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Citation for the original published paper (version of record):
Farjam, T., Themistoklis, C., Wymeersch, H. (2019). Timer-Based Distributed Channel Access in Networked Control Systems over Known and Unknown
Gilbert-Elliott Channels. 2019 18th European Control Conference, ECC 2019: 2983-2989.
http://dx.doi.org/10.23919/ECC.2019.8796177

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Timer-Based Distributed Channel Access in Networked Control Systems over Known and Unknown Gilbert-Elliott Channels

Tahmoores Farjam, Themistoklis Charalambous, and Henk Wymeersch

Abstract— **In this paper, we consider a system consisting of multiple (possibly heterogeneous) decoupled control subsystems which aim at communicating with their corresponding controllers via shared (possibly) time-varying wireless channels. To address the resource allocation problem in a distributed fashion, we propose a timer-based channel access mechanism in which the subsystem with the smallest timer value, in a channel, claims the slot for transmission in that specific channel. The value of the timer is inversely proportional to a cost which is a function of the temporal correlation in the channel variation and the subsystem state. This cost can be calculated individually and does not require explicit communication between the subsystems, since it is based on locally available information only. The temporal correlation in the channel variation may be unknown and, in such cases, each subsystem tries to deduce it via machine learning techniques. The performance of our proposed mechanism is demonstrated via simulations.**

Index Terms— **Wireless networked control systems, distributed channel access, cost of information loss, Gilbert-Elliott channel, Bayesian inference.**

I. INTRODUCTION

The vision of a smart world is to automate whatever is possible, thus offering new applications and services, revolutionizing all aspects of life from politics and economics to cities and factories [1]. It is evident that the improvement of smart devices with advanced sensing, computing and control capabilities makes it possible for our cities, transportation systems, factories and living environments to become more intelligent, energy-efficient, and secure. Typically, such systems are spatially distributed, and communication between smart devices (being sensors, actuators or controllers) is mainly supported by a shared, wireless communication network. These systems are known as Wireless Networked Control Systems (WNCSs); a thorough literature review can be found in a recent survey [2]. The use of a wireless network to connect spatially distributed systems enables flexible architectures with reduced installation and maintenance costs to existing applications, while supporting the development of new applications that would otherwise be impossible.

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The work of T. Charalambous was supported by the Academy of Finland under Grant 317726.

One of the critical challenges in WNCSs is how different dynamical systems, herein called subsystems, access the shared wireless network. Apart from the problem of having limited resources, on one hand there is often no centralized agency to orchestrate the channel access from different subsystems, as it is the case in, e.g., [3]–[8]. On the other hand, wireless channels are unreliable and time-varying. Many of the works in the literature (see, e.g., [9]–[12]) assumed independent and identically distributed (i.i.d.) Bernoulli random variation of the channel state. However, the Bernoulli process is memoryless and it fails to capture the temporal correlation of the variation of channels. The Gilbert-Elliott (GE) channel model [13], [14] is a better model since it accounts for temporal correlation and it allows burst mode in communication channels. The GE model has been considered when studying the stability condition for remote estimation [15]–[17] and more recently for sensor data scheduling [18]. Nevertheless, apart from [18] in which a centralized scheduling mechanism is proposed, no distributed mechanism for WNCSs has considered the temporal correlation of the variation of channels.

In this work, we propose a distributed channel access mechanism for WNCSs, in which each subsystem employs a timer for accessing the channel. The timer for each subsystem is associated with the cost imposed by that subsystem on the entire system and the channel quality; a variation of this mechanism for wired Networked Control Systems (NCSs), in which the timer is a function of the cost only has been proposed by the authors in [19]. This idea is extended to the case of multiple fading, possibly unknown, wireless channels with temporal correlation, where successful reception of transmitted data packets is not guaranteed. The main contributions of this paper are the following:

- First, the distributed channel access mechanism is considered for wireless channels modeled using the GE model and for which the model parameters (i.e., transition probabilities) are known *a priori*.
- Second, on several occasions, the parameters of the GE channel model are not known *a priori*. For this case, we develop an online learning scheme with which the subsystems learn the GE parameters online and, thus, capture the temporal correlation of channel variation and adjust their timers accordingly.

The remainder of the paper is organized as follows. In Section II, we provide the system model and preliminaries necessary for the development of our results. In Section III, we describe the proposed distributed channel access mechanism. The mechanism for known and unknown parameters of the GE channel model is described in Sections IV and V, respectively. In Section VI we demonstrate its performance and discuss its limitations. In Section VII, we draw conclusions and discuss future directions.

