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RESEARCH PAPER

Scheduling tamping operations on railway tracks using mixed integer linear programming

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Abstract A mixed integer linear programming model for the problem of scheduling tamping operations on ballasted tracks is presented and analyzed. The model improves a previously proposed one by introducing disaggregated constraints yielding in a model with stronger lower bound from the continuous relaxation of the binary variables. A more general cost structure is proposed; a structure having the possibility of including setup costs for the tamping operations. The problem considered is shown to be NP-hard. A numerical study is performed to evaluate the performance of the model compared to two simple policies for constructing maintenance schedules. The computational results show that the maintenance costs can be reduced by up to 10 % as compared with the best policy investigated.

Keywords Preventive maintenance · Maintenance scheduling · Railway maintenance · Mixed integer linear optimization · Rail tamping

Mathematics Subject Classification 90C10 · 90B06 · 90B25 · 90B35

1 Introduction

Preserving acceptable conditions of the components in production and infrastructure systems requires that the correct maintenance is performed at the right time. Maintenance is here defined as a set of activities aimed at improving the overall reliability and availability of a system—often categorized into preventive

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maintenance (PM) and corrective maintenance (CM) activities. PM activities consist of scheduled maintenance tasks performed to ensure the safety and to prevent unexpected system breakdowns, while CM activities are performed after failures or breakdowns of the system have occurred and to restore the system to an operational state. Systems with several components differ from single-component systems since dependencies between the components imply that maintenance schedules which are optimal for individual components might be suboptimal or even infeasible for the multi-component system as a whole Dekker et al. (1997). This paper considers a multi-component system in which economic dependencies exist between the components in the form of a setup cost that has to be paid each time at least one of its components is maintained. Although the problem considered is based on a specific application, the mathematical models developed and analyzed can be utilized for general applications of PM scheduling.

An important notion to consider when scheduling PM is what is denoted in the literature as opportunistic maintenance (OM). OM is a strategy in which a mathematical model is utilized to decide whether, at a maintenance occasion, more than the necessary maintenance activities should be performed; this is referred to as PM activities at an opportunity. According to Dickman et al. (1991), Jorgenson and Radner (1960) introduced the opportunistic replacement problem (ORP), in which the objective is to optimally schedule the replacement of components in a system over a finite, discrete time horizon. Each time any maintenance is performed a setup cost is paid, which in turn gives rise to opportunities for PM. Further development and a theoretical analysis of an integer linear programming (ILP) model for the ORP was performed by Almgren et al. (2012). The ORP and the mathematical model described by the authors are generalized by Gustavsson et al. (2014) to the preventive maintenance scheduling problem with interval costs (PMSPIC). The PMSPIC aims at scheduling maintenance activities over a finite time horizon such that the costs for all maintenance intervals are minimized. The interval costs are composed by costs for performing maintenance and costs representing the degradation of the components. In the model presented in this paper, OM is utilized implicitly when scheduling the PM. One assumption of the cost structure of the tamping operations is that whenever any tamping is being performed on the rail a fixed setup cost has to be paid. This cost is generated because the tamping operations limits the traffic on the rail. The setup cost gives rise to opportunities for more than the necessary tamping to be performed, i.e., OM can be utilized.

This paper considers the problem of scheduling PM on a railway system, in particular the scheduling of tamping operations on ballasted tracks [e.g., Esveld (1989)]. The reason for only scheduling PM is that CM is only performed when unexpected breakdowns occur, i.e., when unforeseen events cause the components to break. CM can therefore not be scheduled; it is a forced reaction to the breakdowns that may occur. The system components to be maintained are the track segments, and tamping is the maintenance activity adopted to correct the longitudinal profile, the cross level, and the alignment of ballasted tracks. According to Famurewa et al. (2013), tamping is considered to have a large impact on the effective capacity of a railway network and is therefore an important maintenance activity to schedule optimally; it is also stated that the current decision support tools for scheduling tamping activities do not support the increasing demand for capacity, safety, and cost-effectiveness in the railway industry. Also in Ekberg and Paulsson (2010) the importance of the track geometry as a cost-driving effect for the maintenance cost is stated: one of the highest track maintenance costs for infrastructure managers planning the maintenance of the track is the rectification of a poor track geometry. The model presented here can easily be extended to include more PM activities that are used to mitigate rail geometry deterioration. The reason for limiting the scope of this paper to tamping operations is that the main measure used to correct the rail geometry is rail tamping.

