Thesis for the degree of Licentiate of Engineering

# Realistic prospect for continuous variable quantum computing in circuit-QED

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# Abstract

This licentiate thesis is an extended introduction to the appended papers, which pertain to finding quantum states that are useful for continuous variable quantum computing. The useful states are characterized by a negative Wigner function. This is the underlying motivation for the appended papers, but *why* a negative Wigner function is necessary is not explained in the papers. This is elucidated in this thesis, with an accompanying discussion of which quantum mechanical properties allow quantum computers to surpass the capabilities of classical computers.

**Keywords:** Continuous variable quantum computing, quantum supremacy, quantum advantage, negative Wigner function, quantum state engineering

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Ingrid Strandberg, Göteborg, November 2019

# Publications

**A** **Steady-State Generation of Wigner-Negative States in One-Dimensional Resonance Fluorescence**
Fernando Quijandría, Ingrid Strandberg, Göran Johansson
*Phys. Rev. Lett.* **121**, 263603 (2018)

**B** **Numerical study of Wigner negativity in one-dimensional steady-state resonance fluorescence**
Ingrid Strandberg, Yong Lu, Fernando Quijandría, Göran Johansson
https://arxiv.org/abs/1909.02395
Accepted for publication in *Physical Review A*.

# Contents

# 1 Introduction

In the endless effort to create increasingly powerful computers, quantum mechanics has had, and continues to have, a great influence. Knowledge of quantum mechanics has not only been imperative for the massive increase in computational power over the past 50 or so years, but also opens up the possibility of a completely new paradigm of computing: *quantum computing*.

## 1.1 Quantum mechanics and classical computers

A fundamental limitation to a processor's rate of computation is the speed at which information travels through its circuits. By reducing circuit component sizes as well as the distance between components, processor speed can be improved [1, 2]. Due to this, there has been a continued miniaturization of components like the transistor. This development is described by the famous Moore's law, stating that the number of transistors in an integrated circuit doubles approximately every two years [3]. The first commercial microprocessor released in 1971 had a transistor density of around 190 transistors per square millimeter. Now, modern microprocessors incorporate several million transistors per square millimeter, each with a size on the nanometer scale [4]. Reaching smaller and smaller scales, quantum effects such as tunneling become very important [5]. Not to mention the fact that quantum mechanics describes the behavior of the semiconductor materials of which transistors are made of. In this way, the computers we have today function on the basis of quantum mechanics. This is, however, not what is meant by the term *quantum computer*. To be a quantum computer, a computational device needs to be able to store quantum information and apply quantum operations, not just use quantum effects at the hardware level [6].

## 1.2 Quantum computers

It is convenient to formulate a description of quantum computation which closely resembles the formalism of classical computation. In the classical case, the bit is a fundamental concept. A bit can take two possible values: 0 or 1, represented by e.g. different voltages in the circuits that make up the physical computer. The quantum counterpart of the bit is the *qubit*, which can be observed in one of two states we denote $|0\rangle$ and $|1\rangle$. A qubit can be realized by a variety of physical systems, for example, horizontal and vertical polarization of a photon, excited and ground state of an atom, or spin up and down of a spin-½ particle. What separates the qubit from the classical bit is that the qubit can be in a quantum *superposition* of states:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \quad \alpha, \beta \in \mathbb{C}. \tag{1.1}$$

The quantum state of the qubit is represented by the vector $|\psi\rangle$ in a two-dimensional Hilbert space.[1] Despite the superposition, when measuring the qubit you only obtain either the $|0\rangle$ *or* the $|1\rangle$ state. The coefficients $\alpha$ and $\beta$ are *probability amplitudes*, deciding the statistics of the measurement outcome. A measurement will result in the state $|0\rangle$ with probability $|\alpha|^2$ and $|1\rangle$ with probability $|\beta|^2$.

Another property that separates quantum states from classical states is that quantum states can be *entangled*. An example of an entangled state between two qubits is

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2\right), \tag{1.2}$$

where the subscript denotes qubit 1 and qubit 2. An entangled quantum state incorporates correlations between qubits that are not permitted by classical physics, as demonstrated by the violation of Bell's inequality [7, 8]. Measuring either qubit gives information about it's partner; if you measure either qubit to be in state $|0\rangle$, the other qubit will also be in state $|0\rangle$ with certainty. Similarly when measuring $|1\rangle$.

In general, an entangled state is defined as a state that *cannot* be written as a *product state*:

$$|\psi\rangle_{12} = |\psi\rangle_1 \otimes |\psi\rangle_2\,. \tag{1.3}$$

Such a state is called a separable state, and has no entanglement. In the earlier days of quantum computing, superposition and entanglement were

[1] A Hilbert space (often denoted $\mathcal{H}$) is a complex vector space whose elements are vectors $|\psi\rangle$, corresponding to quantum states.

often assumed to be the quantum properties underlying quantum speedup. But it turns out it is not that simple. In Chapter 2, I will describe the difficulties of finding the origin of the quantum advantage.

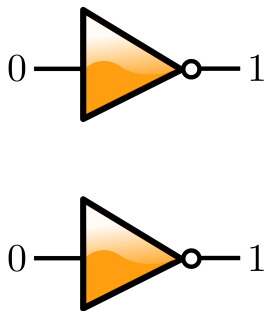
## Gate based model of computation

Just like classical computer processors are constructed of logical gates, a gate based model of quantum computers can be constructed. This is often also called the circuit model of quantum computing. As an example, consider the classical NOT gate, which turns a 0 bit into 1, and vice versa. A corresponding quantum gate is implemented by the Pauli $\sigma_x$ operator, often just denoted $X$ in quantum computing literature. That the effect of this operator is a bit-flip is easily seen by writing the matrix representation of $X$ and the state $|\psi\rangle$ given by (1.1):

$$X|\psi\rangle \iff \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}. \tag{1.4}$$

The classical and quantum NOT gates are illustrated in Fig. 1.1.

(a) Classical NOT gate




(b) Quantum NOT gate

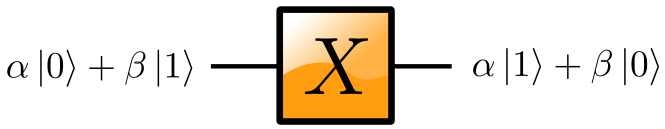



**Figure 1.1:** Unlike classical gates, quantum gates can operate on superposition states.

A quantum logic gate on the input state $|\psi\rangle$ can be described by the transformation $U|\psi\rangle$ where $U = e^{-iHt}$ is a unitary transformation generated by the Hamiltonian $H$, and $t$ is the gate-time.[2] In general, the evolution of a quantum state need not be unitary.[3] In this case, the state cannot in general

[2] We set $\hbar = 1$ throughout this thesis.