II. System Model and Preliminaries

We consider WNCSs consisting of N subsystems that share a wireless channel; see, for instance, Fig. 1 depicting a simple WNCS consisting of 2 subsystems that share the same wireless channel.

A. Subsystems

The dynamics of each subsystem $i \in \{1, \dots, N\}$ are modeled by a linear time-invariant stochastic process governed by the following discrete-time state-space representation:

$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + w_{i,k}, \qquad (1a)$$
$$y_{i,k} = C_i x_{i,k} + \nu_{i,k}, \qquad (1b)$$

where $x_{i,k} \in \mathbb{R}^{n_i}$, $y_{i,k} \in \mathbb{R}^{p_i}$ and $u_{i,k} \in \mathbb{R}^{m_i}$ are the local states of the plant $\mathcal{P}_i$, measurements taken by wireless sensor $\mathcal{S}_i$ and inputs of the actuators computed by controller $\mathcal{C}_i$ at time step k, respectively. Moreover, the stochastic disturbances and measurement noises are assumed to be i.i.d. random sequences described by $w_{i,k} \sim \mathcal{N}(0, W_i)$ and $\nu_{i,k} \sim \mathcal{N}(0, V_i)$, respectively.

We consider minimizing the standard quadratic cost function over the infinite horizon as the objective and design the controller and estimator units accordingly. This cost is defined as

$$J_0 = \mathbb{E}\left\{ \lim_{\kappa\to\infty} \frac{1}{\kappa} \sum_{k=0}^{\kappa-1} \sum_{i=1}^{N} \left(x_{i,k}^T Q_i x_{i,k} + u_{i,k}^T R_i u_{i,k} \right) \right\}, \quad (2)$$

where $Q_i, R_i \succ 0$ are wighting matrices of appropriate dimensions. The control actions for minimizing this cost are computed by $u_{i,k} = L_i \hat{x}_{i,k|k}$, where L_i is a stabilizing feedback matrix of proper dimensions which can be determined independent of the scheduling. Moreover, $\hat{x}_{i,k|k}$ denotes the *a posteriori* state estimates provided by the Kalman filter. Due to the Gaussian nature of disturbances and noises, this setup provides the minimum mean square estimate. Let $P_{i,k|k-1}$ and $P_{i,k|k}$ denote the *a priori* and *a posteriori* covariance matrices at k, respectively. Then, the state estimates can be calculated recursively by the following set of equations [20]

$$\hat{x}_{i,k|k-1} = (A_i + B_i L_i)\hat{x}_{i,k-1|k-1}, \qquad (3a)$$
$$P_{i,k|k-1} = A_i P_{i,k-1|k-1} A_i^T + W_i, \qquad (3b)$$
$$K_{i,k} = P_{i,k|k-1} C_i^T \left(C_i P_{i,k|k-1} C_i^T + V_i \right)^{-1}, \qquad (3c)$$
$$\hat{x}_{i,k|k} = \hat{x}_{i,k|k-1} + \theta_{i,k} K_{i,k}(y_{i,k} - C_i \hat{x}_{i,k|k-1}), \qquad (3d)$$
$$P_{i,k|k} = (I - \theta_{i,k} K_{i,k} C_i) P_{i,k|k-1}, \qquad (3e)$$

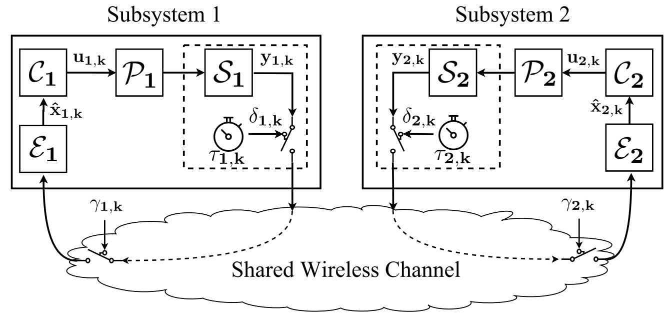

Fig. 1. Example of the WNCS layout where two subsystems compete to access a single wireless channel. $\mathcal{P}_i$ represents the plant of subsystem $i \in \{1, 2\}$, $\mathcal{S}_i$ its sensor, $\mathcal{E}_i$ its estimator, and $\mathcal{C}_i$ its controller. Note that the timer is embedded in the sensor which is capable of local computations.

where $\theta_{i,k} \in \{1, 0\}$ represents the impact of scheduling on computations and is set to 1 only if measurements are received at k.

