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Abstract: Two common methods for predicting the energy usage in vehicles through mathematical simulation, the 'backward' and the 'forward' schemes, are discussed and compared in terms of the longitudinal vehicle behaviour they predict. In the backward scheme, the input driving cycle is initially assumed to be followed perfectly and therefore the vehicle speed is not a dynamic state. In the forward scheme, a driver model controls the vehicle in an attempt to follow the input driving cycle, and the vehicle speed is intrinsically a dynamic state. A theoretical study is made with a simple mathematical vehicle model, where it is shown that the two methods neither predict the same expected energy use nor energy variation. Next, the simulation model that is used for the CO_2 rating of heavy-duty trucks in Europe, VECTO, is used as an example of the backward method, and an equivalent implementation in a forward scheme is attempted. Two numerical experiments are made with these models: a detailed study of the longitudinal vehicle behaviour on a reference mission; and a study of the predicted CO_2 emissions on a family of stochastically generated missions. The conclusion is that the backward method is easier to use but the forward method has a greater potential to predict realistic behaviour.

Keywords: backward simulation, forward simulation, CO_2 emissions, fuel consumption, operating cycle

1. INTRODUCTION

In contemporary vehicle industry and transportation research, numerical simulation with a mathematical model is a standard method to evaluate the energy usage of a vehicle design. As opposed to physical testing, a vehicle manufacturer can evaluate a design early in the development phase before a physical prototype exists, at a fraction of the cost (Weber, 2009). Other institutions, such as: suppliers, governmental agencies and universities, may have limited access to vehicles which leaves simulation as the only option. The numerical method is useful for a wide range of applications; from highly generalised statistical models for evaluating the CO_2 emissions from a transportation network (Grote et al., 2018); through kinematical models common in, for example, microscopic traffic simulations (Donateo et al., 2012); to highly detailed dynamical models for determining the influence of specific components, strategies or functions (Ahlawat et al., 2010).

For development of vehicle systems, the latter kind of models (often called white box or light grey, see e.g. the classification by Zhou et al. (2016)) are the most appropriate to use, because they can resolve more details that are helpful when analysing the vehicle behaviour. Within the context of such dynamical models, there are two common approaches to the simulation process, often referred to as 'backward' (or 'backward facing') simulation and 'forward' (facing) simulation (Wipke et al., 1999; Pettersson, 2017), the choice of which has a profound effect on the model behaviour. The purpose of this paper is to discuss similarities and differences between these two methods, to better understand the advantages and disadvantages of both.

In the backward scheme, a target speed is provided by a driving cycle. The necessary propulsion force is computed from Newton's second law and it, together with the vehicle speed, is propagated from the wheels, through the powertrain, to the prime mover where the necessary input power for the propulsion effort is computed. The approach is called backward because the data flows backwards [1]

[1] What is called backward and forward may seem arbitrary, but our choice is quite natural. With the canonical choice of vehicle coordinate system, defined in ISO 8855 (2011), the positive x-direction is outward from the vehicle along its longitudinal axis. Interpret this as forward. To induce such motion on a flat road, power needs to be produced by the prime mover and transmitted

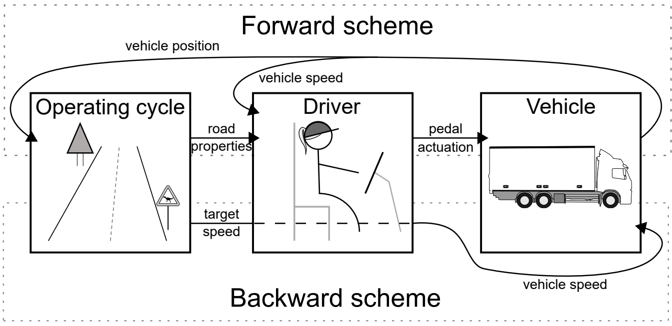

Figure 1. The imagined simulation model topology, with the forward scheme showed top and the backward scheme showed bottom

through the powertrain, but another descriptive name would be 'inverse dynamic' simulation. The arrows in the lower part of Fig. 1 depict the method.