[3] This happens when the quantum system in question is coupled to an environment, and information of the quantum system is lost to the environment.

be represented by a Hilbert space vector $|\psi\rangle$, but instead a *density matrix* $\rho$ which is an operator on the Hilbert space. While a state vector can only describe a *pure* quantum state, density matrices can describe pure as well as *mixed* states. A mixed state density matrix is a sum over an ensemble of pure states:

$$\rho = \sum_i \rho_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{1.5}$$

normalized as

$$\sum_i \rho_i = 1, \tag{1.6}$$

where $\rho_i$ gives the probability of finding the state $\rho$ in the pure state $|\psi_i\rangle$. In the following Chapter we will see some differences between pure and mixed state quantum computing.

Unitary as well as nonunitary operations occurring during the steps of the quantum computation can be modelled as *quantum channels*.[4] A map describing a physical transformation of a density matrix $\rho_{\text{in}}$ into another density matrix $\rho_{\text{out}}$ is called a quantum channel or quantum operation, and denoted $\mathcal{E}$:

$$\rho_{\text{in}} \to \rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}}). \tag{1.7}$$

The quantum channel $\mathcal{E}$ is a superoperator, but by the *Choi-Jamiolkowski correspondence* it also corresponds to an operator $C_{\mathcal{E}}$ [9, 10]. If the input and output states belong to Hilbert spaces $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$, $C_{\mathcal{E}}$ acts in the product space $\mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}}$. The operator $C_{\mathcal{E}}$ is called the dynamical matrix, or *Choi matrix* [11, 12]. The action of the channel (1.7) can then be represented as

$$\rho_{\text{out}} = \text{Tr}_{\text{in}}[C_{\mathcal{E}}(\mathbb{1}_{\text{out}} \otimes \rho_{\text{in}}^{\top})], \tag{1.8}$$

where $\rho_{\text{in}}^{\top}$ is the transposed input density matrix. The Choi matrix is defined as

$$C_{\mathcal{E}} = \sum_{i,j} \mathcal{E}\left(\left|i\right\rangle \left\langle j\right|\right) \otimes \left|i\right\rangle \left\langle j\right|. \tag{1.9}$$

It is defined as the action of the channel $\mathcal{E}$ on the first subsystem of the maximally entangled state in the enlarged Hilbert space. If the Choi matrix is normalized, it is a valid density matrix. This is exact for a finite dimensional Hilbert space. The correspondence needs to be handled with more care in infinite dimension, but it can be done [13].

[4] The terminology comes from quantum communication (communication channel).

## Continuous variables

So far we have talked about qubits, which have two discrete eigenvalues 0 and 1, but there exists an alternative to discrete variable quantum computing: continuous variable (CV) quantum computing [14, 15]. In the CV approach, information is encoded in eigenstates of observables characterized by a continuous spectrum, living in an infinite Hilbert space.[5] The *quadratures* of the electromagnetic field are such observables.

A quantized electromagnetic field mode[6] has a bosonic creation operator $\hat{a}^\dagger$ and destruction operator $\hat{a}$, obeying the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1. \tag{1.10}$$

The creation operator, as the name implies, creates photons. If we consider a Fock state (number state) $|n\rangle$, where $n$ denotes the number of photons in the state, the application of $\hat{a}^\dagger$ gives the state $|n+1\rangle$. Conversely, application of $\hat{a}$ gives the state $|n-1\rangle$. The photon number $n$ is a discrete observable, but as mentioned above, the electromagnetic field also has observables with a continuous spectrum, called quadratures (illustrated in Fig 1.2). The $\hat{x}$ and $\hat{p}$ quadratures are conjugate variables, defined as

$$\begin{aligned} \hat{x} &= \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a}), \\ \hat{p} &= \frac{\mathrm{i}}{\sqrt{2}}(\hat{a}^\dagger - \hat{a}). \end{aligned} \tag{1.11}$$

As the notation implies, they are in some sense analogous to position and momentum, since they obey the same canonical commutation relation

$$[\hat{x}, \hat{p}] = \mathrm{i}. \tag{1.12}$$


**Figure 1.2:** A quantum state of light has both discrete and continuous degrees of freedom.

[5] An observable is an operator associated with a physical measurable quantity. The possible outcomes of the measurement of an observable are the eigenvalues of the operator.

[6] In classical optics, a mode $\boldsymbol{u_k}(\boldsymbol{r})$ is simply a normalized solution to Maxwell's equations, where the wave vector $\boldsymbol{k}$ labels the mode. A multimode quantized field has bosonic operators $\hat{a}_k$, $\hat{a}_k^\dagger$, with different modes commuting: $[\hat{a}_{k'}, \hat{a}_k^\dagger] = \delta_{k'k}$.

Just as $x$ and $p$ are coordinates in classical phase space, the eigenvalues of operators $\hat{x}$, $\hat{p}$ are coordinates in quantum phase space. We can also define the *generalized quadrature*

$$\hat{x}_\theta = \frac{1}{\sqrt{2}}(\hat{a}^\dagger e^{i\theta} + \hat{a}e^{-i\theta}), \tag{1.13}$$

where the phase angle $\theta$ defines a direction in phase space, as illustrated in Fig. 1.3. The generalized quadrature $\hat{x}_\theta$ is related to the original operators (1.11) by $\hat{x}_0 = \hat{x}$ and $\hat{x}_{\pi/2} = \hat{p}$.

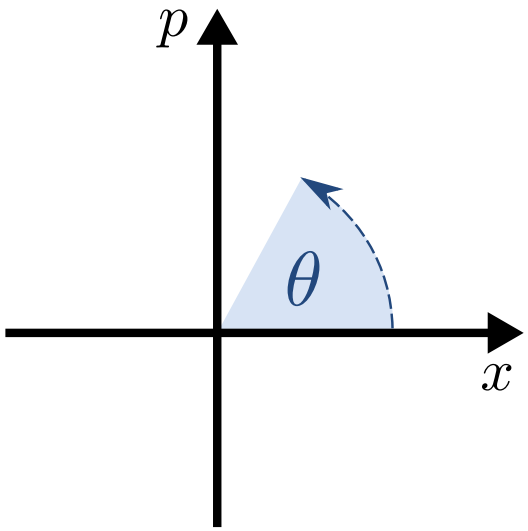



**Figure 1.3:** The generalized quadrature $\hat{x}_\theta$ is defined by the angle $\theta$ in quantum phase space.

We will come back to quantum phase space in the next chapters.

