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BENCHMARK STUDY OF FIVE OPTIMIZATION ALGORITHMS FOR WEATHER ROUTING

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ABSTRACT

Safety and energy efficiency are two of the key issues in the maritime transport community. A sail plan system, which combines the concepts of weather routing and voyage optimization, are recognized by the shipping industry as an efficient measure to ensure a ship's safety, gain more economic benefit, and reduce negative effects on our environment. In such a system, the key component is to develop a proper optimization algorithm to generate potential ship routes between a ship's departure and destination.

In the weather routing market, four routing optimization algorithms are commonly used. They are the so-called modified Isochrone and Isopone methods, dynamic programming, three-dimensional dynamic programming, and Dijkstra's algorithm, respectively. Each optimization algorithm has its own advantages and disadvantages to estimate a ship routing with shortest sailing time or/and minimum fuel consumption. This paper will present a benchmark study that compare these algorithms for routing optimization aiming at minimum fuel consumption. A merchant ship sailing in the North Atlantic with full-scale performance measurements, are employed as the case study vessels for the comparison. The ship's speed/power performance is based on the ISO2015 methods combined with the measurement data. It is expected to demonstrate the pros and cons of different algorithms for the ship's sail planning.

Keywords: Routing optimization, Dynamic programming, Isochrone algorithm, Isopone, Dijkstra, Grid system, Fuel consumption, ETA

1 Introduction

Ship/cargo safety and energy efficiency are two of the most important concerns in the current maritime transport sector. During the past decades, the safety related research has advanced significantly, while more and more attention is on the reduction of fuel burning and air emissions from shipping. Different innovative solutions have been developed in the market to help the shipping industry to increase their energy efficiency. However, large uncertainties associated with investment and payback time of those technologies make ship owners hesitate to implement complex and expensive measures. According to *DNV-GL (2015)*, shipping companies are more willing to invest in simple and cost effective technologies to reduce fuel cost and air emissions from their ships. Among all available energy efficiency solutions, the most recognized measure is the weather routing and voyage optimization system. Its potential benefits are also well studied by e.g. *Chen et al. (1998)*, *ABS (2012)*, etc. Meanwhile, the utilization of such weather routing systems is also in line with the IMO E-navigation recommendation, *IMO (2016)*. In the early stage of weather routing service, the main purpose is to provide guidance to ships in order to reach a ship's destination as soon as possible, based on forecasted weather conditions, and possibly also on a ship's characteristics and operational capabilities (*Bowditch, 2002*). Currently, as the rapid increase (fluctuation) of the oil price and social awareness of air emissions from the transport sector, weather routing is now combined with voyage optimization in order to plan optimum course (waypoints) and speed for ocean voyages with minimum fuel consumption and keeping the expected time of arrival, in addition to consider the ship's safety as constraint for the route planning. The coupling optimization of minimum fuel consumption and expected time of arrival requires speed/power performance of the ship in different operational conditions (*Notteboom and Carriou, 2009*). Another basis for the coupling/multi-objective optimization is the mathematical optimization algorithms. Unlike a ship's traditional routing plan, which often follows the great circle, i.e. shortest distance between departure and destination ports, the mathematical optimization algorithm can help to plan a course (that may differ significantly from the great circle course) associated with certain ship speeds, in order to encounter optimum sea weather. In the maritime industry and research community, various algorithms have been developed and implemented for ship routing optimization. According to *Marie and Courteille (2009)*, these algorithms can be divided into four categories: Isochrone methods, dynamic programming, application of the calculus of variation, and genetic algorithms.

The Isochrone method was proposed to discretize a voyage into equal time stages in order to plan a route with minimum time of sailing (*James 1957*). Both forward and backward optimization approaches were proposed to consider ship route planning, e.g. *Hagiwara (1989)* and *Klompstra et al. (1991)*. The dynamic programming approach is based on the mesh/grid system of the sailing domain and determine the optimum nodes (waypoints) and ship speed according to ship performance models and weather forecast information, see *Chen (1998)* and *Avgouleas (2008)*. In addition, the genetic algorithm has also shown its strong capability for multi-objective optimization, but it often needs long time to get the convergent results, see *Hinnethal (2008)*. Other approaches have been existing mainly in the research community, e.g. the Dividing RECTangles algorithm in *Jones et al.(1993)* and *Larsson et al.(2015)* etc. While for the practical routing optimization, the first two approaches are more mature and widely implemented in current weather routing systems.