In the forward scheme, a target speed is similarly provided by a driving cycle, but it passes through a driver model. The driver controls the longitudinal vehicle interfaces, the accelerator and the brake pedals, based on the (difference between the) target and the vehicle speed. The energy carrier (fuel, charge, etc.) is injected into the prime mover and the torque is propagated forward through the powertrain to the wheels; where the traction sustains the propulsion force. Again, Newton's second law provides the vehicle acceleration which is integrated for speed and position. The position is fed back to the driving cycle to find a target speed, which closes the computation loop. The effort flows in the opposite direction in the powertrain compared to the backward method and the approach is therefore called forward. As the situation is arguably closer to what is happening in reality, a better name would perhaps be 'causal' simulation. Figure 1 shows this method in the upper part.

Both schemes are commonly used in science and engineering. The backward method is often used when treating control problems, for example optimal control of battery management for electric hybrid vehicles (Pourabdollah et al., 2013) or finding the best vehicle configuration (Ghandriz et al., 2017). The model used for the official CO_2 rating of heavy-duty trucks in Europe, VECTO (Fontaras et al., 2013, 2016; European Commission, 2017) (similar to GEM (Franco et al., 2015) in the U.S.), uses the backward scheme. Forward simulation is more common when predicting the influence from the environment (Vepsäläinen et al., 2018), when the driver impact is of concern (Valenti et al., 2018), or when investigating specific components (Ahlawat et al., 2010).

The objective of this paper is to compare the backward and forward simulation schemes with respect to longitudinal vehicle behaviour and energy usage. First, we use a simple vehicle model in a theoretical study to explain the fundamental differences between the backward and forward concepts. Next, we use VECTO (which has been extensively documented and validated (Fontaras et al., 2013; Joint Research Centre, 2019)) as an example of a backward simulation model. Then an equivalent implementation that works in the forward scheme is made, that follows the framework in Pettersson et al. (2018). The two models are used in a numerical study in two parts. First, they are compared in detail on a reference mission, after which the comparison is generalised to a family of missions that have the same statistical properties, generated by a stochastic method (Pettersson et al., 2019). The fitness measure is energy usage in terms of the CO_2 emissions. The two research questions that we try to answer are: 'are there differences in vehicle behaviour between the backward and forward schemes?'; and 'do the backward and forward schemes yield the same expected CO_2 emissions and exhibit the same variance?'.

to the wheels. Then this is the associated positive direction of the powertrain. Hence, we call a flow in the other direction 'backward'.

2. METHOD AND THEORY

The vehicle in this study is a tractor-semitrailer combination with a conventional diesel engine and an automated manual transmission gearbox. The mechanical components include the prime mover, retarder, gearbox, final drive gear (axle gear), the driven wheel and the chassis itself. The governing equations for the reference backward model can be found in the VECTO documentation (Joint Research Centre, 2019) but to simplify the theoretical study, a lumped powertrain-wheel model is used in this section.

The engine torque is transferred to the chassis with an overall ratio R and torque loss T_{loss}:

$$m^*\dot{v} = F_x - mg\,(f_r \cos\theta + \sin\theta) - \frac{1}{2}\rho C_d A v^2, \qquad (1)$$

$$F_x = R\,(T_e - T_{loss}) - F_b, \qquad v = \frac{\omega_e}{R}, \qquad (2)$$

$$P_{out} = \omega_e T_e + P_{aux}, \qquad (3)$$

$$P_{in} = \frac{P_{out}}{\eta_e}, \qquad (4)$$

An explanation of the variables can be found in Table 1. The torque loss, engine efficiency and auxiliary power are all functions of both engine torque and (angular) speed. The output power P_{out} in (3) is limited by the capacity of the engine:

$$P_{out} \leq P_{max}, \qquad (5)$$

where the maximum power $P_{max} = P_{max}(\omega_e)$ depends on the engine speed. The gearshifts are neglect in the theoretical study.

