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# Modelling 3D-woven composites on the macroscale: Predicting damage initiation and inelastic phenomena

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Comparing experimental and simulated shear strain distributions in an Iosipescu test.

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# Abstract

Composites with 3D-woven reinforcement have been slowly making their way into different industries. The interlacement of yarns, not only in-plane but also through-thickness, means that in many applications 3D-woven composites can outperform their laminated counterparts. In particular, this includes increased out-of-plane stiffness and strength, damage tolerance and specific energy absorption capabilities. The widespread adoption of 3D-woven composites in industry however, requires the development of efficient computational models that can capture the material behaviour.

The current work takes a few steps towards the long term goal of developing a phenomenologically based macroscale model to predict how 3D-woven composites deform and eventually fail under mechanical loading. Following a brief introduction to the research field, the feasibility of extending stress-based failure initiation criteria for unidirectional laminated composites, to 3D-woven composites is explored. In particular it is shown that the extension of the LaRC05 criteria presents a number of challenges and leads to inaccurate predictions. Instead strain-based failure criteria inspired by LaRC05 are proposed. They produce results that are qualitatively more reasonable when evaluated numerically for tensile, compressive and shear tests.

As a next step, a thermodynamically consistent framework for modelling the mechanical response of 3D-woven composites on the macroscale is presented. The proposed framework decomposes the stress and strain tensors into two main parts motivated by the material architecture. This allows for the convenient separation of the modelling of the shear behaviour from the modelling of the behaviour along the reinforcement directions. In particular, this division allows for the straightforward addition and modification of various inelastic phenomena observed in 3D-woven composites.

The framework is then used to simulate experimental results of a 3D glass fibre reinforced epoxy composite. A viscoelastic model is incorporated into the framework to capture non-linear behaviour associated with tensile loading along the horizontal weft reinforcement as well as non-linear shear behaviour. In detail, to capture the shear behaviour, a crystal plasticity inspired approach is considered. As such, it is assumed that inelastic strain strictly develops on localised slip planes oriented by the reinforcement architecture. The viscous parameters are calibrated against experimental results, and a preliminary validation of the model is performed for an off-axis tension test.

Keywords: 3D-woven composites, Damage initiation, Anisotropy, Inelasticity

# Preface

Before I was born, my parents spent a few months living and working in Luleå, a small town in northern Sweden. The connections and friendships that they made, gave me the opportunity to move from Canada to Sweden after finishing high school. What I really thought would be one year abroad quickly turned into a three year Bachelors degree at Linnéuniversitetet in Växjö, a two years Masters degree at Chalmers, and now the start of a PhD. Mum, you were right.

After maneuvering my way into the Applied Mechanics Masters Programme at Chalmers (thank you Lennart!), it was mentioned to me that there would be a PhD position opening soon and that I should consider applying. I am extremely happy that I was accepted to be a part of this project entitled *3D Fibre Reinforced Composites with Ductile Properties*. The project is supported by the Swedish Energy Agency under contract 2016-008713 and uses computational resources at C3SE, provided by the Swedish National Infrastructure for Computing (SNIC). Both are gratefully acknowledged.

I would like to thank all of my teachers, past and present. Most importantly though, I would like to thank my supervisors Associate Professor Martin Fagerström and Professor Magnus Ekh for their patience, support and guidance. I would also like to thank my project partners and co-authors Tomas Ekermann, Stefan Hallström and Fredrik Stig whose knowledge and capabilities have made this work possible. To old and new friends, especially those at the division, thank you. Being able to each lunch, have a coffee and drop by your offices everyday is really something special. Finally, I would like to thank my family and most importantly Roeland, who has been next to me through it all.

# Thesis

This thesis consists of an extended summary and the following appended papers:

**Paper A** C. Oddy, T. Ekermann, M. Ekh, M. Fagerström, S. Hallström and F. Stig. Predicting damage initiation in 3D fibre-reinforced composites: The case for strain-based criteria. *Composite Structures* 230 (2019), 111336.

**Paper B** C. Oddy, T. Ekermann, M. Ekh, M. Fagerström, and S. Hallström. A framework for macroscale modelling of inelastic deformations in 3D-woven composites. *To be submitted.*

# Contents

# Part I
# Extended Summary

## 1 Introduction

Composite preforms with three-dimensional (3D) reinforcement first started appearing in the 1970s. Their development was driven by a need for reduced fabrication costs, increased through-thickness mechanical properties and improved impact damage tolerance [1]. Currently, the reported benefits of 3D-woven composites are broad and encompasses aspects relating to not only improved material integrity, but also benefits in manufacturing and in design flexibility.

When it comes specifically to the material's integrity, the inherent nature of the through-thickness reinforcement, suppresses delamination and increases out of plane strength and stiffness properties when compared to traditional laminated composites [2]. Composites with 3D-woven reinforcement have also shown increased fracture toughness and damage tolerance [3]. Furthermore, both Khokar et al. [4] and Kazemahvazi et al. [5] have demonstrated promising energy absorption capabilities. Khokar et al. in particular, compared a 3D-woven carbon fibre reinforced polymer (CFRP) I-beam against a steel I-beam with the same geometry under four-point bending. The results, illustrated in Figure 1.1, showed that the specific energy absorption of the CFRP I-beam was up to three times higher than its steel counterpart. It also showed that the through-thickness reinforcements allowed for a stable and progressive damage growth in a quasi-ductile manner.