In order to give a hint regarding the performance of different weather routing systems with respect to their implemented routing optimization algorithms, this paper will study five commonly used algorithms for routing optimization. In the following section 2, a basic introduction of each algorithm will be presented, as well as their cons and pros for routing optimization. These cons and pros will be further compared based on a case study vessel and her trade route in Section 3 and 4. In Section 5, possible outlook to improve current algorithms and some conclusions will also be given based on the current study.

2 Optimization Algorithms

The choice of algorithm has a crucial effect on the determination of optimum route for a ship's sailing and the computation time to find the optimal route. In general, the routing algorithm often contains two components, i.e. waypoint grid/mesh generation, and path selection and evaluation criteria. In the following, five algorithms will be presented in terms of the two items.

2.1 Isochrone/Isopone algorithms

The isochrone method was firstly proposed by *James (1957)*. The core of the method is the waypoints/grid discretization to generate all potential and reasonable ship route for routing optimization. The method divides a ship voyage into several stages, and each stage is assumed to be sailed at equivalent time. This will help to break a complex optimization problem into many sub route optimization problems. The optimization problem (lowest fuel cost or minimum time of arrival) can be solved by either a forward or backward recursive algorithm for individual sub-routes.

For the forward optimization problem, a modified Isochrone algorithm proposed by *Hagiwara (1989)* has got more practical implementations for ship weather routing systems, where a ship can vary her heading angles at each interim waypoint around the reference route, e.g. the great circle route is often used as the reference course. The range and resolution of heading variation for each waypoint will determine the number of potential ship sub-routes, in addition to the stages, which form the whole ship route. An example of the ship's sub-route grid system generated by the modified Isochrone method is shown in Fig.1. As is shown, the sub-routes/paths generated by the method propagate exponentially as the number of time stages. In order to reduce the number of possible sub-routes, some optimization objective criteria have to been introduced to reduce the potential ship routes. Often a ship's expected time of arrival (ETA) and fuel consumption are used for the selection of potential ship sub-routes. In this case, for each grid/waypoint generated by the method, the Isochrone method assumes that a ship is operated with a constant engine power during the voyage. Therefore, the distance a ship can sail during a time stage depends on the sailing ocean environments (wind, wave and current, etc.) that the ship will encounter at these waypoints. The waypoints of too much delayed ETA, which also means too high fuel consumption during the voyage, will be abandon for the next stage evolution.

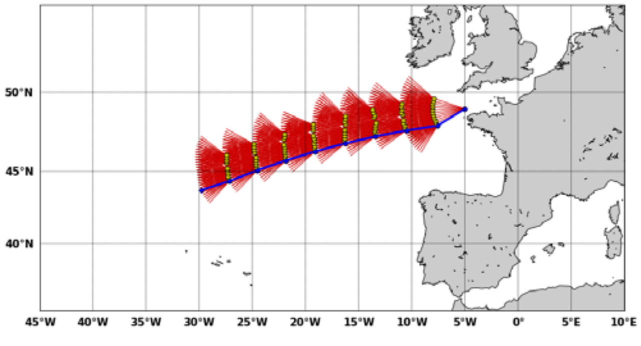

Fig.1. waypoints and potential route paths generated by the Isochrone algorithm

The Isopone method is based on equivalent time-interval stage and grid discretization system, and the great circle route is also used as the reference for grid generation. It is also used for scheduling a ship route with minimum time (or expected time of arrival) sailing. Different from the conventional Isochrone method that takes the distance a ship can reach during a given time period as a criterion, the Isopone considers the distance that a ship can reach using equal fuel consumption. This Isopone method determines the next stage waypoint of a minimum fuel by tracing back the headings and speeds. This means each state in the Isopone method is extended to three dimension by adding a time axis to the position (*Klompstra et al. 1992*).

Since current Isochrone and Isopone methods assume constant engine power operation for a ship's route planning, it is obvious that the fuel consumption may not be optimised to give the minimum consumption, but this method will keep quite accurate ETA planning, which is important for such shipping segments as Liner and Cruise Ferry companies.

2.2 2D/3D dynamic programming

Dynamic programming is based on Bellman's principle of optimality (*Bellman 1952*): "an optimal policy has the property that whatever the initial state and initial decision is, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". In the dynamic programming algorithm, the grid system is often constructed based on the great circle reference path. Along the great circle path, a route is divided into many segments/stages. For each stage/waypoint, several grids/waypoints perpendicular to the great circle sub-route are generated for path selection, as shown in Fig.2. The optimization criteria of a dynamic programming algorithm may consider different settings of ship operations. For example, if a ship's sailing speed is constant, only the path (ship heading, i.e. sailing longitude and latitude locations) can be varying during a route planning by connecting different waypoints at neighbouring stages. It is also named as 2D dynamic programming.

