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Robust MIMO Channel Estimation from Incomplete and Corrupted Measurements

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***Abstract*—Location-aware communication is one of the enabling techniques for future 5G networks. It requires accurate temporal and spatial channel estimation from multidimensional data. Most of the existing channel estimation techniques assume that the measurements are complete and noise is Gaussian. While these approaches are brittle to corrupted or outlying measurements, which are ubiquitous in real applications. To address these issues, we develop a ℓ_p-norm minimization based iteratively reweighted higher-order singular value decomposition algorithm. It is robust to Gaussian as well as the impulsive noise even when the measurement data is incomplete. Compared with the state-of-the-art techniques, accurate estimation results are achieved for the proposed approach.**

I. Introduction

Location-aware communications are the promising technologies for 5G networks [1]. Extensive research has been carried out to tackle the technical challenges. While accurate temporal and spatial channel state information are critical to fulfill these requirements and obtain the location information [2]. Numerous channel estimation techniques have been developed, cf. [3], [4] for overviews. In the presence of white Gaussian noise and complete measurements, optimum performance is obtained for the maximum-likelihood (ML) estimator [5]. However, the computational load is heavy due to the multi-dimensional search. Statistically efficient strategies such as method of direction estimation (MODE) [5] and iterative quadratic ML (IQML) [6] algorithms have been proposed to reduce the computational complexity. Alternatively, subspace method achieves a good balance between complexity and estimation accuracy. Representative subspace methods include multiple signal classification (MUSIC) [7], estimation of signal parameters via rotational invariance technique (ESPRIT) [8], matrix pencil (MP) [9], MODE and principal-singular-vector utilization for modal analysis (PUMA) [10].

Massive MIMO generates massive amounts of multidimensional channel data with multiple aspects. It is convenient and natural to represent the dominant multipath components from MIMO channel measurements using tensors, because they are inherently organized in R-D structure [11]. As an emerging technology, tensors provide a natural and compact representation for such massive multidimensional data via suitable low-rank approximations. Instead of unfolding tensor into matrices and using matrix factorization techniques, tensor models preserve the multi-way structure of the data. Tensor decomposition becomes a powerful technique to capture the intrinsic multi-dimensional structure of the multi-way data. Tucker [12] and CANDECOMP/PARAFAC [13] are two popular models for tensor decomposition. Tensor factorization has emerged as an important method for information analysis [14], [15]. With the development of tensor decomposition, subspace methods are extended to their multi-dimensional variants such as tensor-MUSIC [16], tensor-ESPRIT [14], unitary tensor ESPRIT [14], tensor-MP [17], tensor-PUMA [18], tensor-MODE [19], multi-dimensional folding (MDF) [20], or the R-D rank reduction estimator (RARE) [11]. Two eigenvector-based frequency estimators tensor eigenvector (TEV) and its variant with forward-backward averaging (FB-TEV) are proposed in [21].

In practice non-Gaussian noise is ubiquitous [22], such as the impulsive noise [23]. The performance of the existing ℓ_2-norm minimization based subspace techniques may severely degrade in presence of impulsive noise. To eliminate the adverse effects, ℓ_1-norm minimization based robust model fitting algorithms such as [24] can be used. Recently, ℓ_p-MUSIC is derived in [25] and its tensor version [26], which adopts the ℓ_p-norm of the fitting error matrix or tensor as the objective function to minimize. The underlying idea is to transform the ℓ_p-norm minimization to an iterative ℓ_2-norm minimization. Besides non-Gaussian noise, incomplete measurements make the R-D channel estimation problem more challenging. It might be caused by errors or faults in the process of data transmission, or utilizing low cost irregular sampling schemes [27]. Robust multidimensional channel estimation from incomplete and corrupted measurements is critical and challenging [28], [29].

