(article starts on next page)

PAPER • OPEN ACCESS

Comparison between three mathematical models of three well defined ultrasonic NDT cases

View the article online for updates and enhancements.

Comparison between three mathematical models of three well defined ultrasonic NDT cases

M Semenova, H Wirdelius and G Persson
Department of Industrial and materials science, Chalmers University of Technology, SE-41296 Gothenburg, Sweden

semenova@chalmers.se

Abstract. Ultrasonic nondestructive testing (NDT) is commonly used for in-service inspection in different areas. But reliability of NDT method is highly dependent on the equipment and crack features. Although, it is possible to use thoroughly validated mathematical models to avoid complicated and costly experimental work, when it is necessary to qualify new procedures. Finite Element Model (FEM) is a powerful tool, which is commonly used for such cases. In this paper three mathematical models of three well defined cases will be compared with each other.

Introduction

The propagation and scattering of waves in the elastic solids are important in ultrasound testing and material characterization. Ultrasonic nondestructive testing (NDT) is commonly used for in-service inspection in different areas, e.g. in nuclear and aerospace industries. NDT methods are used to evaluate the integrity of individual components, which might be exposed to different degradation mechanisms (such as fatigue, corrosion and stress corrosion cracking).

However, the reliability of NDT method is highly dependent on how the equipment is adjusted to a specific object and how to anticipate crack features. Their morphology and thus signal response may vary widely between different crack mechanisms and material types, in which crack appear. Due to these reasons it is very complicated and costly to validate (or qualify) new procedures to inspect defects with more complex geometry, e.g. stress corrosion cracks. Such experimental work requires tests on manufactured specimens with well known fabricated defects.

To avoid these difficulties, it is possible to use thoroughly validated mathematical models. Finite Element Model (FEM) is a powerful and comprehensive tool, which is commonly used nowadays. Up to this date only a couple of models have been developed that cover the whole testing procedure, i.e. they include the modeling of transmitting and receiving probes, the scattering by defects and the calibration. Chapman [1] employs geometrical theory of diffraction for some simple crack shapes and Fellinger et al [2] have developed a type of finite integration technique for a two-dimensional treatment of various types of defects. Lhémery et al [3] employs Kirchhoff's diffraction theory that enables their model to handle more complex geometries in 3D. In the literature, Gray et al [4] and Achenbach [5] presents overviews of ultrasonic NDT models.

In this paper an ultrasonic "pulse-echo" inspection situation was modelled by a line-shaped source on the surface of a component. Different defects were introduced beneath the source and the signal response from two FEM models and an analytical model were compared.

Analytical model

The simSUNDT program consists of a windows based pre-processor and postprocessor together with a mathematical kernel (UTDefect) dealing with the actual mathematical modelling. The UTDefect computer code has been developed at Chalmers University of Technology and has been experimentally validated and verified. The analytical model used as kernel in the simSUNDT software [6-8] is completely three dimensional, though the component is two dimensional (infinite plate with finite or infinite thickness) bounded by the scanning surface where one or two probes are scanning the object within a rectangular mesh. The probe is modeled by an assumed effective area beneath the probe, used as boundary conditions in a half-space elastodynamic wave propagation problem. This enables an adaptation to a variety of realistic parameters related to the probe, e.g. wave type, angle, crystal (i.e. size and shape), focus depth and contact conditions. The receiver is modeled by applying a reciprocity argument by Auld [9].

The governing linearized equations for wave propagation in an elastic medium are the equation of motion, Hooke's law and the strain-displacement relation. If time harmonic conditions are assumed (time factor $e^{-i\omega t}$ is suppressed) these three relations can be combined into the elastodynamic equation of motion governing the displacement field **u**:

$$k_p^{-2}\nabla\nabla\cdot\mathbf{u} - k_s^{-2}\nabla\times\nabla\times\mathbf{u} + \mathbf{u} = \mathbf{0} \tag{1}$$

where k_p and k_s are the compressional and shear wave numbers, respectively.

