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MODELLING OF HUMAN LOCOMOTION WITH ARTIFICIAL LOWER EXTREMITIES

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INTRODUCTION

To solve the problems of improvement of the existent and creation of new efficient lower limb prostheses, and to study effect of prosthesis design, as a special technical device, on kinematical dynamical energetical and other characteristics of amputee locomotion it is expedient to use wide capabilities of mathematical modelling of a human walk process on a prosthetic limb. There was proposed earlier mathematical models of biped walk having different degrees of adequacy in multitude of works. These models were used for different purposes: investigation of kinematical dynamical and energetical characteristics of human gait, working out and creation of antropomorphic walking machines, etc.

In the given paper a mathematical model has been proposed for investigation of dynamics of an amputee's locomotor system (ALS) with a below-knee prosthesis. Based on this model computer program (CP) has been composed [1]. The series of problems of dynamics of a two-leggal walking, calculation of consumption of energy on motion and optimization of elastic parameters of the prosthetic foot design has been solved.

This paper is an extension of the research into bipedal locomotion that was undertaken in [2-3].

THE MODEL OF THE BIOTECHNICAL SYSTEM "MAN-PROSTHESIS"

An ALS with below-knee prosthesis under consideration consists of an inertial torso and two legs (Fig. 1).

Each leg consists of three elements. The two elements with weight and inertia model the thigh and shin, while the third weightless and inertia-free element models the foot.

Assume that NXYZ is a fixed rectangular Cartesian coordinate system. It is assumed that the amputee moves in the NXY plane along the NX axis, over a horizontal surface. In addition to the weights of its component parts (the torso, thighs, and shins), the external forces acting on the ALS include the interaction forces between the feet and the surface, which we replace by force R_i applied at the point of the ankle joint, and moment p_i that acts in the same joint (i=1,2). It is assumed that control moments q_i and u_i (i==1,2) act in the hip and knee joints, these moments being treated as internal forces.

Because the feet are weightless and without inertia, we will not concern ourselves with their motion, but will assume that the foot of the shifted leg moves, e.g., parallel to the surface, while the foot of the support leg is fixed in relation to this same surface.

The equations of motion of this controlled system, written in the form of Lagrange equations of the second kind, are as follows [3]:

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{z}_j} - \frac{\partial K}{\partial z_j} = Q\ (z,\dot{z}),\ j=1,\dots,7, \qquad (1)$$

where $z=(x,y,\psi,\alpha_1,\alpha_2,\beta_1,\beta_2)$ - are generalized coordinates (See Fig.1), K - is the kinetic energy of the ALS, Q_j - are external generalized forces including gravity, resistance and control torques.

To receive a complete system of expressions describing the dynamics of the ALS the equations (1) has been added by conditions of a kinethostatic balance of feet under the action of ankle moment and the forces of reactions of the support

$$p_i(t) + (x_{Ri} - x_i)\ R_{iy}(t) + (y_i - y_{Ri})\ R_{ix}(t) = 0 \qquad (2)$$

where R_{ix}, R_{iy} - projections on a horizontal and vertical axes of a main vector of forces of ground reactions acting on the foot of i-th leg, (x_i, y_i) and (x_{Ri}, y_{Ri}) are the Cartesian coordinates of the ankle joint and point of the vector R_i application of the i-th leg (i=1,2). Henceforth the subscript 1 will refer to the prosthetic leg, 2 to the intact leg.

In addition to equations (1), the generalized coordinates obey two kinematic constraints that follow from the geometry of the leg (Fig.1):

$$x(t) = x_i - a_i \sin \alpha_i - b_i \sin \beta_i, \qquad (3)$$
$$y(t) = y_i + a_i \cos \alpha_i + b_i \cos \beta_i,$$

where a_i, b_i are the thigh and shin lengthes of the i-th leg (i=1,2).

Let's note an important feature of the described model. In the frame of this model it is chance directly to take into consideration the principal dynamic difference between an intact and prosthetic limbs of ALS. This will be achieved by the way of refusal from the assumption that the controlling force moments at all the joints of a prosthetic limb are the active ones. In the given paper a mathematical modelling of human gait with a below-knee prosthesis is considered based on a supposition that a force moment at the prosthetic ankle joint, is a passive one. The value of this moment depends not only on the gait pattern of a man but on a prosthetic construction as well. In the algorithm proposed this has been shown in such a way that the moment at the ankle joint of the prosthesis (a function $p_1(t)$) is assumed as preset one in advance. Thus if to select one or another kind of the function $p_1(t)$ it is possible to study the motion of a man with below-knee prosthesis of different construction.