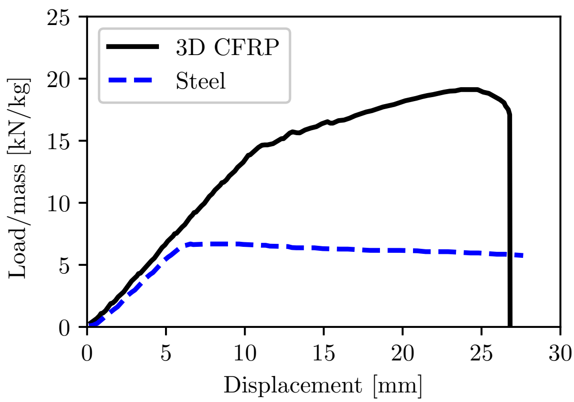



Figure 1.1: *Specific load - displacement curves of a 3D-CFRP and Steel I-Beam with the same dimensions under four point bending. From Khokar et al. [4].*

Along with improved material performance over traditional laminated composites, there are additional benefits to the use of 3D-woven composites. One important advantage being the fact that complex woven preforms can be produced in a near net-shape. As discussed by Mouritz et al. [6], this allows for the reduction of material waste, the need for joining and machining and the amount of material handling during lay-up. The flexibility of the weaving process also creates an impressive design space. As discussed by Whitney and Chou [7] many weave parameters can be changed which affects the overall behaviour of the material. This includes among other things: fibre type, tow size, tension in the yarns, tightness of the tows, number of warp and weft yarns per unit width and length, and the number of warp layers interlaced with each weft yarn. Further, it is possible to produce preforms with various cross-sectional shapes and in fact preforms in which overall shape and weave pattern changes from one part of the structure to the next. When done strategically, it is possible to truly optimise a component to its desired use.

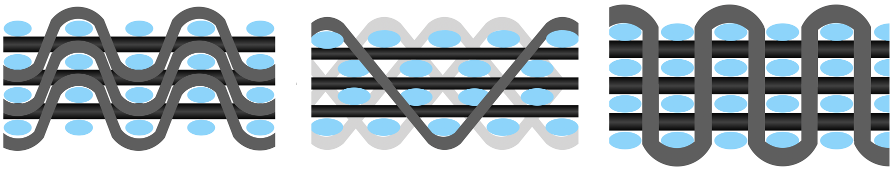
With all of their potential benefits, 3D-woven composites are slowly making their appearance across multiple industries. Within the aerospace industry for example, 3D-woven composites are used as fan blades in engines [8] and in the landing gear braces for the Boeing 787 [9]. Their use has also been reported within marine, civil infrastructure and medical applications [1]. The potential for further applications within the automotive industry also exist, one possibility being in intrusion protection systems. However, in order to further drive the use of 3D-woven composites in industry, efficient modelling techniques are required. The long term goal of this work is to therefore develop a macroscale phenomenologically based model to predict how 3D-woven composites behave under mechanical loading.

# 2 3D-woven composites

## 2.1 Classifying 3D fibre-reinforcements

Many different types of fibre-reinforced composites exist, each having their own benefits and drawbacks. This includes among others, composites with unidirectional (UD) fibre-reinforcements, non-crimp fabrics, 2D textiles and 3D fibre-reinforcements. In the most broad sense, 3D fibre-reinforced composites are characterised by the use of through-thickness reinforcements that improve out of plane properties. According to Tong [1], 3D fibre-reinforced composites can be classified by six main groups based on the manufacturing of their preforms: braided, knitted, stitched, z-pinned, non-woven and woven. Composites with 3D-woven reinforcement, are in many cases further divided into three groups, as discussed by Gereke and Cherif [10]. That is specifically: layer-to-layer angle interlocks, through-the-thickness angle interlocks and orthogonal weaves. They are illustrated in Figure 2.1.

In the case of angle interlocks, single yarns move vertically at specific weave points in order to bind the different layers together. Orthogonal weaves however, in particular those manufactured using the process described by Khokar [11], have three sets of yarns, see Figure 2.2. They are: warp yarns (blue) extending in the weaving direction as well as horizontal weft (red) and vertical weft (green) yarns extending transversely to the weave



(a) *Illustration of layer-to-layer angle interlock.* (b) *Illustration of through-the-thickness angle interlock.* (c) *Illustration of orthogonal weave.*

Figure 2.1: *Illustration of the different types of 3D-woven preforms.*

in the width and thickness directions, respectively. During the weaving process, warp yarns are alternately interlaced with horizontal and vertical weft yarns in a grid-like set.

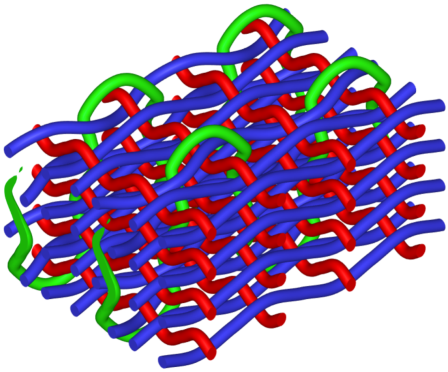


Figure 2.2: *Schematic of 3D-woven preform according to Khokar [4].*

## 2.2 Experimentally observed phenomena

With so much flexibility in the design process, the range of behaviours that 3D-woven composites can show is substantial. An overview of testing campaigns and their results have been presented and discussed by Saleh and Soutis [12]. Three-D woven composites have largely demonstrated prominent matrix driven non-linear behaviours when loaded in shear, cf. Ekermann et al.[13] and Warren et al. [14]. The same general statement however cannot be made when it comes to loading along the reinforcement directions.