B. Shared wireless network

We focus on the case of wireless networks where a time-slotted medium access protocol is implemented. Let $\delta_{i,j,k} \in \{0, 1\}$ describe the transmission status as

$$\delta_{i,j,k} = \begin{cases} 1, & y_{i,k} \text{ is transmitted on channel } j, \\ 0, & \text{otherwise.} \end{cases}$$

We assume that at any given time, each subsystem can only transmit on one channel $j \in \{1, \dots, M\}$ or, equivalently,

$$\sum_{j=1}^{M} \delta_{i,j,k} \leq 1, \qquad \forall i, \forall k. \qquad (4)$$

Furthermore, due to the limited capacity of the communication resources ($M < N$), only a limited number of subsystems can transmit successfully at each slot. Moreover, in case two or more subsystems transmit simultaneously on the same channel, collision occurs and the packets are dropped.

Even by implementing a collision-free channel access mechanism, due to the unreliable nature of wireless channels, possibility of packet dropouts exists. As a result, $\delta_{i,j,k} = 1$ does not necessarily lead to successful reception of the sent packet at its destination. Therefore, we define a binary variable $\gamma_{i,j,k}$ to denote the confirmation of packet reception as

$$\gamma_{i,j,k} = \begin{cases} 1, & y_{i,k} \text{ is successfully received on channel } j, \\ 0, & \text{otherwise.} \end{cases}$$

Using this notation, we define $\theta_{i,k}$ as

$$\theta_{i,k} = \begin{cases} 1, & \text{if } \sum_{j=1}^{M} \gamma_{i,j,k} = 1, \\ 0, & \text{otherwise,} \end{cases}$$

C. The Gilbert-Elliott channel

We model the packet dropout process of the channels as a two-state time homogeneous Markov chain, as introduced by Gilbert [13] and Elliott [14]. This model is widely used for describing error patterns in transmission channels since it captures the existence of temporal correlations between channel conditions. Using this two-state model, the channel can be either in a good (G) or bad (B) state. Successful communication over a channel is guaranteed when its state is G, otherwise the transmitted packet is dropped. Denoting state of the channel by s, the transition probabilities are defined as

$$p = \mathbb{P}\{s_k = B | s_{k-1} = G\}, \tag{5a}$$

$$q = \mathbb{P}\{s_k = G | s_{k-1} = B\}. \tag{5b}$$

Therefore, the probabilities of staying in states B and G are $1-q$ and $1-p$, respectively; see Fig. 2.

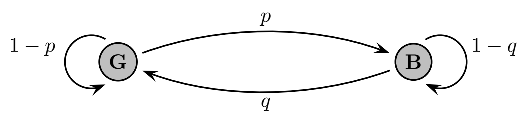

Fig. 2. Two-state Markov chain representing the Gilbert-Elliott model.

The quality of the channel is directly associated with the values of p and q, since a smaller p and a larger q represent a more reliable channel.

III. Distributed Channel Access Mechanism

The distributed resource allocation problem over perfect communication links has been addressed in [19] by introducing a novel channel access mechanism. The proposed method is based on the idea that each subsystem has a local timer, which by appropriate setup, represents the priority for transmission. Here, we slightly modify the proposed method and extend its application to the case where multiple imperfect communication links are available.

Using a similar structure, we assume that every subsystem possesses a separate timer for each available channel $j \in \{1, \ldots, M\}$. For each transmission slot k, the initial value of these local timers, denoted by $\tau_{i,j,k}$, is calculated by

$$\tau_{i,j,k} = \frac{\lambda}{m_{i,j,k}}, \tag{6}$$

where $m_{i,j,k}$ corresponds to a local cost and λ is a constant value. At the beginning of each time slot, the subsystems are synchronized and their timers start. In this setup, λ is identical for all timers and thus their values are inversely proportional to the local cost. Therefore, the timer that expires first corresponds to the largest $m_{i,j,k}$ or, equivalently, the highest priority for transmission.