To address these challenges, in this paper, we focus on robust R-D channel estimation for MIMO systems from incomplete and corrupted measurements. Our contributions are as follows:

- We first formulate robust multidimensional channel estimation as a tensor recovery problem. Our aim is to recover the low rank component $\boldsymbol{\mathcal{L}}_o$ and error component $\boldsymbol{\mathcal{E}}_0$ from complete or incomplete tensor measurements $\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{L}}_o + \boldsymbol{\mathcal{E}}_o$ by optimization,
$$\min_{\boldsymbol{\mathcal{L}},\boldsymbol{\mathcal{E}}} \|\boldsymbol{\mathcal{L}}\|_* + \lambda\|\boldsymbol{\mathcal{E}}\|_p,\ s.t.\ \boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{E}}, \tag{1}$$
where λ is a positive weighting parameter. Under this framework, robustness to Gaussian noise is achieved when $p = 2$ is adopted [30]. In the presence of outliers or corrupted measurements, $0 < p < 2$, can be utilized [31]. Robust principal component analysis algorithm is obtained by setting $p = 1$ [32]. Alternating direction method of multipliers (ADMM) algorithms can be applied to solve (1) [33]. Tensor decomposition is applied on $\boldsymbol{\mathcal{L}}_o$, after obtaining the subspace, the existing subspace algorithms can be utilized for channel estimation.
- We develop an incomplete iteratively reweighted HOSVD (i-IR-HOSVD) algorithm for robust multidimensional channel estimation from partial observation and in impulsive noise environments. Inspired by the tensor completion technique, the main idea is minimizing the ℓ_p-norm of the residual error and recovering the low rank tensor measurements at the same time. It can be applied for robust higher-order tensor decomposition from corrupted and incomplete measurements.

The remainder of this paper is organized as follows. In Section II, the background introduction and problem formulation are provided. In Section III, we present the i-IR-HOSVD. Numerical examples are included to demonstrate the effectiveness of the proposed algorithm in Section IV. Finally, conclusions are drawn in Section V.

II. Background and Preliminaries

Notation: We use $(\cdot)^H$, $(\cdot)^*$ and $(\cdot)^{-1}$ to denote Hermitian transpose, complex conjugate and matrix inverse, respectively. Nuclear norm and ℓ_p-norm are denoted as $\|\cdot\|_*$ and $\|\cdot\|_p$, respectively. The set of unitary matrices of size $m \times n$ is denoted as $\mathcal{O}_{m\times n}$. In this paper, we follow the tensor operations defined in [34]. The $(i_1, i_2, \cdots, i_R)$ entry of two R-D tensors $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{\mathcal{B}}$ are denoted as $a_{i_1,i_2,\cdots,i_R}$ and $b_{i_1,i_2,\cdots,i_R}$, respectively. Scalar product of two tensors is defined as
$$< \boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{B}} >= \sum_{i_1}\sum_{i_2}\cdots\sum_{i_R} b^*_{i_1,i_2,\cdots,i_R} a_{i_1,i_2,\cdots,i_R}. \tag{2}$$

$[\boldsymbol{\mathcal{A}}]_{(r)}$ denotes the rth unfolding of $\boldsymbol{\mathcal{A}}$. $\boldsymbol{\mathcal{I}}_k^R \in \mathbb{R}^{J\times J\times\cdots\times J}$ is a R-D tensor whose $(j, j, \cdots, j)$ entry equals one and zero otherwise, $j = 1, 2, \cdots, J$. The product of a tensor $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1\times I_2\times\cdots\times I_R}$ and a matrix $\mathbf{U} \in \mathbb{C}^{J_r\times I_r}$ along the rth dimension is denoted by $\boldsymbol{\mathcal{A}} \times_r \mathbf{U}$, it is an $(I_1 \times I_2 \times \cdots \times I_{r-1} \times J_r \times I_{r+1} \times \cdots \times I_R)$-tensor and the entries are defined as
$$\begin{aligned}&(\boldsymbol{\mathcal{A}} \times_r \mathbf{U})_{i_1,i_2,\cdots,i_{r-1},j_r,i_{r+1},\cdots,i_R}\\ &= \sum_{i_r} a_{i_1,i_2,\cdots,i_{r-1},i_r,i_{r+1},\cdots,i_R} u_{j_r,i_r}.\end{aligned} \tag{3}$$

The Frobenius norm of a tensor $\boldsymbol{\mathcal{A}}$ is written as
$$\|\boldsymbol{\mathcal{A}}\|_F = \sqrt{< \boldsymbol{\mathcal{A}}, \boldsymbol{\mathcal{A}} >}. \tag{4}$$