## 1.3 Computational complexity and quantum algorithms

Quantum algorithms can solve certain problems faster than any known classical algorithm. This is called *quantum supremacy*, *quantum advantage* or *quantum speedup*. The efficiency of an algorithm is usually measured by the computational complexity. Computational complexity can be evaluated with regard to e.g. the required amount of time or memory. The time complexity, often measured by the number of operations performed by the algorithm, is the relevant factor concerning quantum speedup. Suppose a problem is specified by $n$ input bits. An *efficient algorithm* is one which runs in *polynomial*

*time* in the size of the problem, i.e. the runtime $T$ as a function of the input size $n$ is $T(n) = \mathcal{O}(n^k)$ for some positive constant $k$.[7]

A general pure state of $n$ qubits is specified by $2^n$ parameters. This is *exponential* in $n$, and such a state cannot even be stored efficiently on a classical computer. A quantum computer with $n = 500$ qubits could never be simulated on a classical computer, since $2^{500}$ is larger than the estimated number of atoms in the universe [16].

The most popular quantum algorithm is *Shor's algorithm* for integer factoring [17]. The invention of Shor's algorithm drew a lot of attention to quantum computing because, unlike the best classical factoring algorithm known, it can perform factoring efficiently. This has important ramifications for popular public key cryptosystems such as RSA [18], since the security of this kind of system relies on the presumed difficulty of factoring large numbers [19].

The second most popular quantum algorithm is *Grover's algorithm* for database searches [20]. For an unsorted database of size $N$, a classical computer would need $\mathcal{O}(N)$ queries on average to find a specific item, while Grover's algorithm only needs $\mathcal{O}(\sqrt{N})$ queries. While this is not as a dramatic improvement as provided by Shor's algorithm, it is still a quadratic speedup. In the next chapter we will discuss which quantum features possibly enable the speedup.

## 1.4 Outline of thesis

In the appended papers we study a simple way to generate nonclassical states that could be useful for CV quantum computing. For a detailed and fairly self-contained explanation of the system and methods used for the papers, I refer to my Master's thesis [21]. This Licentiate thesis is more like an extended introduction.

The states that are useful for quantum computing are characterized by a negative Wigner function. This concept will be explained further in Chapter 2, where various nonclassical features and their potential contributions to a quantum advantage are explored. In Chapter 3, the necessity of Wigner-negativity for the possibility of a quantum advantage is proven. After this, different methods of generating nonclassical states are briefly summarized in Chapter 4. At the end, Chapter 5 contains a summary of the appended papers.

[7] The big-O notation $\mathcal{O}(n^k)$ means that $cn^k$ (where $c$ is a constant) is an upper bound of the runtime.

# 2 Nonclassicality and quantum computational advantage

Confirmed to high precision by experiments, quantum physics undoubtedly describes our world [22]. However, limitations in the validity of classical physics only become apparent at very small scales. Where lies the boundary between quantum and classical physics? Understanding what distinguishes quantum physics from classical physics is crucial to fully understand quantum computing. In particular, it is highly desirable to characterize exactly what nonclassical resources are required for quantum computation, quantify them, and understand how to harness them to outperform classical computing. This endeavor is reviewed in the current chapter.

## 2.1 Indications of nonclassicality

Nonclassical properties of light became a subject of interest already in the 1960s. At that time, a later widely used indicator of nonclassicality was introduced in terms of the Glauber-Sudarshan $P$-function [23]. Any quantum state $\rho$ can be represented in terms of the $P$-function. The state $\rho$ is considered nonclassical if the corresponding $P$ is negative or more singular than a Dirac $\delta$-function. The $P$-function is a part of a family of quasiprobability distributions in quantum phase space, which also includes the *Wigner function*. The Wigner function has the benefit of never being singular, and it can also be determined experimentally by measuring generalized quadratures [24]. Negativity of the Wigner function is also regarded as a signature of nonclassicality. The name *quasiprobability* distribution stems from the fact that these distributions are not strictly positive as is required for a probability distribution, but when they are positive they can be interpreted as such. We will talk more about the Wigner function later—it is the main focus of Chapter 3, and will turn out to play an important role in the search for quantum supremacy.

## 2.2 Measures of nonclassicality

Besides classifying states as classical or nonclassical, it can be useful to quantify *how nonclassical* a particular state is. The volume of the negative part of the Wigner function $W$ has been suggested as such a measure [25]. While negativity of the Wigner function plays a special role for quantum computing, classifying and quantifying nonclassicality is not so straightforward. For example, squeezed states are nonclassical, but nevertheless they have a positive Wigner function.[1]

Over the years, there have been a number of proposed measures of nonclassicality. An early suggestion of a measure of nonclassicality named the *nonclassical distance* was introduced in 1987 [27]. It was defined as the distance of a given state from the set of classical states. For this kind of measure there are two decisions that needs to be made: (i) what is the set of classical states, and (ii) what metric should be used to calculate the distance. If we restrict ourselves to pure states, the set of classical states is the coherent states [28]. As for the metric, there are a number of possible choices. Originally in [27] the trace distance was used. Since this is generally difficult to calculate, other options such as the Hilbert-Schmidt distance [29, 30] and Bures metric [31, 32] have been put forth. A problem is that distance-based measures are in general not suitable for quantifying nonclassicality, because the ordering of quantum states with respect to their degree of nonclassicality can be ambiguous [33]. The difficulty with quantifying nonclassicality is that if a state is less nonclassical than another state according to some measure then it might be more nonclassical according to another measure [34]. There are also different nonclassical properties. Below we discuss some of the most prominent ones with respect to quantum supremacy.

## 2.3 Resources for quantum computation

### Superposition

The power of quantum computers is frequently attributed to *quantum parallelism*—meaning that that a quantum algorithm can be performed in parallel on many different inputs when the input is a quantum superposition state—especially in explanations aimed towards laypeople. This is misleading [6]. While superposition inputs can be processed, a single operator is applied to

[1] Squeezed states are a consequence of the Heisenberg uncertainty relation; a state is squeezed when fluctuations in one variable are reduced at the expense of the increased fluctuations in the conjugate variable such that the uncertainty relation is not violated [26].

the full state, not simultaneous operators on multiple separate states. Related to this, a widespread misconception is that a quantum computer can try every possible solution to a problem in parallel, and then pick the correct one. This is plain wrong [35]. Even if the input is a superposition, you can only obtain one output since the result of any quantum computation is given by a measurement that "collapses" any superposition to a single output state [6, 35, 36]. Even worse, the outcome of this measurement is random! It is clear that superposition in itself is not enough to gain a speedup. If superposition is not sufficient, what else is needed? Aaronson [37] says that to ensure that the right outcome is observed with high probability, a quantum algorithm needs to generate an interference pattern in which the computational paths leading to the wrong answer cancel each other out, while paths leading to the right outcome reinforce each other. Entanglement has also been widely believed to be the key to quantum speedup [38], but it is now known not to be the whole answer. Below, we will talk about entanglement, interference, and more.

## Entanglement

For pure states, it has actually been proven by Jozsa and Linden that entanglement is necessary to achieve an exponential speedup. A certain amount of entanglement is needed; it must increase with the input size to the algorithm. That is, if the entanglement is bounded the quantum computation can be efficiently classically simulated [39]. Note that this is required when the state is pure throughout the entire computation. While entanglement is necessary in this case, it is not sufficient to guarantee a speedup. In fact, too much entanglement can destroy speedup [40].

