THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN SOLID AND
STRUCTURAL MECHANICS

Identification and synthesis of components for uncertainty propagation

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ABSTRACT

For automotive structures, built-up of hundreds of components with property spread, knowing the effects of component variability and its propagation through the system assembly is important in order to mitigate noise and vibration problems. To increase the understanding of how the spread propagates into variability in built-up structures, both experimental and computational aspects are considered in this thesis.

In the first part of the thesis, methods to identify models from experimental data are developed. Physical insight is often required for accurate experimental models. To this end, two-phase state-space system identification algorithms are developed where physically motivated residual states are included and physically motivated constraints are enforced. The developed identification algorithms are used together with finite element model updating to investigate the variability in dynamical properties between nominally identical components. Furthermore, the accurate and physical experimental models are used in synthesis with the updated finite element models. It is shown that experimental-analytical synthesis of complex and modally dense structures is possible, and that the component variability can be predicted in such assemblies.

In the second part of the thesis, methods to reduce the computational cost of variability analysis are developed. An efficient multifidelity interface reduction method is developed for component synthesis. It is also shown that modal truncation augmentation vectors can be computed efficiently from the multifidelity interface reduction basis. Lastly, an efficient uncertainty propagation method is developed, based on a second-order modal model. Utilising several approximations, it is shown that industrial-sized models can be handled with small loss in accuracy compared to a purely Monte Carlo based approach.

Keywords: system identification, state-space models, substructuring, interface reduction, experimental methods, uncertainty quantification, uncertainty propagation, Monte Carlo method, surrogate modelling

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PREFACE

This thesis has been carried out from December 2014 to January 2020 in the scope of a Volvo Industrial PhD Project. During this time, I have been located both at Chalmers University of Technology and Volvo Car Corporation (VCC).

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Göteborg, January 2020
Mladen Gibanica

THESIS

This thesis consists of an extended summary and the following appended papers:

Paper A   M. Gibanica, T. J. S. Abrahamsson, and T. McKelvey. State-Space System Identification with Physically Motivated Residual States and Throughput Rank Constraint. *Mechanical Systems and Signal Processing* (In press)

Paper B   M. Gibanica, T. J. S. Abrahamsson, and M. Olsson. Model Updating of Multiple Nominally Identical Car Components. *Experimental Techniques* (In press). DOI: 10.1007/s40799-019-00357-5

Paper C   M. Gibanica and T. J. S. Abrahamsson. Identification of Physically Realistic State-Space Models for Accurate Component Synthesis. *Submitted for international publication* (2019)

Paper D   M. Gibanica, T. J. S. Abrahamsson, and D. J. Rixen. Multifidelity Component Interface Reduction and Modal Truncation Augmentation. *International Journal for Numerical Methods in Engineering* 120.1 (2019), 105–124. DOI: 10.1002/nme.6126

Paper E   M. Gibanica and T. J. S. Abrahamsson. Data-Driven Modal Surrogate Model for Frequency Response Uncertainty Propagation. *Submitted for international publication* (2019)

The thesis has also resulted in the following toolbox:

Software A   M. Gibanica and T. J. S. Abrahamsson. *abraDAQ*. 2019. URL: https://github.com/mgcth/abraDAQ (visited on 02/03/2019)

The appended papers were prepared in collaboration with co-authors. The author of this thesis was responsible for the major progress of the work, i.e. planning the papers, developing the theory, carrying out the numerical simulations, performing the experiments and writing the papers. A MATLAB toolbox, abraDAQ, for vibration measurements has also been developed, in collaboration with Professor Thomas Abrahamsson. The author of this thesis was mainly responsible for implementing the stepped multisine and periodic excitation procedures, along with support functionality.

OTHER PUBLICATIONS

Other publications, not included in, but related to thesis:

Publication I M. Scheel, M. Gibanica, and A. Nord. State-Space Dynamic Substructuring with the Transmission Simulator Method. *Experimental Techniques* 43.3 (2019), 325–340. DOI: 10.1007/s40799-019-00317-z

Publication II M. Gibanica, T. J. S. Abrahamsson, and D. C. Kammer. Redundant Information Rejection in Sensor Localisation Using System Gramians. In: *Topics in Modal Analysis & Testing, Volume 10. Conference Proceedings of the Society for Experimental Mechanics Series*. Cham, Switzerland: Springer, 2016, pp. 325–333. ISBN: 978-3-319-30248-5

Publication III M. Gibanica, T. J. S. Abrahamsson, and M. Olsson. Calibration, Validation and Uncertainty Quantification of Nominally Identical Car Subframes. In: *Model Validation and Uncertainty Quantification, Volume 3. Conference Proceedings of the Society for Experimental Mechanics Series*. Cham, Switzerland: Springer, 2016, pp. 315–326. ISBN: 978-3-319-29753-8

Publication IV M. Gibanica and T. J. S. Abrahamsson. Parameter Estimation and Uncertainty Quantification of a Subframe with Mass Loaded Bushings. In: *Model Validation and Uncertainty Quantification, Volume 3. Conference Proceedings of the Society for Experimental Mechanics Series*. Cham, Switzerland: Springer, 2017, pp. 61–76. ISBN: 978-3-319-54857-9

Publication V M. Gibanica, T. J. S. Abrahamsson, and D. J. Rixen. A Reduced Interface Component Mode Synthesis Method Using Coarse Meshes. *Procedia Engineering. X International Conference on Structural Dynamics, EURODYN 2017* 199 (2017), 348–353. DOI: 10.1016/j.proeng.2017.09.031

Publication VI M. Gibanica, T. J. S. Abrahamsson, and R. J. Allemang. Residual States for Modal Models Identified from Accelerance Data. In: *Topics in Modal Analysis & Testing, Volume 9. Conference Proceedings of the Society for Experimental Mechanics Series*. Cham, Switzerland: Springer, 2018, pp. 195–206. ISBN: 978-3-319-74699-9

Publication VII M. Gibanica, T. J. S. Abrahamsson, and T. McKelvey. Physically Motivated Rank Constraint on Direct Throughput of State-Space Models. *IFAC-PapersOnLine. 18th IFAC Symposium on System Identification SYSID 2018* 51.15 (2018), 329–334. DOI: 10.1016/j.ifacol.2018.09.156

Publication VIII   S. Gumussoy, A. A. Ozdemir, T. McKelvey, L. Ljung, M. Gibanica, and R. Singh. Improving Linear State-Space Models with Additional Iterations. *IFAC-Papers-OnLine*. *18th IFAC Symposium on System Identification SYSID 2018* 51.15 (2018), 341–346. DOI: `10.1016/j.ifacol.2018.09.158`

Publication IX   A. Bylin, M. Gibanica, and T. J. S. Abrahamsson. Experimental-Analytical State-Space Synthesis of Passenger Car Components. In: *Proceedings of ISMA2018 and USD2018*. *International Conference on Noise and Vibration Engineering*. Leuven, Belgium, 2018, pp. 4021–4035

Publication X   G. Dorendorf, M. Gibanica, and T. J. S. Abrahamsson. Variability of Nominally Identical Components and Their Influence in an Assembly - Application to a Volvo XC90. In: *Svenska Mekanikdagar*. Uppsala, Sweden, 2017, p. 32