At a specific time step k, the first timer that reaches zero determines the first claimed channel j^* by subsystem i^* as

$$\{i^*, j^*\} = \arg\min_{i,j} \{\tau_{i,j,k}\}. \tag{7}$$

This subsystem transmits a short duration flag packet on channel j^* immediately, thus informing all other subsystems in the network to stop their timers for this channel and back off. Simultaneously, i^* resets its remaining timers and thus withdraws from competition for the remaining channels and

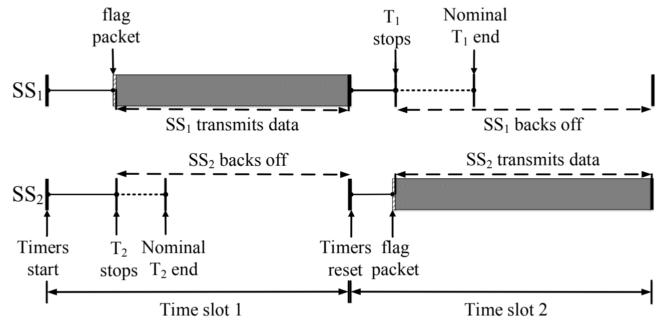

Fig. 3. Example of 2 subsystems competing for one channel in 2 successive time slots. Subsystem 1 (denoted as SS_1) has the largest cost in the first time slot and gets the access to the channel. Subsequently, subsystem 2 (denoted as SS_2), that did not communicate in the first time slot, in the second time slot gets to have a larger cost. Here, $m_{2,1,2} > m_{1,1,2}$ and thus SS_2 gets access to the channel.

starts to transmit on channel j^* without collision. Meanwhile, the rest of the subsystems compete for available resources until all M channels have been allocated for this time slot.

As the duration of the time slot ends, the subsystems are re-synchronized and their timers are updated to new values according to the new $m_{i,j,k}$ and the same procedure is repeated. Given that enough communication resources are available, this method provides collision-free channel access while ensuring stability. Fig. 3 shows a graphical representation of this procedure for this case where two subsystems compete for accessing a single communication channel in two successive time slots.

A. Timer setup

The implementation of this mechanism requires quantification of the parameter $m_{i,j,k}$. In principle, it could be associated with any metric that we choose as a measure for prioritizing transmissions. Here, we define this metric such that it takes into account the control performance as well as the probability of successful transmission.

We adopt the Cost of Information Loss (CoIL), introduced in [10], as the measure of control performance. It can be interpreted as the cost imposed by subsystems in case their measurement updates are not received. For our setup, with the assumption of perfect communication links, CoIL is given by

$$\mathrm{CoIL}_{i,k} = \mathrm{tr}\left(\Gamma_i (P_{i,k|k-1} - P_{i,k|k})\right), \tag{8}$$

where Γ_i is a weighting matrix, and $P_{i,k|k-1}$ and $P_{i,k|k}$ are the *a priori* and *a posteriori* error covariance matrices as defined in (3b) and (3e), respectively.

It is proven in [10] that minimizing the quadratic cost at each time step, is equivalent to granting channel access to subsystems with the largest CoIL. Furthermore, in case of unreliable communication channels, the optimal resource allocation problem (for the *current step*[1]) becomes

$$\max_{\delta_{i,j,k} \in \{0,1\}} \mathrm{CoIL}_{i,k} \bar{q}_{i,j,k} \delta_{i,j,k}, \tag{9}$$

[1]The optimal scheduling of the channels can be obtained by solving the infinite horizon optimization problem; see [21] for details.

where $\bar{q}_{i,j,k}$ denotes the probability of subsystem i transmitting successfully on channel j at time slot k.

The solution to this problem can be provided in a distributed fashion by implementing the timer-based mechanism with the local cost defined as

$$m_{i,j,k} = \mathrm{CoIL}_{i,k}\bar{q}_{i,j,k}. \tag{10}$$

Since the required information for calculating CoIL according to (8) is known locally, by assuming that channel qualities are known, this cost can be computed locally and used in (6) to compute the value of the timer, thus facilitating the distributed implementation of the channel access mechanism. It should be noted that the possibility of multiple links having the same channel statistics has Lebesgue measure zero. However, in the unlikely scenario that the minimum timer is not unique, the subsystem claims one of the best channels randomly, similar to the original method.

IV. Channel access over a known Gilbert-Elliott channel

In this section, for the ease exposition, the subscript for channels have been dropped and we focus on the case of one available wireless channel. However, the results can simply be extended to the case of multiple communication channels.

We assume that the failure rate (transition from G to B) and recovery rate (transition from B to G) of each wireless link, denoted by p_i and q_i, respectively, are known *a priori*. Knowledge of the exact values of these parameters can be exploited for determining the probability of successful transmission over each communication link and thus enable computation of (10).