THE MATHEMATICAL FORMULATION OF THE PROBLEMS STATEMENT

The mechanical system under consideration has nine degrees of freedom. In the general case, therefore, we need to know nine functions of time in order to determine the law of motion of the system. A number of requirements should be imposed on the choice of these functions [2], e.g., rhythmic property of motion, continuity of law of motion, anthropomorphic property of law of motion, requirement that the foot not slide along the surface, constraint on the control moment in the foot, etc.

Obviously, the above requirements to not uniquely specify the law of motion. In view of our assumption $\gamma_1(t) = \gamma_2(t) = \pi/2$ regarding the motion of the feet, only seven of the nine functions remain at our disposal; these can be taken to be the following: $x(t)$, $y(t)$, the coordinates of the suspension point 0 of the legs; $\psi(t)$, the angle that defines the departure of the torso from the vertical drawn through point 0; $x_i(t)$, $y_i(t)$, the coordinates of the ankle joints $(i=1, 2)$.

We will study single-support locomotion, i.e., motion of our system such that only one leg is in contact with the surface at any point in time.

For an arbitrary double step, we specify the Cartesian coordinates of the ankle joints of legs as follows [2]:

$$x_1(t) = \begin{cases} 0 & , t \in [0,T/2] \\ L - L\cos w(t - T/2), & t \in [T/2,T] \end{cases}$$

$$y_1(t) = \begin{cases} 0 & , t \in [0,T/2] \\ a^* \sin^2 w(t - T/2) & , t \in [T/2,T] \end{cases} \tag{4}$$

$$x_2(t) = \begin{cases} -L\cos wt & , t \in [0,T/2] \\ 0 & , t \in [T/2,T] \end{cases}$$

$$y_2(t) = \begin{cases} a^* \sin^2 wt & , t \in [0,T/2] \\ 0 & , t \in [T/2,T] \end{cases} \tag{5}$$

We define the motion of the suspension point of the legs as follows:

$$x(t) = \begin{cases} Vt - S + B_x\sin(2w_xt + \alpha_x) & , t \in [0,T/2] \\ L + Vt_1 - S + B_x\sin(2w_xt_1 + \alpha_x), & t \in [T/2,T] \end{cases} \quad (6)$$

$$y(t) = \begin{cases} h + B_y\sin(2w_yt + \alpha_y) & , t \in [0,T/2] \\ h + B_y\sin(2w_yt_1 + \alpha_y) & , t \in [T/2,T] \end{cases} \quad (7)$$

In (4) -(7), L is the step length; a^* is the maximum elevation of the shifted leg over the surface; S is the distance from the ankle joint of the support leg to the projection of the suspension point 0 onto the surface at t=0; T is the duration of the double step; $w = w_x = w_y = 2\pi/T$, $V = 2L/T$, $t_1 = t - T/2$, h, B_x, B_y, α_x, α_y are arbitrary constants.

In accordance with the requirement of periodicity, we specify the motion of the torso as follows:

$$\psi(t) = -(C_1 + C_2 \sin 2wt + C_3 \cos 2wt), \; t \in [0,T] \quad (8)$$

where C_1, C_2, C_3 are arbitrary constants.

For parameters L, S, V, h, B_x, B_y, α_x, α_y, a^*, C_1, C_2, C_3 have determined the permissible set of its meanings which takes into consideration the requirements on the human gait.

To estimate the energy expenditures in bipedal locomotion we will compute the following functional [1-4]:

$$E = \frac{1}{2L}\sum_{i=1}^{2}\int_0^T \{|q_i(t)(\psi^\cdot(t)-\alpha_i^\cdot(t))| + |u_i(t)(\alpha_i^\cdot(t)-\beta_i^\cdot(t))| + |p_i(t)\,\beta_i^\cdot(t)|\}\,dt \quad (9)$$

Problem 1. Assume that kinematics of legs are given by means of (4)-(7). From among the possible motions of the torso (8), we are to find the energetically optimal law of motion of torso of our system.

