Energy-optimal control of human leg in swing phase

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ENERGY-OPTIMAL CONTROL OF A HUMAN LEG IN SWING PHASE

Victor Berbyuk

Department of Controlled Systems Optimization
Pidstryhach Institute for Applied Problems
of Mechanics and Mathematics
Ukrainian National Academy of Sciences
3-B, Naukova St., Lviv, 290601, Ukraine
Tel.: 007 0322 636000, Fax.: 007 0322 654240
E-mail: Kalyniak%IPPMM.Lviv.UA@LITech.Lviv.UA

ABSTRACT

The mathematical model and efficient algorithm for designing the energy-optimal controlled processes of a human leg in the swing phase are proposed. This algorithm is based on special conversion of the optimal control problem for nonlinear dynamical system model of a human leg into a standard nonlinear programming problem. The objective function for the optimization algorithm is the integral over swing phase's time from the sum of the mechanical power absolute values for all controlling stimuli. A number of the energy-optimal controlled processes of a human leg under different boundary conditions and restrictions on phase coordinates have been obtained. The kinematical and dynamical characteristics of obtained optimal controlled processes are compared with respective characteristics of a human leg's swing phase during normal gait.

INTRODUCTION

To understand how inertial interaction between body segments and musculotendon dynamics coordinates a human movement it is very usefull to solve the modelling and optimization problems of controlled motions of a human locomotor system or some of its parts [1-8]. At present the neuro system's laws which control human motions are not completely recognized. It is important to study the types of optimal control problems which, probably, are solved by

a human neuro system and to find the histories of optimal controlling forces, angular displacements, etc. Above-mentioned is one of the reasons for the attempts to solve the different optimization problems for biodynamical systems [5, 9-13].

In this paper the energy-optimal control problem for human leg in swing phase is considered. The controlled motions are investigated within the frame of mechanical model for dynamical system of two rigid bodies moving in the vertical plane. In contradistinction [9] the optimization procedure in the phase space is used to obtain the energy-optimal controlled processes. The objective function for the optimization problem takes into account not only mechanical work of the hip and knee controlling forces but also mechanical work of the principal vector of forces acting at the hip joint during swing phase of a human leg. It is assumed that on the initial and final instants only geometrical states of a human leg are given in advance. The velocities of a human leg on above-mentioned instants are determined by optimization procedure. Results are presented in the form of control laws and angular displacements histories during the motion.

STATEMENT OF THE PROBLEM

The system under consideration is the lower limb consisting of two rigid bodies: the thigh HK and shank with foot KA. This system is depicted diagrammatically in Fig. 1. The bodies HK and KA are connected by ideal cylindrical hinge with the centre at point K. On mechanical system considered the gravitational forces, external force R and controlling moments $\mu_1(t)$ and $\mu_2(t)$ are acting. The force R and moment $\mu_1(t)$ are, respectively, the principal vector and principal moment of the reaction forces of human body, refered to the point of the hip joint H. Control moment $\mu_2(t)$ acts at the knee joint and is treated as internal stimulus. The muscles dynamics is not taken into account in this paper. The different models of the muscle dynamics can be found in [3, 5, 10, 11, 13-15].

The average weight of the foot is about 1%-1,5% of the body and its dimensions are also small compared to the thigh or the shank. In swing phase the foot is locked with the shank at an approximately constant angle [10]. Hence, the inertial and gravitational effects of the foot during swing phase are small in comparison with those of the shank or the thigh and in this study they will be neglected.

All movement of the lower limb is restricted to the sagittal plane NXY of a fixed rectangular Cartesian coordinate system $NXYZ$ (Fig. 1).