A. System Model

The entries of multipath channel measurement $\boldsymbol{\mathcal{X}}$ are given by
$$x_{m_1,m_2,\cdots,m_R,n} = \sum_{l=1}^{L} \gamma_l(n) \prod_{r=1}^{R} e^{j\omega_{r,l}m_r} + v_{m_1,m_2,\cdots,m_R,n} \tag{5}$$
where $m_r = 1, 2, \cdots, M_r$, $r = 1, 2, \cdots, R$, $n = 1, 2, \cdots, N$, $l = 1, 2, \cdots, L$ and $v_{m_1,m_2,\cdots,m_R,n}$ denotes the noise component. The R and N denote the numbers of dimensions and snapshots, respectively. Numbers of frequencies L is known. $\omega_{r,l} \in (-\pi, \pi)$ are the unknown R-D channel parameters to be estimated. $\gamma_l(n)$ denotes the complex amplitude of the lth tone at the nth snapshot. The tensor dimension is R+1 together with the snapshots.

According to (5), $\boldsymbol{\mathcal{X}}$ can be written as
$$\boldsymbol{\mathcal{X}} \approx \boldsymbol{\mathcal{I}}_L^{R+1} \times_1 \mathbf{A}_1 \times_2 \mathbf{A}_2 \cdots \times_{R+1} \mathbf{A}_{R+1}, \tag{6}$$
where for $r = 1, 2, \cdots, R$,
$$\mathbf{A}_r = \begin{bmatrix}\mathbf{a}_{r,1} & \mathbf{a}_{r,2} & \cdots & \mathbf{a}_{r,L}\end{bmatrix} \in \mathbb{C}^{M_r\times L} \tag{7}$$
with $\mathbf{a}_{r,l} = \begin{bmatrix}e^{j\omega_{r,f}} & e^{j2\omega_{r,f}} & \cdots & e^{jM_r\omega_{r,l}}\end{bmatrix}^T$, while
$$\mathbf{A}_{R+1} = \begin{bmatrix}\mathbf{a}_{R+1,1} & \mathbf{a}_{R+1,2} & \cdots & \mathbf{a}_{R+1,L}\end{bmatrix} \in \mathbb{C}^{N\times L} \tag{8}$$
with $\mathbf{a}_{R+1,l} = \begin{bmatrix}\gamma_l(1) & \gamma_l(2) & \cdots & \gamma_l(N)\end{bmatrix}^T$.

For MIMO systems with N_T transmit and N_R receive antennas, the steering vectors $\mathbf{a}(\theta, \varphi) = \mathbf{a}_{az}(\theta) \otimes \mathbf{a}_{el}(\varphi)$ are functions of both the azimuth angle θ and elevation angle φ. In general, the frequency domain channel response of a MIMO channel with N_p paths can be described as
$$\mathbf{H}(t, f) = \sum_{\ell=1}^{N_p} \alpha_\ell e^{j2\pi(v_\ell t-\tau_\ell f)} \mathbf{a}_R(\theta_{R,\ell}, \varphi_{R,\ell})\mathbf{a}_T^*(\theta_{T,\ell}, \varphi_{T,\ell}), \tag{9}$$
where for each path ℓ, it is described by direction of arrival $(\theta_{R,\ell}, \varphi_{R,\ell})$, direction of departure $(\theta_{T,\ell}, \varphi_{T,\ell})$, delay τ_ℓ, complex gain α_ℓ and Doppler shift v_ℓ [35].

B. Problem Formulation

Our target is estimating R-D channel parameters from complete or incomplete noisy tensor measurements. For HOSVD, it decomposes a given tensor $\boldsymbol{\mathcal{X}}$ into a core tensor $\boldsymbol{\mathcal{C}}$ multiplied by a factor matrix $\mathbf{U}_r$ along each mode r as follows:
$$\boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r \tag{10}$$
where $\boldsymbol{\mathcal{C}} \in \mathbb{C}^{J_1\times J_2\times\cdots\times J_R}$, $\mathbf{U}_r \in \mathbb{C}^{I_r\times J_r}$, and $\times_r$ denotes the mode-r tensor-matrix multiplication. Since J_r is in general much smaller than I_r, the core tensor $\boldsymbol{\mathcal{C}}$ can be though of as a low rank version of $\boldsymbol{\mathcal{X}}$.

