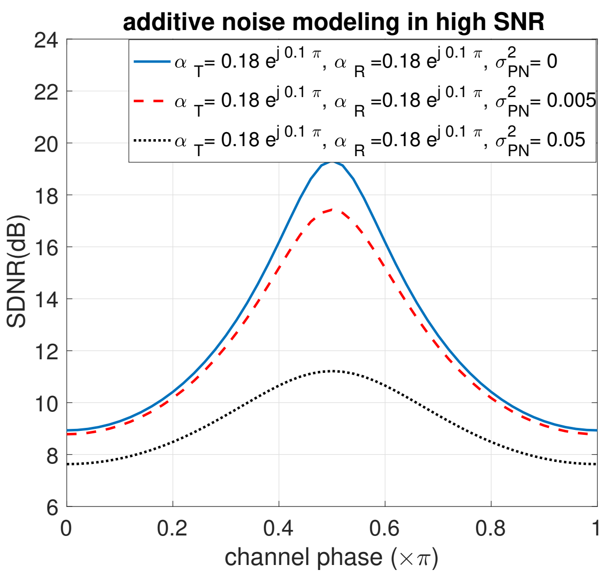
IV. Additive noise modeling for other impairments

The additive noise modeling framework can also be applied to other impairments in the same way as we did in (4)-(9). As we talk about RF communications, other effects of hardware impairments such as timing jitter, power amplifier non-linearity, CFO, and DC offset should be considered as they can degrade the system performance, considerably. In this section we present the additive noise modeling for some of these impairments without proof. More details on proofs and also further analytical studies will be published in an extended transaction version of this paper.

In case of timing jitter, which can introduce inter-symbol interference (ISI), we have:



(a)



(b)

Fig. 4: SDNR vs. channel phase in: a) different values of η and b) different variances of residual phase noise

$$y_k = \underbrace{Ax_k f(\tau + \delta_0)}_{\text{Desired-signal}(\Lambda)} + \underbrace{\sum_{l=k-M, l\neq k}^{k+M} Ax_l g((k-l)T + \delta_0 + \tau) + w_k}_{\text{ISI signal and noise}(\Delta)}, \tag{11}$$

where $\delta_0 + \tau$ is the timing jitter, $g(t)$ is the matched filter function and M is the number of effecting taps defined according to the pulse-shaping filter design, and f is given by:

$$f(t) = \int_{-\infty}^{\infty} g(t - \upsilon) g(\upsilon) d\upsilon. \tag{12}$$

We can follow the same procedure as we do in (7)-(9) to define the SDNR metric and analyze the impairments using additive noise modeling. Such derivations can also be made for CFO and power amplifier nonlinearity by exploiting additive noise modeling technique.

It is also possible to apply the additive noise modeling framework with different settings on multiple-input multiple-output (MIMO) scenarios for both the point-to-point MIMO (such as back-haul links) and the multi-user MIMO (cellular networks) to analyze the hardware impairment effects. At one hand, many effects can be described as multiple SISO imperfections, while other depends quite a lot on the MIMO setting. We leave this for future work.

V. Conclusion

In this paper, we have introduced a method for statistical study of hardware impairment effects in RF communications by exploiting an additive noise modeling technique. As the main advantage in comparison with other statistical methods, the proposed modeling can accurately represent the effects of hardware impairments by providing a term-by-term description of their individual and combined effects. As a case study, we have exploited additive noise modeling for joint residual phase noise and IQI and we have discussed how similar steps can be utilized for studying the impacts of other hardware impairments such as timing jitter, CFO and power amplifier nonlinearities.

References

[1] "http://silika-project.eu/," .

[2] C. Studer, M. Wenk, and A. Burg, "MIMO transmission with residual transmit-RF impairments," in *Smart Antennas (WSA), 2010 International ITG Workshop on*. IEEE, 2010, pp. 189–196.

[3] X. Zhang, M. Matthaiou, M. Coldrey, and E. Björnson, "Impact of residual transmit RF impairments on training-based MIMO systems," *IEEE Trans. Commun.*, vol. 63, no. 8, pp. 2899–2911, 2015.

[4] S. Jacobsson, U. Gustavsson, G. Durisi, and C. Studer, "Massive MU-MIMO-OFDM uplink with hardware impairments: Modeling and analysis," *arXiv preprint arXiv:1812.02078*, 2018.

[5] J. J. Bussgang, "Crosscorrelation functions of amplitude-distorted gaussian signals," 1952.

[6] A. Georgiadis, "Gain, phase imbalance, and phase noise effects on error vector magnitude," *IEEE Trans. Veh. Technol*, vol. 53, no. 2, pp. 443–449, 2004.

[7] R. Hamila, Ö. Özdemir, and N. Al-Dhahir, "Beamforming OFDM performance under joint phase noise and I/Q imbalance," *IEEE Trans. Veh. Technol*, vol. 65, no. 5, pp. 2978–2989, 2016.

[8] B. Selim, P. C. Sofotasios, S. Muhaidat, G. K. Karagiannidis, and B. Sharif, "Performance of differential modulation under RF impairments," in *Proc. IEEE Int. Conf. Commun.(ICC)*, 2017, pp. 1–6.

[9] P. Yang, Y. Xiao, Y. Li. Guan, K.V.S. Hari, A. Chockalingam, S.a Sugiura, H.d Haas, M. Di Renzo, C. Masouros, Z. Liu, et al., "Single-carrier SM-MIMO: A promising design for broadband large-scale antenna systems," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 3, pp. 1687–1716, 2016.

[10] G. Fettweis, M. Löhning, D. Petrovic, M. Windisch, P. Zillmann, and W. Rave, "Dirty RF: A new paradigm," *International Journal of Wireless Information Networks*, vol. 14, no. 2, pp. 133–148, Jun 2007.

[11] S. D. Blostein T. Eriksson G. K. Karagiannidis H. Mehrpouyan, A. A. Nasir and T. Svensson, "Joint estimation of channel and oscillator phase noise in mimo systems," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4790–4807, 2012.