It has been argued that a computation that involves a restricted amount of entanglement can be efficiently simulated classically [41], but there are also examples of pure-state quantum computing where little or no entanglement is present and the computation speed is still better than anything classically achievable, although the advantage in that case is smaller than exponential [42]. One issue is that there are many measures of entanglement, many of them unsuitable to assess the power of quantum computers, as they can be very small and even tend to zero during a quantum computation [43]. Also, whether one state is more or less entangled compared to another state depends on which entanglement measure is used [44, 45]. This indicates that entanglement does not account fully for the quantum advantage, although it still plays a role in getting an exponential advantage with Shor's algorithm, for which entanglement is necessary and correlated with the speedup [46, 47]. The question remains whether entanglement must be present for a less than

exponential speedup with pure states [48]. With mixed states, entanglement is not necessarily needed for higher quantum efficiency. This is expanded on below.

## Discord

While entanglement is the archetypal example, it does not account for all non-classical correlations. For mixed states, even states that are not entangled usually contain correlations that have no classical counterpart [49]. Quantum *discord* was introduced by Ollivier and Zurek as a measure of quantum correlations including but not limited to entanglement [50]. Discord is an entropy based measure, defined in terms of the *mutual information*. Mutual information is an information theoretic concept first introduced for classical systems. It is a measure of correlation between two variables. Consider two classical random variables $X$ and $Y$. The mutual information between the variables is

$$I(X:Y) = S(X) - S(X|Y), \tag{2.1}$$

where $S(X)$ is the Shannon entropy of the random variable $X$, quantifying the uncertainty of $X$. The conditional entropy $S(X|Y)$ is the entropy of $X$ given $Y$. For classical variables, the mutual information can also be written as

$$J(X:Y) = S(X) + S(Y) - S(X,Y), \tag{2.2}$$

where $S(X,Y)$ is the entropy of the joint probability distribution of $X$ and $Y$. For a classical system, the expressions (2.1) and (2.2) are equivalent: $I(X:Y) = J(X:Y)$ [51]. For a bipartite quantum system $\rho_{AB}$, density matrices are used instead of random variables and the von Neumann entropy is used instead of the Shannon entropy. In this case, the two expressions for mutual information are no longer equivalent, and the quantum discord $\mathcal{D}$ is defined as the difference[2]

$$\mathcal{D} = I(\rho_{AB}) - J(\rho_{AB}). \tag{2.3}$$

For pure states, discord reduces to entanglement. Since entanglement isn't necessarily a resource for mixed state quantum computing, discord has been suggested to be the actual resource [54, 55]. There is a particular example called "the power of one qubit" [56, 57] that uses highly mixed states, where there is no entanglement, but large amounts of discord [58, 59], that achieves an advantage.

[2]Besides this definition, which was the original one, there now exists different types of discord (for example geometric discord) as well as other quantum correlation measures [52, 53].

On the other hand, almost all quantum states possess discord, similarly to how almost all pure states possess entanglement [60]. If discord was responsible for the quantum speedup, then almost all quantum states would be useful resources! This has been shown not to be the case for pure states, where random states are not useful for quantum computation [61]. This is a reason why discord is unlikely to be the reason behind speedup [62]. Additionally, discord can be created by classically mixing discord-free states, another indication that it is not a resource for quantum computing [63]. Basically, discord is an indicator of nonclassicality but might not be a resource for quantum computing [64]. Still, there is no general proof that computations without discord can be efficiently simulated, although there are known cases [65, 66].

## Purity

Can quantum computing with mixed states achieve the same speedup as the best algorithms using pure states? The example of "the power of one qubit" mentioned above that uses highly mixed states is more powerful than classical computing, but not as powerful as pure-state quantum computing [56]. It can also be noted that while there is a register of mixed qubits, one pure qubit is present.

For Grover's algorithm, evidence was presented that the amount of mixedness as quantified by the von Neumann entropy can impose limitations of the efficiency of the search [67]. But another study showed that entropy is not a good measure for the usefulness of Grover's algorithm; for practically every value of entropy, there exist initial states that are good and initial states that are bad [68]. This is another example of the difficulty of finding suitable measures to quantify nonclassicality.

For Shor's algorithm, factorization can be achieved efficiently with just one initial pure qubit and a supply of initially maximally mixed qubits. Still, at least one pure qubit is still needed [69]. While the highly mixed states allow an efficient implementation of the algorithm, entanglement is an intrinsic part of of it, and if the computer is in a highly mixed state any attempt to remove entanglement by further mixing of the algorithm results in a significant decrease in its efficiency. Overall, entanglement is lower in the mixed-state algorithm, and thus it is less efficient [70]. It should be noted that while entanglement may decrease with the mixedness of the state, as is appears to do in Shor's algorithm, the two are not necessarily related concepts. For example, a mixed state has more entanglement than a completely pure but disentangled state [71].

## Interference

Interference is in general not a purely quantum effect. Interference of classical waves such as those of an electromagnetic field is a well known-phenomena, but there are interference effects that distinguish the quantum nature of light from the wave nature [72]. In particular, the interference of probability amplitudes can give rise to peculiar outcomes, such as the *Hong-Ou-Mandel effect* [73]. Consider two indistinguishable photons, one in mode $\hat{a}$ and one in mode $\hat{b}$, impinging on a 50:50 beam splitter. The beam splitter transforms the input mode operators into output modes $\hat{c}$ and $\hat{d}$ by

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}. \tag{2.4}$$

There are four different ways for the two photons to interact at a beam splitter, illustrated in Fig 2.1. The probability amplitudes for two of the paths cancel each other by destructive interference, leaving only two possible outputs. Subsequently, when performing the experiment one photon in each output mode will never be observed; two photons will always be detected in one of the modes.