Let $b_{i,k}$ denote the belief, i.e., the probability that the communication link of subsystem i is in the good state at current time step k. In case this subsystem claims the channel at current transmission slot ($\delta_{i,k} = 1$), according to (5), the belief for the next step becomes

$$b_{i,k+1} = \begin{cases} p_i, & (\delta_{i,k} = 1) \wedge (\gamma_{i,k} = 1) \\ q_i, & (\delta_{i,k} = 1) \wedge (\gamma_{i,k} = 0). \end{cases} \tag{11}$$

However, if no transmission attempt is made ($\delta_{i,k} = 0$), the belief evolves according to

$$b_{i,k+1} = b_{i,k}(1 - p_i) + (1 - b_{i,k})q_i. \tag{12}$$

As a result, the probability of being in state G at the next step can be determined regardless of the transmission status at current state. Consequently, although due to the limited communication resources state of the wireless link is partially observed, the probability of successful transmission can be determined according to (11) or (12). Therefore, the current belief can be regarded as a quantified measure of the channel quality. Hence, this belief is used to rewrite (10) for the general case as

$$m_{i,j,k} = \mathrm{CoIL}_{i,k}b_{i,j,k}. \tag{13}$$

Algorithm 1 shoes how this mechanism is implemented for each subsystem.

Algorithm 1: Timer-based channel access mechanism for subsystem i

Input: number of channels M, GE parameters of each wireless link $p_{i,j}$ and $q_{i,j}$, constant value for timer setup λ

Set initial belief $b_{i,j,1} = 0.5, \forall j \in \{1, \dots, M\}$
for $k = 1, 2, \dots$ **do**
  calculate local cost $m_{i,j,k}$ (13) and start timers $\tau_{i,j,k} = \frac{\lambda}{m_{i,j,k}}$ (6)
  initiate set of flags $\mathcal{F}_i = \{f_{i,j} = 0 | \forall j \in \{1, \dots, M\}\}$
  update $b_{i,j,k+1}$ according to (12)
  while $\sum_{j=1}^{M} f_{i,j} < M$ **do**
    for $j \leftarrow 1$ **to** M **do**
      if $\tau_{i,j,k} \neq 0$ *and timer is running* **then**
        listen for signals
        if *signal is received in channel* j **then**
          freeze $\tau_{i,j,k}$ and set $f_{i,j} = 1$
        end
      else if $\tau_{i,j,k} = 0$ **then**
        send flag signal and set $f_{i,j} = M$
        freeze all running timers
        transmit data on channel j
        re-update $b_{i,j,k+1}$ according to (11)
      end
    end
  end
end

Remark 1: It should be noted that if the state is not observed for a large number of consecutive steps, the belief converges to its stationary distribution, given by

$$b_{i,\infty} = \frac{q_i}{p_i + q_i}. \tag{14}$$

V. Channel access over an unknown Gilbert-Elliott channel

Implementing the timer-based mechanism according to (13) assumes complete knowledge of the transition probabilities of the underlying Markov chain. However, this is a strong assumption and such information is not known *a priori* in practice. This assumption can be relaxed by adopting a Bayesian learning method which maintains a probability distribution over the possible settings of each unknown parameter. In this section, we first formulate our problem as a Bayesian adaptive partially observable Markov decision process. Then, we propose a novel method to adopt a heuristic posterior sampling algorithm in our timer-based mechanism in order to take into account the control performance during the learning phase.

A. Bayesian framework

In the Bayesian approach, an initial prior distribution is assumed over the unknown parameters, and posterior distribution is updated using the Bayes' rule. In our setup, the parameters p and q of wireless links are unknown and due to the limited resources, they are partially observed. These parameters have independent Bernoulli distributions, thus we assume the priors to have independent Beta distributions since it is the conjugate prior for Bernoulli distribution. Let $\Phi = [\phi_1\ \phi_2\ \phi_3\ \phi_4] \in \mathbb{Z}_+^4$, then

$$\mathbb{P}(p; \phi_1, \phi_2) = \frac{p^{\phi_1 - 1}(1-p)^{\phi_2 - 1}}{B(\phi_1, \phi_2)}, \tag{15}$$

$$\mathbb{P}(q; \phi_3, \phi_4) = \frac{q^{\phi_3 - 1}(1-q)^{\phi_4 - 1}}{B(\phi_3, \phi_4)}, \tag{16}$$

where B denotes the Beta function. We assume that the initial priors have a uniform distribution parametrized by $\Phi = [1, 1, 1, 1]$ and easily update the posterior after each observation. We denote the observation at time step k by $O_k \in \{G, B, V\}$ where V corresponds to no transmission attempt and thus no information about the current state. Moreover, the observation history up to k is denoted by O^k.