The total displacement field is given by the sum of the incident field (ui) and the scattered field (us). Let us expand the incident field in terms of regular spherical partial vector waves (Re$\boldsymbol{\Psi}_n$) and the scattered field in corresponding outgoing spherical partial vector waves ($\boldsymbol{\Psi}_n$), i.e.

$$\begin{cases} \mathbf{u}^i = \sum_n a_n \mathrm{Re}\,\boldsymbol{\Psi}_n \\ \mathbf{u}^s = \sum_n f_n\,\boldsymbol{\Psi}_n \end{cases} \tag{2}$$

Then it is possible to find a linear relationship between the expansion coefficients for the incident and scattered field and this entity is known as the transition matrix T

$$f_n = \sum_{n'} T_{nn'}\, a_{n'} \tag{3}$$

All information about the scattered field is contained in its transition matrix and the characterization of the probe acting as a transmitter is encapsulated in the expansion coefficients for the incident field (an). To evaluate its behaviour as a receiver we use an electromechanical reciprocity argument by Auld [9]. Then the change in the electrical response of probe b, due to the presence of a defect (enclosed by a control surface S), is found as

$$\delta\Gamma \sim \sum_{nn'} a_n^{\mathrm{b}}\, T_{nn'} a_{n'}^{\mathrm{a}} \tag{4}$$

Numerical models

The discrete modeling is made by using two different approaches, FE-modeling by use of COMSOL and a finite difference method by k-Wave.

COMSOL Multiphysics is a simulation software for a wide array of applications, which included several modules, categorized according to the applications areas: Electrical, Mechanical, Fluid, Chemical, Multipurpose, and Interfacing.

k-Wave is an open source acoustics toolbox for MATLAB and C++. The software is designed for time domain acoustic and ultrasound simulations in complex and tissue-realistic media [10,11]. The equations are solved using a k-space pseudospectral method, where spatial gradients are calculated using a Fourier collocation scheme, and temporal gradients are calculated using a k-space corrected finite-difference scheme.
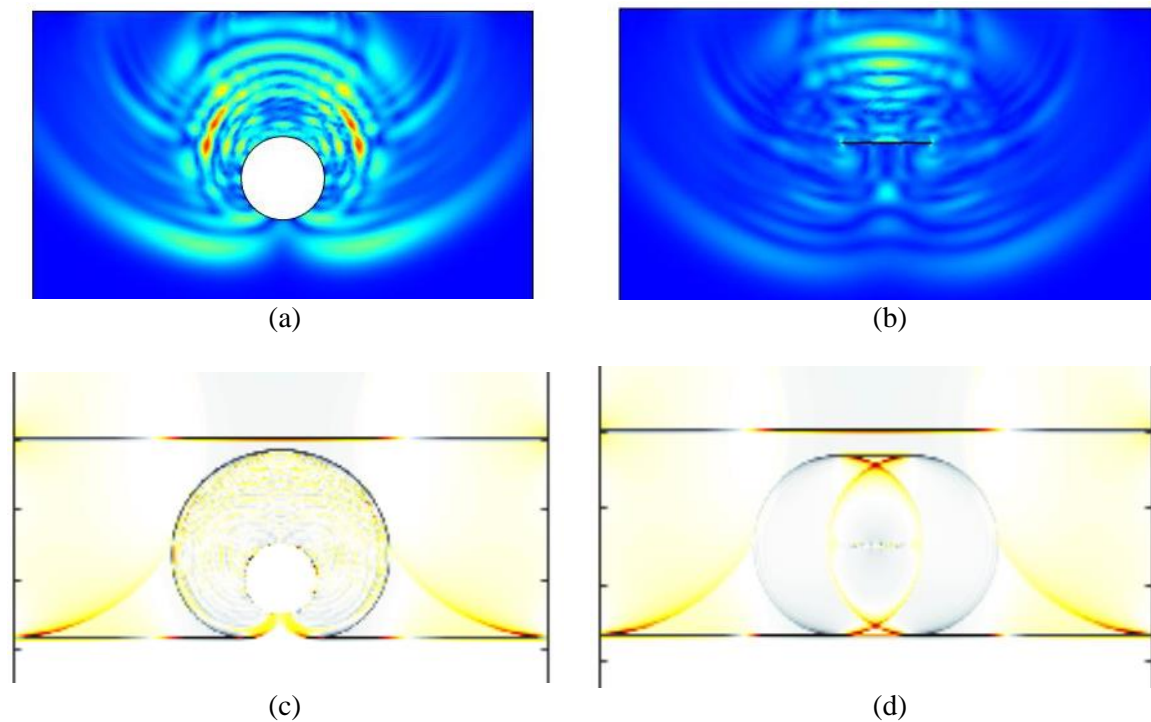

(a)   (b)

(c)   (d)

Figure 1. Stress field at t=2.9 µs for SDH and parallel SC in COMSOL (a, b) and in k-wave (c, d)

In this paper a 2D time-dependent problem is considered. For the geometry, a half infinite solid structure with a defect is used. The perfectly matched layer (k-wave) and low-reflected boundary (COMSOL) are used to allow a free-field simulation at the borders of the computational grid. The ultrasonic 6 mm line-source with a center frequency of 2MHz and 50% bandwidth is placed on the surface and modelled as a pressure distribution with values for specific line. For the defects side-drilled hole (SDH), perpendicular and parallel strip-like cracks (SC) are considered. The diameter of SDH and length of SC are 5 mm, and the distance between probe and the defect is 7,5mm. The resulting stress field can be seen in Figure 1.