In the case of that reported by Marcin [15], 3D-woven composites showed only light non-linearity up to failure when loaded in tension along both the warp and weft directions. Dahale et al. [16] however, demonstrated that the degree of the non-linear response can be tied to the weave architecture. In particular a denser weave pattern gives a nearly linear stress-strain response when specimens are loaded in tension along the reinforcement directions. In contrast however, a composite with a less dense weave pattern showed more significant non-linearity. This was likely due to the straightening of heavily misaligned yarns. In a similar fashion, Stig and Hallström [17] noticed that crimp can be another factor influencing the overall behaviour. Specimens with nominally straight yarns behaved linearly until failure when loaded along the reinforcements. Those with higher crimp again exhibited increased non-linearity due to a phenomenon denoted by Cox et al. [18], as plastic tow straightening

# 3 Existing modelling approaches

The structure of 3D-woven composites is hierarchical in nature, with the possibility to distinguish between three different scales, illustrated in Figure 3.1. The finest scale relevant for a continuum model, the microscale, describes the yarns as single fibre filaments embedded in a matrix material. The mesoscale, on the other hand describes the woven architecture of the yarns. Finally, the macroscale describes the material on a component level.

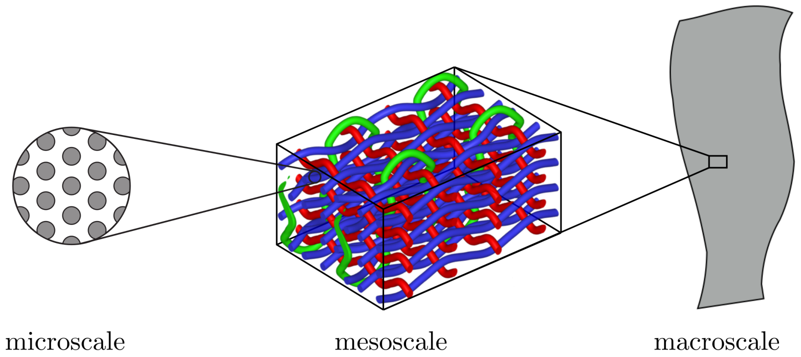



Figure 3.1: *An illustration of the hierarchical nature of 3D-woven composites.*

The nature of the micro and mesoscales affects the overall behaviour of the material on the macroscale. Many modelling approaches up to this point have largely focused on capturing and understanding the material behaviour and phenomena at the mesoscale. The overall goal in particular being to predict the link between preform geometry and macroscale mechanical properties. Some of the earliest approaches focused initially on predicting the elastic material parameters and material strength through analytical approaches. For example, Whitney and Chou [7] presented a model to predict in-plane elastic properties by dividing a unit cell into smaller microcells to form an inclined laminate. An analytical approach to predict elastic stiffness in all three directions and capture anisotropy of the reinforcements was later presented by Yushanov and Bogdanovich [19]. Their method, known as the Generalised Modified Matrix method is based on local spatial averaging of the reinforcement paths. An example of an analytical model to predict the strength of 3D-woven composites, is the work carried out by Tong et al. [20]. In this case a beam model with an elastic foundation was used to predict the effect of yarn waviness on tensile strength.

Finite Element (FE) approaches on the mesoscale allow for the prediction of material behaviours beyond simply elastic stiffness. The creation of a representative volume element (RVE) is then required. Bogdanovich [3] uses what is referred to as a Mosaic method to produce a simplified mesomodel. This RVE is built out of multiple 3D anisotropic bricks each having their own elastic and strength properties. Creating RVEs which discretely model the yarn paths within a matrix material is also possible. It should be noted however, that the creation of an RVE, is in itself a challenging task. This is especially the case for increasingly complex weave architectures. See for example Lomov [21] as well as Stig and Hallström [22].

Once the RVE has been generated, various phenomena can be considered. This can include viscoelastic behaviour of the matrix and yarns, as shown by Hirsekorn et al. [23], as well as progressive damage. When modelling progressive damage, most authors then follow a similar algorithm based on continuum damage mechanics, see for example Lomov et al.[24], Green et al.[25]. First they predict homogenised elastic stiffness properties, then damage is initiated based on failure criteria for the matrix and yarns. The latter is commonly described by criteria for UD composites. Subsequently, the constituent stiffness properties (matrix and yarns respectively) are progressively degraded according to the prevailing damage mode through damage evolution laws. As an alternative to continuum damage modelling, others such as Espadas-Escalante [26], turn to techniques such as a phase field approach to brittle fracture in order to capture complex crack scenarios.

Explicitly considering the mesoscale allows for the careful consideration of important subscale behaviours. Modelling large structural components with so much detail however, is computationally costly. Another possibility is then to turn to to macroscale models, which consider an anisotropic but homogeneous material response. In particular, macroscale models are more industrially applicable, as discussed by Marcin [15]. Current examples include the ONERA Damage Model for Polymer Matrix Composites, cf. Hurmane et al. [27], Marcin et al. [28], and Marcin [15].

# 4 Research scope

Approximating the structural domain to consist of a homogeneous material of orthotropic nature allows for a computationally efficient manner to model the mechanical behaviour of the material. The long term goals of the presented research work are summarised as follows:

- Develop a phenomenologically based macroscale model to predict how 3D woven composites deform and eventually fail under loading. In particular it must predict the inelastic processes that lead to energy absorption.

- Implement the proposed model in a commercial FE software.

- Calibrate the model against experimental results carried out in an ongoing parallel project at KTH Royal Institute of Technology. The considered material in particular being a glass fibre reinforced epoxy system.

- Asses the predicative capability of the proposed model against component testing on a structural level.

Emphasis has until this point been given to evaluating possible damage initiation criteria (appended **Paper A**) and the development of a general constitutive model framework for 3D-woven composites (appended **Paper B**). This framework in particular allows for the inclusion of various experimentally observed inelastic phenomena in a modular fashion. As a starting point, viscoelastic behaviour has been incorporated into the framework to capture reinforcement and shear related inelasticity.