Publication XI   M. Gibanica, T. J. S. Abrahamsson, and D. J. Rixen. Component Mode Synthesis Interface Reduction and Modal Truncation Augmentation Using Coarse Meshes. In: *Svenska Mekanikdagar*. Stockholm, Sweden, 2019, p. 41

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# CONTENTS

Part I

EXTENDED SUMMARY

1 INTRODUCTION

Noise and vibration problems in vehicles can cause serious driver and passenger discomfort and are often used by customers as a measure of the perceived vehicle quality, e.g. see Cerrato [21] and Griffin [46]. Many external and internal sources can contribute to both noise and vibration phenomena, e.g. Sheng [87] identifies wind structure interaction, engine excitation and tire-road contact among others. A commonality between all these phenomena is the automotive structure, which is involved in either transmitting the vibrations, e.g. vibrations felt through the seat, or generating sound from panels. Such problems are addressed during the development phase of new vehicles. Finite element (FE) models, e.g. see Bathe [15], are most often used for analysis work, based on computer-aided design (CAD) geometry, and allow for a fast design iteration cycle. Structural design modelling targets the quasi-static and low-frequency behaviour. A major challenge in the development of FE models for low-frequency vibration analysis is to avoid modelling errors and wrongly set model parameters that can lead to predictions that deviate much from experimental data. This is especially true for mass-produced components, such as nominally identical manufactured cars, where a nominal FE model is often used to predict the fleet's dynamic behaviour. For automotive structures, built-up of hundreds of components with uncertain properties, knowing the effects of component variability and its propagation through the system assembly is important from a decision making perspective in order to mitigate noise and vibration problems. For example, it is possible that component variability can affect the assembly such that individuals in a production series are of insufficient quality. It is, therefore, of interest to know how many such individuals that can be expected for some given component variability, and also which property variations that cause most spread in the assembled system. To identify these cases, and mitigate them, it is important to take uncertainty into account during the development process. However, to use the results from a variability analysis of built-up structures as support for better decision making, accurate models are necessary. It is even possible that using inaccurate models for such analyses could lead to faulty decisions. Therefore, to increase the understanding of how uncertainties propagate in built-up structures, both experimental and computational aspects are considered in this thesis.

A CAD drawing of the two considered components, a Volvo XC90 (2015) body-in-white (BIW) and rear subframe, and their assembly can be seen in Figure 1.1. A hypothesis is that, given accurate models of the BIW and several rear subframes, their assemblies should, in general, match the reference measurements, and in particular, the variability between the assembled systems should be captured. It is assumed that the large BIW model is easier to model experimentally, and that the smaller and simpler rear subframe model is easier to model with FE. To test that hypothesis, several developments are necessary. These developments, and the thesis overview and paper connections, can be seen in Figure 1.2. In that figure, dashed paths and boxes indicate other relevant activities. Some of these activities are discussed in Chapter 7. First, a measurement
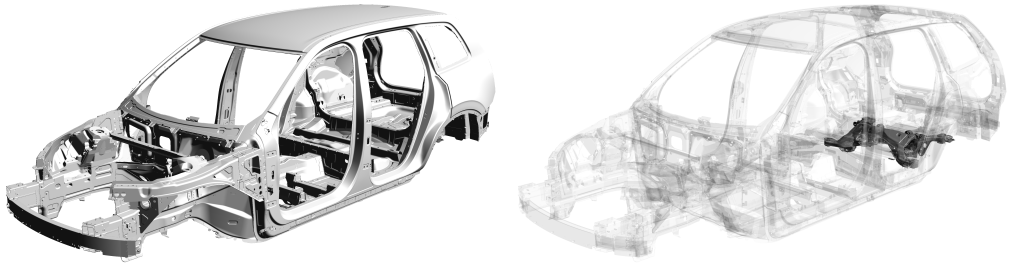
(a) *BIW*   (b) *BIW and rear subframe assembly*

Figure 1.1: *The BIW and rear subframe of a Volvo XC90 and their assembly.*

software is developed in *Software A* to obtain accurate experimental data. Then, in *Paper A* and *Paper C*, system identification methods are developed with physically motivated constraints. These methods are used in *Paper B* to update the rear subframe FE models, which are then used for experimental-analytical synthesis in *Paper C*.

To test the initial hypothesis, however, requires a lot of effort, and availability of real components. During the development phase of new products, enough resources might not be available to create experimental models usable for synthesis, but FE models might be available. There is, usually, still a strong interest in obtaining information about the fully assembled system's uncertain response using probabilistic analysis. It is also important to use accurate models and know something about the distribution of the model input parameters, which is why model updating and uncertainty quantification are important tools, see Figure 1.2. The remaining challange with probabilistic analysis is then that many model evaluations are required, e.g. by using the Monte Carlo method. For such analyses to be feasible for industrial models, the solution time for a single evaluation must be as small as possible. Therefore, the last part of this thesis is related to computational cost reduction for variability analysis.

In *Paper D*, a substructuring method with interface reduction is developed using multiple model fidelities. It is also shown that modal truncation augmentation vectors (MTAs) can be computed at a low cost, and that their inclusion in the reduction basis improves the model accuracy substantially. However, even with efficient solution methods, industrial-sized models cannot be directly used for Monte Carlo simulations without extreme computational resources, but can be used as an ingredient to reduce the overall computational time, e.g. to cheaper sample the design space, necessary for most Monte Carlo alternatives. Lastly, in *Paper E*, a computationally efficient surrogate model for propagating uncertainties into frequency responses is developed.
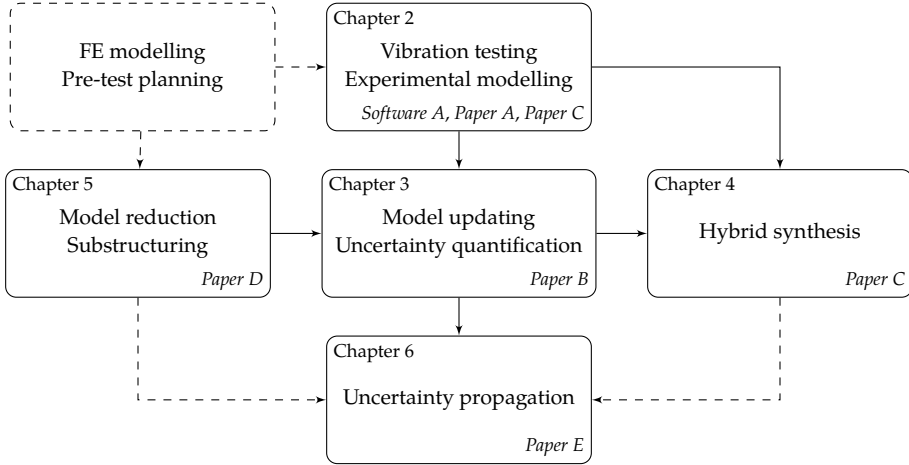
Figure 1.2: *Overview of thesis and connections between papers.*

1.1 AIMS OF THESIS

The three main aims of this thesis can be concisely stated as:

1. *Development of tools to validate and update computational models* used in computer-aided development. These tools are general purpose and address a specific task, e.g. identifying experimental models and doing experimental synthesis.
2. *Investigate how well the measured variability in assemblies can be predicted* by using calibrated component models. Of particular interest is to find out whether experimental-analytical synthesis is feasible for complex and modally dense structures, and if such hybrid models can provide better results than a pure FE model.
3. *Reduce the computational cost of variability analysis* to enable its use for better decision making in the early phase of development.