In Figure 2 a comparison of the results from simSundt and k-wave is presented. For each defect two simulations are made: with and without defect. As a result, only pure signal from the defect is used, since the signal from the case 'without defect' is subtracted. It should be considered, that presented results are normalized to SDH, which means that all values are divided by the maximum value for SDH case.
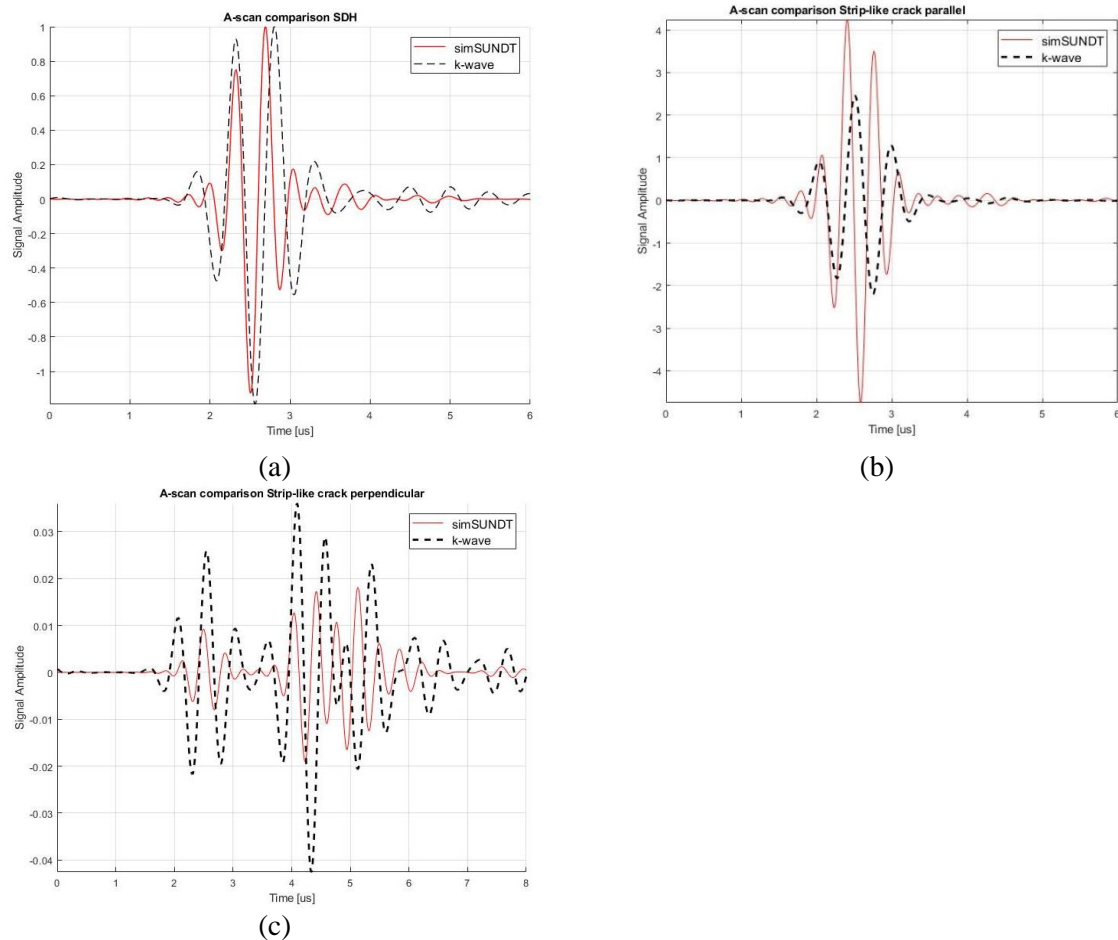
Figure 2. Comparison of A-scan for: (a) SDH, (b) SC parallel, (c) SC perpendicular in SimSUNDT and k-wave

Results

In k-wave simulation it is impossible to use materials with large differences in density. Hence, properties of steel (density, pressure and shear wave speeds) is used for the solid structure, and for the defects values of these properties are reduced by 100. In comparison, in SimSUNDT and COMSOL it is possible to have a void with zero material properties.