A 3D (three-dimensional) Dynamic Programming algorithm can consider the time variable for route planning. It uses the voyage progress as stage variable through voluntary or involuntary speed/power control. In this case, every stage is composed of many states, while a state is defined by a location (grid waypoint) and a discretized time. Because ship path (i.e. headings and grid waypoints) are predefined on the grid system, ship speed becomes the only variable to be optimized.

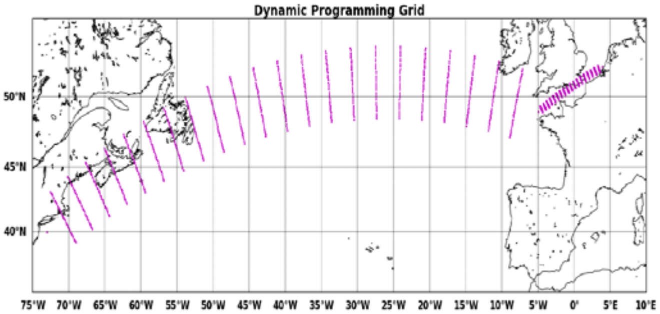

Fig.2, the grid system used in the dynamic programming algorithms

It should be noted that both 2D and 3D Dynamic Programming algorithms use predefined grid/waypoint systems that are saved in a computer memory, theoretically speaking the two methods require less computation effort than the isochrone algorithm for routing optimization. However, their accuracy will greatly depend on the resolution of the grid system, which should also allow ship operators to take into account their ships' navigation boundaries, such as no-go zones, land avoidance etc.

2.3 Dijkstra algorithm

The algorithm is based on two principles (*Dijkstra 1959*), i.e. 1, a sub-route within a shortest route is also a shortest sub-route, 2, for a given shortest distance between points ***A*** and ***C***, a path going from point ***A*** to ***C*** through a third point ***B*** will always be a distance greater than the direct distance from ***A*** to ***C***.

The basic concept of this algorithm is presented in Fig.3. There are many potential routes sailing from the "Start" point to the "Destination" point. The algorithm will first divide a whole route/path into a series of sub-routes. The starting point of a sub-route is connected to all of its neighbouring points by paths, which are associated with some cost values. For the ship routing problem, the cost values may be, e.g. sailing distance, fuel consumption or a weighted combination relative to certain objective functions.

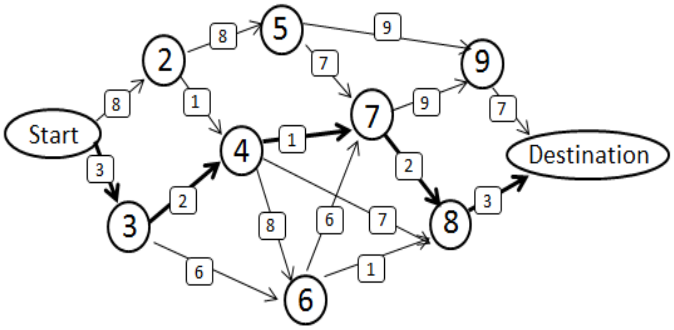

Fig. 3, illustration of the Dijkstra algorithm.

In this algorithm, the first principle is that the optimum sub-route follows the path with the lowest cost value. This path and its associated cost value will be taken as reference and used for the following tests. When a point is reached and its corresponding paths have already been tested, the current reference cost value will be compared with her preceding sub-paths. If the newly tested route is smaller than the reference, the new cost value and associate path is assigned to that point. Consequently, the previous reference path is dismissed and the new path will be taken as reference. Following this procedure, paths will evolve towards the destination point. During the whole process, only one optimum route remains. As an example of such a route optimization in Fig. 3, the bold lines mark the optimum route with the smallest value/distance from the "Start" point to the "Destination" point.

This algorithm has been implemented for ship weather routing with respect to fuel cost reduction, with the focus on studying the impact of environmental uncertainties to a ship's optimization planning. Different strategies have been incorporated into the optimization algorithm to consider a ship's speed and power performance for various operational and environmental conditions, e.g. Chu et al. (2015), Padhy et al. (2008).

