Extension and Optimization of the Local Geodetic Network at the Onsala Space Observatory

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Cornelia Eschelbach [1], Michael Lösler [1], Rüdiger Haas [2], Henrik Fath [3]

Abstract Since May 2017, the Onsala Space Observatory (OSO) has hosted the Onsala twin telescopes (OTT), two identical telescopes fulfilling the VGOS-specifications. The local geodetic ground network has to be extended to the area around the OTT to provide local tie vectors for combining different geodetic space techniques at the observatory. Furthermore, this network is essential for monitoring the temporal and spatial stability of the new radio telescopes. Both network configuration and measurement uncertainties of the terrestrial observations have a strong impact on the obtainable accuracy of the reference points. Network optimization procedures help to avoid misconfigurations and provide suitable network configurations. For OSO, an extended ground network and an optimal observation schedule are derived that fulfill the accuracy requirements for monitoring processes. The observation schedule, derived by a second order design optimization, focuses on a practical experience when using modern geodetic instruments.

Keywords Second Order Design, Criterion Matrix, Integer Least-Squares, Network Optimization, Local Ties

1. Frankfurt University of Applied Sciences, Laboratory for Industrial Metrology
2. Chalmers University of Technology, Onsala Space Observatory
3. Freistaat Bayern, Körperschaft des öffentlichen Rechts, Amt für Ländliche Entwicklung Unterfranken

1 Introduction

In May 2017, the Onsala Space Observatory (OSO) inaugurated the Onsala twin telescopes (OTT). Both VLBI radio telescopes are identical in design and construction and fulfill the VGOS-specifications. For combining VLBI results of the OTT with other geodetic space techniques, i.e. for deriving the local tie vectors, as well as for monitoring the temporal and spatial stability of the new radio telescopes, the local geodetic ground network at OSO must be extended. The obtainable accuracies for, e.g. the reference points depend on the network configuration and the measurement uncertainties of the observations. The natural environment and especially the rough terrain limit the selection of locations for markers or pillars and their metrological connections. Network optimization procedures help to avoid misconfigurations and provide suitable network configurations. The optimal selection of locations and the optimization of the required observation weights are known as first order design (FOD) and second order design (SOD), respectively. Whereas — in most cases — the FOD cannot be optimized by analytical or numerical methods, the required weights of the SOD are estimable. For OSO, an extended ground network and an optimal observation schedule are derived that fulfill the accuracy requirements for monitoring processes. The observation schedule focuses on practical experience when using modern geodetic instruments.

2 Optimization of Geodetic Networks

A geodetic network is designed to observe topographic properties of the landscape or geometrical phenomena, e.g. deformations on buildings. The configuration depends on the purpose of the network. Usually, four design states are distinguished. The definition of an ideal datum of the network is called zero-order design (ZOD). The optimization of the network configuration and the observation schedule are called first-order design (FOD), where the uncertainties of the observations are known. ZOD and FOD mainly depend on the topography and the realizable metrological connections. In most cases, ZOD and FOD cannot be optimized completely by numerical methods. The second-order design (SOD) is characterized to optimize the uncertainties of the observations, while the network configuration is kept unchanged. The evaluation of the improvement of further points and observations, which are introduced to an existing network, is summarized as third-order design (TOD). Thus, TOD combines the optimization techniques used by FOD and SOD.

The optimization of a single stage cannot be done independently of the prior states, e.g. the optimization of the uncertainties of the observations (SOD) requires a meaningful configuration (FOD) and a proper datum (ZOD) of the network. Table 1 summarizes the parameters which are assumed to be known and the parameters to be optimized during the optimization process. Here, the network configuration is given by the design matrix $\mathbf{A}$, the positive-definite dispersion matrix of the observations $\mathbf{l}$ is denoted by $\mathbf{P}^{-1}$, and $\mathbf{Q} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-}$ is the dispersion matrix of the unknown parameters $\mathbf{x}$, e.g. the coordinates of the points. The generalized inverse is denoted by $(\ )^{-}$ and depends on the nullity of $\mathbf{A}$, i.e. the definition of the geodetic datum. Due to the natural

Table 1 Characterization of the parameters that will be optimized during the optimization process w.r.t. the parameters that are assumed to be known (e.g. [12]).