With complete measurements $\boldsymbol{\mathcal{X}}$, the higher-order orthogonality iteration (HOOI) algorithm [12] solves the following Frobenius norm minimization problem,

$$\begin{aligned} \min_{\boldsymbol{\mathcal{C}},\mathbf{U}} \quad & \left\|\boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r - \boldsymbol{\mathcal{X}}\right\|_F^2 \\ \text{s.t.} \quad & \mathbf{U}_r \in \mathcal{O}_{I_r \times J_r},\ r = 1, 2, \cdots, R, \end{aligned} \tag{11}$$

where $\mathbf{U}_r$ has orthonormal columns for all J_r.

Assuming that the measurements are incomplete, which is denoted as $\boldsymbol{\mathcal{M}}$. Then iHOOI [36] algorithm is formulated as:

$$\begin{aligned} \min_{\boldsymbol{\mathcal{C}},\mathbf{U}} \quad & \left\|\mathcal{P}_\Omega\left(\boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r - \boldsymbol{\mathcal{M}}\right)\right\|_F^2 \\ \text{s.t.} \quad & \mathbf{U}_r \in \mathcal{O}_{I_r \times J_r},\ r = 1, 2, \cdots, R. \end{aligned} \tag{12}$$

where Ω is the observed entry index, $\mathcal{P}_\Omega$ is a projection operator that keeps the entries in Ω and zeros out others [37].

Recently, a ℓ_p-norm minimization based IR-HOSVD algorithm is proposed in [26]. It solves the following optimization problem,

$$\begin{aligned} \min_{\boldsymbol{\mathcal{C}},\mathbf{U}} \quad & \left\|\boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r - \boldsymbol{\mathcal{X}}\right\|_p^p \\ \text{s.t.} \quad & \mathbf{U}_r \in \mathcal{O}_{I_r \times J_r},\ r = 1, 2, \cdots, R, \end{aligned} \tag{13}$$

where $\|\cdot\|_p$ denotes the element wise ℓ_p-norm. IR-HOSVD is robust to the complete measurements with outliers. While it can not handle the incomplete measurements. To address the issue, incomplete IR-HOSVD (i-IR-HOSVD) is proposed.

III. Proposed Incomplete IR-HOSVD Method

In this section, we consider robust R-D channel estimation from incomplete measurements with impulsive noise. It is well known that the ℓ_2-norm minimization based approaches such as HOOI and iHOOI are not robust to the corrupt data with outliers. Replacing the squared residuals by ℓ_p-norm is one of the commonly used candidates to deal with the impulsive noise [38] . For example, ℓ_1-norm minimization based algorithm has been studied in [24].

Given incomplete measurement $\boldsymbol{\mathcal{M}}$, i-IR-HOSVD solves the following ℓ_p-norm minimization problem,

$$\begin{aligned} \min_{\boldsymbol{\mathcal{C}},\mathbf{U}} \quad & \left\|\mathcal{P}_\Omega\big(\boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r - \boldsymbol{\mathcal{M}}\big)\right\|_p^p \\ \text{s.t.} \quad & \mathbf{U}_r \in \mathcal{O}_{I_r \times J_r} \end{aligned} \tag{14}$$

Introducing an auxiliary variable $\boldsymbol{\mathcal{X}}$, (14) is reformulated as

$$\begin{aligned} \min_{\boldsymbol{\mathcal{S}},\mathbf{U},\boldsymbol{\mathcal{X}}} \quad & \left\|\boldsymbol{\mathcal{C}} \times_{r=1}^{R} \mathbf{U}_r - \boldsymbol{\mathcal{X}}\right\|_p^p \\ \text{s.t.} \quad & \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{M}}) \text{ and } \mathbf{U}_r \in \mathcal{O}_{I_r \times J_r} \end{aligned} \tag{15}$$

For $r = 1, 2, \cdots, R$, the ℓ_p-norm based objective function is reformulated as

$$\mathcal{J}_r(\boldsymbol{\mathcal{X}}, \mathbf{P}_r, \mathbf{Q}_r) = \left\|\boldsymbol{\mathcal{X}}_{[r]} - \mathbf{P}_r\mathbf{Q}_r\right\|_p^p. \tag{16}$$

Instead of directly minimizing the ℓ_p quasi-norm, which most likely ends up with one of its many local minimizers, reweighted ℓ_1/ℓ_2 algorithms were proposed to solve a sequence of smoothed subproblems [39], [40].