2.4 Cons/Pros of these algorithms

Each optimization algorithm has its own benefits and deficiencies, and may be more suitable for specific routing planning problems. For example, some liner ships may demand very strict arrival time, while other ships may need serious consideration of ship motions to protect important/dangerous cargoes. Sometime, even for the same ship, if she is commanded for different voyage and cargo properties, the optimization requirements may also be different. For example, a voyage with expensive and dangerous cargoes may require prioritizing safety and less motion for her route planning, while other voyages may be more prone to choose route with certain ETA and minimum fuel cost. Therefore, a hybrid routing optimization through the combination of several optimization algorithms can be more flexible to fulfil various routing planning purposes. For the above-mentioned algorithms, the Isochrone method has been widely used for ETA preferred routing plans, but may have limited value for other constraints and other operation variables.

The Dijkstra and Dynamic programming algorithms could find the most optimal route from the given grid. This makes the accuracy of the optimization results highly dependent on the grid resolutions, which directly connects with the required computation effort. For cases of constant speed and handling of ship motions through reduction curves only, the algorithm is very fast. The predefined grid system can easily handle impassable areas, finding the shortest route around or between e.g. islands and no-go zones. In particular, the Dijkstra method is very well suited for the coastline route planning.

The original versions of the Isochrone, Dijkstra and 2D dynamic programming methods are more suitable for the single objective routing optimization since only the constant speed is considered in these methods. The 3D dynamic programming method is more capable to handle dynamic weather, essential for involuntary speed reduction, and voluntary speed variation. It can deal with cases of ETA and minimum fuel consumption combined objectives.

3 Case studies for routing optimization algorithm comparison

In this study, a merchant vessel with length of 202m, width of 35m, service speed of 20 knots and main engine power of 25MW is used for the study. She is assumed to be operated in the North Atlantic route between Rotterdam and NewYork, as shown in Fig.4. For the case study ship, the full-scale measurements of her speed, heading, motion response and power performances are collected and used to derive her actual speed/power performance (under different operational and environmental conditions) in the optimization study.

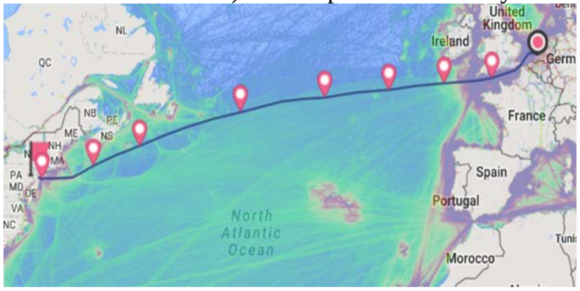

Fig.4, Case study sailing route for the merchant vessel between Rotterdam and New York.

3.1 Basic weather routing plan concept

Traditionally, a weather routing system is used by the ship traffic officer to design a ship routing schedule, according to which the ship should reach her destination as fast as possible based on the received weather forecast information. The new discipline of current weather routing/voyage optimization is to produce the most favourable route of certain passing waypoints, ship speeds, heading and engine power profiles etc. with respect to ETA, passenger comfort and ship/cargo safety etc. In particular, the most interesting objective of today's weather routing systems is to minimize the fuel/operation costs while arriving on time, taking into account safety of crew, ship, and cargo, as well as air emission in certain control areas. A general description of the routing optimization concept is shown in Fig.5. In principle, it can be broken down into several comprehendible categories and sub-components.

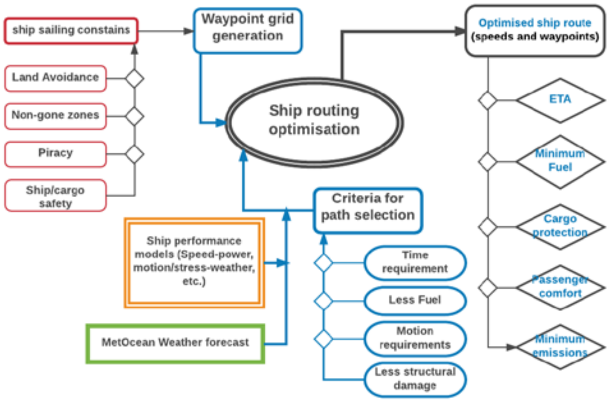

Fig.5, Overall structure of ship routing optimization

The core part of a voyage optimization system is the routing optimization algorithms, which here are composed of waypoint grid generation and criteria for path selection. For the optimum planning, it will require inputs from a ship's sailing constrains (e.g. land avoidance, piracy, no-go zones from local maritime authorities, etc.), weather forecast information at the future sailing region and certain ship performance models. The type of required ship models for routing optimization depends on the optimization objective function determined by the shipping companies.