Under the influence of dynamic track loads the track geometry deteriorates. This deterioration is typically quantified as track irregularities of the longitudinal track profile. According to Esveld (1989), tamping can only correct irregularities with wavelengths between 3 and 25 m. Therefore, the condition of a track segment with respect to the track geometry is measured as the standard deviation of the longitudinal profile over wavelengths from 3 to 25 m. To forecast the condition of track segments, a deterioration and restoration model for the track irregularities needs to be employed. The models available can be classified into two types: (a) deterministic and (b) stochastic. The deterministic models found in the literature are based on linear (Vale et al. 2012), polynomial (Jovanovic 2004), exponential (Veit 2007; UIC 2008), or multi-stage linear (Chang et al. 2010) approximations of the rate of the deterioration of the track irregularities. For an example of a model in which the deterioration of the track geometry is modelled as a stochastic process, see Lyngby et al. (2008), and for a model based on a Gamma process, see Meier-Hirmer et al. (2006) and Meier-Hirmer et al. (2009). Quiroga and Schnieder (2012) analyze thoroughly several models, both deterministic and stochastic. In this paper, a deterministic model which is described in detail in Sect. 3 is utilized.

The aim of this paper is to present an extension of the mathematical optimization model developed in Vale et al. (2012). The authors are the first to present an integer linear programming model for the problem of scheduling tamping operations on ballasted tracks. The model is, however, not thoroughly analyzed from a mathematical point of view. Some of the constraints in the proposed model can be strengthened to reduce the computation time for solving the model. The model proposed in this paper includes economic dependencies between the components (i.e., the track segments) of the railway system, as well as generalizes the objective function to be minimized in the model. The model is validated through a numerical study comparing the performance of the model with that of simple policies for constructing maintenance schedules.

The remainder of this paper is organized as follows. In Sect. 2, a literature review of maintenance scheduling for railway systems is presented. Section 3 introduces and analyzes a mathematical model for the problem of scheduling tamping operations on a railway system. In Sect. 4, two simple policies are presented for constructing tamping schedules, and in Sect. 5 a numerical study of the model is performed. The paper is concluded in Sect. 6 where some notes regarding possible further developments of the model are expressed.

2 Literature review

The problem of scheduling maintenance activities for mitigating the railway track irregularities has been studied in the past. Miwa (2002), and later Oyama and Miwa (2006) develop an integer linear programming (ILP) model in which the degradation of the surface irregularities of the track is taken into account. A restoration model explaining the effects of the maintenance activities on the track irregularities is also developed and incorporated into the ILP model. The objective of the model is to maximize the total expected improvement obtained from the track maintenance scheduled. Another study concerning scheduling maintenance to improve track irregularities is performed by Vale et al. (2012). The authors consider the objective to minimize the total number of tamping operations performed on the system over a finite time horizon. An ILP model of the scheduling problem is developed and solved using optimization software. One of the latest contributions to the area is by Famurewa et al. (2013) where the authors develop a model and present a case study for the scheduling of tamping activities on a line section in the network of the Swedish Transport Administration. The model utilizes an exponential degradation model which is calibrated to fit historic measurements on the line section studied. Another paper considering the scheduling of tamping activities is Zhang et al. (2012) where the authors develop a mathematical optimization model to solve a condition-based scheduling problem using a genetic algorithm.

Ferreira and Murray (1997) and Soh et al. (2012) present two surveys within the area of railway maintenance scheduling. For a more technical analysis of the possible defects of the components in the rail industry and how the components can be maintained, see Cannon et al. (2003). For thorough surveys within the area of maintenance scheduling for general systems, see Nicolai and Dekker (2008), Pintelon and Gelders (1992), Sharma et al. (2011) and Wang (2002). For the special case of maintenance of production systems, the survey by Budai et al. (2008) analyzes the relation between the maintenance costs and the production benefits of the system.

3 Mathematical model

In this section, a mathematical optimization model for the problem of scheduling tamping activities on a railway system is proposed. The model extends the one presented by Vale et al. (2012); the similarities and differences between the two models will be clarified.

3.1 System and component degradation

Consider a rail network consisting of a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of track segments and a set $\mathcal{T} = \{0, \ldots, T\}$ of time steps at which tamping can be performed. The condition of a track segment is measured as the standard deviation of the longitudinal profile over wavelengths from 3 to 25 m. With this notion, a high (low) value of the standard deviation represents a bad (good) track condition, and the lower bound of a track segment's standard deviation is zero.

One assumption in this paper is that the condition of a segment at the beginning of a time step depends only on its condition in the previous time step and on whether or not tamping was performed on the segment in the previous time step. This assumption is supported in UIC (2008) where the authors provide a degradation model only depending on the track quality after the last tamping operation. Letting the continuous variable s_{it} denote the standard deviation of track segment $i \in \mathcal{N}$ at the beginning of time step $t \in \mathcal{T}$, the standard deviation of the segment will evolve according to

$$s_{i,t+1} = \begin{cases} (1+\alpha_i)(s_{it} - r_{it}) + h_i, & \text{iftampingwasperformedon} \\ & \text{tracksegment}i\text{attimestep}t, \\ (1+\alpha_i)s_{it} + h_i, & \text{otherwise,} \end{cases} \quad t \in \mathcal{T} \setminus \{T\}, \tag{1}$$

where the parameter $h_i \geq 0$ denotes a constant increase of the standard deviation of track segment $i \in \mathcal{N}$ between two consecutive time steps, the parameter $\alpha_i \geq 0$ describes the increase of the degradation rate of track segment $i \in \mathcal{N}$ when the condition deteriorates, and the parameter $r_{it} \geq 0$ denotes the recovery of the condition obtained by performing tamping on track segment $i \in \mathcal{N}$ at time step $t \in \mathcal{T}$.