The residual error matrix of the rth dimension is defined as

$$\boldsymbol{\Delta}_r^{(k)} = \boldsymbol{\mathcal{X}}_{[r]}^{(k)} - \mathbf{P}_r^{(k)}\mathbf{Q}_r^{(k)}, \tag{17}$$

with $\delta_{r_{m,n}}^{(k)}$ be the (m, n) entry . Its ℓ_p-norm minimization problem can be expressed as an equivalent iteratively reweighted Frobenius norm minimization problem,

$$\mathcal{J}_r^{(k)} = \left\|\mathbf{W}_r^{(k)} \circ \boldsymbol{\mathcal{X}}_{[r]} - \mathbf{W}_r^{(k)} \circ \left(\mathbf{P}_r^{(k)}\mathbf{Q}_r^{(k)}\right)\right\|_2^2, \tag{18}$$

where $\mathbf{W}_r$ is the weighting matrix, and its (m, n) entry is defined as

$$w_{m,n}^{(k)} = \begin{cases} \left(\left|\delta_{r_{m,n}}^{(k)}\right| + \epsilon\right)^{(p-2)/2}, & \text{if } (m, n) \in \Omega \\ 0, & \text{if } (m, n) \notin \Omega \end{cases} \tag{19}$$

where $\epsilon > 0$ being a regularization parameter to ensure that $w_{m,n}^{(k)}$ is well defined [41] and $w_{m,n}^{(0)} = 1$.

Let

$$\mathbf{G}_r^{(k)} = \mathbf{W}_r^{(k)} \circ \boldsymbol{\mathcal{X}}_{[r]}. \tag{20}$$

Note that $\mathbf{W}_r$ is a function of $\mathbf{P}_r$ and $\mathbf{Q}_r$. We cannot immediately obtain the optimal solution by performing SVD on $\mathbf{G}_r$ only once. As the IR-HOSVD algorithm in [26], an iterative procedure is adopted to solve the problem.

The global minimum of (18) is obtained via the truncated SVD of $\mathbf{G}_r^{(k)}$ [42]:

$$\mathbf{G}_r^{(k)} = \mathbf{U}_r^{(k)}\boldsymbol{\Sigma}_r^{(k)}\left(\mathbf{V}_r^{(k)}\right)^H, \tag{21}$$

where $\mathbf{U}_r^{(k)} \in \mathbb{C}^{I_r \times J_r}$ contains the first J_r principal left singular vectors of $\mathbf{G}_r^{(k)}$. The global optima of $\mathbf{P}_r^{(k)}$ and $\mathbf{Q}_r^{(k)}$ are given by [42]:

$$\mathbf{P}_r^{(k+1)} = \mathbf{U}_r^{(k)}, \quad \mathbf{Q}_r^{(k+1)} = \boldsymbol{\Sigma}_r^{(k)}\left(\mathbf{V}_r^{(k)}\right)^H. \tag{22}$$

With available $\mathbf{U}^{(k+1)}$, $\boldsymbol{\mathcal{X}}^{(k+1)}$ is recovered as

$$\begin{aligned} &\boldsymbol{\mathcal{X}}^{(k+1)} = \mathcal{P}_\Omega(\boldsymbol{\mathcal{M}}) \\ &+\mathcal{P}_{\Omega^c}\left(\boldsymbol{\mathcal{X}}^{(k)} \times_{r=1}^{R} \mathbf{U}_r^{(k+1)}\left(\mathbf{U}_r^{(k+1)}\right)^H\right), \end{aligned} \tag{23}$$

where Ω^c is the absolute complement of Ω.

The Algorithm is terminated if they reach the maximum number of iterations [37]. If stopping criterion is satisfied, then return $\mathbf{U}_r^{(k)}$ for $r = 1, 2, \cdots, R$. The pseudocode of the proposed i-IR-HOSVD algorithm is summarized in Algorithm 1.

After obtaining $\mathbf{U}_r^{(k+1)}$ for all r, and $\boldsymbol{\mathcal{X}}^{(k+1)}$, ESPRIT can be used to estimate the R-D parameters.

A comparison of the introduced four algorithms is given in Table I. IR-HOSVD [26] is a special case of i-IR-HOSVD, provided that the observation is complete. Meanwhile, iHOOI [37] is obtained by specifying ℓ_p-norm to Frobenius norm. Furthermore, HOOI [12] is obtained by setting ℓ_p-norm to Frobenius norm and the data is complete.