We have a problem of parametric optimization of functional (9) with free parameters: C_1, C_2, C_3. The law of motion and controls are subject to the constraints (1)-(3).

Let the construction of a prosthetic foot is designed in such a way that a moment at the ankle joint of a prosthetic leg may be represented as

$$p_1(t) = p_0 + c\beta_1(t) + k\dot{\beta}_1(t)$$

where c, k are the parameters characterizing concentrated elasticity and visco-elasticity of the prosthesis, respectively, p_0 is free parameter.

Problem 2. Assume that $x_i(t), y_i(t)$, (i=1,2) and $x(t), y(t)$ are given by (4)-(7). Find generalized coordinates $\psi(t)$, $\alpha_i(t), \beta_i(t)$, control moments $q_i(t)$, $u_i(t)$, $p_i(t)$, components of the support reactions $R_{ix}(t)$, $R_{iy}(t)$, (i=1,2) and parameters p_0^*, c^*, k^*, which satisfy equations (1), constraints (2), (3) and minimizes functional

$$\Phi = \max_{t\in[0,\,T/2]} |p_1(t,p_0,c,k) - p_1^N(t)|$$

In (11) function $p_1^N(t)$ is control moment in ankle joint and is determined by solution of the problem 1.

RESULTS AND DISCUSSION

There has been set up the algorithms to solve of the problem 1 and problem 2. Based on these algorithms computer program (CP) has been composed in C-language. This CP makes it possible:

1) to simulate a human locomotion both normal and on ar-

tificial lower extremities;

2) based on preset antropometric data and kinematics of human's locomotor system to calculate the forces of ground reaction and the moments of forces acting at the joints of legs during the motion of a man along a horizontal surface;

3) to calculate the energy costs required for the preset motion of a human's locomotor system both normal and on a below-knee prosthesis;

4) to solve the problems of optimization of elastic and visco-elastic parameters of prosthetic foot.

The CP developed makes it possible to display the film of motion of a man, graphic dependencies of kinematic dynamic and energetic characteristics of a two-leggal walking.

Let us describe some results of solution of problem 1 and problem 2 for $L = 0.7m$, $S = 0.35m$, $h = 0.967m$, $B_x = 0.00m$, $B_y = 0.016m$, $a^* = 0.05m$, $\alpha_x=-1.570rad$, $\alpha_y=-1.571rad$, $V = 1.10m/sec$, $M = 80kg$, $H = 1.8m$, where M and H are mass and height of human's locomotor system, respectively.

Fig.2 shows the way in which the angular coordinate of torso change in time over a double step. The longitudinal and vertical support reactions of the obtained laws of motion are shown in Fig.3 and Fig.4. Figures 5 and 6 show the control moments in ankle and knee joints of the prosthetic leg. On Fig.2-6 curves i correspond solution of the problem i (i=1,2); ψ in rad, R_{1x}, R_{1y} in N, time t in % of duration of double step.

The maximum value of $R_{1x}(t)$ for prosthetic leg is ≈ 20% of the weight of biomechanical system "Man-Prosthesis". The maximum value of vertical component of the support reaction for prosthetic leg differs from the weight of system by not more than 20%.

Energy expenditure for laws of motion of a system which have finded in problem 1 and problem 2 are 289.6J/m and 303.0J/m, respectively. Optimal meaning of constructive parameters of prosthetic foot for solution of the problem 2 are: $p_0^* = 207.06$ Nm, $c^* = 914.24$ Nm, $k^* = 0.0$ Nmsec.

With the help of the CP developed a problem of optimization of resilient characteristics of prosthetic foot had solved for a number of gait patterns. The ankle moment at prosthetic foot was chosen as $p_1(t) = c\beta_1(t) + k\dot{\beta}_1(t)$. The analysis of numerical results has shown that kinematical dynamical and energetical characteristics of biped locomotion are strongly change when we change the concentrated elasticity and visco-elasticity of the ankle joint of the prosthetic foot. At a preset gait there exist optimal meanings of the parameters c, k, which require minimal energy costs during ambulation of a man on a below-knee prosthesis.

For example, for a gait of a kind (4)-(7) at V = 0.83m/s, T = 1.22s, h = 0.8m, S = 0.23m, B_x = 0.021m, B_y = 0.016m, a^* = 0.03m, α_x= 0.77rad, α_y=-1.62rad and for amputee with mass 70kg and height 1.8m the optimal meanings of these parameters are c^*= 245 Nm, k^*= 2 Nms. The required energy expenditures on a unit of way are equal to 271 J/m.