The equations of motion for the controlled mechanical system in question, written in the form of Lagrange equations of the second kind, are as follows:

$$M\ddot{x} + K_a(\dot{\varphi}_1 \cos \varphi_1)\dot{} + K_b(\dot{\varphi}_2 \cos \varphi_2)\dot{} = R_x(t), \qquad (1)$$

$$M(\ddot{y} + g) + K_a(\dot{\varphi}_1 \sin \varphi_1)\dot{} + K_b(\dot{\varphi}_2 \sin \varphi_2)\dot{} = R_y(t), \qquad (2)$$

$$J_a\ddot{\varphi}_1 + K_a[\ddot{x} \cos \varphi_1 + (\ddot{y} + g)\sin \varphi_1] +$$

$$\qquad\qquad\qquad\qquad\qquad (3)$$

$$+ J_b[\ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + \dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2)] = \mu_1(t) - \mu_2(t),$$

$$J_2\ddot{\varphi}_2 + K_b[\ddot{x} \cos \varphi_2 + (\ddot{y} + g)\sin \varphi_2] +$$

$$\qquad\qquad\qquad\qquad\qquad (4)$$

$$+ J_b[\ddot{\varphi}_1 \cos(\varphi_1 - \varphi_2) - \dot{\varphi}_1^2 \sin(\varphi_1 - \varphi_2)] = \mu_2(t),$$

where

$$K_a = m_1 r_1 + m_2 l_1, \quad K_b = r_2 m_2, \quad M = m_1 + m_2,$$

$$J_a = J_1 + m_2 l_1^2, \quad J_b = l_1 K_b.$$

In equations (1)-(4): x and y are the Cartesian coordinates of the hip joint H; φ_1, φ_2 are the angles that specify the position of the thigh and shank relative to the vertical (Fig. 1); m_1 are the masses, l_1 are the lengths, r_1 are the distances of the centres of mass from the

proximal joint and J_1 are the moments of the inertia of thigh ($i=1$) and shank with foot ($i=2$), respectively; $R_x(t)$, $R_y(t)$ are the horizontal and vertical components of the principal vector of the reaction forces at the hip joint; and g is the acceleration due to gravity.

Let $z(t)=\{x, \dot{x}, y, \dot{y}, \varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2\}$ be a vector of the phase state, $u(t)=\{R_x, R_y, \mu_1, \mu_2\}$ be a vector of the controlling stimuli of human leg, and τ be the duration of swing phase.

Consider the next optimal control problem.

Problem 1. It is required to determine the controlled process $\{z(t), u(t)\}$, $t\in[0,\tau]$, which satisfies the equations of motions (1)-(4), boundary conditions

$$x_2(0)=x_{02}, \quad x_2(\tau)=x_{\tau 2}, \tag{5}$$

$$y_2(0)=y_{02}, \quad y_2(\tau)=y_{\tau 2}, \tag{6}$$

given restrictions on the phase coordinates

$$x(t)=x_H(t), \quad y(t)=y_H(t), \qquad t\in[0,\tau], \tag{7}$$

$$\varphi_1(t)\geq\varphi_2(t), \qquad\qquad t\in[0,\tau], \tag{8}$$

$$y_2(t)\geq 0, \qquad\qquad t\in[0,\tau], \tag{9}$$

$$(x-x_2)^2+(y-y_2)^2\leq(l_1+l_2)^2, \quad t\in[0,\tau], \tag{10}$$

and reaches minima of the functional

$$E[z(\cdot), u(\cdot)]=\int_0^\tau\left[|R_x(t)\dot{x}(t)|+|R_y(t)\dot{y}(t)|+\right.$$

$$\left.+|\mu_1(t)\dot{\varphi}_1(t)|+|\mu_2(t)(\dot{\varphi}_1-\dot{\varphi}_2)|\right]dt. \tag{11}$$

In expressions (5)-(10): $x_2(t)$, $y_2(t)$ are the Cartesian coordinates of the ankle joint (point **A**); $x_H(t)$, $y_H(t)$ are the Cartesian coordinates of the hip joint given from the experimental data of human locomotion [13, 16];

x_{02}, y_{02} and $x_{\tau 2}$, $y_{\tau 2}$ are given parameters determining the position of the point A on the initial $t=0$ and final $t=\tau$ instants, respectively. In functional (11) the absolute values are used for the integrands since they represent effort that is not recoverable.