In Vale et al. (2012), the degradation rate does not increase with the deterioration of the track condition (i.e., linear degradation). The parameters $\alpha_i \geq 0$, $i \in \mathcal{N}$ are here introduced because they allow us to utilize a combination of a linear and an exponential degradation model for the track irregularities; the usefulness of an exponential model is justified in Quiroga and Schnieder (2010) through a numerical study. The recovery of the track condition is assumed to depend linearly on the value of the condition at the time of tamping, i.e.,

$$r_{it} = \gamma_i s_{it} + b_i, \quad i \in \mathcal{N},\ t \in \mathcal{T}, \tag{2}$$

where $\gamma_i \in [0, 1]$ and $b_i \in \mathbb{R}$ are called the recovery parameters for track segment $i \in \mathcal{N}$. This assumption is validated in Office for Research and Experiments (1988), which states that the recovery effectiveness of the longitudinal level depends on the quality of the track at the time of maintenance. For a set of track segments on the Portuguese Railway Northern Line, Vale et al. (2012) employed the values $\gamma_i = 0.4257$ and $b_i = -0.153$, $i \in \mathcal{N}$; Famurewa et al. (2013) employed $\gamma_i = 0.5445$ and $b_i = -0.889$, $i \in \mathcal{N}$, for a set of track segments on a line section in the network of the Swedish Transport Administration. In Fig. 1, the condition of a track section between time steps $t = 0$ and $t = 50$ is illustrated for a constant degradation rate, i.e., $\alpha_i = 0$. In Fig. 2, the degradation rate is assumed to increase with deteriorating condition, i.e., $\alpha_i > 0$. In both figures, tamping is performed at time steps $t = 18$ and $t = 42$, and the recovery parameters are set to $\gamma_i = 0.8$ and $b_i = 0$.

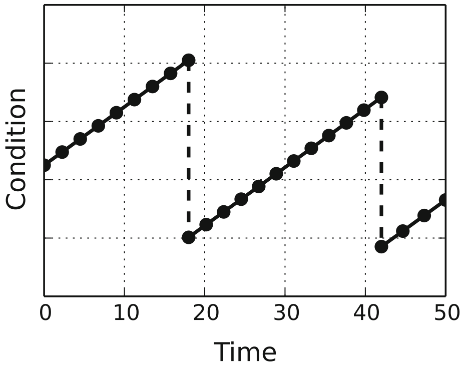

Fig. 1 The condition of a track segment assuming linear degradation ($\alpha_i = 0$) and tamping performed at time steps $t = 18$ and $t = 42$

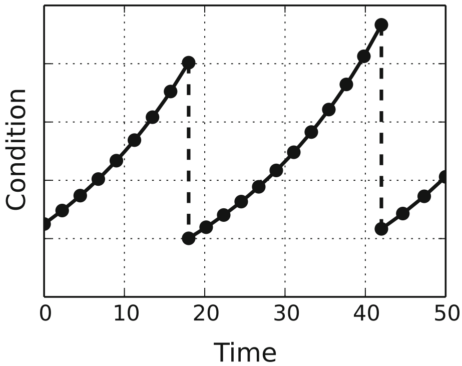

Fig. 2 The condition of a track segment assuming increasing degradation ($\alpha_i > 0$) and tamping performed at time steps $t = 18$ and $t = 42$

3.2 The model

To formulate the mathematical model for optimally scheduling the tamping activities, the following binary decision variables are introduced:

$$x_{it} = \begin{cases} 1, & \text{iftampingisperformedon} \\ & \text{tracksegment}i\text{attimestep}t, \\ 0, & \text{otherwise}, \end{cases} \qquad i \in \mathcal{N},\ t \in \mathcal{T}, \tag{3}$$

$$z_t = \begin{cases} 1, & \text{iftampingisperformedonatleast} \\ & \text{onetracksegmentattimestep}\, t, \\ 0, & \text{otherwise}, \end{cases} \qquad t \in \mathcal{T}. \tag{4}$$

The variables x_{it} denote whether or not tamping is performed on track segment $i \in \mathcal{N}$ at time step $t \in \mathcal{T}$, while the variables z_t denote whether any tamping is performed at time step t. A time step $t \in \mathcal{T}$ such that $z_t = 1$ is denoted a maintenance occasion.