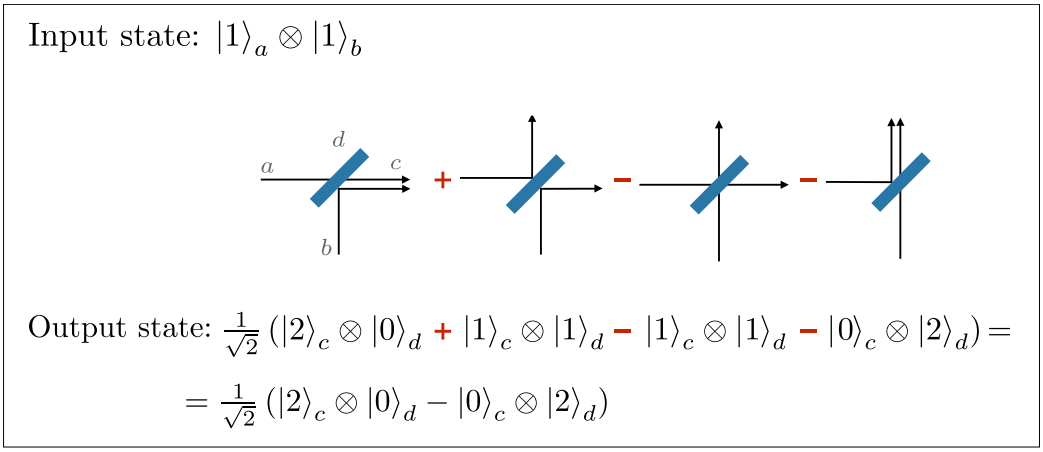



**Figure 2.1:** Hong-Ou-Mandel effect: there are four different ways of interaction for two input photons that impinge on each side of a beam splitter. The two middle options result in the same state, but with opposite probability amplitudes—meaning that they interfere destructively and cancel each other.

Similarly to how the Hong-Ou-Mandel setup enhances the probability to observe two photons in the same mode and suppresses the probability to observe a photon in each mode, quantum computers use the quantum interference of different computational paths to enhance correct outcomes and

suppress erroneous outcomes of computations [74]. The crucial quantum property is that while probabilities are always positive, probability *amplitudes* can be negative, with the square of the amplitude corresponding to a probability.[3]

The importance of interference for Shor's and Grover's algorithm has been studied. In both algorithms the destruction of interference goes hand in hand with the reduction of success probability. This reinforces the idea that interference is an necessary ingredient for quantum speedup. However, a large amount of interference is not sufficient. In many cases, additional interference can even reduce the success probability; altering the algorithm by adding random gates is likely to increase the interference, but this addition also destroys the algorithm. Not surprisingly, interference needs to be exploited in particular ways to be useful. Introducing a measure for interference, it is found that both Shor's and Grover's algorithms generate an exponential amount of "potentially available interference". But while Shor's algorithm utilized the exponential (in the number of qubits) amount of interference, Grover's algorithm used the same amount of interference independent of the number of qubits. It can be speculated if this differing usage of interference gives rise to the different speedups for the two algorithms [75, 76]. Nevertheless, no amount of increased interference could improve Grover's algorithm, as it is already optimal [77].

## 2.4 Classical simulatability and universal quantum computation

It appears that there is no single resource that grants speedup for all quantum algorithms, and the origin of quantum speedup remains largely unknown. Since it is difficult to figure out what provides the quantum advantage, we can instead approach the problem from the opposite side—by determining what is sufficient for efficient classical simulation of quantum circuits. The *Gottesman-Knill* theorem gives a class of quantum circuits that can be simulated on classical computers in polynomial time. Quantum circuits belonging to this class are constructed of what Gottesman named *Clifford gates*. There are Clifford operations and a corresponding theorem for both qubit and CV circuits, but here we focus on the CV version.

The Gottesman-Knill theorem states that a quantum computation can be efficiently simulated classically if the input to the quantum circuit is a

[3] In fact, probability amplitudes are in general complex numbers.

Gaussian state,[4] the circuit only applies gates belonging to the Clifford group, and if the measurements are Gaussian [78].[5]

Clifford operations are linear operations corresponding to Gaussian quantum channels, i.e., they transform Gaussian states into Gaussian states.[6] Gaussian channels and Gaussian states are fully characterized by their first two moments: the mean and covariance. For a system with $n$ modes, the covariance matrix for a Gaussian state has dimension $2n \times 2n$, and the moment vector has $2n$ elements. The Choi matrix for a Gaussian channel is determined by its $4n \times 4n$ covariance matrix and $4n$-dimensional vector of first moments [80]. Keeping track of the values requires evaluation of $\mathcal{O}(n^2)$ parameters, which is efficient.

## Universal computation

A universal computer is a machine that can perform any physically possible computation. A universal quantum circuit has the ability to simulate any Hamiltonian $H$, meaning it can perform an arbitrary quantum computation by implementing unitary operators $U = \exp(-\mathrm{i}Ht)$. Not only are Clifford circuits classically simulatable, they are also not universal.[7] Clifford gates are generated by Hamiltonians that are at most quadratic in the operators $\hat{x}$ and $\hat{p}$, and the effect of these gates are linear transformations of $\hat{x}$ and $\hat{p}$ [13, 79, 81]. A nonlinear operation, i.e. a non-Clifford operation, is needed for for universal CV quantum computation [14]. A nonlinear operation would provide a non-Gaussian state, but not all such states are sufficient for universal computing. A particular kind of non-Gaussian state is needed—a state with a negative Wigner function. For pure states, *Hudson's theorem* states that Gaussian states are the only states with a positive Wigner function [82]. But for mixed states, a distinction can be made between Gaussian and Wigner-positive states. Still, Gaussian states always have positive Wigner functions, but there are mixed states which are not Gaussian but still have a positive Wigner function [83]. In the next chapter, we show how states with positive Wigner functions allow quantum circuits to be efficiently simulated classically.

[4] Gaussian states are defined as states with a Gaussian Wigner function.

[5] This means that the Choi matrix corresponding to the measurement channel has a Gaussian Wigner function.

[6] The Clifford operations are unitary representations of the inhomogeneous symplectic group $ISp(2n, \mathbb{R})$, which is a semidirect product of the real symplectic group in $2n$ dimensions $Sp(2n, \mathbb{R})$ (squeezing operations) and the Heisenberg-Weyl group $HW(n)$ (phase space displacements), where $n$ is the number of modes in the CV system [79].

[7] A universal quantum computer would be able to perform algorithms that are not classically simulatable.

# 3 Wigner-negativity for quantum computational advantage

Any quantum circuit that can be efficiently simulated classically offers no quantum advantage. Here, by following papers [83–85] we show that quantum computations represented by positive Wigner functions can be efficiently simulated on a classical computer, under certain conditions.[1] To show this, we begin by reviewing some properties of the Wigner function.

The Wigner function provides a way to represent and visualize operators with a continuous spectrum, i.e. CV systems. Defining a Wigner function for discrete systems is not straightforward, and multiple different definitions can be found [86]. In this thesis, we focus on CV systems.