SC might be modelled as a rectangular crack with very small thickness compare to the length. In the case of a SC parallel to the surface, the thickness does not matter, since the largest amount of received energy is due to reflection from the parallel surface of the defect. A rule of thumb is that crack-tip diffraction tends to be about 20 to 30 dB less than corresponding surface reflection.
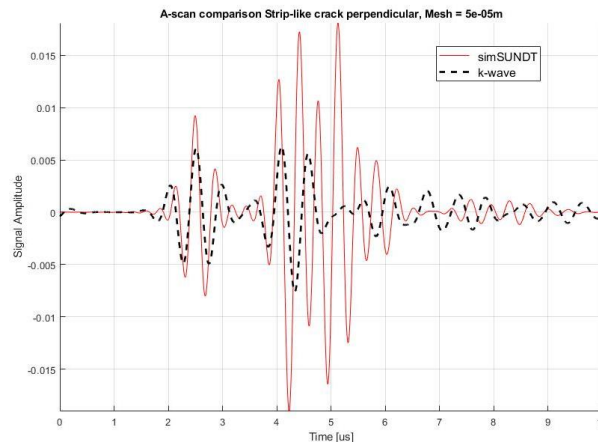
Figure 3. Comparison of A-scan for perpendicular SC in SimSUNDT and perpendicular rectangular SC in k-wave

For the perpendicular SC case the dimension of crack tip is important. In k-wave smallest possible mesh, element size 50µm is used. Accordingly, the smallest possible width of the perpendicular SC is also 50µm. In this case the impact of the parallel surface and the corners of the defect is significant. In Figure 3 you can see that the signal from rectangular SC in k-wave looks like signal from the parallel SC. To avoid this influence, instead of rectangular looking crack there is used a line, located on the grid points under the centre of the probe. That is not physically correct, and thus the differences in results seems bigger than in other cases, see Figure 2, c). However, the agreement between analytical and simulation results in SimSUNDT and k-wave can be considered quite good, as the position and shape of the signal are same, and differences in values are less than 10 dB. Although even if this software has good correlation with analytical solution, there are some features, which may limit further use. One essential parameter that may limit its application in wave propagation is the wavenumber dependence on grid coordinates.

COMSOL results differ a lot from other software. Even if the stress distribution on the pictures looks correct, see Figure 1, the values of stress and displacement are very noisy. Such simulations require big CPU of the computer to be able to use finest mesh and relatively small time-step. However, the good-looking animation of stress distribution can be achieved even with the fine-sized mesh.

Conclusions
The next step of the research is investigation of possibility to model complexed shaped defects (SCC) or defects surrounded by strongly anisotropic material using existing FEM software. This paper describes a comparison between analytical and two numerical solutions for a 2D time-dependent problem. Both mathematical models have advantages and disadvantages, which must be considered for each simulation purpose.

Acknowledgements

References
[1]   Chapman R K 1990, A System Model for the Ultrasonic Inspection of Smooth Planar Cracks, *J. Nondestr.*

                      Eval. **9**, pp 197-211

[2]   Fellinger P, Marklein R, Langenberg K J and Klaholz S 1995, Numerical Modelling of Elastic WavePropagation and Scattering with EFIT-Elastodynamic Finite Integration Technique, *Wave Motion* **21**, pp 47-66

[3]   Lhémery A, Calmon P, Levær-Taïbi I, Raillon R and Paradis L 2000, Modeling Tools for Ultrasonic

          Inspection of Welds, *NDT & E International* **33**, pp 499-513

[4]   Gray J N, Gray T A, Nakagawa N and Thompson R B 1989 *Metals Handbook* **17** ASM International,

          Metals Park, Ohio, p 202

[5]   Achenbach J D 1993, *Evaluation of Materials and Structures by Quantitative Ultrasonics*, Springer, Wien

[6]   Boström A and Wirdelius H 1995, Ultrasonic probe modeling and nondestructive crack detection,

          J. Acoust. Soc. Am. **97**, pp 2836 – 48

[7]   Wirdelius H 1995, User Guide to SUND, *Swedish Nuclear Power Inspectorate Report* 00:29, Stockholm

[8]   Bövik P and Boström A 1997, A model of ultrasonic nondestructive testing for internal and subsurface

          cracks, *J. Acoust. Soc. Am.* **102**, pp 2723 – 33

[9]   Auld B A 1979, General electromechanical reciprocity relations applied to the calculation of elastic wave

          scattering coefficients, *Wave Motion* **1**, pp 3-10

[10]   Treeby B E, Jaros J, Rohrbach D and Cox B T 2014, Modelling elastic wave propagation using the

          k-Wave MATLAB toolbox, *Int. Ultrasonics Symp.*, pp. 146-149

[11]   Treeby B E and Cox B T 2010, Modeling power law absorption and dispersion for acoustic propagation

          using the fractional Laplacian, *J. Acoust. Soc. Am.* **127**, no. **5**, pp 2741-48