The modeling of the evolution of the track condition defined in (1) using the track condition variables s_{it} and the binary variables introduced in (3) can be made through the intuitive equations

$$s_{i,t+1} = x_{it}\underbrace{((1+\alpha_i)(s_{it}-r_{it})+h_i)}_{\text{tamping}}+(1-x_{it})\underbrace{((1+\alpha_i)s_{it}+h_i)}_{\text{notamping}}, \quad i \in \mathcal{N}, t \in \mathcal{T} \setminus \{T\}.$$

These equations are, however, nonlinear and would therefore lead to a computationally intractable model. Since the aim of the paper is to formulate a more tractable ILP model, the following constraints will instead be utilized:

$$s_{i,t+1} \geq (1+\alpha_i)s_{it} + h_i - x_{it}s_i^{\max}, \qquad i \in \mathcal{N},\ t \in \mathcal{T} \setminus \{T\}, \tag{5}$$

$$s_{i,t+1} \geq (1+\alpha_i)(s_{it}-r_{it}) + h_i - (1-x_{it})s_i^{\max}, \quad i \in \mathcal{N},\ t \in \mathcal{T} \setminus \{T\}, \tag{6}$$

where the constants $s_i^{\max} \geq 0$, $i \in \mathcal{N}$, are large enough (to be defined below). Provided that the objective function strives to minimize the value of s_{it} for all $i \in \mathcal{N}$, $t \in \mathcal{T}$, the constraints (6) will be the bounding constraint on $s_{i,t+1}$ if tamping is performed on component $i \in \mathcal{N}$ at time step $t \in \mathcal{T}$ (i.e., if $x_{it} = 1$). If there is no tamping on component $i \in \mathcal{N}$ at time step $t \in \mathcal{T}$, (i.e., if $x_{it} = 0$) then the constraints (5) will be bounding. Note that the reason for the constraints being bounding is that the objective is to minimize the cost for maintenance, meaning that a lower track condition is always favorable over a higher one. This implies that the condition variables s_{it} will always be set to the smallest feasible values, given the values of x_{it}. Another important note is that the term $(1-x_{it})s_i^{\max}$ can be removed from the constraints (6), since the value of the right-hand side of (6) is always lower than that of the value of the right-hand side in (5) when $x_{it} = 0$.

The reason for performing any tamping on the system is the fact that the track condition of track segment $i \in \mathcal{N}$ shall not exceed its predefined limit $s_i^{\max}$. These conditions are based on riding quality and safety assessments and are formulated by the constraints

$$s_{it} \leq s_i^{\max}, \quad i \in \mathcal{N},\ t \in \mathcal{T}. \tag{7}$$

The track conditions at time zero are defined by the constraints

$$s_{i0} = s_i^{\text{init}}, \quad i \in \mathcal{N}, \tag{8}$$

where s_i^{init} denotes the initial condition of track segment i. I assume that the track conditions are non-negative scalars, which is represented by the constraints $s_{it} \geq 0$, $i \in \mathcal{N}$, $t \in \mathcal{T}$. These constraints will in most practical applications be redundant because $\gamma_i \leq 1$ and $b_i \leq h_i$ in a realistic deterioration model.

Vale et al. (2012) assume that tamping operations have to begin and end on straight alignment segments (UIC 2008); this assumption is modelled by the constraints

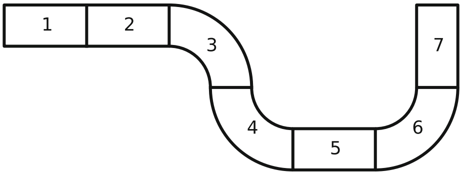

Fig. 3 A railway system consisting of seven track segments. For this example, $\mathcal{I}_1 = \{1\}$, $\mathcal{I}_2 = \{2\}$, $\mathcal{I}_3 = \mathcal{I}_4 = \{2,3,4,5\}$, $\mathcal{I}_5 = \{5\}$, $\mathcal{I}_6 = \{5,6,7\}$, and $\mathcal{I}_7 = \{7\}$

$$\sum_{i \in \mathcal{I}_k} x_{it} \geq |\mathcal{I}_k| x_{kt}, \quad k \in \mathcal{N}, \ t \in \mathcal{T}, \tag{9}$$

where $\mathcal{I}_k$ denotes the smallest set of consecutive track segments such that $k \in \mathcal{I}_k$ and the first and last segment in $\mathcal{I}_k$ are straight alignment segments, and $|\mathcal{I}_k|$ denotes the number of elements in the set $\mathcal{I}_k$. For an illustration of the sets $\mathcal{I}_k$, $k \in \mathcal{N}$, for a railway network consisting of seven track segments, see Fig. 3. A stronger formulation of the constraints (9), in the sense of the strength of the continuous linear relaxation (Nemhauser and Wolsey 1988), is given by

$$x_{it} \geq x_{kt}, \quad i \in \mathcal{I}_k, \ k \in \mathcal{N}, \ t \in \mathcal{T}, \tag{10}$$

since the constraints (9) constitute an aggregation of the constraints (10). In this paper, the disaggregated constraints (10) will be employed since they yield a model with a stronger lower bound from the continuous relaxation of the binary variables than with the use of the constraints (9). A stronger lower bound means that when applying a branch-and-bound algorithm for solving the proposed problem, the lower bounds obtained in the branching nodes will be of higher quality. Hence, using the constraints (10) instead of (9), the efficiency of the branch-and-bound algorithm can be improved. For a numerical comparison between the two formulations, see Sect. 5.3.