## 3.1 Wigner function

While the Heisenberg uncertainty principle forbids the definition of a joint probability distribution at a point $(x, p)$ in phase space, since the corresponding operators do not commute, it is possible to define quasiprobability distributions. As already mentioned, one such distribution is the Wigner function. It is defined as

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}y\, e^{-\mathrm{i}py/\hbar}\, \langle x + \frac{y}{2}|\rho|x - \frac{y}{2}\rangle\,. \tag{3.1}$$

Among all quantum phase space distributions, the Wigner function is special in a few ways: (i) it is always real-valued, (ii) integrating it along a direction in phase space defined by an angle $\theta$ gives a genuine probability distribution of the orthogonal generalized quadrature $x_{\theta+\pi/2}$. For example, integrating over $p$ gives a probability distribution in $x$. And (iii), the expectation value

[1] Still, efficient classical simulation might still be possible using other methods even if these conditions are not satisfied.

of an operator $A$ can be expressed in a form that mirrors expectation values for classical distributions [87]:

$$\langle A \rangle = \mathrm{Tr}[A\rho] = 2\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_\rho(x,p) W_A(x,p)\, \mathrm{d}x\, \mathrm{d}p. \tag{3.2}$$

In Eq. (3.2) we denote the regular Wigner function given by (3.1) as $W_\rho$. But what is $W_A$? There is a general relation between operators in Hilbert space and functions in phase space called the Weyl correspondence. In fact, the regular Wigner function is the Weyl transform of the density matrix $\rho$. Equivalently to Eq. (3.1), the Weyl transform of any operator $A$ is given by [88]

$$W_A(x,p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}y\, e^{-\mathrm{i}py/\hbar} \left\langle x + \frac{y}{2} | A | x - \frac{y}{2} \right\rangle. \tag{3.3}$$

From here on, we will refer to all Weyl transforms as Wigner functions. The subscript will show which operator it belongs to.

The final important property of the Wigner function is the following [89]:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x,p)\, \mathrm{d}x\, \mathrm{d}p = 1, \tag{3.4}$$

which means that when $W$ is non-negative it can actually be interpreted as a probability distribution (the sum of all probabilities is one). This will be the foundation of the following proof.

## 3.2 Efficient classical simulation with positive Wigner functions

The output of a quantum computer after running a quantum algorithm is a string $k$ of measurement outcomes. These outcomes are random, and we can assign the random variable $X$ to be the algorithm output. The algorithm can then be considered as a way of sampling from the probability distribution $\Pr(X = k)$. This means that a classical algorithm that can sample from this distribution, i.e. $\Pr(X_{\text{classical}} = k) = \Pr(X_{\text{quantum}} = k)$, can simulate the quantum algorithm. We can divide the procedure of a quantum computation into three parts:

1. Preparation of the input state.
2. Application of gates.
3. Measurement to obtain output.

At least one of these three parts has to contain a negative Wigner function. If the computation begins with a product state having positive Wigner function, followed by quantum gates that have a positive Wigner function, and finishes with a measurement associated with a positive Wigner function, the circuit can be simulated efficiently by classical sampling. To build up to the proof, we start by defining the Wigner functions of the steps of a quantum computation as listed above.

### The input state

We consider the input product state

$$\rho_{\text{in}} = \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n \tag{3.5}$$

consisting of $n$ subsystems (modes). For a multimode system in a product state the Wigner function is simply the product of the Wigner functions of the individual subsystems:

$$W_{\rho_{\text{in}}}(x,p) = W_{\rho_1}(x_1,p_1)W_{\rho_2}(x_2,p_2)\ldots W_{\rho_n}(x_n,p_n). \tag{3.6}$$

### The quantum gates

For simplicity, we consider only one gate (channel), but the approach is easily generalized to an arbitrary number of gates. Since the Choi matrix of a quantum channel is a density matrix after normalization, it can be assigned a Wigner function. Looking back to chapter 1, we determine that the Wigner function corresponding to the output state (1.8) is

$$W_{\rho_{\text{out}}}(\boldsymbol{x}',\boldsymbol{p}') = (2\pi)^n \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} W_{C_{\mathcal{E}}}(\boldsymbol{x}',\boldsymbol{p}',\boldsymbol{x},\boldsymbol{p}) W_{\rho_{\text{in}}}(\boldsymbol{x},-\boldsymbol{p})\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}\boldsymbol{p} \tag{3.7}$$

where the Wigner function of the transposed input density matrix $\rho_{\text{in}}^{\top}$ is obtained by changing the sign of the $p$ quadratures [90]. Since we consider a multimode input product state, the position and momentum variables in Eq. (3.7) are vectors $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$, $\boldsymbol{p} = (p_1, p_2, \ldots, p_n)$ and the integral is $2n$-dimensional.

### The final measurement

A general quantum measurement can be described by a set of measurement operators $M_k$. Considering only one subsystem, a quantum measurement performed on the state $\rho$ produces outcome $k$ with probability $\Pr(k) = \mathrm{Tr}[M_k\rho]$.

From Eq. (3.2), we can write this as

$$\Pr(k) = \mathrm{Tr}[M_k \rho] = 2\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{M_k}(x,p) W_\rho(x,p)\, \mathrm{d}x\, \mathrm{d}p. \tag{3.8}$$

The complete measurement outcome with $n$ subsystems will be a list $\boldsymbol{k} = (k_1, k_2, \ldots, k_n)$, with one outcome $k_i$ for each measured subsystem $\rho_i$. The probability of a given list of outcomes after the gates have been applied is

$$\Pr(k_1, k_2, \ldots, k_n) = \mathrm{Tr}[(M_{k_1} \otimes \ldots \otimes M_{k_n})\rho_{\mathrm{out}}] \tag{3.9}$$

Using Eq. (3.8) and (3.9), and also simplifying the notation by denoting phase space points $\boldsymbol{r} = (x_1, p_1, x_2, p_2, \ldots, x_n, p_n)$, we get

$$\Pr(\boldsymbol{k}) = (2\pi)^n \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} W_{M_k}(\boldsymbol{r}') W_{\rho_{\mathrm{out}}}(\boldsymbol{r}')\, \mathrm{d}\boldsymbol{r}', \tag{3.10}$$

with $W_{M_k} = W_{M_{k_1}}(\boldsymbol{r}'_1) W_{M_{k_2}}(\boldsymbol{r}'_2) \ldots W_{M_{k_n}}(\boldsymbol{r}'_n)$. Inserting $W_{\rho_{\mathrm{out}}}$ from Eq. (3.7) we get

$$\Pr(\boldsymbol{k}) = (2\pi)^{2n} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} W_{M_k}(\boldsymbol{r}') W_{C_\mathcal{E}}(\boldsymbol{r}', \boldsymbol{r}) W_{\rho_{\mathrm{in}}}(\Lambda \boldsymbol{r})\, \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}' \tag{3.11}$$

where $\Lambda = \mathrm{diag}(1, -1, \ldots, 1, -1)$ gives the transposed input density matrix.

The quantum circuit is shown in Fig. 3.1.