METHOD

Central in the approach proposed for solving the problem 1 is the idea that any optimal control problem can be converted into a standard nonlinear programming problem by parameterizing each of the free variable functions. A key feature of the method is that it dispenses with the need to solve the two-point, boundary-value problem derived from the necessary conditions of optimal control theory. We shall use an inverse-dynamics approach for converting the problem 1 into the corresponding nonlinear programming problem. An important advantage of the inverse-dynamics formulation is that it does not require the system of differential equations to be numerically integrated.

From analysis of equations (1)-(4) and constraints (7) it follows that there are only two independently variable functions in problem 1. Taking into account the boundary conditions (5), (6) and the restrictions (8), (9), the functions $x_2(t)$, $y_2(t)$ will be chosen as independently variable functions.

We have the next kinematical constraints (see Fig. 1):

$$x_2(t)=x_H(t)+l_1\sin\varphi_1+l_2\sin\varphi_2,$$

$$y_2(t)=y_H(t)-l_1\cos\varphi_1-l_2\cos\varphi_2. \tag{12}$$

If functions $x_2(t)$, $y_2(t)$, $x_H(t)$, $y_H(t)$ are given then from (12) next expressions can be obtained:

$$\varphi_1(t)=\text{arctg}\left[\frac{\theta_2}{\theta_1}\right]+\text{arctg}\left[\frac{4l_1^2(\theta_1^2+\theta_2^2)}{(\theta_1^2+\theta_2^2+l_1^2-l_2^2)^2}-1\right]^{1/2} \tag{13}$$

$$\varphi_2(t)=\text{arctg}\left[\frac{\theta_2}{\theta_1}\right]-\text{arctg}\left[\frac{4l_2^2(\theta_1^2+\theta_2^2)}{(\theta_1^2+\theta_2^2+l_2^2-l_1^2)^2}-1\right]^{1/2},$$

where

$$\theta_1(t)=y_H(t)-y_2(t),$$

$$\theta_2(t)=x_2(t)-x_H(t).$$

Hence the state vector $z(t)$ is known and from equations (1)-(4) the control histories can be computed.

The function $x_2(t)$ is approximated by the sum of a finite number of terms defining a Fourier-type series

$$x_2(t)=a_0/2+\sum_{k=1}^{N}(a_k\cos k\omega t+b_k\sin k\omega t), \qquad (14)$$

where $\omega=2\pi/T$, T is the duration of human double step, a_0, a_k, b_k, $(k=1,\ldots,N)$ are parameters.

Taking into account the boundary conditions (6) and restrictions (9), the independently variable function y_2 is parameterized by introducing a number of nodal points $y_2(t_j)$, $\dot{y}_2(t_j)$, $(j=0,\ldots,N1;\ t_0=0,\ t_{N1}=\tau)$ in the interval $[0,\tau]$.

The function $y_2(t)$ is reconstructed at points $y_2(t_j)$, $\dot{y}_2(t_j)$ using the third-order spline approximation.

Therefore, choosing the functions $x_2(t)$ and $y_2(t)$ as the independently variable functions and using the Fourier and spline approximations the continual energy-optimal control problem of a human leg (problem 1) can be converted into the following parameter optimization problem.

Problem 2. Find the parameters

$$C=\{a_0,\ a_k,\ b_k,\ y_2(t_j),\ \dot{y}_2(t_j)\},$$

$$k=1,\ldots,N;\ j=0,1,\ldots,N1,$$

which minimize the performance criterion

$$E = Q(C),\tag{15}$$

subject both to the equality constraints

$$f_i(C) = 0, \quad i = 1,2,3,4,\tag{16}$$

and to the inequality constraints

$$g_m(C) \geq 0, \quad m = 1,2.\tag{17}$$

In expressions (15)-(17) the functions Q, f_i and g_m are determined by means of formula (11), the boundary conditions (5), (6) and restrictions (9) and (10).