1.2 LIMITATIONS

The thesis is purposely narrowed in scope to these relevant limitations:

• *Only two substructure components are considered*, i.e. a car BIW combined with a rear subframe.
• *Component variability is not quantified*, because only three individual subframes are considered, too few to estimate meaningful statistical data.
• *Response sensitivity is not quantified* with respect to individual model parameters.
• *Only low-frequency behaviour*, below 300 Hz, is studied because the structural design is usually set based on this behaviour.
• *Passivity constraint* on the experimentally identified models is not considered.

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2 SYSTEM IDENTIFICATION

To test the hypothesis stated in Chapter 1, and for uncertainty propagation in general, accurate models are necessary. Experience from validation of FE models with given experimental data shows that FE models of practical use do not always adequately model the dynamics of complex structures. In such situations, FE model updating, e.g. see Friswell and Mottershead [33], can be used to update FE models to better represent experimental data. Such updating is the topic of *Paper B*. Updating procedures often require experimentally identified models. However, FE model updating procedures can fail as they assume a properly parametrised model which can, in practice, be difficult to achieve for more complex structures. Another possible modelling strategy is to rely partly on experimentally derived models. To derive such models for synthesis applications, as in *Paper C*, well planned and conducted experiments and data processing are required. In addition, the experimentally derived models must often be equipped with properties that only proper physical insight can provide. Aspects of these procedures are briefly described here, and form the basis for much of the results in this thesis as shown in Figure 1.2.

2.1 VIBRATION EXPERIMENTS

Obtaining experimental data is the natural first step in deriving an experimental model. Usually, the in-band dynamics (the frequency response in between cut-on and cut-off frequencies) is measured directly, and out-of-band dynamics estimated or approximated through known physical laws. To support such approximation, it is important to know the structure's boundary conditions during the test, e.g. see Ewins [31]. It is often easiest to approximate a free-free boundary condition by isolating the structure from its environment by best practices. In this thesis, air springs have been used to isolate the heavy BIW, see Figure 2.1a, while the lighter subframe components were hung in thin high-strength lines, see Figure 2.1b. Both isolation cases render the approximate rigid body modes with eigenfrequencies well below that of the first flexible mode. In an experimental modal analysis, accelerometers are most commonly used to measure the system response excited by some excitation signal. The excitation is most commonly provided by an impact hammer or from an electrodynamic shaker.

Pre-test planning is a crucial step in order to increase the information content in the gathered experimental data. Most importantly, in testing, all modes in the frequency range of interest must be observable and controllable. For practical reasons, the system inputs are usually limited to easily reachable locations or positions related to known force inputs during operating conditions. However, accelerometer placement is usually easier, and many methods are available for their optimal placement. These methods usually rely on available FE models, that must be fairly accurate. A well known and often used

(a)   (b)

Figure 2.1: *Experimental set-up of a BIW on air springs in (a) and rear subframe hanging in lines in (b).*

method is the method of effective independence (EfI) by Kammer [57]. In this thesis, in particular in *Paper A*, *Paper B* and *Paper C*, a modified variant [41] of the extended EfI method for triaxial accelerometers [58] has been used, where redundant information is rejected. Another redundant information rejection approach based on the EfI method was proposed by Stephan [93]. A stochastic variant of the EfI method has been developed by Kim et al. [62], and several stochastic variants compared by Castro-Triguero et al. [20].

2.2 NONPARAMETRIC IDENTIFICATION

The goal of nonparametric system identification is to obtain the frequency response function (FRF) matrix $G(\omega) \in \mathbb{C}^{n_y \times n_u}$ from noisy measured input-output time (t) data $\{u(t), y(t)\}$ as

$$Y(\omega) = G(\omega)U(\omega) \tag{2.1}$$

with ω the angular frequency and $Y(\omega)$ and $U(\omega)$ the Fourier transform of the system outputs and inputs, respectively. Here, n_y and n_u denote the number of outputs and inputs, respectively. A MATLAB toolbox *Software A* [40] for nonparametric identification has been developed. Procedures for both periodic and stepped multisine excitation signals have been implemented, and used to obtain test data for *Paper A*, *Paper B* and *Paper C*. For periodic excitation, e.g. chirp or impact hammer excitation, techniques found in Pintelon and Schoukens [81] and Maia and Silva [95] can be used to estimate the FRF matrix. For the stepped multisine excitation, multiple frequency components simultaneously excite the system. Through a harmonic regression, e.g. see Kay [60], the FRF matrix is estimated, for the known frequency components in each stepping. In this method, it is possible to impose a check on the system stationarity by analysing consecutively recorded time blocks of response data for each stepping. This stationarity check has been implemented in *Software A*. Such a procedure, for simulations, has been described by Andersson [10] in more detail.

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In parametric identification, a model with some predefined structure, e.g. a first-order or a second-order differential in time structure, describing the experimental data is sought. An overview of parametric identification methods used in experimental modal analysis can be found in Ewins [31] and Maia and Silva [95]. For parametric state-space system identification, see Ljung [69] and van Overschee and De Moor [100]. A frequency domain subspace state-space (N4SID) algorithm developed by McKelvey et al. [72] is used in this thesis to initially obtain the first-order system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.2a}$$
$$y(t) = Cx(t) + Du(t) \tag{2.2b}$$

with $x(t)$ the state vector, $u(t)$ the input vector and $y(t)$ the output vector. Matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$ and $D \in \mathbb{R}^{n_y \times n_u}$ denote the state, input, output and direct throughput matrices, respectively, for n states. The dot notation is used for time differentiation. With $i^2 \triangleq -1$ the imaginary number i, the frequency response from the first-order form can be obtained as

$$G(\omega) = C (i\omega I - A)^{-1} B + D \tag{2.3}$$

When no physical insight has been used in the identification of the four system matrices in Equations (2.2) from frequency response data, the identification is commonly called black-box modelling. In contrast, white-box modelling is based on first principles only. Grey-box modelling, however, combines the two and can often increase the black-box model's accuracy at the expense of some added complexity. In experimental modal analysis of flexible and approximately free-free hanging structures, it is common to include mass and stiffness residuals to model out-of-band behaviour, usually not measured directly in experiments, but affecting the in-band behaviour, e.g. see Ewins [31]. In *Paper A*, mass and stiffness residuals are reformulated for possible inclusion in the state-space description in Equations (2.2). In addition, a complementary residual is proposed to model some particularly difficult energy conjugated input-output pairs. Such pairs are almost always used in system synthesis of experimentally derived models. Another strategy for mass and stiffness residual inclusion on state-space form was recently developed by Waimer [104]. In El-Kafafy et al. [55] another method for residual inclusion was proposed for second-order form systems.