To clarify further why the formulation (10) constitute a tighter description of the convex hull of feasible solutions, consider the example in Fig. 3. First, assume that the integrality requirements on the variables x_{it} are relaxed and that the formulation (9) is used for $k = 6$ and $t = 1$. Then the variable values $x_{5,1} = 0$, $x_{6,1} = 0.5$, and $x_{7,1} = 1$ are feasible with respect to that constraint. If the tighter formulation (10) is utilized, this solution would not be feasible since $x_{5,1} < x_{6,1}$, which violates constraint (10) for $i = 5$, $k = 6$, and $t = 1$. In general, any feasible solution with respect to the constraints (10) is also a feasible solution with respect to the constraints (9). The reverse, however, does not hold in general as the numerical example just presented illustrates. So, when utilizing branch-and-bound techniques for solving the optimization model presented, the formulation (10) always provides a linear relaxation which is tighter than the one obtained using the formulation (9).

As in the opportunistic replacement problem (ORP) described in Almgren et al. (2012) the binary variables, z_t, $t \in \mathcal{T}$, are included denoting whether or not any maintenance shall be performed in time step t. To connect these variables with the variables, x_{it}, $i \in \mathcal{N}$, $t \in \mathcal{T}$, the following constraints are introduced:

$$x_{it} \leq z_t, \quad i \in \mathcal{N},\ t \in \mathcal{T}. \tag{11}$$

Note that also for these constraints the aggregated formulation $\sum_{i \in \mathcal{N}} x_{it} \leq |\mathcal{N}| z_t$, $t \in \mathcal{T}$ (Dickman et al. 1991) yields a model with a weaker linear relaxation than the one presented in this paper.

To formulate a general optimization model, the function that is to be minimized is given by

$$\sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} c_{it} x_{it} + \sum_{t \in \mathcal{T}} d_t z_t,$$

where $c_{it} \geq 0$ is the cost of performing tamping on track segment $i \in \mathcal{N}$ at time step $t \in \mathcal{T}$, and $d_t \geq 0$ is the cost associated with a maintenance occasion at time step $t \in \mathcal{T}$. The special case presented in Vale et al. (2012), in which the objective is to minimize the number of tamping operations, is obtained by letting $c_{it} = 1$, $i \in \mathcal{N}$, $t \in \mathcal{T}$, and $d_t = 0$, $t \in \mathcal{T}$. The complete optimization model can then be formulated as the problem to

$$\underset{x,z,s,r}{\text{minimize}} \quad \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} c_{it} x_{it} + \sum_{t \in \mathcal{T}} d_t z_t, \tag{12a}$$

$$\text{subject to} \quad s_{i,t+1} \geq (1 + \alpha_i) s_{it} + h_i - x_{it} s_i^{\max}, \quad i \in \mathcal{N},\ t \in \mathcal{T} \setminus \{T\}, \tag{12b}$$

$$s_{i,t+1} \geq (1 - \alpha_i)(s_{it} - r_{it}) + h_i, \quad i \in \mathcal{N},\ t \in \mathcal{T} \setminus \{T\}, \tag{12c}$$

$$r_{it} = \gamma_i s_{it} + b_i, \quad i \in \mathcal{N},\ t \in \mathcal{T}, \tag{12d}$$

$$s_{it} \leq s_i^{\max}, \quad i \in \mathcal{N},\ t \in \mathcal{T}, \tag{12e}$$

$$s_{i0} = s_i^{\text{init}}, \quad i \in \mathcal{N}, \tag{12f}$$

$$x_{it} \geq x_{kt}, \quad i \in \mathcal{I}_k,\ k \in \mathcal{N},\ t \in \mathcal{T}, \tag{12g}$$

$$x_{it} \leq z_t, \quad i \in \mathcal{N},\ t \in \mathcal{T}, \tag{12h}$$

$$x_{it}, z_t \in \{0, 1\}, \quad i \in \mathcal{N},\ t \in \mathcal{T}. \tag{12i}$$

The objective (12a) is to minimize the total maintenance cost over the time period defined by the time steps $\mathcal{T}$. As discussed above, the constraints (12b) and (12c) ensure a correct modeling of the conditions of the track segments with respect to the timing of the tamping activities. The restoration model described in (2) is modelled by the constraints (12d). The track conditions are bounded from above through the constraints (12e), and the conditions at time zero are initialized through the constraints (12f). The constraints (12g) ensure that a tamping operation begins and ends at a straight alignment track, and the constraints (12h) ensure that the cost of a maintenance occasion is paid every time step in which any tamping is being performed. Finally, the constraints (12i) ensure that the maintenance decision variables are binary.

Note that the values of the standard deviation variables s_{it} and r_{it} are fully determined by the values of the decision variables x_{it} and z_t, since the conditions of the components can always be computed as long as the tamping occasions are known.