RESULTS AND DISCUSSION

To solve the above parameter optimization problem (problem 1) the computational algorithm based on Rosenbrock's method [17] has been devised. Using this algorithm the computer programme has been composed in C-language. The computer programme developed makes it possible to simulate a human leg's motion in swing phase and to determine the near-energy-optimal control of the considered motion. A number of energy-optimal controlled processes of a human leg for different boundary conditions and restrictions on phase coordinates and controlling forces have been obtained.

Let us describe some numerical results of solutions of the problem 1.

In the model a subject height of 1.76 m, mass of 73.2 kg and next parameters of the limb: $m_1 = 7.08$ kg, $l_1 = 0.41$ m, $r_1 = 0.16$ m, $J_1 = 0.082$ kg·m^2, $m_2 = 5.04$ kg, $l_2 = 0.5$ m, $J_2 = 0.053$ kg·m^2, $r_2 = 0.203$ m are assumed [1].

Following the works [10, 13], the pelvis is assumed to progress forwards (in the x-direction) at constant velocity V during swing phase. The vertical motion (in the y-direction) is modelled as a sinusoidal movement. Hence, the functions $x_H(t)$ and $y_H(t)$ in restrictions (7) are

specified in the next form:

$$x_H(t)=Vt, \quad y_H(t)=h+B\sin(4\pi t+\beta), \qquad (18)$$

where V, h, B, β are parameters.

For the swing phase of normal gait the amplitude B is of order 0.0254 m, the phase β is equal to 0.1 rad. [1,13].

Three examples of solution of the problem 1 for the gaits with slow, natural and fast cadences are represented below. The input data are given by Table 1 (cadence in step/min, the others in SI units). In examples the duration of swing phase is determined by formula $\tau=0.4T$ [16].

Table 1

	Example 1	Example 2	Example 3
Cadence	86.8	105.3	123.3
T	1.383	1.1396	0.9733
V	0.998	1.325	1.685
h	0.85	0.85	0.85
x_{02}	-0.4	-0.4	-0.4
$x_{\tau 2}$	1.03	1.122	1.2
y_{02}	0.1	0.1	0.1
$y_{\tau 2}$	0.096	0.0949	0.1

Solution of problem 1 yielded energetically optimal laws of motion of ankle joint of a human leg in the swing phase. These laws of motion are specified by formula (14) and the values of free parameters in Table 2 and by the function $y_2(t)$ which is reconstructed using the third-order spline approximation at nodal points $y_2(t_j)$, $\dot{y}_2(t_j)$ ($t_j=j\tau$, $j=0,1,2,3,4$). All values in Table 2 are given in SI units.

Some results of the obtained energetically optimal motions of a human leg in the swing phase are also represented in Fig. 2 -Fig. 8 (curves 1, 2 and 3 correspond to slow, natural and fast cadence, respectively).

Table 2

	Example 1	Example 2	Example 3
a_0	-0.305656	1.107789	0.740580
a_1	-0.709509	-0.921005	-0.763508
a_2	0.493035	-0.050685	0.030840
a_3	0.025509	0.015340	-0.031458
a_4	-0.022945	-0.000832	-0.011800
a_5	-0.033263	0.003287	0.005636
b_1	0.906408	-0.090967	0.199931
b_2	0.011020	0.153752	-0.037729
b_3	-0.102752	0.025650	0.039181
b_4	-0.066881	-0.007999	-0.012714
b_5	0.006433	0.003614	0.014439
$\dot{x}_2(0)$	1.748345	1.541342	1.70075
$\dot{x}_2(\tau)$	2.979572	3.501934	3.468549
$\dot{y}_2(0)$	-0.043187	1.571520	1.589000
$y_2(t_1)$	0.082853	0.144629	0.124435
$\dot{y}_2(t_1)$	-0.192005	-0.429568	-0.692177
$y_2(t_2)$	0.084807	0.083680	0.043989
$\dot{y}_2(t_2)$	-0.023957	-0.427519	-0.647529
$y_2(t_3)$	0.015845	0.036819	0.026243
$\dot{y}_2(t_3)$	-0.244236	-0.164971	0.101602
$\dot{y}_2(\tau)$	1.131966	1.399498	1.088870

Let us describe in more detail the obtained energetically optimal swing motions of a human leg.