Design	Known parameters	Optimizable parameters
ZOD	A, P	x, Q
FOD	P, Q	A
SOD	A, Q	P
TOD	Q	A, P

environment and the local terrain there is only a limited number of suitable locations for survey pillars and ground markers, which determines ZOD and FOD of the network. In most cases, the optimizations are carried out without numerical efforts. For SOD, several approaches are known for deriving optimal uncertainties of the observations w.r.t. the required and idealized dispersion matrix $\mathbf{K}$ (e.g. [4, 10, 16]).

2.1 Procedure of second order design

The goal of the SOD is to find reliable weights $\mathbf{P}$, so that

$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A}\right)^{-} = \mathbf{Q} \doteq \mathbf{K}. \tag{1}$$

The matrix $\mathbf{K}$ is often called *criterion matrix*. Such a criterion matrix contains a homogeneous-isotropic structure of the point uncertainties and their dependencies and can be expressed by a so-called Taylor-Karman matrix (cf. [6])

$$\tilde{\mathbf{K}} = \sigma_0^2 \left(\phi_t\left(s\right)\mathbf{E} + \frac{\phi_t\left(s\right) - \phi_l\left(s\right)}{s^2}\mathbf{D} \right). \tag{2}$$

Here, the transversal and longitudinal correlation functions are given by $\phi_t\left(s\right)$ and $\phi_l\left(s\right)$, respectively; σ_0^2 is the variance of unit weight, and $s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$ is the distance between the points in the network. The identity matrix is denoted by $\mathbf{E}$ and the symmetric matrix $\mathbf{D}$ reads

$$\mathbf{D} = \begin{pmatrix} \Delta x^2 & \Delta x \Delta y & \Delta x \Delta z \\ \Delta y \Delta x & \Delta y^2 & \Delta y \Delta z \\ \Delta z \Delta x & \Delta z \Delta y & \Delta z^2 \end{pmatrix}. \tag{3}$$

Using the modified Bessel function of the second kind, the longitudinal and transversal correlation functions are given by

$$\phi_l\left(s\right) = \frac{4d^2}{s^2} - 2K_0\left(\frac{s}{d}\right) - \frac{4d}{s}K_1\left(\frac{s}{d}\right), \tag{4}$$

$$\phi_t\left(s\right) = \frac{2s}{d}K_1\left(\frac{s}{d}\right) - \phi_l\left(s\right) \tag{5}$$

where K_0 and K_1 are the modified Bessel function of zero and first order, and d is the so-called characteristic distance. There are several approaches for defining a suitable characteristic distance d (e.g. [11, 14]). According to Yazji [15], the characteristic distance is set to

$$d = \sqrt{2}\min s. \tag{6}$$

The derived criterion matrix $\tilde{\mathbf{K}}$ is unrelated to the specified ZOD datum. Hence, $\tilde{\mathbf{K}}$ is transformed to the defined datum via an S-transformation, i.e.

$$\mathbf{K} = \mathbf{S}\tilde{\mathbf{K}}\mathbf{S}^{\mathrm{T}} \tag{7}$$

where $\mathbf{S}$ is the specific transformation matrix (cf. [1]).

In this distribution, a modified approach of the direct approximation of the inverse criterion matrix is applied (e.g. [7]). The inverse representation of Equation 1 is given by

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A} = \mathbf{Q}^{-} \doteq \mathbf{K}^{-}. \tag{8}$$

The corresponding normal equation system reads

$$\left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)\mathbf{p} = \mathbf{M}^{\mathrm{T}}\mathbf{k}, \tag{9}$$

where

$$\mathbf{M} = \left(\mathbf{A}^{\mathrm{T}} \odot \mathbf{A}^{\mathrm{T}}\right). \tag{10}$$

Here, $\odot$ denotes the Khatri-Rao product, $\mathbf{p} = \operatorname{diag} \mathbf{P}$, and $\mathbf{k} = \operatorname{vec} \mathbf{K}^{-}$ (e.g. [12])[1,2]. To avoid negative weights during the estimation process, convex optimization with inequality constraints are recommended (e.g. [10]). The estimated weights $\mathbf{p}$ are optimal in the least-squares sense for $\mathbf{K}^{-}$ but not necessarily for $\mathbf{K}$. According to Illner and Müller [7] a correction factor

$$\lambda_u = \frac{\operatorname{tr}(\mathbf{QQ})}{\operatorname{tr}(\mathbf{QK})} \tag{11}$$