3.3 Special cases and computational complexity

For the special case when $c_{it} = 1$, $i \in \mathcal{N}$, $t \in \mathcal{T}$, $d_t = 0$, $t \in \mathcal{T}$, and $\alpha_i = 0$, $i \in \mathcal{N}$, the model (12) is equivalent to that presented by Vale et al. (2012), except that the constraints (9), which are presented in Vale et al. (2012), are here replaced by the stronger formulation (12g).

A problem which also arises as a special case of the model (12) is the ORP analyzed in Almgren et al. (2012), which models the maintenance planning for a multi-component system with the objective function (12a). For each component $i \in \mathcal{N}$ a maximum interval T_i between any two consecutive replacements is given, and the problem is to find a minimum cost schedule for the system such that, for each component $i \in \mathcal{N}$, no period without maintenance is longer than T_i. Consider an instance of the model (12) with $\alpha_i = 0$, $h_i = 1$, $s_i^{\max} = T_i$, $s_i^{\text{init}} = 0$, $b_i = 0$, $\mathcal{I}_i = \emptyset$, and $\gamma_i = 1$, $i \in \mathcal{N}$. Any feasible schedule for the model (12) will then only include maintenance intervals of length $\leq T_i$, $i \in \mathcal{N}$. The optimal solution to the model (12) is thus also an optimal solution to the ORP. As was shown in Almgren et al. (2012), the ORP is NP-hard [see, e.g., Garey and Johnson (1979) and Papadimitriou (2003)], which implies that also the scheduling problem considered in this paper is NP-hard.

4 Policies

As mentioned above the condition variables, s_{it}, are fully determined by the values of the decision variables, x_{it} and z_t. A tamping schedule can therefore be defined by a set of binary values x_{it}, $i \in \mathcal{N}$, $t \in \mathcal{T}$, and z_t, $t \in \mathcal{T}$. A policy is defined as a rule or an algorithm that constructs a tamping schedule given the input parameters of the specific problem instance.

The reason for introducing the following two policies is to evaluate the maintenance costs generated by the PM schedules obtained when solving the model (12) compared to the costs generated by schedules obtained from simpler policies.

The first policy is a simple greedy rule for constructing tamping schedules. One version of the simple policy is described in (Almgren et al. (2012), Definition 2) and is there denoted the non-opportunistic maintenance policy. The algorithm starts at time zero and then steps forward in time and schedules only the necessary tamping in each time step.

4.1 Greedy policy

- Step 0: initialize the schedule and the component conditions: Let $x_{it} := 0$, $i \in \mathcal{N}$, $t \in \mathcal{T}$, and $z_t := 0$, $t \in \mathcal{T}$. Let $s_{i0} := s_i^{\text{init}}$, $i \in \mathcal{N}$, and $t := 0$.

- Step 1: perform tamping on any component that will reach its maximum standard deviation in the next time step: For all $i \in \mathcal{N}$ such that $(1 + \alpha_i)s_{it} + h_i > s_i^{\max}$, let $x_{it} := 1$ and $x_{kt} := 1$, $k \in \mathcal{I}_i$.
- Step 2: check whether a maintenance occasion has occurred: If $x_{it} := 1$ for any $i \in \mathcal{N}$, let $z_t := 1$.
- Step 3: update the standard deviation of the track segments according to (1):
- Step 4: check termination criteria: If $t \geq T$, terminate. Otherwise, let $t := t + 1$ and go to Step 1.

One drawback of the greedy policy is that it does not take into account the opportunities generated at the time steps at which tamping is performed. Regardless of the cost for a maintenance occasion, the policy will produce the same tamping schedule. This is also the reason for including the policy in the evaluation in this paper. Because no OM is utilized, the effect of OM can easily be evaluated when comparing the schedules created by the greedy policy and the ones created by solving the model (12).

In the maintenance literature [e.g., (Barlow and Proschan (1965), Chapter 3)] age replacement policies are common. For a thorough review of age replacement policies, see Wang (2002). One such policy (introduced in Crocker and Kumar (2000) where the application was maintenance of aero-engines, and utilized in Almgren et al. (2012) and Gustavsson et al. (2014) is the age policy, in which at each time step when any maintenance is being performed, one also maintains all components with a remaining life (number of time steps until the next necessary maintenance occasion) below a threshold $\eta > 0$. The remaining life, $T_i(s_{it})$, for component i given a condition s_{it} is defined as the number of steps of the iteration formula $s_{i,t+1} = (1 + \alpha_i)s_{it} + h_i$ required until it reaches a state $s_{it} \geq s_i^{\max}$.