Apart from increasing the experimentally derived model's accuracy, physical insight can be used to constrain the model for physical consistency. If the experimental model is to be used for synthesis, physical consistency constraints are usually required for accuracy. For system synthesis using a state-space formulation of systems with rigid body motion, Sjövall [89] states the following physical constraints:

- *Stability*
- *Passivity*
- *Reciprocity*
- *Displacement-velocity consistency*

Passivity is not treated in this thesis, but has been addressed by, e.g. McKelvey and Reza Moheimani [73] and Liljerehn [67]. However, an additional constraint is proposed here for systems possessing rigid body motion:

- *Constrained rigid body motion*

In a first phase of state-space model identification, to obtain the system in Equations (2.2), a stable system can be directly obtained from the method by McKelvey et al. [72]. However, the reciprocity and rigid body motion constraints must be enforced in a second phase. The constraint on displacement-velocity consistency can be fulfilled with additional states after the second phase, as proposed by Liljerehn [67].

The second phase of the identification consist of an optimisation problem, with various constraints, see *Paper A* and *Paper C*. For some experimental FRF matrix $\bar{G}(\Omega_k)$ with Ω_k the discrete frequency with $k = 1, \dots, K$, the corresponding unconstrained minimisation problem can be written as

$$\hat{B}, \hat{C}, \hat{D} = \arg\min_{B,C,D} \sum_{k=1}^{K} \left\| \bar{G}(\Omega_k) - C \left(i\Omega_k I - A \right)^{-1} B - D \right\|_{\mathrm{F}} \tag{2.4}$$

with $\| \cdot \|_{\mathrm{F}}$ the Frobenius norm. It can be seen that the frequency response in Equation (2.4) is bilinear, i.e. linear in $\{C, D\}$ for fixed $\{A, B\}$ and linear in $\{B, D\}$ for fixed $\{A, C\}$. Based on this, it has been observed in Gumussoy et al. [47] that a good solution strategy for the unconstrained problem in Equation (2.4) is an alternating least squares estimation algorithm

$$C^{(l+1)}, D^{(l+1)} = \arg\min_{C,D} \sum_{k=1}^{K} \left\| \bar{G}(\Omega_k) - D - C \left(i\Omega_k I - A \right)^{-1} B^{(l)} \right\|_{\mathrm{F}} \tag{2.5a}$$

$$B^{(l+1)}, D^{(l+1)} = \arg\min_{B,D} \sum_{k=1}^{K} \left\| \bar{G}(\Omega_k) - D - C^{(l+1)} \left(i\Omega_k I - A \right)^{-1} B \right\|_{\mathrm{F}} \tag{2.5b}$$

for $l = 0, \dots, L - 1$ iterations.

In *Paper A* and *Paper C*, it is shown that a similar solution strategy can be used for constrained problems with good results. A rank constraint on the rigid body modes partition of the direct throughput matrix is considered in *Paper A* for systems on acceleration output form. A similar constraint for systems on displacement output form is developed in *Paper C*. In addition, in *Paper C*, a reciprocity constraint is also developed for models on state-space form. All these constraints are enforced using the alternating least squares solution method, and are hence fast, in comparison to non-linear optimisation problems. Physically motivated constraints have also been studied by, e.g. El-Kafafy et al. [56], Sjövall [91] and Liljerehn [67].

3 MODEL UPDATING AND UNCERTAINTIES

While computational models allows for rapid prototyping of new design ideas, experience shows that FE models can often give erroneous results as obvious when validation is tried towards experimental data. There often exists a lack-of-knowledge or uncertainty about the model structure, the model parameter values or the model parameter distribution for nominally identical components. If the computational model deviates from the assumed true test data and important decisions are taken based on this model, actions must be taken. For large deviations, it is possible that the model structure is wrong and must be improved. If the component to be modelled is large and complex, with hundreds or thousands of possible uncertain properties, experimental modelling can be a better option for an accurate model, see Chaper 2. Note that, an experimental model will in general not give access to underlying physical processes providing the data. However, the experimental models can be synthesised with FE models, to provide hybrid models. For smaller deviations, model updating can be a tool to calibrate model parameters, improving the model fit to test data, e.g. see Friswell and Mottershead [33]. Because test data is usually contaminated by noise, the calibrated model parameters cannot be determined with perfect precision. Uncertainty quantification methods, e.g. see Smith [92], can be used to quantify uncertainty in the updated model parameters. For successful model updating and uncertainty quantification, high-quality experimental data or experimentally derived models are often necessary, again see Chaper 2. In Figure 1.2, the connection between model updating and system identification is clearly shown. In addition, model updating procedures can require many model evaluations, for which efficient solution methods are required, further discussed in Chaper 5. Aspects of model updating and uncertainty quantification are described below.

3.1 TYPES OF UNCERTAINTIES

Uncertainties can be categorised into reducible uncertainty, also called lack-of-knowledge or epistemic uncertainty and irreducible uncertainty, also called random or aleatory uncertainty, e.g. see Der Kiureghian and Ditlevsen [26]. However, the line between them is not always sharp. For example, take variability between components. If tolerances in production cannot be controlled, the uncertainty is irreducible, however, if the tolerances can be tightened the uncertainty is reducible. For this reason, an explicit discussion between these two types of uncertainties is not considered in this thesis. Instead, the following classification of model uncertainties by Kennedy and O'Hagan [61] are considered:

- *Parametric uncertainty* describes the statistical knowledge associated with model input parameters when considering test data from a single individual from a population. Multiple experiments will never produce exactly the same results due to noise, and the estimated parameters will, therefore, be uncertain to some extent.

- *Model adequacy* refers to how well the model at hand is capturing reality. Many times simplifications are introduced, which might be overly crude and make the model inadequate. In situations of model inadequacy, it is important not to update model parameters to be biased by modelling simplifications.
- *Residual variability* means that for a real process with well specified repeated inputs, the outputs are not necessarily always identically repeated. This can result from two sources of uncertainties. The process can be inherently random or the variability can also be a result of some unexpected stimulus not being specified, making this variability another form of model inadequacy.
- *Observation errors* are caused due to measurement noise, e.g. electronic noise in equipment or temperature fluctuations, and can be hard to separate from residual variability.
- *Parametric variability* represents the actual variability of the physical properties, manifested in mass-produced components from manufacturing tolerances. Such variation can be captured as parameter variation after an update of model parameters that represent such physical properties.
- *Code uncertainty* is associated with implementation errors in complex computer codes, causing erroneous results resulting in uncertainties.

In this thesis, parametric variability is simply denoted by variability between nominally identical, but in reality slightly different, component individuals. In the uncertainty quantification step, mainly parametric uncertainty is quantified, due to observation errors, residual variability and model inadequacy. Code uncertainty is not considered. Note that many of these uncertainties can apply to both FE and experimentally derived models. For example, experimentally identified models lacking the out-of-band residuals described in Chapter 2 and *Paper A* can be seen as a form of model inadequacy.

3.2 MODEL UPDATING

When model validation reveals a discrepancy between the FE model and test data, model calibration can be used to decrease that discrepancy by updating some model parameters. The used parametrisation is often based on convenience, and shell thickness and material stiffness parameters are commonly used. However, Mottershead et al. [74] and Friswell et al. [34] have argued for geometric and joint parameters being more realistic model parameters. Geometrical parametrisation is more difficult to implement, and is not considered in this thesis. However, in *Paper B*, the bushing stiffness parameters are considered. Bushing stiffness parameters are often used in the automotive industry as tunable parameters to achieve certain car dynamics. There is, therefore, interest in the possibility to update such parameters from experimental data of components with mounted bushings. In *Paper B*, two bushing modelling techniques are also compared.