Today's shipping market has shifted its focus into the energy efficient related issues, which aim at reducing fuel consumption and air emissions. Therefore, a ship weather routing system should implement a reliable model to describe a ship's energy performance at sea. In addition, the conventional safety and ETA issues etc. should also be properly taken into account. In such a case, a ship's speed-power characteristics should be established as input to the weather routing system. In this study, two schemes of speed-power performance prediction models are used in the voyage optimization framework. One is to predict the required ship engine power for a given ship speed based on the encountered MetOcean conditions, while the other is to predict the ship's speed for a given ship power for practical ship operation. To make it complete, the details of the two models are presented in the following section.

3.2 A ship's speed-power performance modelling for routing optimization

Even though each ship is an individual complex energy system by itself, the prediction of a ship's energy performance can be divided into some of the main elements or energy sub-components, such as a ship's calm water resistance, added resistance in wind and waves, propeller-engine load diagram and ship hull & propulsive efficiency etc. The estimation of energy performance for each sub-component may be created with different levels of complexity differing from the most basic theories to the complex combination of long-term on-board monitoring measurements and computational fluid dynamics (CFD) methods. The complexity may be determined by the amount of available information. The precision of estimated fuel consumption is highly dependent on the degree of detail, but as a relative optimization, this may be obsolete. A comprehensive ship speed-power prediction model can be constructed in two schemes.

Figure 6 presents a typical workflow for the first scheme, i.e. an estimation procedure to predict the fuel consumption rate using input parameters of encountered weather information, the ship's characteristics, and operational profiles etc. The first step in the fuel cost estimation is to get the calm water resistance based on the ship characteristics. For a ship operated in open sea, there are also added resistance due to wind and wave loads. The summation of the calm water resistance and added resistance is called the total resistance, which needs to be compensated by the thrust force provided by the engine and propellers. Dependent on the engine type and propeller properties used on a specific ship, their work efficiency is often provided by the manufactures and can be used to calculate the final fuel consumption needed for marine engines to push the ship forward.

Another estimation scheme is presented in Fig.7. It is assumed that a ship's engine power for sailing at sea will be set up first. There are different ways to reach the goal of speed prediction. In this study, we first make an initial guess of ship speed, say service ship speed. Then the required power to allow the ship to sail at service speed can be computed at encountered sea conditions. If the required power is larger than the allowed engine power, ship speed has to be reduced. An iteration method will be used to predict the speed with required power convergent to the input engine power. Finally, by taking into account the water depth for shallow water effect and side forces caused by current/wind on drifting, we could get the ship's final speed over ground.

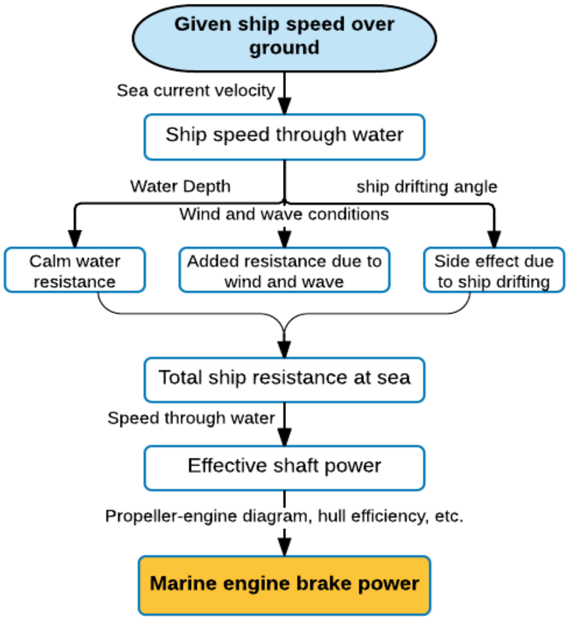

Fig.6, Speed based routing optimization

As is shown by both schemes, the methods to compute a ship's total resistance are the most important issues for the speed/power prediction approaches. For the calm water resistance, several simple models exist, e.g. *Holtrop and Mennen (1982)*. With limited available ship specific information, the modified version of *Kristensen and Lützen (2012)* can be used to account for the effects of bulbous bows, correction for hull form and position of the longitudinal centre of buoyancy. The added resistance caused by encountered waves may be modelled through extensive model testing or strip theory, to determine heave and pitch, and from sea spectrums obtain added resistance. Another option is to describe the loss of speed due to wind and waves based on main ship parameters in *Kwon (2008)*, which thus makes for a more generic method. The accuracy may though be limited. It should be noted that a ship's energy performance is also affected by hull and propeller fouling. As fouling is both dependent on time and places the ship has sailed, precise predictions may be difficult. However, since the voyage optimization may require the model that properly describe a ship's performance in short-term time, the fouling effect to the voyage planning is neglected here.