# 5 Predicting damage initiation

The ability to accurately predict failure in fibre-reinforced composites is an ongoing research area. The substructural aspects of failure are complex and initiate due to different mechanisms, in different material constituents (fibre and matrix), under different loading conditions. In the case of UD fibre-reinforced composites, homogenised material descriptions are commonly used in an FE analysis. Again in most cases, failure initiation criteria are used to predict damage onset and continuum damage mechanics approaches are applied in order to degrade the material properties. An important first step is then determining appropriate criteria for failure initiation. Predicting failure initiation in UD composites, when compared to their 3D fibre-reinforced counterparts, is however a more mature research area.

The most straightforward methods for predicting failure initiation in UD composites are maximum stress or strain criteria. This however does not account for stress interactions. Some of the earliest proposed criteria that were able to account for combined stress states were Tsai-Hill [29] and Tsai-Wu [30]. While different failure fractions, according to the orthotropic nature of the material, are calculated, they all accumulate to a single failure criterion. The ability to not only predict failure but also predict the failure mode (fibre tension, fibre compression, matrix tension and matrix compression) came next with models proposed by e.g. Hashin [31]. Capturing more complex phenomena such as the matrix fracture angle under compression and the influence of transverse compression on shear strength then became possible with criteria proposed by Puck and Schürmann [32]. Failure initiation criteria have continued to grow in complexity and completeness, and address specific failure modes. They can predict fibre tensile failure, matrix related failure, and compressive fibre kinking, cf. Cuntze and Freund [33], Pinho et al. [34], and Carrere [35]. Further, the criteria presented by Pinho et al., commonly referred to as LaRC05 can, for example, account for various phenomena such as pressure dependence, non-linear shear behaviour, and in-situ effects. More recently, Camanho et al. [36] have proposed a stress invariant based criteria for damage initiation.

As previously discussed, when it comes to mesoscale modelling of 3D-woven composites, many use failure initiation criteria designed for UD composites inside the yarns with success. Extending such criteria for the application to 3D fibre-reinforced composites on the macroscale, has also been carried out by e.g. Böhm et al.[37] and Juhasz et al. [38]. In the case of Böhm et al., the criteria proposed by Cuntze and Freund [33] were considered and applied to 3D fibre-reinforced composites in which the majority of the reinforcement lay in-plane. Similarly the considered material in the work carried out by Juhasz et al. was dominated by the in-plane reinforcement. Extending UD failure initiation criteria to 3D fibre-reinforced composites in other cases however, can present a number of challenges, discussed further in the following section.

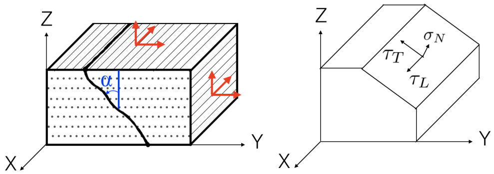
## 5.1 Paper A: Predicting damage initiation in 3D fibre-reinforced composites - The case for strain based criteria

The success of failure initiation models employing stress based criteria, are well established when it comes to UD fibre-reinforced composites. The extension of such failure criteria to 3D fibre-reinforced composites is explored in **Paper A**. Focus in particular is given to LaRC05 [34], which has gained wide spread familiarity and popularity. This is largely due to their performance in exercises such as the World Wide Failure Exercise [39], their computation efficiency and relative ease of use.

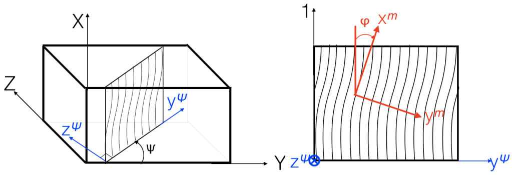
LaRC05 uses three failure indices to predict tensile fibre failure, matrix dominated failure and compressive fibre failure due to fibre kinking or splitting. One possible way to extend these criteria to 3D fibre-reinforced composites is to assume three independent, superimposed fibre directions, which can each be simultaneously analysed for fibre tensile failure, matrix failure and fibre kinking/splitting. It becomes apparent quite quickly however, that the lack of a clear plane of material isotropy dominated by the properties of the matrix makes the extension of the LaRC05 criteria challenging. Both the failure indices used to predict matrix failure and compressive fibre failure require that stresses be computed in rotated frames of reference, illustrated in Figure 5.1a and 5.1b respectively. The anisotropic nature of 3D-woven composites, simply produce erroneous predictions as the failure indices are checked in each plane.

In fact, the direct use of any stress-based failure criteria imposes the general assumption that homogenised stress states are representative of the material's loading response. In order to investigate this, a mesoscale model of the reinforcement architecture, shown in Figure 5.2, is considered. How the stress and strain distributions vary between material constituents within the composite is analysed. Elementary load cases i.e. tension along the warp and weft yarns as well as in-plane shear, were applied to the RVE. The stress distributions varied noticeably between the different material constituents. The strain fields on the other hand showed a more comparable distribution that would be better represented by an assumed homogeneous material response.

With this in mind, a set of strain-based failure indices were formulated in the spirit of LaRC05. For tensile and compressive failure in the reinforcement directions, maximum strain criteria were introduced along with three indices to predict matrix dominated failure. Initial results indicated that the predictive capability of strain-based criteria are qualitatively more reasonable than their stress-based counterparts.