For clarity, we will argue in terms of the input power P_{in} instead of fuel mass flow $\dot{m}_f = cP_{in}$ (whose integral is really the measure of interest) because they are directly related by the heating value c. The mathematical vehicle model is of course the same independent of whether the computation scheme is backward or forward: the difference appears when considering the flow of data in the simulation loop.

2.1 Backward scheme

In the backward scheme, there is a set speed v_{set} provided by the driving cycle and it is initially assumed that the vehicle speed follows this identically

$$v \equiv v_{set}. \qquad (6)$$

The set speed is given as a function of time or distance and hence all its derivatives are known as well. Therefore, the vehicle speed is not a dynamic state variable. In fact,

Table 1. Notation, variables and operators

Symbol	Explanation
A	Vehicle frontal area
C	(Co-) variance remainder
C_d	Drag coefficient
F_b	Brake force
F_{max}	Maximum brake force
F_x	Propulsion force
P_{aux}	Auxiliary power request
P_{bwd}	Power in backward scheme
P_{in}	Engine input power
P_{max}	Maximum engine power
P_{out}	Engine output power
R	Total transmission ratio
T_e	Engine torque
T_{loss}	Transmission torque loss
e_k, ε_k	Noise in segment k
f_d	Driver model
f_r	Rolling resistance coefficient
g	Gravitational acceleration
m	Gross combination mass
m^*	Effective gross mass (with rotational inertia)
m_f	Fuel mass
p	Pedal actuation
v	Vehicle speed
v_{set}	Set speed from driving cycle
η_e	Engine efficiency
θ	Road gradient angle
ρ	Air density
σ_v, σ_θ	Noise amplitude
ω_e	Engine angular speed
$\mathbb{E}(X)$	Expectation of X
$\mathrm{Var}(X)$	Variance of X
$\mathrm{Cov}(X, Y)$	Covariance between X and Y
$\bar{x}$	Arithmetic mean of x

we will use this criterion as the definition of a backward method. Equations (1) to (4) can be solved one after another for the required input power and the fuel mass flow. It needs to be emphasised that there is no feedback to the driving cycle, since the vehicle speed is identical to the target speed. With this simple vehicle model, the problem turns into a fully algebraic one, as there are no longer any unknown derivatives.

In practice, there are several technical problems that need to be solved. For instance, sometimes maintaining the set speed requires a higher engine power than allowed by (5). If so, then the mathematical problem has no solution and a strict backward simulation breaks down. This must be solved by deviating from the set speed, for which there are many methods [2]. Depending on the interpretation of the set speed from the driving cycle, it is also possible introduce modifications based on heuristic criteria. These may be seen as actions from a driver, represented by the dashed line in Fig. 1, as is done in VECTO. Such modifications are typically implemented to remove unrealistic actions (like excessively high decelerations) or introduce otherwise wanted behaviour (like coasting and eco driving).

[2] One way to handle it is to make the vehicle speed a dynamic state variable at these instances and let the engine work at full power, then switch back once the inequality in (5) is re-established. This has been called a hybrid or combined backward/forward method (Wipke et al., 1999). No such distinction is made here.