Example 1: Assume that in the beginning $\Phi = [1, 1, 1, 1]$ which can also be interpreted as $\mathbb{P}(p, q) = 1$. We observe the state transitions for 4 consecutive steps and find $O^4 = \{G, G, G, B\}$. Therefore, the channel has stayed in state G for 3 steps (transition from G to G with probability $1-p$) and then transitioned to the bad state (G to B with probability p). Hence, the posterior count is easily found to be $\Phi = [1+1, 1+3, 1, 1]$.

We should emphasize that the state of the Markov chain evolves independently of the action of each subsystem, since the evolution of the channel conditions are governed by other factors, such as fading and shadowing. Since for every $O_k = \{V\}$ there are two possibilities for the state at that step, we can define all possible state histories of O^k as

$$S(O^k) = \{s^k | s_t = O_t, \forall t \in \{\tau | O_\tau \neq V\}\}. \tag{17}$$

Moreover, we define appearance count denoted by $\Psi(\Phi, S^{k-1}, s_k)$ as the number of state histories that lead to the same posterior count Φ. As a result [22],

$$\mathbb{P}(s_k, p, q | O^{k-1})\mathbb{P}(O^{k-1}) = \sum_{\Phi} \Psi(\Phi, S(O^{k-1}), s_k) p^{\phi_1 - 1}(1-p)^{\phi_2 - 1} q^{\phi_3 - 1}(1-q)^{\phi_4 - 1}, \tag{18}$$

where $\mathbb{P}(O^{k-1})$ is the normalization term. Upon making a new observation O_k, the posterior is updated by

$$\begin{aligned}&\mathbb{P}(s_{k+1}, p, q | O^k)\mathbb{P}(O_k | O^{k-1}) \\ &\quad = \sum_{s_k} \mathbb{P}(s_k, p, q | O^{k-1})\mathbb{P}(s_{k+1}, O_k | s_k, p, q).\end{aligned} \tag{19}$$

It can be deduced from (18) and (19) that if the subsystem transmits over the channel ($O_k \in \{G, B\}$), the total number of possible posteriors does not change during update. However, in case of no available state information ($O_k \in \{V\}$), this number increases by a factor of less than or equal to two. The following example illustrates how the learned belief evolves according to the observations and taken actions.

Example 2: Assuming that no observation has been made at the current time step ($O_k = V$), we try to find the posterior update for $s_{k+1} = G$. This is equivalent to finding the probability of success in the next time step, when no transmission attempt has been made at the current step. Using (19) and then (18) we find

$$\begin{aligned}&\mathbb{P}(G, p, q | O^k)\mathbb{P}(O_k | O^{k-1}) \\ &= \sum_{s_k \in \{G, B\}} \mathbb{P}(s_k, p, q | O^{k-1})\mathbb{P}(G | s_k, p, q) \\ &= \Bigg[\sum_{\Phi} \Psi(\Phi, S(O^{k-1}), G) p^{\phi_1 - 1}(1-p)^{\phi_2} q^{\phi_3 - 1}(1-q)^{\phi_4 - 1} \\ &+ \sum_{\Phi} \Psi(\Phi, S(O^{k-1}), B) p^{\phi_1 - 1}(1-p)^{\phi_2 - 1} q^{\phi_3}(1-q)^{\phi_4 - 1}\Bigg] \\ &/\mathbb{P}(O^{k-1}).\end{aligned} \tag{20}$$

Therefore, the number of possible posterior counts grows in case of no transmission. The maximum growth is when the appearance counts remain unchanged and thus the updated number of possibilities is twice the number of current possibilities. The transition from time t in Fig. 4 illustrates the case where the appearance count is updated and the growth factor is $3/2$.

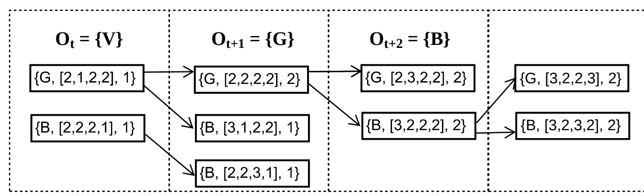

Fig. 4. Graphical representation of Bayesian update for three successive steps where the contents of solid rectangular blocks are $\{s_k, \Phi, \Psi\}$.

B. Online learning through the timer-based mechanism

The parameter $b_{i,j,k}$ in (13) is equivalent to the posterior of subsystem i being in state G over channel j, given the observation history O^{k-1}. However, only a limited number of subsystems transmit at each time step. Furthermore, each of these subsystems can observe the state of only one channel at each time step. Consequently, the number of posterior possibilities grows inevitably and can go to infinity. To reduce the computational complexities that stem from this growth, an approach based on approximate belief monitoring [23] is adopted. More precisely, we adopt a heuristic posterior sampling where only K number of posterior counts with the largest appearance counts are kept for the Bayesian update; see [22] for a more detailed discussion. Hence, the belief can be computed efficiently regardless of the observation history.