is introduced and the final weights are given by

$$\mathbf{P}_u = \lambda_u \mathbf{P}. \tag{12}$$

The estimated weights $\mathbf{P}_u$ of the SOD optimization are unrelated to instruments used during the field work, and the necessary number of repetitions of the observations are simply derived w.r.t. the a-priori uncertainty of a single measurement (e.g. [10, 13]). Practical experiences when using modern instrumentation are not included in the optimization process. For example, modern automated total stations register slope distances, directions, and zenith angles in parallel. A single element of a polar measurement triple cannot be observed without further ado. Moreover, a repetition of a full set of a standpoint can be carried out automatically by the total station. To limit the effort of the measurement procedure, the number of observed target points in a set should be kept unchanged. To take these practical aspects into account an adapted approach is needed.

[1] Function diag extracts the main diagonal of a matrix.

[2] Function vec vectorizes a matrix by stacking its columns.

2.2 Adapted Approach of SOD

The a-priori uncertainties of a single polar measurement triple can be evaluated by metrological experience and knowledge about the measurement instrument. It is the deterministic part of the weights $\mathbf{P}_{\text{det}}$. The unknown part is the number of necessary repetitions $\mathbf{P}_{\text{rep}}$ of the grouped observations. Groupings are applied to force full polar triples and to force the repetition of a full set of target points of a standpoint using specific grouping matrices $\mathbf{G}_{\text{trip}}$ and $\mathbf{G}_{\text{set}}$, respectively. A grouping matrix (e.g. [13])

$$\mathbf{G} = \left[\mathbf{g}_1 \ \dots \ \mathbf{g}_i \ \dots \ \mathbf{g}_g\right] \tag{13}$$

is setup by g group vectors $\mathbf{g}_i^{\mathrm{T}} = \left(0 \dots 0 \ 1 \dots 1 \ 0 \dots 0\right)$, where ones indicate observations of the i-th group. Equation 10 is extended by the deterministic part of the weights $\mathbf{P}_{\text{det}}$ and the specific grouping matrix $\mathbf{G}$, i.e.

$$\mathbf{M}_{\text{rep}} = \mathbf{M}\mathbf{P}_{\text{det}}\mathbf{G}. \tag{14}$$

Substituting Equation 14 into Equation 9 provides the repetition part $\mathbf{p}_{\text{rep}} = \operatorname{diag} \mathbf{P}_{\text{rep}}$ via

$$\left(\mathbf{M}_{\text{rep}}^{\mathrm{T}}\mathbf{M}_{\text{rep}}\right)\mathbf{p}_{\text{rep}} = \mathbf{M}_{\text{rep}}^{\mathrm{T}}\mathbf{k}, \tag{15}$$

and the weights of the observations, which are scaled by Equation 11, are given by

$$\mathbf{P}_u = \lambda_u \mathbf{P}_{\text{det}}\mathbf{P}_{\text{rep}}. \tag{16}$$

Whereas the estimated repetition part results in real numbers, only integer numbers are useful for practical application. As demonstrated in [9], the solution of an integer least-squares problem and the rounded solution of the corresponding real least-squares solution differ. For example, letting

$$\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}, \ \mathbf{k} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

the real solution is

$$\mathbf{p}^{\mathrm{T}} = \left(-\tfrac{1}{3}\ \tfrac{1}{3}\right).$$

Rounding the entries of $\mathbf{p}$ to their nearest integer value yields

$$\lceil \mathbf{p}^{\mathrm{T}} \rfloor = \left(0\ 0\right),$$

whereas the optimal integer least-squares solution reads (cf. [9])

$$\mathrm{p}^{\mathrm{T}} = \left(-2\ 1\right).$$

In geodesy, integer least-squares are used, for example, to estimate the GNSS double-difference integer ambiguity. The algorithm to solve integer least-squares problems is based on a tree search (cf. [2]). Therefore, the numerical effort and the runtime are larger than for the ordinary real least-squares algorithm. According to Chang et al. [3], a real least-squares solution can be transformed to its optimal integer solution. To get a useful solution of the SOD, finally, integer least-squares techniques are applied to transform $\mathbf{P}_{\mathrm{rep}}$ to its optimal integer representation.