Algorithm 1: Incomplete IR-HOSVD

Initialize $\mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}^{(0)}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{M}})$
for $r = 1, 2,$ **to** R **do**
  Initialize $\mathbf{P}_r^{(0)}$, $\mathbf{Q}_r^{(0)}$ and $\mathbf{W}_r^{(0)}$.
end
for $k = 0, 1,$ **to** $\max$ **do**
  for $r = 1, 2,$ **to** R **do**
    Construct mode-r unfolding $\boldsymbol{\mathcal{X}}_{[r]}^{(k)}$.
    Compute $\boldsymbol{\Delta}_r^{(k)}$ using (17).
    Compute $\mathbf{W}_r^{(k)}$ using (19).
    Update $\mathbf{P}_r^{(k+1)}$ and $\mathbf{Q}_r^{(k+1)}$ using (22).
  end
  Update $\boldsymbol{\mathcal{X}}^{(k+1)}$ by (23).
  if *terminated* **then**
    Return $\boldsymbol{\mathcal{X}}^{(k+1)}$ and $\mathbf{U}^{(k+1)}$.
  end
end

TABLE I
A COMPARISON OF TENSOR DECOMPOSITION AND COMPLETION METHODS

	Complete Data	Incomplete Data
Frobenius norm	HOOI [12]	iHOOI [37]
ℓ_p-norm	IR-HOSVD [26]	i-IR-HOSVD

IV. Simulation Results

We consider the following system setup. Base station is equipped with a uniform rectangular array (URA) with $(M_1 \times M_2)$ elements and mobile node is equipped with a uniform linear array (ULA) with M_3 elements. The coordinate of the (m_1, m_2)-th and m_3-th antenna elements are $(\frac{\lambda}{2}m_1, 0, \frac{\lambda}{2}m_2)$ and $(\frac{\lambda}{2}m_3, 0, 0)$ in three dimensional Cartesian coordinate systems, where λ is the wavelength of the carrier frequency. The origin is the array reference point. Here we evaluate the performance of the proposed approach in the presence of impulsive noise and incomplete measurements. It is evaluated in terms of the root mean square error (RMSE). For each snapshot, the data set dimension is $(16 \times 16 \times 16)$, that is, $M_1 = M_2 = M_3 = 16$, and $N = 5$ snapshots are collected. The RMSE performance is obtained by averaging over all the number of sources and 100 independent runs. Tensorlab is used for tensor computations [43].

The three unknown channel parameters $(\theta_R, \varphi_R, \varphi_T)$ are $(6°, 17°, 53°)$, $(30°, 53°, 24°)$ and $(37°, 30°, 12°)$. Generalized Gaussian (GG) model is one of the widely used distributions, that can well model the phenomenon in the presence of both Gaussian thermal noise and outliers. As shown in Fig. 1, the PDF of GG [44] in terms for different shape parameter β, mean $\mu = 0$ and standard deviation $\sigma = 1$. Impulsive noise can be described by setting shape parameter to $1 < \beta < 2$ and Gaussian noise is obtained for $\beta = 2$. Here GG noise mode is used to describe the impulsive noise. Sample ratio (SR) is utilized to describe the data completeness. For example, SR $= 0.8$ means that 20% of the entries are missing and they are randomly chosen along all the dimensions with equal probability. Channel estimation using complete measurements (SR = 1) is chose as the benchmark to compared with.

The proposed i-IR-HOSVD algorithm is compared with T-ESPRIT [14], TEV and TEV-FB [21]. Since T-ESPRIT, TEV and TEV-FB are developed for complete measurements, the missing measurements are assigened to zero. Maximum number of iterations is 50. Fig. 2-4 show the RMSE results versus signal-to-noise ratio (SNR) in the presence of GG noise by considering different shape parameters $\beta = 0.4$, $\beta = 1$ and $\beta = 1.6$. For the proposed i-IR-HOSVD algorithm, $p = 1$ is used. We observe that it outperforms the other three ℓ_2-norm minimization based methods. Angles are estimated accurately in the presence of impulsive noise and incomplete channel measurements.