There are many approaches to FE model updating, e.g. updating towards eigenfrequencies and mode shapes or FRFs, see Friswell and Mottershead [33] for an overview of

methods. FRF based modal updating has been considered by, e.g. Visser [101], Balmès [12], Lin and Ewins [68], and Grafe [45]. In this thesis, an FRF based FE model updating procedure proposed by Abrahamsson and Kammer [2] is used, which circumvents the problem of mode pairing by imposed equalised damping. This is possible because the updating is performed towards synthesised FRF data from an experimentally identified model. Hence, the experimentally identified model's accuracy is of high importance, as discussed in Chapter 2. Successful results have been obtained in previous studies using this method on various industrial components, e.g. see [3, 1, 66, 30].

In Abrahamsson and Kammer [2] the following deviation function is minimised

$$p^* = \arg\min_{p \in \mathbb{P}} \sum_{k=1}^{K} \left\| \log_{10}\left(\text{vec}\left(G^{\text{FE}}\left(\Omega_k, p \right) \right) \right) - \log_{10}\left(\text{vec}\left(G\left(\Omega_k \right) \right) \right) \right\|_2 \qquad (3.1)$$

with $\|\cdot\|_2$ the Euclidean norm and $\text{vec}(\cdot)$ the vectorisation operation in which a matrix is transformed into a vector by stacking its columns. The physical parameters are denoted by $p \in \mathbb{R}^{P \times 1}$ with P the number of parameters. The parametrised FRF matrix $G^{\text{FE}}\left(\Omega_k, p \right)$ can be set-up as in Equation (2.3). In the model calibration step, FRF data $G\left(\Omega_k \right)$ is from the experimentally identified model.

3.3 UNCERTAINTY QUANTIFICATION

From a model calibration procedure, usually, one calibrated parameter setting p^* is obtained by minimisation. However, calibration towards datasets from multiple tests of the same structure will not yield the same calibrated parameter settings due to experimental uncertainties described in Section 3.1. Similarly, variability between components will cause calibration outcome for data stemming from different individuals to vary. Therefore, to increase the confidence in the updated model parameters, the model parameter uncertainty must be quantified. In this thesis, only uncertainty due to experiments is quantified in *Paper B*, and not variability between components. For this purpose, stochastic methods for FE model updating have been proposed, e.g. see Mares et al. [71], Mottershead et al. [75] and Govers and Link [44]. Also, perturbation methods have been developed, e.g. see Mottershead et al. [76]. Gautier and co-workers [36, 37] developed a sub-space fitting approach for FE model updating, accounting for FE model parameter uncertainties through the identification procedure. Bayesian methods have been developed, see Beck and Katafygiotis [16], and are driven mainly by more efficient Markov chain Monte Carlo methods, e.g. see Vakilzadeh et al. [97]. A review of probabilistic and non-probabilistic uncertainty quantification methods for model updating can be found in Simoen et al. [88].

Using FRF data for uncertainty quantification has the advantage that uncertainties in the raw FRFs can be directly mapped to the FE model parameters, without intermediate steps mapping to modal data first. Vakilzadeh et al. [98] developed a stochastic model updating procedure, based on bootstrapping, for the FE model updating method by Abrahamsson and Kammer [2]. However, it has been observed that for large data sets,

that method can be infeasible due to high computational cost. Instead, in *Paper C*, a linear-in-parameters surrogate model of the deviation metric between the FE model around the calibrated parameter setting p^* and raw FRF data is created and bootstrapping used to quantify the parameter uncertainties.

4 HYBRID SYNTHESIS AND MODELLING

It has been mentioned previously that accurate FE modelling can be difficult to obtain for some structures, but are necessary for uncertainty propagation studies. In such situations, an experimentally derived model can be easier to obtain. For structures like cars, consisting of many components or substructures, some hard-to-model component can be modelled experimentally and other easier-to-model components with FE. The component models can then be synthesised, to form the assembled system's model. System synthesis is straightforward using FE models, but is considerably harder for experimental models. Most commonly, three methods are used for experimental substructuring:

1. *Component mode synthesis (CMS)* methods are used in the modal domain, e.g. see Allen and Mayes [6].
2. *Frequency-based substructuring (FBS)* approaches are applied in the frequency domain, e.g. see Jetmundsen et al. [54] and de Klerk et al. [24].
3. *State-space* approaches generally assume a first-order state-space form, e.g. see Su and Juang [94] and Sjövall and Abrahamsson [90].

For a review of substructuring methods, see de Klerk et al. [25] and Voormeeren [103].

Substructuring is domain-independent, and requires compatibility constraints and force equilibrium conditions for synthesis. However, obtaining good experimental models can be difficult for several reasons:

- *Accurate measurements* are necessary, but can be difficult to obtain, e.g. avoiding noise, non-linearities, obtaining reciprocity and existence of many hard-to-measure locations.
- *Coupling locations* most often need many input-output pairs, which can be physically hard to reach and measure.
- *Accurate system identification* is necessary for both CMS and state-space methods, such that the derived model does not deviate much from measured FRFs.
- *Physical consistency* must be enforced on the experimentally identified model without any detrimental effect of the model's similarity to test data, which can be difficult to achieve.

Much recent research has tried to overcome many of these problems. Chapter 2 touched upon the problem of obtaining accurate measurements and experimentally identified models, and the approach taken in this thesis was described.

The different substructuring methods have different requirements on the experimental data or identified models. For example, the FBS method can be conveniently used on measured FRFs directly (without parametric identification), but has to deal with inverting a noisy transfer function matrix. Effects of these uncertainties have been studied closer by Voormeeren et al. [102]. A review of uncertainties in substructuring was carried out by Allen et al. [5]. The CMS and state-space methods instead rely on parametrically identified experimental models, which are noise-free, but can be hard to identify for modally dense structures. Also, physical consistency conditions can be required, depending on the

model type. For example, for state-space coupling, many physical constraints can be necessary to enforce, see Chapter 2. In *Paper A*, a system identification procedure was developed to first obtain accurate state-space models by inclusion of residual states. Then, in *Paper C* that method was extended to include a reciprocity constraint and a rigid body mode constraint for displacement output systems. The reciprocity constraint is necessary for accurate synthesis, but it is also used to expand the identified model for unmeasured moment inputs which are also necessary for synthesis of components that have point-like connections to other components.

Another difficult obstacle relates to sensor and actuator positioning on the tested structure. Approximating node connection points can, in reality, be difficult as it is almost impossible to excite and measure responses for the necessary six degrees-of-freedom (DOFs) in exactly one point. Therefore, methods have been developed to overcome these problems. In the transmission simulator method for CMS, see Allen et al. [7] and Allen et al. [4], the interface is mass loaded by a simple-to-model component. That simple-to-model component is later subtracted by an FE representation. In this way, the contact dynamics can be implicitly modelled in the experimental model and accelerometers can be positioned on the transmission simulator instead of the actual coupling positions. The transmission simulator method was extended to the state-space domain by Scheel et al. [86]. Expansion methods have also been proposed to merge numerical and experimental models, e.g. see O'Callahan et al [77] for the system equivalent reduction expansion process method and Klaassen et al [63] for the system equivalent model mixing (SEMM) method. Other methods are based on obtaining virtual points from several response measurements around the coupling locations, e.g. see van der Seijs et al. [99]. Recently, the SEMM method was used together with the virtual point method for assembly of two experimental substructures, see Pasma et al [78]. In *Paper C*, the virtual point method has been used to create what is denoted the coupling points. In addition, a type of transmission simulator resembling an actual part in the FE substructure has been used at each coupling location. Removing that part from the FE model was trivial, and the techniques developed in [7, 4, 86] did not have to be used. Furthermore, as mentioned above, the identified model is expanded for the necessary, but unmeasured, moment inputs.