## The classical simulation

Now we assume all Wigner functions to be positive. To understand the simulation, we can consider the Wigner functions $W_{C_\mathcal{E}}(\boldsymbol{r}', \boldsymbol{r})$ and $W_{M_k}(\boldsymbol{r}')$ as conditional probability densities. The quantum channel Wigner function is a distribution for the output coordinates $\boldsymbol{r}'$ conditioned on the input coordinates $\boldsymbol{r}$: $W_{C_\mathcal{E}}(\boldsymbol{r}', \boldsymbol{r}) = W_{C_\mathcal{E}}(\boldsymbol{r}'|\boldsymbol{r})$. The measurement Wigner function is a distribution for the measurement outcomes $\boldsymbol{k}$, conditioned on $\boldsymbol{r}'$: $W_{M_k}(\boldsymbol{r}') = W_{M_k}(\boldsymbol{k}|\boldsymbol{r}')$. We rewrite Eq. (3.11) as

$$\Pr(\boldsymbol{k}) = (2\pi)^{2n} \int W_{M_k}(\boldsymbol{k}|\boldsymbol{r}') W_{C_\mathcal{E}}(\boldsymbol{r}'|\boldsymbol{r}) W_{\rho_{\mathrm{in}}}(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}'. \tag{3.12}$$

Assuming it is possible to efficiently draw phase space points according to these probability distributions, sampling from the distribution $\Pr(\boldsymbol{k})$ can be done by the composition method in the following way [91, 92]:

1. Draw phase space points $\tilde{\boldsymbol{r}}$ according to the input distribution $W_{\rho_{\mathrm{in}}}(\boldsymbol{r})$, i.e. draw samples $\tilde{r}_i$ from $W_{\rho_i}(r_i)$ for $i = 1, \ldots, n$.

2. Draw a phase space point $\tilde{\boldsymbol{r}}'$ according to the distribution $W_{C\mathcal{E}}(\boldsymbol{r}'|\tilde{\boldsymbol{r}})$.

3. Draw a measurement outcome according to the distribution $\Pr(\boldsymbol{k}) = (2\pi)^n W_{M_k}(\boldsymbol{k}|\tilde{\boldsymbol{r}}')$.

To understand step 3, consider adding a summation over all measurement outcomes $k$ on both sides of Eq. (3.10). The left-hand side of course becomes a vector $(1, 1, \ldots, 1)$ of size $n$. Since the integral of $W_{\rho_{\text{out}}}$ over all of phase space is one, we must have

$$(2\pi)^n \sum_{\boldsymbol{k}} W_{M_k}(\boldsymbol{r}') = (1, 1, \ldots, 1), \tag{3.13}$$

for all phase space points $\boldsymbol{r}'$. Hence, $(2\pi)^n W_{M_k}$ is a probability distribution over $\boldsymbol{k}$.

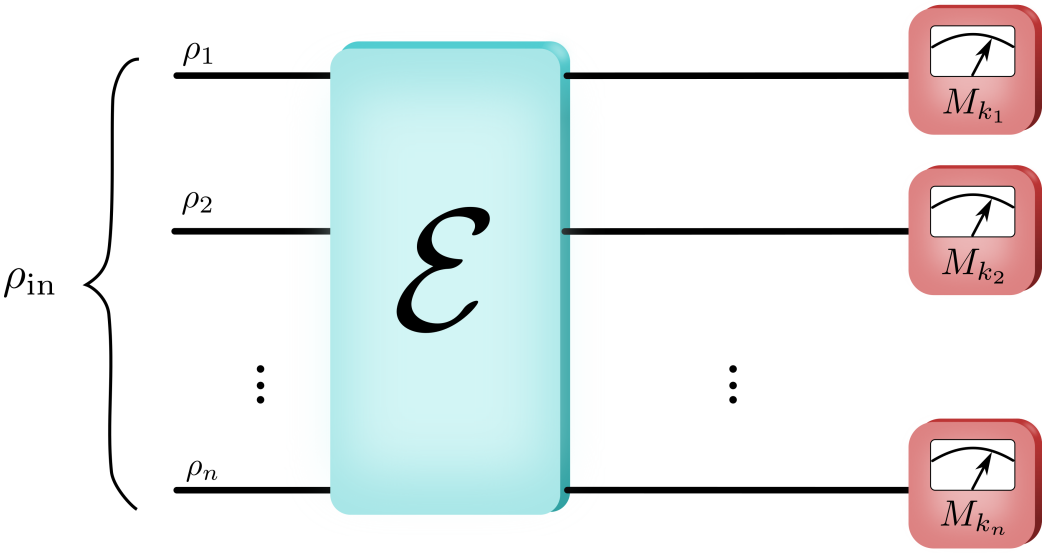



**Figure 3.1:** The input state $\rho_{\text{in}}$ is processed by a quantum gate $\mathcal{E}$, producing the output state $\rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}})$. The output is given by local measurements of each subsystem, with output probability distribution $\Pr(k) = \text{Tr}[M_k \rho_{\text{out}}]$. At least one of the three parts—input state, gates, or measurements—must have a negative Wigner function to obtain any quantum advantage.

### Efficiency

We consider the sampling efficient if it can be performed in polynomial time in $n$ and $t$, where as before $n$ is the number of modes, and the parameter

$t$ is introduced as the number of gates.[2] Clearly, the sampling is linear (polynomial of order one) in $t$. Considering the system size $n$, the product assumption is sufficient for efficient sampling in steps 1 and 3. This is the assumption of an initial product state (3.5) and the final measurements being local, meaning the total measurement operator can be written in product form like in Eq. (3.9). For step 1, you draw $n$ independent phase space points corresponding to each subsystem $\rho_i$. Also for step 3; since the measurements are local, you draw $n$ independent outcomes $k_i$.

The efficiency of step 2 is obtained by different assumptions in References [83] and [84]. The former restricts the operations $\mathcal{E}$ to be linear optical transformations. This ensures that the evolution of the Wigner function follows the Liouville equation, which describes classical evolution. It preserves non-negativity and probability, so the Wigner function will remain a classical probability distribution throughout the evolution. As mentioned at the end of chapter 2, linear transformations correspond to Gaussian transformations which can be efficiently simulated. Despite linear transformations not providing universal quantum computing, restricting to this type of linear gates does not necessarily limit the computational power—if the initial state is prepared nonlinearly, or if a nonlinearity is introduced in the measurement step, linear optical transformations can provide universal quantum computation [93]. This means that if Wigner negativity is not provided by the gates, it must be provided by the input state or final measurement.

The reference [84] makes the assumption of local gates, i.e. gates that act nontrivially on at most $m < n$ subsystems, where $m$ is a fixed number. In this case, the Wigner function can be efficiently sampled [94] since it is independent of $n$. This assumption does also not restrict the computational power. In fact, we can go as low as $m = 2$ and still have universal quantum computing [95–97].

This derivation has assumed infinite numerical precision, as well as possibility to sample efficiently from all involved probability distributions. The result still holds for finite-precision classical simulations and sampling errors. We have showed that a negative Wigner function is a necessary ingredient to a quantum advantage, but it is not yet known whether it is sufficient.