(a) *Illustration of the stress components in an arbitrary matrix failure plane.*




(b) *Illustration of the stress components in an arbitrary reference frame for the kinking failure criterion.*

Figure 5.1: *Illustration of the failure planes where X denotes the considered reinforcement direction.*

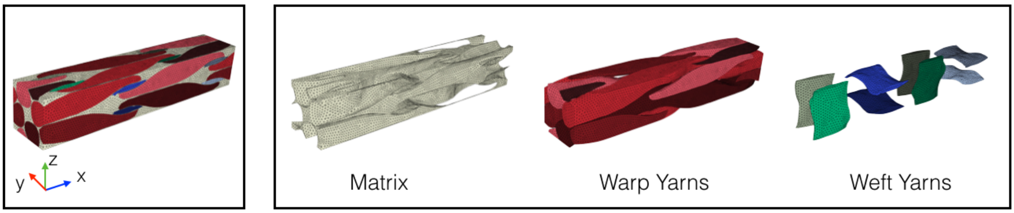



Figure 5.2: *The Mesoscale model and its constituents.*

# 6 Predicting inelastic behaviour

The weaving techniques associated with 3D-woven composites allow for the direct manufacturing of complex fibre preforms. These preforms can be strategically tailored to the overall needs of a desired component, from material choice to geometry and to weave architecture. This of course means that the range of possible macroscopic material behaviours is broad and can vary drastically from one 3D-woven material to the next. Developing a flexible foundation that allows for straightforward model additions and extensions to capture various experimentally observed behaviours would therefore be a promising step.

In the following sections, structural tensors and a structural tensor based formulation of the orthotropic stiffness tensor are briefly reviewed. The proposed framework, based on work initially proposed by Spencer [40] is then introduced. The framework in particular is flexible, thermodynamically consistent and allows for various inelastic behaviours to be added in a modular fashion. Focus is given to capturing reinforcement and shear related viscoelasticity. In particular, a crystal plasticity inspired approach is introduced in order to consider localised viscoelastic slip on planes determined by the reinforcement architecture.

## 6.1 A general modelling framework for 3D-woven composites

In traditional laminate theory, the stress and strain states within each ply are transformed between the global and local (reinforcement defined) coordinate systems using a set of transformation tensors. While this is one possible method, another elegant option is to use structural tensors. This allows for global material properties to be described based on the orientation of the local reinforcement architecture in each point. Further, it makes it possible to couple anisotropy development to rotating structural tensors when considering large deformations.

For the case of the 3D-woven architecture shown in Figure 6.1, this requires the definition of three mutually orthogonal vectors. These vectors $\boldsymbol{a}^1$, $\boldsymbol{a}^2$ and $\boldsymbol{a}^3$ then describe the nominal direction of the warp, horizontal weft and vertical weft, respectively. Three structural tensors, one for each reinforcement direction can in turn be defined as

$$\boldsymbol{A}^1 = \boldsymbol{a}^1 \otimes \boldsymbol{a}^1, \quad \boldsymbol{A}^2 = \boldsymbol{a}^2 \otimes \boldsymbol{a}^2 \text{ and } \boldsymbol{A}^3 = \boldsymbol{a}^3 \otimes \boldsymbol{a}^3.$$

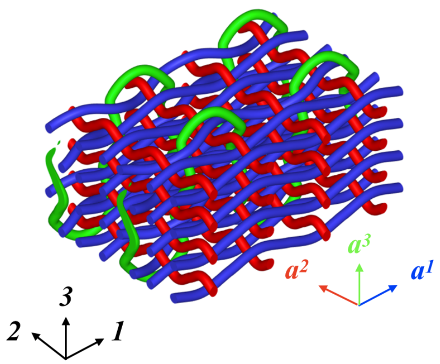



Figure 6.1: *Idealized mesoscopic 3D fibre-reinforcement structure with the definition of the structural reinforcement vectors* $\boldsymbol{a}^I$.

The nature of the reinforcement creates a material with three orthogonal preferred directions, associated to the reinforcement architecture. This implies that the elasticity should be described by an orthotropic stiffness tensor. In the case when the reinforcement architecture does not align with the global coordinate system, a structural tensors based description of the orthotropic stiffness tensors is a convenient choice. In this case the fourth order orthotropic stiffness tensor can be expressed as

$$\mathbb{E} = \sum_{I=1}^{3} \varphi_I \, \mathbb{A}^I + \sum_{I=1}^{3} \sum_{J=1}^{3} \phi_{IJ} \boldsymbol{A}^I \otimes \boldsymbol{A}^J, \tag{6.1}$$

where the fourth order tensor $\mathbb{A}^I$ is given by [1]

$$\mathbb{A}^I = \frac{1}{2} (\boldsymbol{A}^I \bar{\otimes} 1 + 1 \bar{\otimes} \boldsymbol{A}^I). \tag{6.2}$$

While this structural tensor-based representation of the orthotropic elastic stiffness does not directly involve the elastic engineering parameters, it is possible to relate the coefficients $\varphi_I$ and $\phi_{IJ}$ to them. This is possible by considering the case where the three reinforcement directions correspond to the main coordinate axes.

As previously stated, developing a flexible foundation that allows for various experimentally observed phenomena to be captured and modelled, would be a promising first step. Work initially carried out by Spencer [40], which focused on continuum theories to describe the macroscopic behaviour of fibre-reinforced materials, provides such a basis. In particular he proposes that the stress tensor be subdivided into different parts related to different material constituents. His proposed decomposition method has more recently been used by Nedjar [41], Vogler et al. [42] and Camanho et al. [36] in their own work. In each case, the authors took the decomposition and used it to develop a viscoelasticity model, a plasticity model as well as three-dimensional failure criteria for transversely isotropic unidirectional composites.