With tools available to accurately create experimental models with sufficient accuracy for experimental synthesis, it is possible to perform hybrid or experimental-analytical synthesis. Simpler components such as the rear subframe, on which FE model updating can be applied can be modelled with FE. For more complex structures with possibly too many model parameters, such as the BIW, experimental models can be a better alternative. It is then possible to couple these two models, obtaining a more accurate model of the assembled system. This is the topic of *Paper C*, where a successful assembly is reported. Because such synthesis still allows for easy parametrisation of the FE component, three updated rear subframe FE models are coupled and compared to reference measurements. It is found that the assembly variability, due to variability between the nominally identical subframes, is well captured by the experimental-analytical model.

5 MODEL REDUCTION

This chapter marks the shift to computational methods, which seek to obtain compact reduced-order models and more efficiently compute the eigenvalue problem for structural dynamic models. Commonly, in industry, one high-fidelity FE model is often built and shared across engineering disciplines such as stress analysis, crash simulations and dynamics. In dynamics, two main problems exist related to high-fidelity models:

1. *High computational cost* is associated with solving the dynamic eigenproblem.
2. *Large models* are inefficient when often only the low-frequency dynamics is of interest.

The computational time, in particular, becomes important when many model evaluations are necessary. This is the case for both uncertainty propagation and quantification studies which require many model evaluations at different parametrisations, see Chapter 3 and Chapter 6, respectively. Substructuring and model reduction are possible techniques to overcome both of the above stated problems, e.g. see Craig and Kurdila [23] and Geradin and Rixen [38]. In substructuring, the model is decomposed into smaller substructures for which the solution can be computed cheaper, and in parallel, and assembling the substructure solutions to form the global solution. It is often accompanied with a model reduction step to reduce the computational cost further, and to reduce the model size. A widely adopted method is the Hurty-Craig-Bampton (HCB) method [51, 50, 14]. However, the HCB method can have decreased efficiency for high-fidelity models with many interface DOFs, as described below.

5.1 COMPONENT MODES

Consider the undamped EOMs for a linear system on second-order form

$$M\ddot{q}(t) + Kq(t) = f(t) + g(t) \tag{5.1}$$

with the excitation vector split into an external excitation vector $f(t)$ and an interface force vector $g(t)$ containing counteracting forces from neighbouring structures. It is possible to partition the DOF vector in Equation (5.1) into a boundary (or interface) set b and an internal set i, with m_b and m_i DOFs each. The substructures can then be split into boundary and internal representations

$$\begin{bmatrix} M_{bb} & M_{bi} \\ M_{ib} & M_{ii} \end{bmatrix} \begin{bmatrix} \ddot{q}_b \\ \ddot{q}_i \end{bmatrix} + \begin{bmatrix} K_{bb} & K_{bi} \\ K_{ib} & K_{ii} \end{bmatrix} \begin{bmatrix} q_b \\ q_i \end{bmatrix} = \begin{bmatrix} f_b \\ f_i \end{bmatrix} + \begin{bmatrix} g_b \\ g_i \end{bmatrix} \tag{5.2}$$

The reduced-order model is built from a set of static constraint modes, also known as Guyan-Irons (GI) modes [48, 52, 53], which describe the static deformation at component interfaces, and fixed interface vibration modes (denoted HCB modes) approximating the system dynamics. The GI modes can be derived by decomposing the internal DOF

vector into a static and dynamic contribution $q_i \triangleq q_{i,\text{static}} + q_{i,\text{dynamic}}$. Assuming zero acceleration in Equation (5.2) and $g = 0$, the EOMs are reduced to the boundary DOFs

$$q_{i,\text{static}} = -K_{ii}^{-1}K_{ib}q_b + K_{ii}^{-1}f_i \triangleq \Psi q_b + K_{ii}^{-1}f_i \tag{5.3}$$

with $\Psi \in \mathbb{R}^{m_i \times m_b}$ denoting the GI modes.

The HCB modes are obtained from the truncated eigenvalue problem of the internal partition of the EOMs in Equation (5.2) as

$$K_{ii}\Phi_i = M_{ii}\Phi_i\Lambda_i \tag{5.4}$$

with $\Phi_i \in \mathbb{R}^{m_i \times m_v}$ the truncated eigenvector matrix and $\Lambda_i \in \mathbb{R}^{m_v \times m_v}$ the diagonal eigenvalue matrix in ordered sequence from smallest to largest with m_v modes.

For HCB models, built with GI and HCB modes, all interface DOFs are kept as physical DOFs while the internal DOFs of each substructure are reduced and represented as generalised DOFs. Often, this allows for significant model order reduction, and the reduced-order models so defined allow for easy synthesis. However, for models with high mesh density and large interface regions between substructures, unnecessarily high-dimensional reduced-order models are obtained from the HCB method. It is a consequence of the exact static condensation to the interface, which preserves all interface DOFs. The GI modes then tend to be expensive to compute due to the solution of m_b problems of size m_i. This has motivated research in interface reduction methods.

5.2 INTERFACE REDUCTION

Interface reduction methods can be categorised into roughly three categories:

1. *System-level* methods compute an interface reduction basis from the assembled system.
2. *Local-level* methods compute an interface reduction basis found for each substructure independent of the other substructures, e.g. see Hong et al. [49].
3. *Hybrid methods* compute the interface reduction basis for each interface, e.g. see Aoyama and Yagawa [11] and Wu et al. [105].

A review of all three approaches is given in [65, 103, 25]. In this thesis, a system-level method is developed. System-level methods provide the best accuracy of the three methods, but are also the most computationally expensive. The system-level interface reduction method was first proposed by Craig and Chang [22]. That method was rediscovered by Balmès [13] and later by Castanier et al. [19], where the interface modes were named characteristic constraint (CC) modes.

In *Paper D*, a computationally cheap and accurate system-level method is presented. Computing the CC modes requires the computation of GI modes for each substructure, which is expensive for high-fidelity models with many interface DOFs. To reduce that cost, a low-fidelity model is used to find the interface reduction basis from which the CC

(a) *High-fidelity model*   (b) *Low-fidelity model*

Figure 5.1: *High-fidelity model in (a), and low-fidelity model in (b) with coarsened internal mesh and intact interface DOFs. White nodes denote internal nodes associated to $\boldsymbol{q}_i$ and black nodes denote interface nodes associated to $\boldsymbol{q}_b$.*

modes are computed. An illustration is shown in Figure 5.1, where a low-fidelity model is created with intact interface DOFs but coarsened on the internal DOFs. Computing the interface reduction basis $\boldsymbol{\Theta} \in \mathbb{R}^{m_b \times m_r}$ from an assembly of statically condensed low-fidelity models is generally cheap. That reduction basis can be applied to the high-fidelity model, due to the kept interface DOFs in the low-fidelity model, reducing the high-fidelity model's number of interface DOFs from m_b to m_r. Hence, only m_r problems of dimension m_i must be solved to compute the CC modes, with $m_r < m_b$.