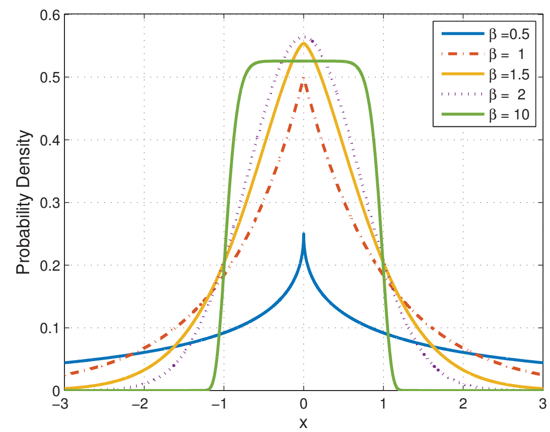

Fig. 1. Probability density function of generalized Gaussian distribution with different shape parameter β.

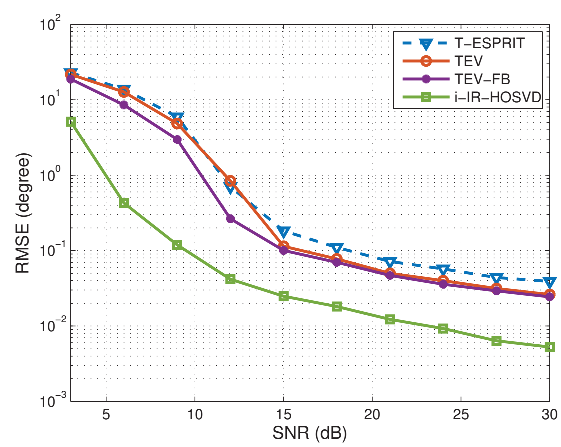

Fig. 2. RMSE of angle versus SNR, data dimension is $(16 \times 16 \times 16)$, number of measurements $N = 5$, SR $= 0.8$ and $\beta = 0.4$.

V. Conclusion

We propose an i-IR-HOSVD algorithm for robust multi-dimensional channel estimation from partial observation and

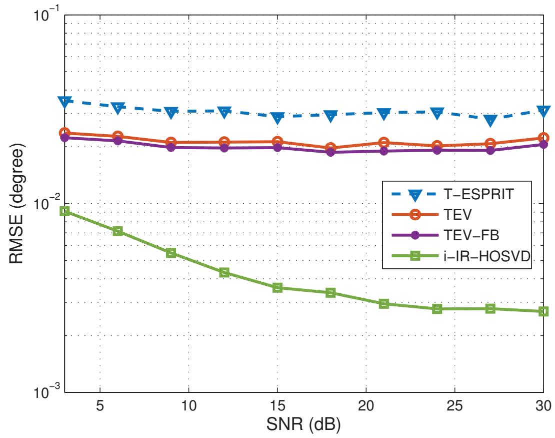

Fig. 3. RMSE of angle versus SNR, data dimension is $(16\times16\times16)$, number of measurements $N=5$, SR $=0.8$ and $\beta=1$.

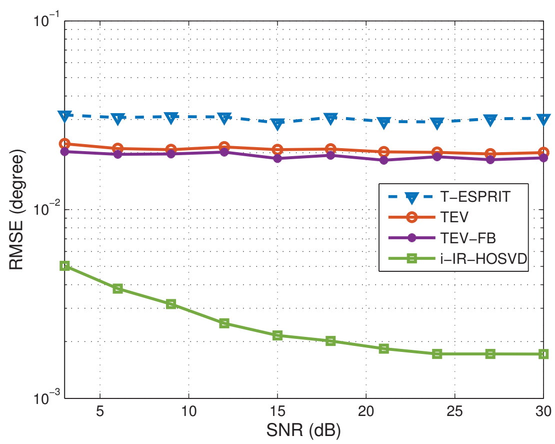

Fig. 4. RMSE of angle versus SNR, data dimension is $(16\times16\times16)$, number of measurements $N=5$, SR $=0.8$ and $\beta=1.6$.

in impulsive noise environments. Inspired by the tensor completion technique, the key idea is to minimize the ℓ_p-norm of the residual error instead of ℓ_2-norm and recover the low rank tensor measurements at the same time. i-IR-HOSVD can be applied for robust higher-order tensor decomposition from crossly corrupted and incomplete measurements. It achieves accurate channel estimation performance in impulsive noise environments, even with incomplete measurements.

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