Following the methodology set out by Spencer, and extending it to decompose the stress and strain tensors for a 3D-woven composites then gives the following,

$$\boldsymbol{\sigma} = \boldsymbol{s} + \sum_{I=1}^{3} \left( \boldsymbol{\sigma} : \boldsymbol{A^I} \right) \boldsymbol{A}^I \text{ and } \boldsymbol{\epsilon} = \boldsymbol{e} + \sum_{I=1}^{3} \left( \boldsymbol{\epsilon} : \boldsymbol{A^I} \right) \boldsymbol{A}^I. \tag{6.3}$$

In order to conceptualise this, consider again the case where $\boldsymbol{a}^1 = [1\ 0\ 0]^T$, $\boldsymbol{a}^2 = [0\ 1\ 0]^T$ and $\boldsymbol{a}^3 = [0\ 0\ 1]^T$. The stress decomposition (and in a synonymous way the strain) can be expressed as

$$\boldsymbol{\sigma} = \underbrace{\begin{bmatrix} 0 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & 0 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & 0 \end{bmatrix}}_{\boldsymbol{s}} + \underbrace{\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\left(\boldsymbol{\sigma}:\boldsymbol{A^1}\right)\boldsymbol{A}^1} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\left(\boldsymbol{\sigma}:\boldsymbol{A^2}\right)\boldsymbol{A}^2} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}}_{\left(\boldsymbol{\sigma}:\boldsymbol{A^3}\right)\boldsymbol{A}^3}. \tag{6.4}$$

Even more favourably however, is that it can be shown that by using these decompositions and the orthotropic stiffness tensor in Equation (6.1), a direct relationship between $\boldsymbol{s}$ and $\boldsymbol{e}$ can be formulated, without couplings to any other term. From a thermodynamics perspective, it can also be shown that $\boldsymbol{s}$ and $\boldsymbol{e}$ are energy conjugated to one another. The full constitutive stress strain relationship is then given by

$$\boldsymbol{\sigma} = \mathbb{E}_{\mathrm{m}} : \boldsymbol{e} + \mathbb{E}_{\mathrm{f}} : \boldsymbol{\epsilon}, \tag{6.5}$$

[1] The nonstandard $\bar{\otimes}$ operator expresses the operation $(\boldsymbol{A} \bar{\otimes} \boldsymbol{B})_{ijkl} = A_{ik} B_{jl}$.

where

$$\mathbb{E}_\mathrm{m} = \sum_{I=1}^{3} \varphi_i \, \mathbb{A}^I, \text{ and } \mathbb{E}_\mathrm{f} = \sum_{I=1}^{3} \left( \varphi_I \, \boldsymbol{A}^I \otimes \boldsymbol{A}^I + \sum_{J=1}^{3} \phi_{IJ} \, \boldsymbol{A}^I \otimes \boldsymbol{A}^J \right). \tag{6.6}$$

From a physical standpoint, this means that the first term in Equation (6.5) contains shear related behaviour. Similarly, the second term contains the reinforcement related behaviours. This provides a convenient basis from which it is possible to add experimentally observed inelastic behaviours in a modular style approach.

## 6.2 Modelling inelasticity

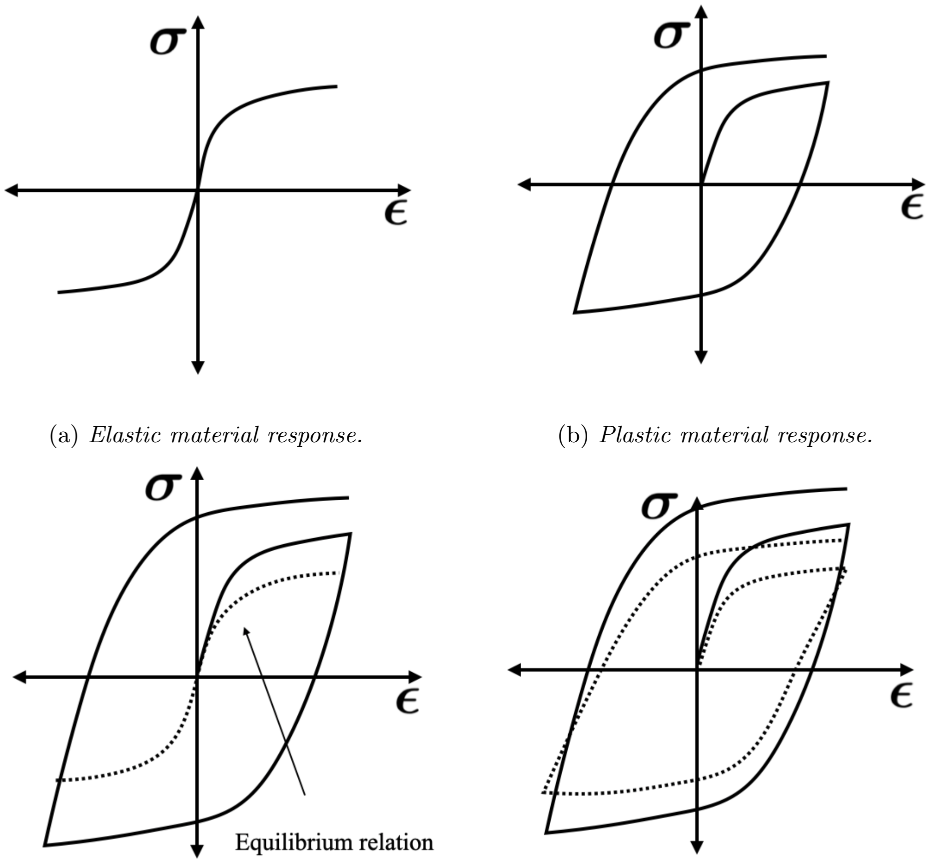
The experimental behaviour of fibre-reinforced composite materials often shows various non-linear phenomena. These non-linearities can be due to, for example, damage mechanisms or the behaviour of the polymer matrix. According to Haupt [43], material behaviours (excluding damage) and their corresponding constitutive models can generally be classified into four different groups: elasticity, plasticity, viscoelasticity and viscoplasticity. They are illustrated in Figure 6.2 and characterised as follows:

- *Elasticity*: rate-independent material behaviour without hysteresis.
- *Plasticity*: rate-independent material behaviour with hysteresis.
- *Viscoelasticity*: rate-dependent material behaviour without equilibrium hysteresis.
- *Viscoplasticity*: rate-dependent material behaviour with equilibrium hysteresis.