Figure 2 shows the displacements of the ankle joint of a leg along the direction of motion. It can be seen that horizontal motions are almost uniform (e.g. curve 2 for natural cadence) and without reversing cycles ($\dot{x}_2(t)>0$, $t\in[0,\tau]$).

The vertical displacements of the ankle joint of lower limb for energetically optimal swing phase are shown in Fig. 3. It can be seen that in all examples 1-3 the phase restriction given by formula (9) is satisfied. Figures 2 and 3 show that except near initial and final instants the velocities of horizontal motions of the ankle joint of a leg are an order of magnitude greater than the velocities of the vertical motions.

The variations in the angular coordinates characterizing the position of elements of a leg over the swing phase are shown in Fig. 4 and Fig. 5. The following notations are employed: $\varphi_1(t)$ is the hip angle specifying the mutual position of the pelvis and thigh; $\varphi_1(t)-\varphi_2(t)$ is the knee angle specifying the mutual position of the thigh and shank.

It can be seen from Fig. 5 that the phase restriction given by formula (8) is satisfied for all found energy-optimal controlling processes.

Figure 6 and Figure 7 show graphs of the control moments in the joints for the obtained energetically optimal laws of motions of a leg in the swing phase ($\mu_1(t)/M$ is the hip moment; $\mu_2(t)/M$ is the knee moment, where M is the total mass of human body).

The way in which the function

$$Rh=\left[F_x^2(t)+F_y^2(t)\right]^{1/2}/Mg$$

varies (Fig. 8) indicates that the magnitude of the principal vector of the reaction forces acting at hip joint has an order of the total weight of human body (dashed

curve in Fig. 8) and differs from the weight of the body by no more than 45%.

In Fig.4 -7, for comparison purposes, the hip and knee angles and moments obtained from experiments for a human gait with natural cadence are shown (dashed curves) [16].

Comparison of these experimental curves with modelling results indicates that obtained near-energy-optimal characteristics are within reasonable proximity to the corresponding characteristics of the motion of a human leg in the swing phase during gait with natural cadence.

A number of energetical characteristics of the obtained optimal controlled processes of a human leg in the swing phase are represented in Table 3 (in J/m). There are next notations in Table 3:

$$ES = E/L, \qquad\qquad\qquad (19)$$

$$EH = \left[E - \int_0^\tau |\mu_2(t)\,(\dot\varphi_1 - \dot\varphi_2)|\,dt \right]/L, \qquad (20)$$

$$EU = \frac{1}{L} \int_0^\tau \left[|\mu_1(t)\,\dot\varphi_1(t)| + |\mu_2(\dot\varphi_1 - \dot\varphi_2)| \right] dt. \qquad (21)$$

In expressions (19)-(21) E is a performance index given by formula (11), $L = x_{\tau 2} - x_{02}$ is the stride length which is equal to the sum of two steps' lengths.

Table 3

	Example 1	Example 2	Example 3
ES	70	56	67
EH	64	50	54
EU	9	8	20

The functional (20) estimates all energy expenditure per unit of distance travelled only at hip joint of the

lower limb. The functional (21) estimates the energy expenditure per unit of distance travelled only of oscillation of human lower extremity during swing phase.

The analysis of data of the Table 3 shows that for any of the functionals (19)-(21) the natural cadence of gait (Example 2) gives a minimum of the energy expended per unit of distance travelled. This result is in agreement with the conclusion that there is an ideal cadence for an individual that will give minimum energy expended per unit of distance travelled [9].

Comparison of the values *EH* and *EU* (Table 3) indicates that the energy expenditure at hip joint is approximately 3-7 times greater than that of oscillation of a human lower extremity during the swing phase.