The learning algorithm does not inherently resolve the optimal resource allocation problem. However, by implementing the resulting belief in Algorithm 1, the learning outcome improves the overall performance with respect to reducing (2). In the beginning, since the prior distributions are uniform, the observations are made purely with respect to CoIL. As the posteriors are updated, the learned channel parameters are exploited for increasing the probability of successful transmission.

VI. Numerical Results

In this section, we first demonstrate the performance of our algorithm in learning the transition probabilities associated with GE channel model. Then the effect of known and unknown parameters on timer setup and consequently on the performance of the WNCS is studied.

A. Learning The Parameters of Gilbert-Elliott Model

Here, we demonstrate the impact of resource limitations and properties of the involved subsystems on performance of the learning algorithm. First, we consider the case of a WNCS consisting of two subsystems where $A_1 = 0.9$ and $A_2 = 1.5$. These subsystems share a single wireless channel where p and q for both communication links are assumed to be 0.3 and 0.6, respectively. The timer-based channel access mechanism can be implemented by rewriting (6) as

$$\tau_{i,j,k} = \frac{\lambda}{\mathrm{CoIL}_{i,k} b_{L_{i,j,k}}}, \tag{21}$$

where b_L denotes the belief learned with Algorithm 1.

Fig. 5 shows how the learned parameters of GE channel model evolve over 2000 time steps. In the beginning, both subsystems have the initial belief of 0.5. Due to its unstable nature, Subsystem 2 has a larger CoIL and thus claims the channel and observes its state more frequently. As a result, the underlying parameters of the channel are learned more accurately for this subsystem. On the other hand, due to the limited number of observations made by Subsystem 1, the unknown parameters are learned less accurately.

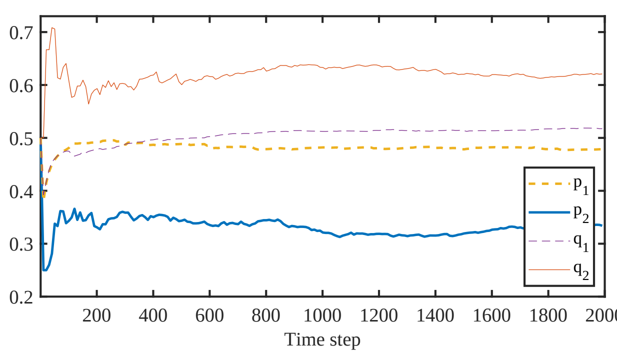

Fig. 5. Performance of the algorithm in learning the unknown parameters when one stable ($i \in \{1\}$) and on unstable ($i \in \{2\}$) subsystems compete over one channel. The true values of p and q are 0.3 and 0.6, respectively.

Fig. 6 shows the results for a similar setup where Subsystem 1 is marginally stable with $A_1 = 1$. As expected, since subsystem 1 claims the channel more frequently compared to the previous case, the learning algorithm provides a better estimate of the parameters. However, Subsystem 2 learns the unknown parameters more accurately again, since it claims the channel more frequently.

B. Cost Reduction

In this subsection, we analyze the impact of utilizing different channel models in (10) on the performance of the timer-based channel access mechanism. The performance is measured in terms of the reduction in the quadratic cost defined in (2) compared to the case where the well-known round-robin scheme is implemented. This scheme provides decentralized collision-free channel access; however, it is not deterministic and subsystems transmit in a random order decided upon initiation.

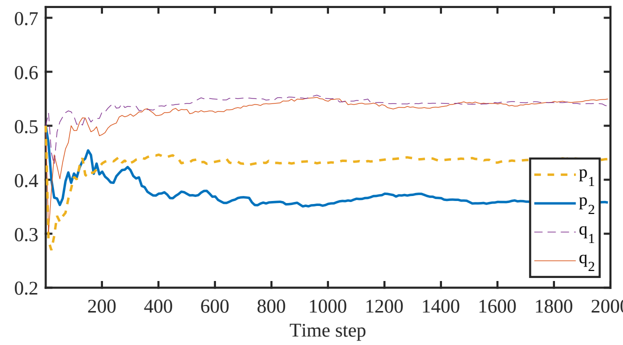

Fig. 6. Performance of the algorithm in learning the unknown parameters when a marginally stable ($i \in \{1\}$) and an unstable ($i \in \{2\}$) subsystem compete for a single communication channel. The channel quality is assumed to be similar for all subsystems with $p = 0.3$ and $q = 0.6$.