3 Results and Conclusion

Two different network configurations were simulated mainly differing in the number of pillars near the OTT and in the number of connecting standpoints. Whereas the pillars surround the new telescopes and are predestined for reference point determinations, the standpoints are necessary to combine the network around the 20-m telescope with the newly designed network of the OTT. The first configuration forms the network of six pillars and 23 further standpoints, and the second configuration uses eight pillars and 24 standpoints. A detailed description is given in [5]. The results of both configurations are quite similar; therefore, we restrict the discussion to the eight pillar configuration.

The uncertainties of the terrestrial instrument are derived by an absolute uncertainty term σ_{c} and a distance-dependent uncertainty term σ_{v}. The combined uncertainty of a slope distance σ_{SD}, a horizontal direction σ_{HD}, and a vertical angle σ_{VA} measurement, respectively, are (cf. [8])

$$\sigma_{\mathrm{SD}} = \sqrt{\sigma_{\mathrm{c,s}}^2 + (s \cdot \sigma_{\mathrm{v,s}})^2}, \tag{17}$$

$$\sigma_{\mathrm{HD/VA}} = \sqrt{\sigma_{\mathrm{c,a}}^2 + (\rho \cdot \frac{\sigma_{\mathrm{v,a}}}{s})^2}, \tag{18}$$

where s denotes the distance and ρ is the conversion factor between radian and gon. Using Equations 17 and 18 and the values given in Table 2, the deterministic part of the weights $\mathbf{P}_{\mathrm{det}}$, cf. Equation 16, is defined.

Table 2 A-priori uncertainties of the polar measurement elements used for defining the stochastic model of the SOD. The constant uncertainties σ_{c} as well as the distance-dependent uncertainties σ_{v} are presented for polar observations.

Uncertainty	Distance	Direction	Vertical angle
σ_{c}	1.0 mm	0.3 mgon	0.3 mgon
σ_{v}	2.5 ppm	0.75 mm	1.0 mm

The criterion matrix is set up by Equation 2, using a target uncertainty $\sigma_0 = 0.5$ mm of a coordinate component, and S-transformed by Equation 7 to the geodetic datum of the ZOD process. Here, the geodetic datum is defined by marked points, i.e. ground markers and pillars.

The SOD process is carried out sequentially. In the first step, $\mathbf{G}_{\mathrm{trip}}$ is introduced in Equation 14 to identify the necessary polar observation triples that form the network with the specified point uncertainties. Using the remaining observation triples of the first analysis step and the extended grouping matrix $\left(\mathbf{G}_{\mathrm{trip}}\mathbf{G}_{\mathrm{set}}\right)$, which ties a full set of target points of a standpoint for repetition, the optimal number of repetition in real number representation is derived by solving Equation 15. Finally, using integer least-squares techniques the real least-squares result is transformed to the optimal integer solution, to get a useful solution. The final solution contains 22 standpoints and requires 137 grouped polar observation triples, cf. Figure 1. The repetition numbers are one, two and one-time nine. The large repetition number of nine is derived for a close range distance of about $s = 5.2$ m, which results in large angle uncertainties, cf. Equation 18.

In comparison to the spatial confidence intervals of the points derived by $\mathbf{K}$, the corresponding intervals derived by $\mathbf{C}$ are always smaller, cf. Figure 1. Thus, the solution derived by $\mathbf{P}_{\mathrm{rep}}$ reaches the specified target uncertainty of the points.

Summarizing, the optimization of geodetic networks allows for an evaluation of expected point

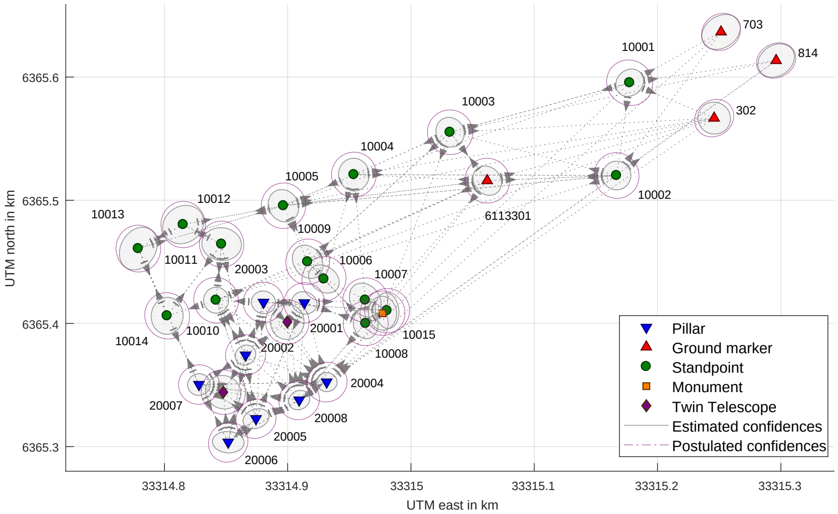

Fig. 1 Resulting network configuration and postulated and estimated confidences, as well as the SOD optimization observation plan.