CONCLUSION

In this paper the analysis of a human leg motions in the swing phase is based on solution of energy-optimal control problems for the plane mechanical system of two rigid bodies connected by ideal cylindrical hinge. The performance index used is the mechanical work spent to transfer a human leg from the initial position into the final one over the given time.

To solve the arisen nonlinear optimal control problem under given boundary conditions and restrictions on the phase coordinates the parameter optimization method has been proposed.

This method is based on Fourier and spline approximations of the independently variable functions and inverse-dynamics approach. The method proposed makes it possible to satisfy the boundary conditions and some of equality and inequality constraints on the phase coordinates automatically and accurately.

It is expected that the result derived for human legs' swing phase using parameter optimization method represents the global optima since the algorithm converged to the same solution irrespective of the initial guess.

One of the important possible practical applications

of the results of the present study may be the optimal design of artificial lower limbs.

ACKNOWLEGMENTS

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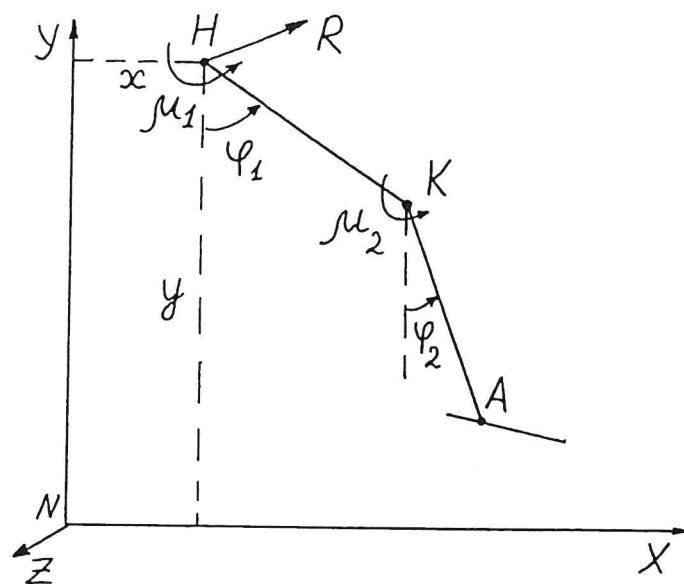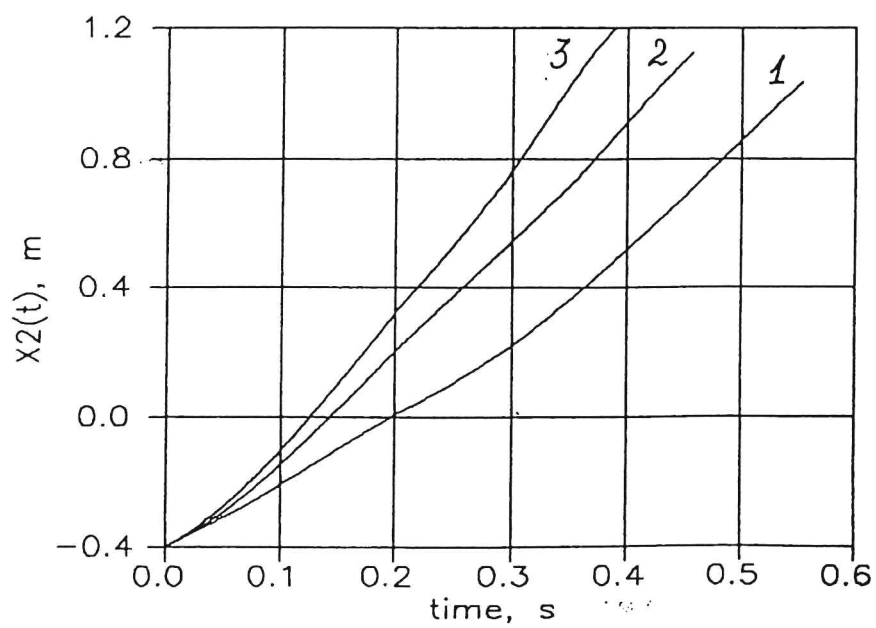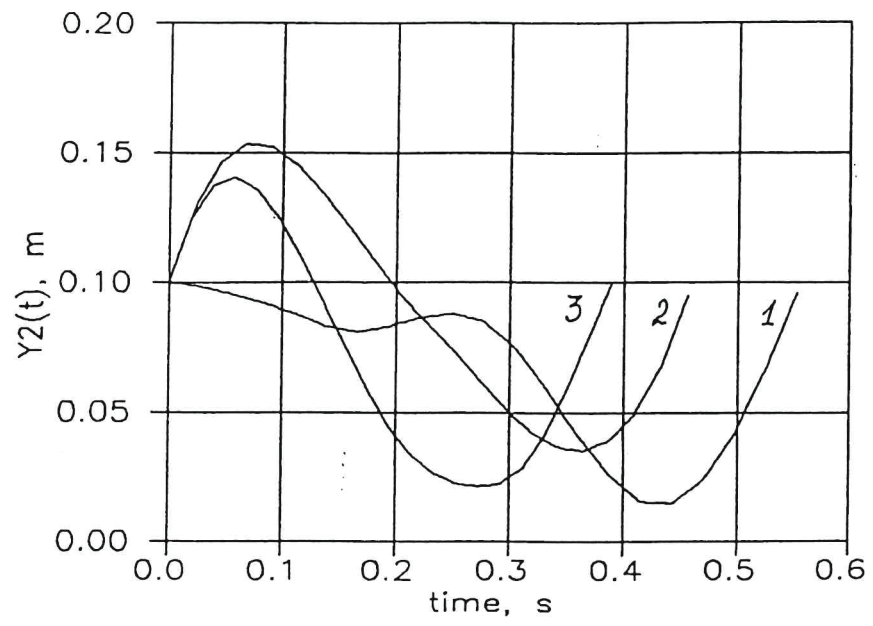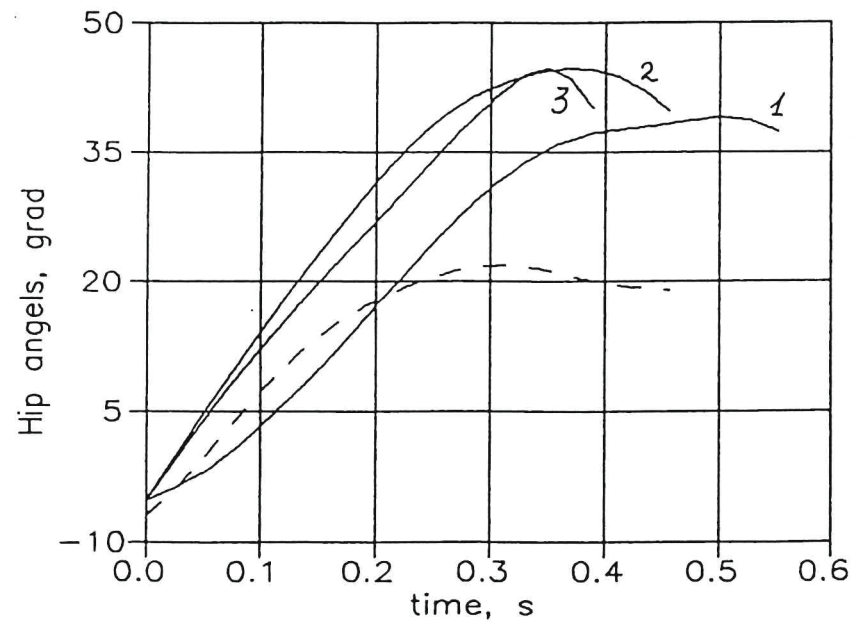
Fig.1

Fig.2

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Fig.6

Fig.7

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