5.3 MODAL TRUNCATION AUGMENTATION

The response convergence of HCB reduced-order models to the response of the full model as more modes are added to the reduction space can be viewed in two terms, see Voormeeren [103]:

1. *Spectral coverage* implies that the eigenfrequency of the reduced model well embrace the frequency range of the external loading.
2. *Spatial coverage* implies that the reduced-order models should be built from a basis of modes with strong spatial correlation to the distribution of the excitation.

In the HCB method, the GI modes, describing the exact static behaviour, are used to increase the spatial coverage. Another method is to use MTAs, see Dickens et al. [28], Dickens and Stroeve [29]. Higher order MTAs $\boldsymbol{X}_j \in \mathbb{R}^{m_i \times m_b}$ with orders $j = 1, \dots, N$ for component synthesis were developed by Rixen [83, 84]. A drawback with the MTAs is that for high-fidelity interfaces they can be expensive to compute as every MTA order results in m_b MTA vectors. MTAs are, therefore, often employed with interface reduction techniques. *Paper D* shows that the MTAs can be cheaply computed from the developed multifidelity interface reduction basis, with good accuracy. Furthermore, replacing some HCB modes with MTAs in the reduction basis for a fixed reduced-order dimension is shown to increases the reduced model's in-band accuracy, and reduce the overall solution time. Also, in *Paper D*, an analysis is performed on the selection between HCB modes and MTAs, which is found to be consistent with findings in Dickens and Stroeve [29], Rixen [84] and Voormeeren [103].

6 UNCERTAINTY PROPAGATION

During the development phase, knowledge of frequency response sensitivity to model input parameters can be important for better decision making for noise and vibration problems. When undertaking uncertainty propagation analyses, it is important to use accurate models and have good knowledge about the distribution of the model parameters. Without accurate models, support from such analyses can be detrimental to the decision making process as more faulty decisions can be made. Learning the distribution of the model parameters would require many tests on nominally identical components, and has not been of main interest in this thesis. Instead, a great deal of effort has gone into obtaining accurate models, which were used for experimental-analytical synthesis in *Paper C*. More research is necessary for uncertainty propagation in such assemblies, see Chapter 7. Here, only FE models are considered for use in uncertainty propagation. For that analysis, the three updated components in *Paper B* have been used to obtain a rough estimate of the parameter uncertainties, later used in *Paper E*. This might describe a best-case scenario in industrial settings, where large resources are invested in deriving accurate FE models and some information about parameter uncertainties is known.

Even with accurate models, a remaining obstacle for uncertainty propagation and global sensitivity analysis with pure FE models is the large computational time. Efficient solution methods, as investigated in *Paper D*, are often not fast enough for industrial-sized models to be directly used for Monte Carlo simulations, without very significant computational resources. More efficient procedures have been proposed for uncertainty propagation that can be roughly categorised as:

1. *Intrusive* methods require new computational procedures to be developed fully or be embedded in existing code.
2. *Non-intrusive* methods can use existing code bases and are often preferable in industry due to less code uncertainty.

Examples of intrusive methods are, e.g. the stochastic finite element method, see Ghanem and Spanos [39], or the perturbation finite element method, see Kleiber and Hien [64]. Non-intrusive methods are usually used together with, or based on, the Monte Carlo method, see Robert and Casella [85]. Some examples of Monte Carlo methods are the control variate Monte Carlo method [85] and the multilevel Monte Carlo method, see Giles [42]. The multilevel Monte Carlo method has recently been applied to structural dynamics problems by Blondeel et al. [18] and Unwin [96]. However, only simple problems were considered, and the method might be less suited for models with complex geometries and many modes, for which low-fidelity models capturing similar dynamic behaviour as high-fidelity models can be hard to construct. Parametric reduced-order methods can be more suitable for complex dynamical problems, e.g. see Amsallem and Farhat [8, 9] for an interpolation-based method. For a review of parametric reduced-order models see Benner et al. [17]. Parametric reduced-order methods can often be too general, and in

effect still too computationally expensive. Instead, to propagate uncertainties into FRFs, more specialised methods can be used. In *Paper E*, a data-driven surrogate modelling approach is proposed, briefly described below.

6.1 SURROGATE MODELLING

Much research has been devoted to propagating uncertainties into FRFs, e.g. see Kammer and Krattiger [59], Pichler and Schuëller [79], Pichler et al. [80], Goller et al. [43], Gallina et al [35], Fricker et al. [32], DiazDelaO et al. [27], Yaghoubi et al. [106] and Lu et al. [70]. Many methods are based on the modal representation of a proportionally damped second-order mechanical system

$$G(\omega, p) = \sum_{r=1}^{m} \frac{R^{(r)}(p)}{\omega_r^2(p) + 2\xi_r \omega_r(p)\mathrm{i}\omega - \omega^2} \tag{6.1}$$

with ω_r the eigenfrequencies, ξ_r the non-parametrised damping ratio and $R^{(r)} \in \mathbb{R}^{n_y \times n_u}$ the matrix of residue elements for mode r. It is important to note that m eigenfrequency surrogate models and $mn_y n_u$ residue element surrogate models must be created, in total $m(1 + n_y n_u)$ surrogate models. In the literature, various simplifications have been introduced to reduce the computational cost of creating surrogate models for the eigenfrequencies and residue matrix elements for each mode. For example, Lu et al. [70] recently proposed the use of a multi-output Gaussian process (GP) [82] surrogate model of the modal quantities in Equation (6.1). A mode dominance method was proposed to reduce the number of modal quantities necessary for the surrogate model. However, they noted that due to the used mode dominance approach mode crossing and veering phenomena could reduce the surrogate model's accuracy.

In *Paper E*, a method is developed and approximations introduced to reduce the computational cost, while retaining the surrogate model's ability to handle mode veering and crossing. Three approximations are proposed:

1. *Few high-frequency residuals* are used to model potentially many out-of-band modes.
2. *Principal component analysis (PCA)* is used to reduce the residue matrix dimension.
3. *Multiple surrogate model structures* are used with leave-one-out cross-validation (LOOCV) based model selection.

The two first approximations are reductions of the problem. First, the number of modes are reduced by inclusion of only two high-frequency residual modes to model the out-of-band modes, shown to introduce almost no error compared to the full problem. Second, the columns of the residue matrix are reduced for each mode, separately. The third approximation is data-driven. Cheap second-order multivariate polynomial models are trained for all surrogate models, and LOOCV used to decide if more expensive GP surrogate models are necessary. In addition, two covariance functions (kernels) are used for the GP models, with different computational cost.