Herein, we assume that the WNCSs consist of equal number of subsystems from two classes of homogeneous dynamical systems. The unstable subsystems, form the first class, denoted by I while the second class, denoted by II, consists of marginally stable subsystems. The state-space representation of the considered cases is determined by

$$A_{\mathrm{I}} = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.1 \end{bmatrix} A_{\mathrm{II}} = \begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix}, B = C = I_{2\times 2},$$

where I denotes the identity matrix. Furthermore, state estimates and feedback control law are determined as discussed and we intend to minimize the quadratic cost defined in (2) with $Q = I_{2\times 2}$ and $R = 0.01 I_{2\times 2}$. The communication resources are limited such that at most only 75% of subsystems can transmit successfully at each time step. Transition probabilities are assumed to be known and unique for each subsystem and are chosen randomly while ensuring $p \in [0.2, 0.4]$ and $q \in [0.3, 0.5]$. Furthermore, the number of the posterior counts kept for Bayesian update is set to 20.

Fig. 7 shows the percentage of reduction in the quadratic cost achievable by four setups in WNCSs consisting of $N \in \{8, 12, 16, 20, 24\}$ subsystems. Case I corresponds to the case when the exact parameters of the GE model are known and the local cost in timer setup is given by (13). As expected, this setup leads to the best results reducing the quadratic cost from 44.98% up to 55.78% depending on the size of the WNCS. In case II, it is assumed that the stationary distribution is known and the belief (14) is implemented in setup. As it can be seen, this setup can even increase the cost compared to round-robin. This is due to ignoring the memory in channel for resource allocation by using b_∞ as a measure. In this scenario, a subsystem transmits on one of the available channels that has the largest stationary distribution of belief. However, if the transmission is not successful, since $q < 1 - q$, it is more likely that transmission over the same channel would be unsuccessful in the next slot.

In the absence of information about the channel statistics, we study two setups referred to as case III and case IV. The former corresponds to the scenario where the original timer-based mechanism is implemented and the subsystem transmits over one of the available channels randomly once its timer reaches zero. Although this setup outperforms round-robin, Fig. 7 indicates that adopting the learning algorithm improves performance. This scenario is denoted by case IV where the timers are set to (21). As expected, using the transmission history for learning the channel parameters results in noticeable cost reduction.

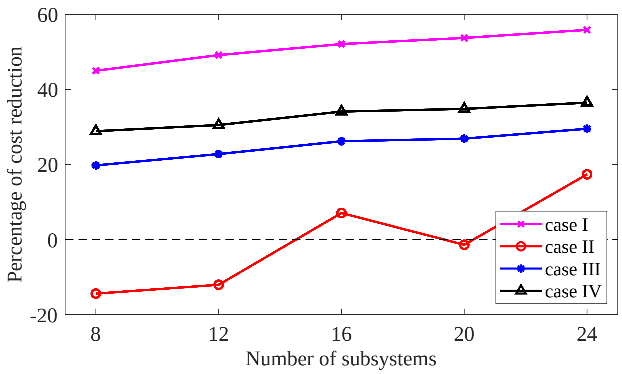

Fig. 7. The cost reduction achieved with four different channel models used in the timer setup (eq. (10)) compared to scheduling with round-robin. Case I and case II represent the setups employing the known temporal correlation in channel variation and the known steady state distribution of belief, respectively. Case III corresponds to to assuming constant 50% belief of successful transmission over all links, while in case IV the introduced Bayesian learning algorithm is implemented in the timer-based mechanism.

VII. Conclusions and Future Directions

A. Conclusions

We have presented a novel distributed method for the resource allocation problem in WNCSs consisting of subsystems capable of local computations. We utilized the concept of local timers as a measure to prioritize the communication of subsystems that have a higher probability to reduce the overall cost of the system. In our setup, CoIL was used as a measure of control performance and various channel models were used to gauge the quality of available wireless channels. The simulation results show that the best performance is achieved with the GE channel model with known parameters. Moreover, the introduced learning algorithm can improve the performance when compared to the case with no memory in the channel. Furthermore, it results in considerable improvements compared to the case where only the known steady state distribution is adopted in the timer setup.

B. Future Directions

Interesting future research directions include investigating the case where the duration of flag packet is not negligible. Therefore, the possibility of collisions in the flag signals can lead to collision during data transmission. Furthermore, the impact of Markov chain mixing times on performance when the resources are more limited can be incorporated in the channel access scheme.

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