There was considered the problem of modelling of human locomotion with an above-knee prosthesis. It was assumed that in equations (1) the knee and ankle moments of artificial leg are passive ones. These moments were chosen as $p_1(t) = c\beta_1(t) + k\dot{\beta}_1(t)$, $u_1(t) = c_u(\alpha_1 - \beta_1) + k_u(\dot{\alpha}_1 - \dot{\beta}_1)$. The parameters c, c_u, k, k_u are concentrated elasticity and visco-elasticity of ankle and knee joints, respectively.

The problem of optimization of resilient characteristics of above-knee prosthesis had solved for a number of gait patterns. The analysis of numerical results has shown that at a preset gait there exist optimal meanings of the parameters c, c_u, k, k_u, which require minimal energy costs during ambulation of a man on an abowe-knee prosthesis. For example, for a gait with L = 0.7m, T = 2s, a^*= 0.03m, and for amputee with mass 80kg and height 1.7m the optimal meanings of these parameters are c^*= 238 Nm, k^*= 10 Nms, c_u^*= 54 Nm, k_u^*= 15 Nms. The required energy expenditures on a unit of way are equal to 290 J/m.

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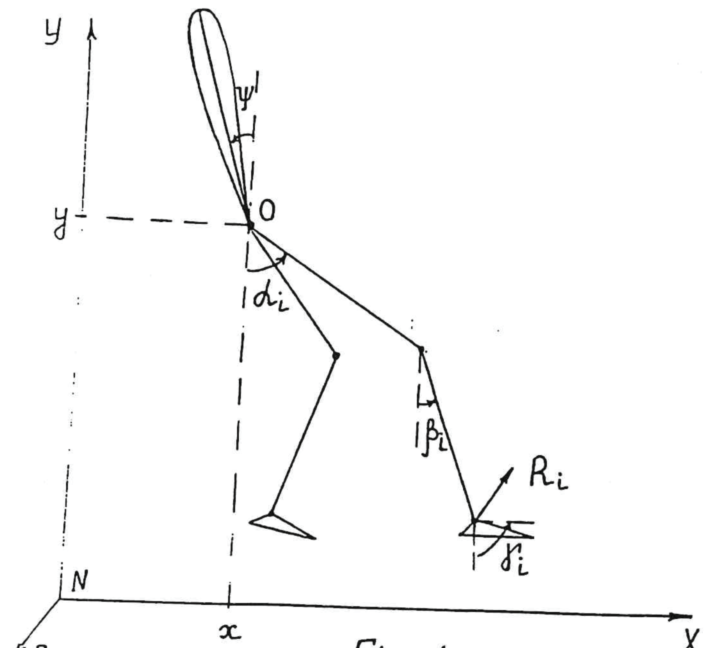