7 CONCLUSION AND FUTURE WORK

In Chapter 1, three aims were outlined. The first aim was the development of tools for model validation and updating. In *Software A*, a measurement system was developed with stationarity checks for a stepped multisine excitation signal. With this software, high-quality experimental data could be gathered, for use in system identification. An improvement for this software could be to also include test procedures for non-linear modelling. In *Paper A*, a system identification method was developed that could model some difficult input-output pairs, often seen in experiments, by inclusion of one extra residual mode. That method was successfully demonstrated on a modally dense BIW structure. That identification method, together with the data gathered with *Software A*, was used in *Paper B* to validate and update a rear subframe model towards three nominally identical subframe components. The novelty in *Paper B* is the mass loading of the bushings, shown to augment the test data so that it could be successfully used for model updating of most bushing stiffness parameters in the rear subframe. However, it was found that some translational stiffness parameters in the bushings could not be reliably updated due to the rather simple mass loading on the bushings, which could be addressed in future studies. Because of the equalised damping, the calibration method cannot update damping parameters. One, sub-optimal, approach to this problem could be in two phases. First, calibrate the model with equalised damping, and then start from the calibrated parametrisation and perform a calibration in which damping parameters are let free.

The second aim regards the experimental-analytical synthesis. This is addressed in *Paper C*, which builds upon the identification method developed in *Paper A* and the updated models in *Paper B*. Experimental data of the BIW were acquired with *Software A*. For successful synthesis, a physically constrained identification procedure was developed. The results in *Paper C* indicate that with accurate models, and with correct model structure and parameter settings for the specific components, accurate assemblies of highly complex structures are possible. This is indeed expected, and has been verified in this thesis. In addition, it was shown that a purely state-of-the-art FE model assembly gave poorer accuracy. From the successful results with one assembly, it is expected that several assemblies, with accurate FE models of each nominally identical subframe, can be accurate. Then, the variability in these assemblies, caused by variability in the subcomponent, should also be captured. Not much previous research has been found which considers how component variability propagates into variability of experimental-analytical assemblies in relation to FE model accuracy. Therefore, one of the main contributions of this thesis is evidence that variability from components can indeed be accurately predicted by experimental-analytical assemblies under reasonable but demanding circumstances. However, the undertaking of such studies, while rewarding when done correctly, are likely too time consuming and resource intensive to be undertaken in industry in the immediate future, and more problems related to physical consistency must be overcome for the models to be fully reliable. For example, because a passivity constraint was not enforced on the

identified BIW model the synthesised model could be unstable, demonstrated in assembly with the nominal rear subframe parametrisation. Enforcing passivity for modally dense models should be in the focus of future work, which could allow experimental-analytical assembly models to be more reliably used for uncertainty propagation. It cannot be overstated that, as support for decision making, model accuracy is necessary. However, a possible middle-ground for industry, not requiring such large resources and creation of experimental models with strict physical consistency constraints, could be to use updated FE models and learn a rough estimate of the parameter distribution from a few nominally identical components. That approach is taken in *Paper E*, for which efficient solution methods are required.

Computational efficiency for variability analysis was listed as the last aim of this thesis. Such methods are important both for uncertainty quantification and propagation studies, requiring many model evaluations at different parameter settings. In *Paper D*, an efficient solution method was proposed for the eigenvalue problem for structural dynamic models using multifidelity interface reduction. It was shown that MTAs could be accurately computed using this interface reduction technique. It was also shown that for a fixed reduced model order the model was more accurate with inclusion of these MTAs compared to only including the HCB modes. Extensions of this work could be to cast the method in a multi-level or local-level formulation.

In *Paper E*, a frequency response uncertainty propagation method was developed. It was shown that through several reductions and approximations, a cheap and accurate surrogate model could be established. Updated parameters from *Paper B* were used to decide on the input parameter distribution. When the available FE model structure is correct, such rough estimates of the model parameter uncertainties could be feasible in industrial settings, and that information used for variability analysis. An interesting extension of the work in this thesis would be to develop a method to propagate model uncertainties through an experimental-analytical model, i.e. combine the work in *Paper C* and *Paper E*, see Figure 1.2. However, that requires the development of an improved mode matching strategy compared to the strategy used in *Paper E*. Another important aspect not considered here is the quantification of the response sensitivity to the input parameters in order to identify which parameters are responsible for the seen variability in the assembly. Such information could be obtained with global sensitivity analysis methods, and cheaply with the developed method in *Paper E*.

8 SUMMARY OF APPENDED PAPERS

Paper A: State-Space System Identification with Physically Motivated Residual States and Throughput Rank Constraint

A two-phase state-space system identification method is developed. A physically motivated residual is introduced, complementary to the mass and stiffness residuals, to improve hard-to-model energy conjugated input-output pairs. A method to place the complementary residual's pole is derived. Furthermore, for systems on acceleration output form and with rigid body modes, it is shown that a partition of the direct feedthrough matrix must be rank constrained for physical consistency. The constraint is enforced in a re-estimation phase of the state input, output and direct feedthrough matrices. It is shown that accurate experimental models can be obtained for a variety of structures, even with high modal density.

Paper B: Model Updating of Multiple Nominally Identical Car Components

Model updating is used to calibrate finite element models of a car rear subframe to three experimental data sets from nominally identical components. The developed method from *Paper A* is used for system identification. Special attention is given to bushing parameters, for which two modelling techniques are compared. It is shown that good models can be obtained for the simpler generalised spring element bushing, compared to a solid bushing model. Parameter uncertainty with respect to measurement noise is also quantified. Furthermore, it is shown that relatively stable parameters are obtained for the three components.

Paper C: Identification of Physically Realistic State-Space Models for Accurate Component Synthesis

In *Paper C*, the built-up structure of a car body-in-white and rear subframe is considered and modelled as an experimental-analytical assembly. The method developed in *Paper A* is used, and extended with a reciprocity constraint and another additional constraint on the rigid body modes for systems on displacement output form. The body-in-white is modelled experimentally and assembled to the three updated finite element rear subframe models from *Paper B* through four rubber bushings. It is shown that good agreement to reference measurements is obtained, and that the assembly variability, stemming from the different subframes, can to a large extent be captured using accurate experimental and updated finite element models.

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Paper D: Multifidelity Component Interface Reduction and Modal Truncation Augmentation

Paper D marks the shift to computational methods. An efficient component mode synthesis method is proposed, based on substructuring and interface reduction. It is shown that an accurate interface reduction basis can be computed cheaply from a coarse mesh derived from the original high-fidelity mesh. From this basis, the static constraint modes can be reduced, and characteristic constraint modes computed at a small cost. Small loss in accuracy is associated with the coarse mesh reduction basis, mainly for the low-frequency range where the error is small. The method can be used to decrease the computational cost of computing the dynamic solution and to obtain a compact reduced-order representation of finite element models.

Paper E: Data-Driven Modal Surrogate Model for Frequency Response Uncertainty Propagation

A data-driven modal surrogate model is developed for efficient uncertainty propagation of large FE models. The surrogate model builds local surrogate models of the eigenfrequencies and residue matrix elements for each mode. The cheapest surrogate model structure is adaptively selected, out of second-order multivariate polynomial and Gaussian process models, for each modal quantity using leave-one-out cross-validation. Out-of-band modes are approximated with previously introduced residuals, from *Paper A*. Dimensionality reduction of the residue matrix is performed with principal component analysis. Using these approximations, it is shown that Monte Carlo analysis can be run on large scale industrial models for moderately many parameters at a small cost, with negligible loss in accuracy. Updated parameters from *Paper B* are used in this paper.

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