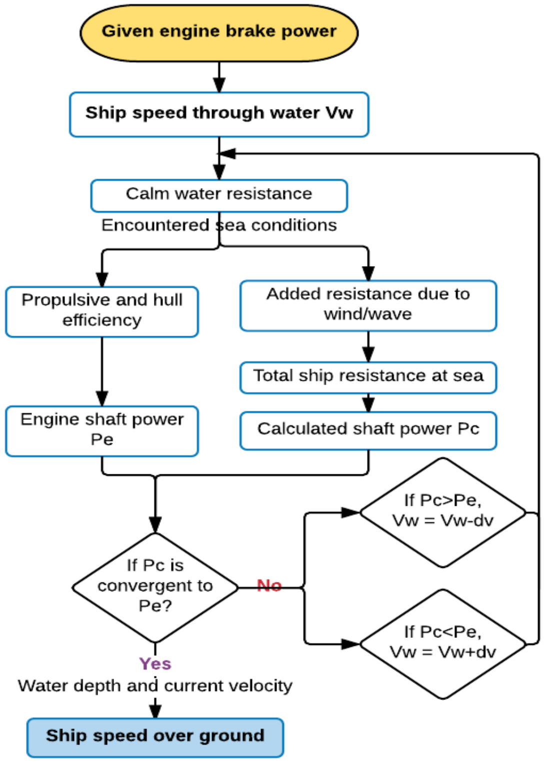

Fig.7, Ship power based optimization

In this study, all the performance related sub-components are estimated by the explicit formulas given in *ISO 2015*. Since for the two case study vessels, many energy performance data and their encountered MetOcean conditions are collected, the difference between the calculated ship energy performance and their actual measurements will be captured through the statistical regression analysis as in *Mao et al. (2016a).*

4 Comparison of different optimization algorithm for routing plan

In this study, the five optimization algorithms have been investigated for voyage planning with respect to minimum fuel cost and air emissions, while the objective of ETA will be taken as a constrain for the study. In the routing optimization, semi-theoretical explicit formulas combined with the full-scale monitoring performance data are used to derive the ship's speed/power relationship. Another important element for the routing optimization is the weather information along a ship's future sailing region and time. The weather forecasts within a 2-5 days time span are often sound enough for the route planning, while forecasts ranging further than 7-14 days most often do not directly include the parameters needed for routing and may only include pressure distributions in the atmosphere and are thus not applicable for routing purposes. Because of lack of reliability on weather forecast, in particular weather after 5 days, the optimized routing should be updated every 12 or 24 hours based on new accepted weather forecast information. Furthermore, the lack of reliable weather forecast may lead to the situation that part of a ship's journey has to be calculated based e.g. statistical weather data, e.g. *Mao (2014)* and *Mao and Rychlik (2016b).*

Since the focus of the current study is to benchmark various optimization algorithms and compare their capabilities for routing plan at different regions and for different sailing purposes, the weather information input to the optimization system is from the European Centre for Medium Range Weather Forecast (ECMWF) reanalysis model as shown in Fig.8. The use of reanalysis data means that the weather input is regarded as accurate for route planning. Since the weather routing application is more beneficial for winter navigation to save fuel and enhance safety at the northern sailing regions *Mao et al.(2012)*, the following study will only investigate the sailing during the January month.

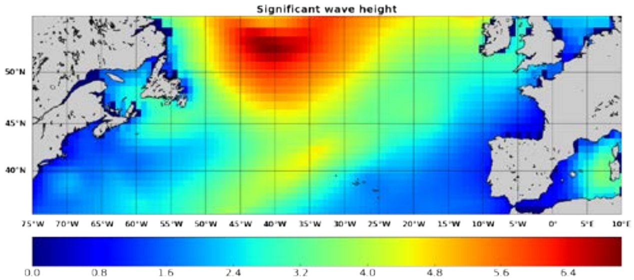

Fig.8, Significant wave height H_s in January 2015 from ECMWF reanalysis data source

In this study, if the ship speed input is predefined for the routing plan, scheme 1 is adopted for performance modelling. It will be named as "Ship speed based optimization" in Section 4.1. While scheme 2 is corresponding to the "Engine power based optimization" in Section 4.2 for the Ischrone/Isopone, Dijkstra and 2D/3D dynamic programming algorithms. It should be noted that the variation of both speed and power is used in the 3D dynamic programming method.