When it comes to common choices of polymer matrices many have been classified as viscoelastic, cf. Woo et al. [44], Saseendran et al. [45] and Bardella [46]. Some, for example Gerbaud et al. [47] and Hurmane et al. [27], also attribute experimentally observed non-linearities in fibre-reinforced composites to the viscoelastic nature of the matrix. In order to model viscoelasticity, the most elementary choice is a Maxwell element. Rheologically, a Maxwell model can be represented in one dimension as a combination of an elastic spring with stiffness $E$ and a dashpot with viscosity $\mu$, shown in Figure 6.3. The strain is split additively into an elastic and an inelastic part, while the stress is equal in both component. Therefore, the development of the strain can be written as

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\mu}. \tag{6.7}$$

The simple nature of the Maxwell model does however have its drawbacks. As a linear viscoelastic model and given that only one viscous parameter can be modified, the range of possible loading and unloading behaviours that it can capture are limited. One remedy is to chain additional elastic springs and dashpots together in series and/or parallel. Another possible starting point is to introduce different non-linear behaviour of




(a) *Elastic material response.* (b) *Plastic material response.*

(c) *Viscoelastic material response.* (d) *Viscoplastic material response.*

Figure 6.2: *The four material classifications according to Haupt [43].*

the dashpot. One alternative (among many) is the Norton power-law model [48], which for the one dimensional case is

$$\dot{\epsilon}_i = \frac{1}{t_*}\left(\frac{|\sigma|}{\kappa}\right)^n \operatorname{sgn}(\sigma)\,. \tag{6.8}$$

In this respect the viscosity of a Norton style model is dependent on the viscous stress and gives the possibility to tune both $t_*, \kappa^n$ and $n$ to fit the available experimental results.

Returning to the constitutive stress-strain relationship of the proposed framework given in Section 6.1, it is again possible to additively split the strain into an elastic part and an inelastic part to model non-linear behaviour associated to loading along the reinforcements or loading in shear. In order to, for example, model inelasticity observed

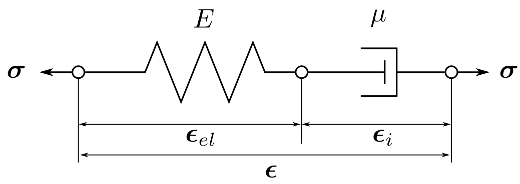


Figure 6.3: *A Maxwell element.*

strictly due to loading along the direction defined by $\boldsymbol{a}^I$, then the following is proposed

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_{el} + \boldsymbol{\epsilon}_i, \tag{6.9}$$

where

$$\boldsymbol{\epsilon}_i = \epsilon_v \boldsymbol{A}^I. \tag{6.10}$$

Turning again to a Norton style model for viscoelasticity, the evolution of the inelastic strain, $\dot{\epsilon}_v$ is expressed as

$$\dot{\epsilon}_v = \frac{1}{t_*^r} \left( \frac{|\boldsymbol{\sigma} : \boldsymbol{A}^I|}{\kappa^r} \right)^{n^r} \operatorname{sgn}(\boldsymbol{\sigma} : \boldsymbol{A}^I). \tag{6.11}$$

Due to the nature of the construction, Equation (6.5) now can be rephrased,

$$\boldsymbol{\sigma} = \mathbb{E}_m : \boldsymbol{e} + \mathbb{E}_f : (\boldsymbol{\epsilon} - \epsilon_v \boldsymbol{A}^I). \tag{6.12}$$

Further, to also allow for the development of inelastic strains due to shear loading, it is possible to decompose $\boldsymbol{e}$. As such

$$\boldsymbol{e} = \boldsymbol{e}_{el} + \boldsymbol{e}_i, \tag{6.13}$$

and the stress is obtained as

$$\boldsymbol{\sigma} = \mathbb{E}_m : (\boldsymbol{e} - \boldsymbol{e}_i) + \mathbb{E}_f : (\boldsymbol{\epsilon} - \epsilon_v \boldsymbol{A}^I). \tag{6.14}$$

A crystal plasticity inspired approach, discussed in the following section, is specifically proposed to model inelasticity due to shear loading.

## 6.3 A crystal plasticity inspired approach

The mechanical properties of engineering materials, are affected by the substructure of the material. In the case of metals, their substructures are built up of individual crystals. The nature of these crystal structures causes plastic deformations to occur by slipping along preferred planes defined by their atomic structure. Crystal plasticity [49] is a physically based theory which describes the development of inelastic strain in these slip systems driven by the projected stress.

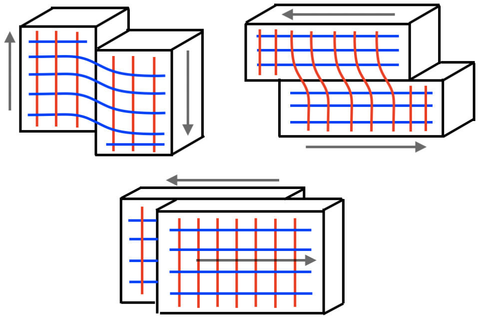


Figure 6.4: *Illustration of shear loading and deformation in planes oriented based on the reinforcement.*

Crystal plasticity also provides an interesting starting point for considering the shear behaviour of 3D-woven composites. In particular it allows for the modelling of localised slip behaviour in planes defined by the reinforcement architecture. This is illustrated in Figure 6.4.