[2] The letter $t$ is chosen because of the correspondence between the number of gates in the circuit and the runtime of the algorithm.

# 4 Quantum state engineering: a quick review

Over the years there have been considerable efforts to engineer quantum states, not only for quantum computing but also for quantum communication and quantum sensing. Qubit states have been implemented on a multitude of platforms, for example trapped ions, neutral atoms in optical lattices, crystal defects in diamonds, semiconductor quantum dots and superconducting circuits. While CV states have been realized by oscillatory motional states of trapped ions [98], it is most commonly done by electromagnetic field states as mentioned in the introduction. Sought-after Wigner-negative states include Fock states (photon number states) and Schrödinger's-cat states (coherent superpositions of coherent states). These states can be confined to a cavity, or freely propagating. While arbitrary Fock states and their superpositions can be produced in superconducting cavities [99, 100], engineering travelling fields is particularly difficult because of their transient nature. Propagating Fock state superpositions can only be produced up to the two-photon level [101–103].

## State engineering

There are several different ways to engineer nonclassical states. One way is by *conditional measurement*, which is realized in two steps: first, the quantum system of interest is entangled with another auxiliary system. Next, a measurement is performed on the auxiliary system, projecting the state of the system of interest to a state correlated with the measurement outcome. This can result in highly nonlinear dynamics [104, 105]. Some measurement outcome should project into the desired target state, and measurements that do not are discarded. This method was developed in the 1990s to produce Fock states and their superpositions [106–108]. Lately, this method has been used to prepare cat states [109, 110] as well as single-photon-added thermal

and coherent states [104, 111], all with negative Wigner functions. However, since this method depends on detecting a particular auxiliary state, there is some probability of failure to prepare the correct state.

Another way to prepare a nonclassical state is to construct a quantum system with a Hamiltonian whose time evolution drives an initial state to the target state. [112–114]. However, for open quantum systems, the time evolution is not purely unitary due to irreversible loss into the surrounding environment. Interaction with the environment is in general detrimental, since it tends to turn quantum states into classical states [115, 116]. But the system-environment coupling can be engineered to act in our benefit [117, 118], so that the steady-state is a desired quantum state. This is often called *Reservoir engineering*. Entanglement between ions [119] and superconducting qubits [120] has been produced by engineered dissipation, as well as cat states in superconducting cavities [121, 122]. Wigner-negative states can also be generated by feedback-controlled systems [123].

## Engineering states to perform gates

As just mentioned above, one of the methods for state engineering is applying a Hamiltonian that drives a quantum state to another, wanted state. Unfortunately, this is a lot easier said than done. Clifford operations as described in Chapter 2.4 can be implemented experimentally relatively easy. Non-Clifford operations are more difficult. The lowest-order Hamiltonian that can generate the necessary nonlinearity to obtain Wigner-negativity is cubic in $\hat{x}$ or $\hat{p}$, and the corresponding gate is the so-called *cubic phase gate* [14]

$$U(t) = \exp(\mathrm{i}t\hat{x}^3). \tag{4.1}$$

In optical settings, a major difficulty is that currently available optical media only provide weak nonlinearities [81]. Circuit-QED setups (superconducting circuits) have the advantage that the Josephson junction provides a strong nonlinearity [124]. Still, a gate such as the cubic phase gate is yet to be realized. The cubic phase gate can be implemented if one can create the cubic phase state. One approach in the attempts to do this is by distillation.

### State distillation

The word *distillation* in general language means to purify a liquid. Here we are not talking about liquids, but instead quantum states. For example, entanglement distillation turns a large number of weakly entangled states into a smaller number of highly entangled states [125]. Also, there is a type of

quantum states called *magic states*, or *resource states*. These magic states are special states that, combined with Clifford operations, produce a universal set of gates by effectively implementing a non-Clifford gate. Multiple copies of an approximate magic state is prepared, and then the magic state can be *distilled* from the faulty states by Clifford operations [126]. The initial states need to have a negative Wigner function, otherwise they are not useful for magic state distillation [127].

## 4.1 Summary

Previous Chapters discussed the origins of quantum speedup, which is still a bit of a mystery. One thing that is known, though, is that negative Wigner functions are a requisite. Here we summarized different methods for obtaining Wigner-negative states.

The next Chapter briefly describes the appended papers and their results. The aim was to investigate whether a particularly simple system could provide Wigner-negative states. While we have obtained such a state, we do not yet know how, or if, it can be utilized for CV quantum computation. One possibility is that it could possibly be used to distill a state that is known to be useful, for example the cubic phase state.

# 5 Summary of papers

As we saw in the previous Chapter, there are a multitude of more or less complicated ways to generate nonclassical states. Our aim was to generate a nonclassical state (in the sense of Wigner negativity) in the least complicated way possible.

The simplest fully quantized model of light-matter interaction is a two-level atom interacting with a single quantized electromagnetic field mode. When the atom is driven on resonance, it will absorb and re-emit radiation through the process of resonance fluorescence. This is a typical quantum optics textbook setup, and the resonance fluorescence radiation field is since long known to exhibit nonclassical properties such as antibunching (the tendency to emit photons one by one) and squeezing [128, 129]. Nevertheless, a characterization of this radiation field in terms of the Wigner function has not been performed previously.

As already mentioned, quantum states with negative Wigner functions are generated by nonlinear interactions. In this setup, the input state is a coherent drive field, and the measurement is homodyne detection. Both of these have positive Wigner functions. The nonlinearity is provided by the two-level atom. In order to utilize this nonlinearity to create nonclassical states of light, strong coupling between the electromagnetic field and the atom is needed. As opposed to real atoms that emit radiation in all spatial directions, artificial superconducting atoms can be coupled to a one-dimensional waveguide. This facilitates strong coupling by confining the radiation energy in a small volume, and also avoids spatial mode mismatch between incident and scattered fields [130]. For this reason, we investigate the resonance fluorescence in the one-dimensional setup.

Wigner-negative states such as the single photon Fock state can be generated by a pulsed drive on a two-level atom, but we look at the system driven to steady-state. In optics, the most common method to produce Wigner-negative states are by conditional measurement of an auxiliary state. As mentioned in the previous chapter, this method is probabilistic. An advantage of our steady-state setup is that it can deterministically generate Wigner

negative states. We found that if the atom is placed in front of a mirror, this setup can indeed generate states with a negative Wigner function. But if the the waveguide is open on both ends, no negativity is obtained.

Following our results, experiments with a qubit (two-level system) in front of a mirror have recently been performed with superconducting circuits, and the results are consistent with the numerical simulations. It seems like it is indeed a prospect for CV quantum computing.

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# Paper A

**Steady-State Generation of Wigner-Negative States in One-Dimensional Resonance Fluorescence**

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# Paper B

**Numerical study of Wigner negativity in one-dimensional steady-state resonance fluorescence**

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