uncertainties before markers are installed or measurements are carried out. For optimizing the second order design (SOD) numerical methods are well-known.

In this distribution, an extended network at the OSO was derived, which surrounds the new OTT. During the SOD optimization, the deviation between a Taylor-Karman based criterion matrix and the derived dispersion matrix was minimized. By grouping observations during the optimization process, practical experiences when using modern geodetic instruments are considered. Finally, integer least-squares techniques are applied to get a practically usable solution of the SOD.

References

1. W. Baarda: S-transformations and criterion matrices. *Netherlands Geodetic Commission*, Publications on Geodesy, 5(1), Delft, 2nd edn.
2. M.A. Borno, X.-W. Chang, X.H. Xie: On decorrelation in solving integer least-squares problems for ambiguity determination. *Surv. Rev.*, 46(334), 37–49, 2014.
3. X.-W. Chang, X. Yang, T. Zhou: MLAMBDA: A modified LAMBDA method for integer least-squares estimation. *J. Geod.*, 79(9), 552–565, 1997.
4. P.A. Cross: Numerical methods in network design. In: Grafarend W, Sansõ F (eds.): *Optimization and Design of Geodetic Networks*, Springer, Berlin, 132–168, 1985.
5. H. Fath: Planning of the monitoring network of the VGOS twin telescopes at the Onsala Space Observatory in Sweden (in German). *Master Thesis*, Frankfurt University of Applied Sciences, Chalmers University of Technology, unpublished, 2017.
6. E. Grafarend, B. Schaffrin: Kriterion-Matrizen I - zweidimensionale homogene und isotrope geodätische Netze. *zfv*, 104(4), 133–149, 1979.
7. M. Illner, H. Müller: Gewichtsoptimierung geodätischer Netze – Zur Anpassung von Kriteriumsmatrizen bei der Gewichtsoptimierung. *avn*, 91(7), 253–269, 1984.
8. M. Lösler, R. Haas, C. Eschelbach: Automated and continual determination of radio telescope reference points with sub-mm accuracy – Results from a campaign at the Onsala Space Observatory. *J. Geod.*, 87(8), 791–804, 2013.
9. S. Qiao: Integer least squares: Sphere decoding and the LLL algorithm. In: *Proceedings of the 2008 Canadian Conference on Computer Science and Software Engineering (C3S2E) conference*, 23–28, 2008.
10. L. Roese-Koerner, W.-D. Schuh: Convex optimization under inequality constraints in rank-deficient systems. *J. Geod.*, 88(5), 415–426, 2014.
11. G. Schmitt: Second order design of a free distance network, considering different types of criterion matrices. *Bull. Geodesique*, 54(4), 531–543, 1980.
12. G. Schmitt: Review of Network Designs: Criteria, Risk, Functions, Design Ordering. In: W. Grafarend, F. Sansõ (eds.): *Optimization and Design of Geodetic Networks*, Springer, Berlin, 6–10, 1985.
13. G. Schmitt: Second order design. In: W. Grafarend, F. Sansõ (eds.): *Optimization and Design of Geodetic Networks*, Springer, Berlin, 74–120, 1985.
14. H. Wimmer: Ein Beitrag zur Gewichtsoptimierung geodätischer Netze. *Dissertation*, DGK-C 269, Munich.
15. S. Yazji: The Effect of the Characteristic Distance of the Correlation Function on the Optimal Design of Geodetic Networks. *Acta Geod. Geoph. Hung*, 33(2-4), 215–234, 1998.
16. M. Yetkin, C. Inal, C.O. Yigit: Use of the particle swarm optimization algorithm for second order design of levelling networks. *J. Appl. Geod.*, 3(6), 171–178, 2009.