4.1 Ship speed based optimization

For simple consideration of the routing optimization problem, the input speed here, the speed is input as the ship's service speed and 85% of the brake engine power for the routing optimization. When encountering harsh weather environment, it is assumed that the ship will use her 15% sea margin power to catch up with her service speed. If the encountered sea is too severe, the ship is assumed to use all her engine power and operate as fast as she can.

The planned ship routes/courses optimised by the optimization algorithms are presented in Fig.9, and the corresponding power calculated for each waypoint is presented in Fig.10. Figure 11 shows the encountered wave conditions (significant wave height) and ship speeds achieved during the voyage for these optimizations. The optimised ship courses generated by all algorithms except the Isochrone method are quite similar.

This is because the grid system used in dynamic programming, Dijkstra etc. are the same but differ significantly from the Isochrone method. On the other side, the required power for each waypoint is quite close for the investigated algorithms except the 3D dynamic programming method. This is because all the other four algorithms take the fixed speed as input for the initial optimization. The sailing speed is only involuntary reduced due to too harsh ocean conditions and not enough engine power (all 15% sea margin is used). In the 3D dynamic programming method, the speed can be optimised according to encountered ocean environments. It will lead to the situation that the ship's speed will be voluntary reduced for harsh environments to save fuel. This fact is shown clearly in Fig.11.

It can be concluded that there are quite large involuntary speed reduction along the ship voyage. As is shown in Fig.10 of required power distribution along the ship voyage, the 3D dynamic programming that accounts the speed variation during the routing optimization will require significantly lower fuel cost to complete the voyage. It is also indicated that for the routing optimization with respect to minimum fuel consumption, the capability to optimise a ship's speed would be an extremely important element for an efficient weather routing system.

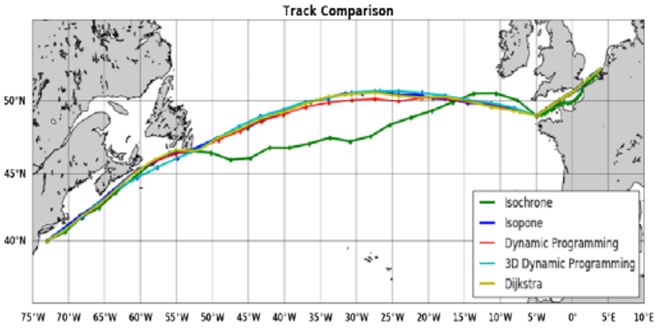

Fig.9, Optimised ship route/course generated by the five optimization algorithms.

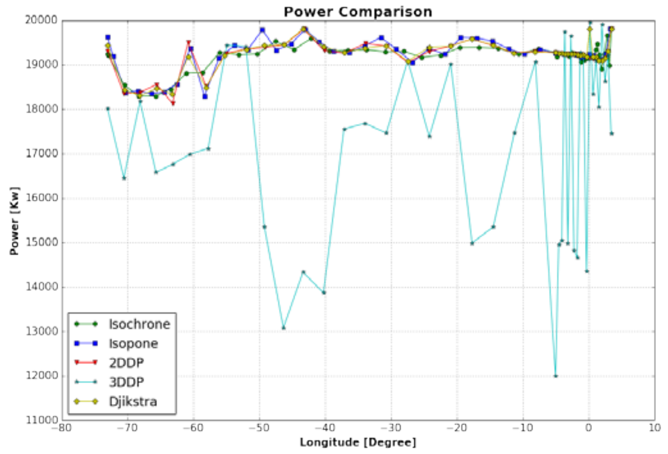

Fig.10, The required power distribution along all optimised ship routes/courses for the five investigated optimization algorithms.

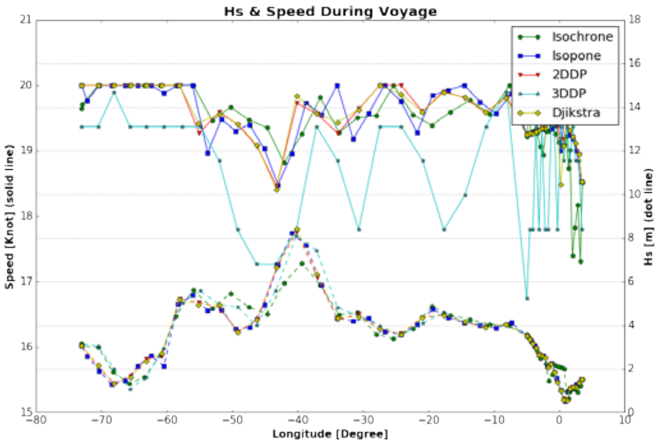

Fig.11, The achieved ship speed during the optimised ship course (shown as solid lines) and the encountered wave conditions (significant wave heights H_s shown as dashed lines).