Fig. 1

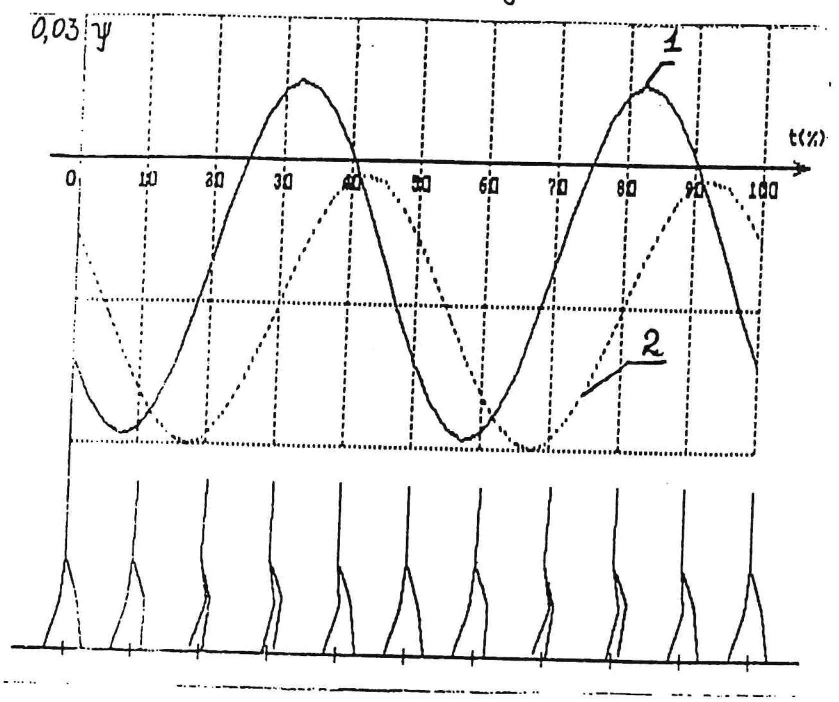
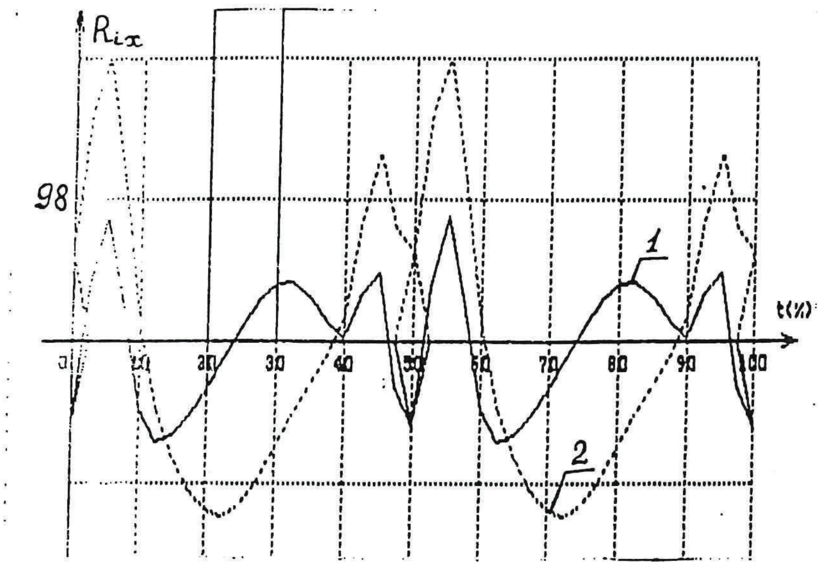
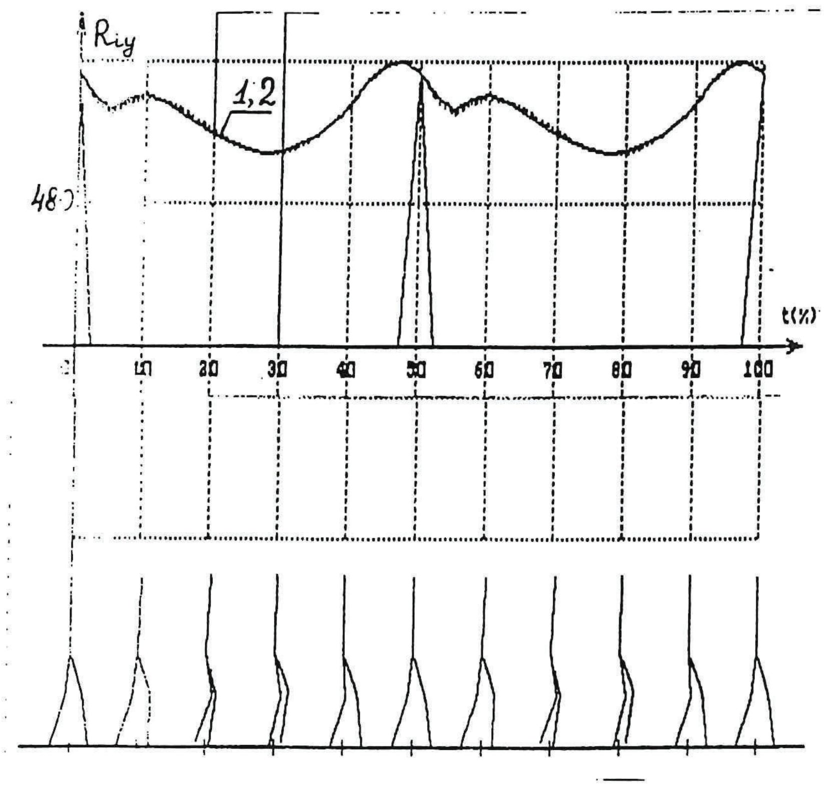