4.2 Age policy

- Step 0: initialize the schedule and the component conditions: Let $x_{it} := 0$, $i \in \mathcal{N}$, $t \in \mathcal{T}$, and $z_t := 0$, $t \in \mathcal{T}$. Let $s_{i0} := s_i^{\text{init}}$, $i \in \mathcal{N}$ and $t := 0$.
- Step 1: perform tamping on any component that will reach its maximum condition in the next time step: For all $i \in \mathcal{N}$ such that $(1 + \alpha_i)s_{it} + h_i > s_i^{\max}$, let $x_{it} := 1$ and $x_{kt} := 1$, $k \in \mathcal{I}_i$,
- Step 2: check whether a maintenance occasion has occurred: If $x_{it} = 1$ for any $i \in \mathcal{N}$, let $z_t := 1$, and also let $x_{kt} := 1$ and $x_{jt} := 1$, $j \in \mathcal{I}_k$, for all $k \in \mathcal{N}$ such that $T_k(s_{kt}) < \eta$.
- Step 3: update the standard deviation of the track segments according to (1):
- Step 4: check termination criteria: If $t \geq T$, terminate. Otherwise, let $t := t + 1$ and go to Step 1.

In the numerical study below the threshold η is varied for each problem instance and the threshold yielding the lowest maintenance cost for each specific problem instance is chosen.

5 Numerical study

The following numerical study shows the usefulness of the model (12) for computing schedules for tamping activities on a rail system.

5.1 Problem instances and implementation

The planning horizon for the instances generated is 2 years and the discrete time steps represent two weeks, meaning that the number of time steps is $T = 52$. To analyze the usefulness of the model and the performance of the policies presented in Sect. 4, ten problem instances are generated each with $n = 10, 15$, and 20 track segments, respectively. For each problem instance the setup cost is varied as $d_t = 0, 1$, and 10, $t \in \mathcal{T}$, to evaluate the performance of the heuristics depending on the setup costs. In all instances $c_{it} = 1$, $i \in \mathcal{N}$, $t \in \mathcal{T}$, and the limit $s_i^{\max} = 2.4\,\text{mm}$, $i \in \mathcal{N}$, for the standard deviation of the longitudinal level (Vale et al. 2012).

The random instances are then generated as follows. The initial standard deviation of the track segments are uniformly distributed random variables in the range 0.5–2.4 mm, i.e., $s_i^{\text{init}} \sim U(0.5, 2.4)$ mm. In Vale et al. (2012), the constant degradation rate of the standard deviation is for their example mostly distributed between 0.0005 and 0.0025 mm per day. Since each time step in my model consists of 14 days, the constant degradation rate is distributed as $h_i \sim 14 \times U(0.0005, 0.0025)$ mm per time step. To evaluate the effect of an exponential degradation of condition of the track segments, two cases for each problem instance are considered: (i) constant degradation ($\alpha_i = 0$, $i \in \mathcal{N}$), and (ii) increasing degradation ($\alpha_i = 0.01$, $i \in \mathcal{N}$).

The mathematical model (12) is implemented in AMPL (version 12.1) and solved using CPLEX (version 12.1). The policies described in Sect. 4 are implemented in MATLAB (version R2010b). The age policy is applied for each of the values of $\eta \in \{1, \ldots, T\}$; for each problem instance the value yielding the lowest objective value is chosen.

5.2 Computational results

In Table 1, the average total maintenance costs resulting from the three maintenance principles [greedy policy, age policy, and optimal solution to the model (12)] are presented for the two cases of constant degradation ($\alpha_i = 0$) and increasing degradation ($\alpha_i = 0.01$), respectively. As expected, the age police clearly produces maintenance schedules with lower maintenance costs than the schedules generated by the greedy policy. Since the greedy policy does not take into account the opportunities generated at time steps at which maintenance is being performed, the performance of the policy gets worse when the setup costs, d_t, increases. This behavior can be seen for both when the degradation is constant and when it is increasing. When employing a high setup cost ($d_t = 10$), the greedy policy produces, on average, schedules with maintenance costs almost three times the costs of the schedules obtained by solving the model (12).

The schedules generated by the age policy have on average maintenance costs which are a few percent higher than the schedules obtained by solving the model (12). For the worst cases, when there is no setup costs and the degradation is increasing, the schedules produced by the age policy are on average ten percent more expensive than the optimal schedules. There is no clear difference in the performance of the age policy between the constant and the increasing degradation case, and neither any difference when varying the setup costs.

The benefit of the model (12) compared to both the greedy policy and the age policy is apparent. Even though the cost reduction obtained using the model compared to the age policy is only a few percent on average, it is still quite a large improvement considering the huge costs involved in tamping operations. The average computation time for solving the model to optimality was approximately 20 s. Some problem instances, however, took up to 10 min to solve; the majority of the time being spent on verifying optimality of the solutions found.

5.3 Comparison of computation times

As stated in Sect. 3.2, one of the main differences between the model proposed in this paper and the one presented by Vale et al. (2012) is the formulation of the constraints ensuring that tamping operations begin and end on straight alignment track segments. In this paper, the stronger formulation (10) is utilized instead of (9) which was proposed by Vale et al. (2012). Since the formulation (10) is stronger in the sense that the lower bounds obtained in the branching nodes in a branch-and-bound framework is higher, the efficiency of the branch–and–bound algorithm is expected to be higher.