The shear behaviour in these planes can then be assumed to be driven by the total projected shear stress $\boldsymbol{t}_s^I$, cf. Figure 6.5. This is similar to the ideas presented by Larijani et al. [50] for pearlitic steel. Turning to a Norton style viscoelastic model for the development of the inelastic strain then gives that

$$\dot{\boldsymbol{e}}_i = \sum_{I=1}^{3} \frac{1}{t_*^I} \left( \frac{|\boldsymbol{t}_s^I|}{\kappa^I} \right)^{n^I} \boldsymbol{m}^I. \tag{6.15}$$

Further, if the direction of the viscoelastic strain evolution is chosen to be of associative type, then

$$\boldsymbol{m}^I = \frac{\partial |\boldsymbol{t}_s^I|}{\partial \boldsymbol{s}} = \frac{\mathbb{A}^I : \boldsymbol{s}}{\sqrt{\boldsymbol{s} : \mathbb{A}^I : \boldsymbol{s}}}. \tag{6.16}$$

Although in the simplest case, $t_*^I$, $\kappa^I$ and $n^I$ would be the same on each plane, it is highlighted here that this model framework allows for different inelastic behaviour in the individual planes.

## 6.4 Paper B: A framework for macroscale modelling of inelastic deformations in 3D-woven composites

In the second appended paper, the framework in Section 6.1 is formulated and presented in greater detail. In order to demonstrate the capability of the proposed framework, a 3D-woven composite with orthogonal glass fibre reinforcement is considered. The glass fibre reinforced epoxy system shows a number of noteworthy experimental behaviours. Tensile loading along the warp yarns, the strongest and stiffest direction,

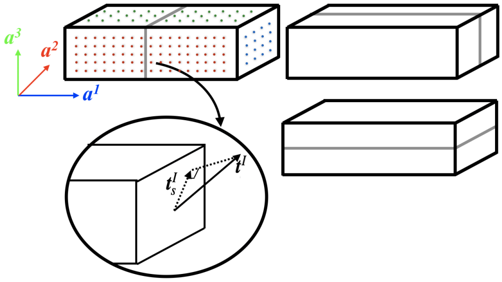



Figure 6.5: *Illustration of the considered traction vector in each plane.*

produces a nearly linear response until final failure. This however, is not the case when the composite is loaded in tension along the horizontal weft yarns. Loading the material in this manner produces a noticeable non-linear response. Loading the material in shear also produces prominent non-linearities.

In order to capture the observed non-linear behaviours the strain is split additively, and Norton style viscoelasticity models are introduced as discussed in Section 6.2 and 6.3. This makes it possible to capture the tensile loading behaviour of the material along the horizontal weft $\boldsymbol{a}^2$ and in shear. The viscous parameters are calibrated against experimental tensile and shear tests. The model is then validated against a tensile test with the reinforcements oriented off-axis to the loading direction.

# 7 Conclusions and outlook

Composite materials with 3D-woven reinforcements have shown a number of promising characteristics. In order to encourage their widespread adoption in industry however, efficient modelling techniques are required. Many authors turn to the mesoscale to predict the behaviour of 3D-woven composites. This allows for a detailed description of important subscale behaviours. Modelling large structural components with such detail however, is computationally costly. A macroscale approach, that describes the composite as an anisotropic but homogeneous material allows for a more computationally efficient approach. The long term goal of this work is to develop a phenomenologically based macroscale model to predict how 3D-woven composites deform and eventually fail under mechanical loads. Emphasis until this point has been given to evaluating possible damage initiation criteria and the development of a general model framework for 3D-woven composites.

As a first step, in **Paper A**, the possibility of extending damage initiation criteria for laminated composites to 3D-woven composites was explored. The direct extension of stress-based criteria for laminated composites, such as LaRC05 however, produces a number of challenges. Composites with 3D-woven reinforcements have no clear plane of material isotropy dominated by the behaviour of the matrix. This complicates (or even disqualifies) the direct extension of failure criteria based on the identification of a critical plane determined from the homogenised stress state. Further, the use of stress-based criteria implies the assumption that a homogeneous stress state can be used to predict the risk for different failure mechanisms. An analysis of the weave architecture on the mesoscale indicated that strain-based criteria may be more appropriate for predicting failure initiation. With this in mind, strain-based criteria inspired by LaRC05 were proposed. They provided results that were qualitatively more reasonable.

In **Paper B** a framework for modelling 3D fibre-reinforced composites was presented. Based on the work by Spencer [40], it is proposed that the stress and strain tensors be divided into two part based on the material architecture. Specifically, one term consists of the normal stresses and strains along the directions of the reinforcement, while the other holds the shear related components. The framework allows for various inelastic behaviours to be added in a modular fashion. Focus was given to capturing reinforcement and shear related non-linear behaviours using a Norton style viscoelasticity model. Further, a crystal plasticity inspired approach was introduced in order to model shear related localised viscoelastic slip on planes determined by the reinforcement architecture.

The presented framework is thermodynamically consistent, general and flexible. As more experimental results become available, the proposed framework allows for the modification and extension of the model to better capture various phenomena. While for the time being, a viscoelastic material response is considered, this can, without difficulty be modified to plasticity or viscoplasticity. The framework opens the door for future model developments to include progressive damage modelling and a formulation in large strains.

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