Table 1: Sailing time estimated by different optimization algorithms for both speed based and engine power based routing plan.

Time (Hours)	Isochrone	*Isopone*	2DDP	3DDP	Dijkstra
Speed based	171.8	166.4	166.2	173.4	166.38
Power based	165.6	166.0	165.7	173.4	165.55

4.2 Engine power based optimization

In the following, it is assumed that the ship will be operated with the predefined/constant power. Therefore, the ship speed is involuntary reduced when sailing at harsh ocean environment with high wave/wind conditions. All the relevant optimization results are presented in Fig.12 and 13. Different from the "speed based optimization", all the routing plan methods give similar sailing courses for the current voyage optimization. The arrival time/sailing speed at each waypoint differ a bit from various methods because of the slightly different encountered sea conditions. It should also be noted that the 3D dynamic programming method gives quite different speed variations since the speed and power used in the 3D optimization are varying simultaneously.

The required sailing times for each optimised ship courses are listed in Table 1. It looks like all algorithms can plan a ship's route with quite similar expected time of arrival, since those candidate routes whose potential time of arrival is too far from the expected are dismissed during the optimization process. Meanwhile the uncertainty of weather forecast is not considered in the current study. Otherwise, large scatter of ETA will be expected for the actual ship operation.

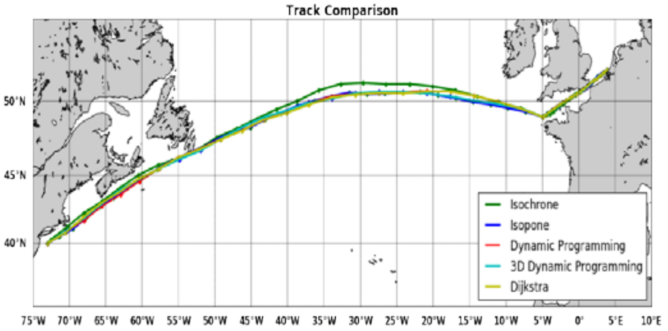

Fig.12, Optimised ship route/course generated by the five optimization algorithms.

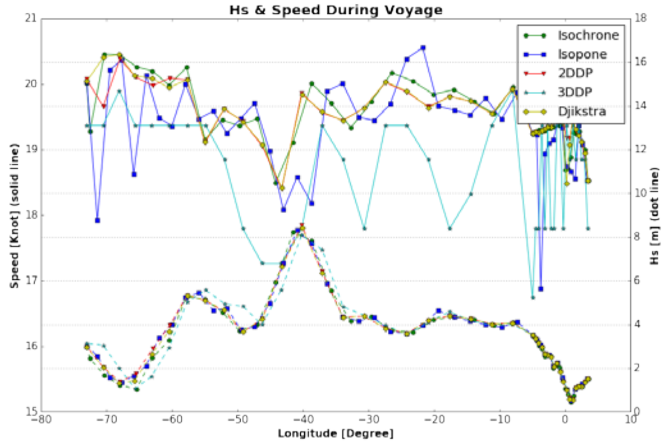

Fig.13, The required power distribution along all optimised ship route/course for the five investigated optimization algorithms.

5 Summary and Conclusion

Weather routing is an important tool for the shipping industry to reduce fuel consumption and air emissions, and at the same time to consider ship/cargo/crew safety with an expected time of arrival. In this study, five state-of-the-art weather routing algorithms have been investigated. Their cons and pros are briefly discussed based on the theoretical construction of these algorithms. The capabilities of these algorithms for optimizing speed or power profiles along a journey are tested by a merchant case study ship sailing in the North Atlantic. It is found that all the optimization algorithms will produce quite similar ship course since the grid/waypoint generation systems are quite similar among these methods and they are all based on the great circle reference route. The 3D dynamic programming algorithm takes one more parameter, i.e. speed/power variation in the optimization, for routing plan than other conventional algorithms. It shows more capabilities (voluntary speed reduction during harsh weather conditions) and better results (less fuel) for planning. For the future development of routing optimizations, all the other algorithms could also be upgraded with methods to account for one or more variables for the ship routing optimization. It will provide more flexible solutions to routing plan in the shipping market.

Acknowledgments

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