To evaluate the difference in computation times between the formulations, the two models are solved for 10 random instances for each of $N = 3, 4, \ldots, 10$. The average computation times for the two models are illustrated in Fig. 4. As one can clearly see in the figure, the computation time for the model using the stronger formulation (10) is shorter than when using the formulation (9). When $N = 10$, the average computation time for the model using (9) is more than 50 % longer (95.2 vs. 62.7 s) than when using (10). One reason for this large difference between the computation times is that most of the computation time when solving the problems with CPLEX is dedicated to proving optimality of the solutions found. Hence, a model that can provide a better lower bound in the branching nodes will be superior.

6 Conclusions and future research

A mathematical optimization model for the problem of scheduling tamping operations on ballasted tracks is proposed. The model previously developed in Vale et al. (2012) is improved by introducing disaggregated constraints yielding in a model with stronger lower bound from the continuous relaxation of the binary variables. The model in Vale et al. (2012) is also extended to include a more general cost structure yielding in a model having the possibility of including setup costs for the tamping operations. The problem considered is shown to be NP-hard.

Table 1 The average total maintenance costs of the different schedules over ten random test instances for both a constant degradation ($\alpha = 0$) and an increasing degradation ($\alpha = 0.01$)

	Greedy policy		Age policy		Optimal cost
	Cost	% From opt.	Cost	% From opt.	
$\alpha = 0$					
No setup cost ($d_t = 0$)					
$n = 10$	17.0	16.4	15.2	4.1	14.6
$n = 15$	21.0	9.4	19.6	2.1	19.2
$n = 20$	29.0	16.0	26.4	5.6	25.0
Medium setup cost ($d_t = 1$)					
$n = 10$	23.2	38.1	17.2	2.4	16.8
$n = 15$	29.0	35.5	22.6	5.6	21.4
$n = 20$	40.6	50.4	29.0	7.4	27.0
High setup cost ($d_t = 10$)					
$n = 10$	79.0	127.0	35.2	1.2	34.8
$n = 15$	101.0	157.6	41.4	5.6	39.2
$n = 20$	145.0	222.2	47.4	5.3	45.0
$\alpha = 0.01$					
No setup cost ($d_t = 0$)					
$n = 10$	28.0	4.5	27.0	0.8	26.8
$n = 15$	42.2	14.7	40.6	10.3	36.8
$n = 20$	51.8	13.1	46.8	2.2	45.8
Medium setup cost ($d_t = 1$)					
$n = 10$	37.0	23.3	30.6	2.0	30.0
$n = 15$	59.6	46.1	44.4	8.8	40.8
$n = 20$	69.4	41.6	50.0	2.0	49.0
High setup cost ($d_t = 10$)					
$n = 10$	118.0	108.5	60.8	7.4	56.6
$n = 15$	216.0	219.8	73.2	8.3	67.6
$n = 20$	227.8	200.5	77.0	1.6	75.8

The optimal cost is the maintenance cost for the schedule obtained by solving the model (12)

The model (12) is evaluated through a numerical study where the costs of the maintenance schedules obtained from solving the model with schedules obtained using simple policies are compared. Using the model (12) reduces the maintenance costs by up to 10 % as compared with the best policy investigated.

Future research include trying to include the cost structure utilized in the PMSPIC [see Gustavsson et al. (2014)] where the cost for a maintenance operation on a component depends on the time since the last maintenance operation. My intention is to evaluate the usefulness of the model (12) by performing a case study where I compare the schedules generated with the ones utilized in practice. I also

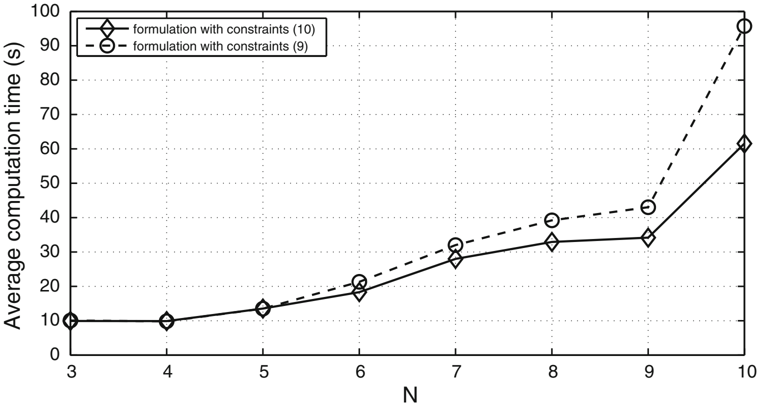

Fig. 4 The average computation times for 10 random instances for each of $N = 3, 4, \ldots, 10$ when using the formulation (9) and (10) as constraints in the model (12)

intend to include deterioration and restoration models which can vary over time [e.g., Quiroga and Schnieder (2012)].

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