Fig. 2



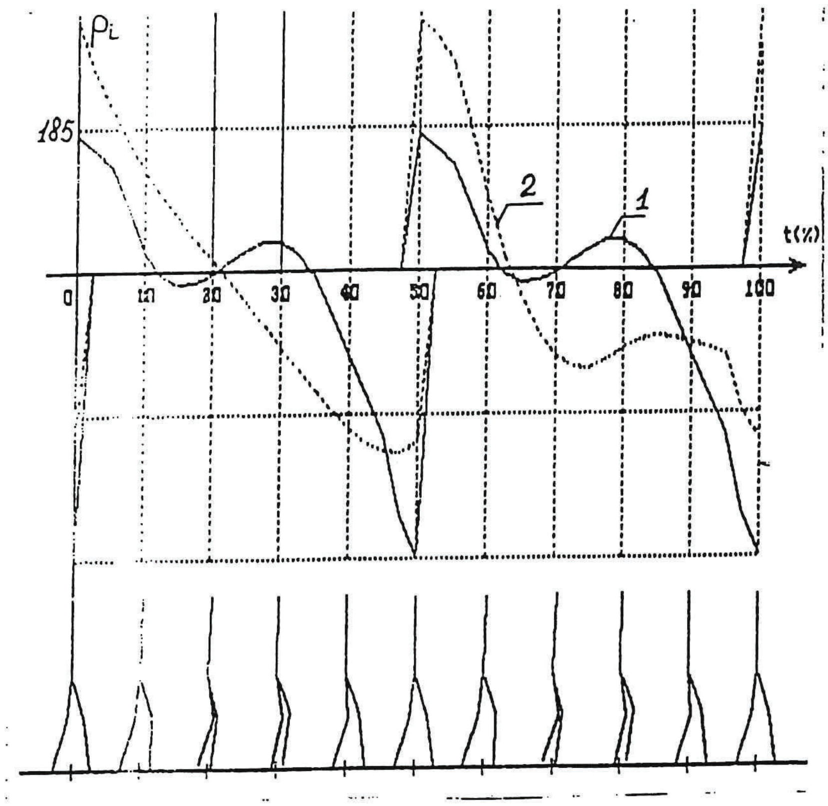
R_{ix}
98
t(%)
1
2
0
10
20
30
40
50
60
70
80
90
100

Fig. 3



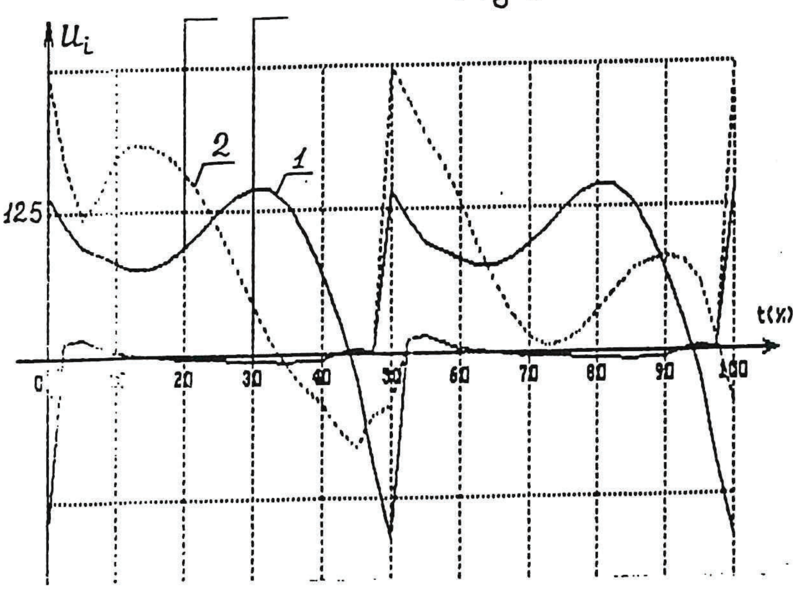
R_{iy}
1,2
48
t(%)
10
20
30
40
50
60
70
80
90
100

Fig. 4



ρi
185
2
1
t(%)
0
10
20
30
40
50
60
70
80
90
100

Fig. 5



ui
125
2
1
t(%)
0
20
30
40
50
60
70
80
90
100

Fig. 6