To see how the backward approach works in practice, assume that the driving cycle is given as a function of position. Let the set speed be constant, but also consider a road gradient angle θ. Let the angle be discretised such that it is constant over short road segments (2 5 m or so), and assume that it varies around a mean $\bar{\theta}$ with some noise e_k, so that in segment k

$$v_k = v_{set} = \bar{v}, \tag{7}$$

$$\theta_k = \bar{\theta} + e_k, \qquad \mathbb{E}(e_k) = 0, \qquad \mathrm{Var}(e_k) = \sigma_\theta^2. \tag{8}$$

Even with these simple models, it is not possible to derive an analytical expression for the expected input power and its variance. Thus, assume that θ_k is small, that the combined resistance forces in (1) are large enough to require a positive input power in (4), that the torque loss, auxiliary power and engine efficiency vary slowly enough around their work points to be approximately constant, and neglect the air resistance term. Provided that the constraint in (5) is satisfied, (1) to (4) combine to

$$P_{in} = \frac{v\left(m^*\dot{v} + mgf_r + mg\theta + rT_{loss}\right) + P_{aux}}{\eta_e}, \tag{9}$$

on segment k. The expected value and variance of (9) take the closed form expressions:

$$\mathbb{E}(P_{in}) = \frac{\bar{v}\left(mgf_r + mg\bar{\theta} + rT_{loss}\right) + P_{aux}}{\eta_e} = \frac{\bar{P}_{bwd}}{\eta_e}, \tag{10}$$

$$\mathrm{Var}(P_{in}) = \left(\frac{mg\bar{v}\sigma_\theta}{\eta_e}\right)^2. \tag{11}$$

The result is intuitive: the expected value of the input power is simply what is needed to overcome the rolling resistance, the average gradient and the losses in the transmission. For the dispersion (standard deviation), the only source of variation comes from the road gradient. The variation in input power is the variance of the gradient scaled by the average required power per radian $mg\bar{v}/\eta_e$.

2.2 Forward scheme

In the simplest forward scheme, the same driving cycle can provide a target speed. However, no assumption is made about the vehicle speed. Instead, a driver model f_d controls the longitudinal actuator p (e.g. accelerator and brake pedals) based on the current and target speeds:

$$p = f_d(v_{set}, v), \tag{12}$$

where f_d may be a PID controller based on the speed error and bounded in some interval, e.g. $p \in [-1, 1]$. To connect the pedal actuation to the input power, we use a linear function

$$P_{in} = \frac{P_{max}}{\eta_e} p, \qquad F_b = 0, \qquad \text{if } p > 0, \tag{13}$$

$$P_{in} = 0, \qquad F_b = F_{max}|p|, \qquad \text{otherwise}. \tag{14}$$

The output power can be computed with (4). The resulting vehicle speed and position are unknown and must be solved for through (3), (2) and (1). The feedback (the closed loop system, that is) consists of the vehicle speed to the driver model in (12), and the vehicle position to the driving cycle. So, the forward approach is intrinsically a system of differential algebraic equations. We take this criterion as the definition of a forward model: if the vehicle speed is always a dynamic state, then the simulation model follows a forward scheme. The method can be made considerably more advanced than described here, see, for example, Pettersson (2017); Pettersson et al. (2018).

To derive an expression for the expected input power and its variance, we proceed in the same way as before. The same assumptions are made, and the driving cycle is discretised in the same way as in (8). The difference is that the vehicle speed will change even though the set speed is constant: the varying topography causes a fluctuating resistance force and the controller in the driver model introduces some delay. To treat the problem without specifying the driver model further (the methods of stochastic calculus would be needed even with only a P controller), we invoke the assumption that the resistive forces are moderate and model the speed in the same way as the topography: by a small variation ε_k around its mean $\bar{v}$

$$v_k = \bar{v} + \varepsilon_k, \quad \mathbb{E}(\varepsilon_k) = 0, \quad \mathrm{Var}(\varepsilon_k) = \sigma_v^2, \tag{15}$$

where $\bar{v} = v_{set}$ as before. Taking the expectation of the input power in (9) again,

$$\mathbb{E}(P_{in}) = \frac{\left(\bar{P}_{bwd} + m^* \ \mathrm{Cov}(\dot{v}, v) + mg \ \mathrm{Cov}(v, \theta)\right)}{\eta_e}. \tag{16}$$

Compared to the backward power (10), two new terms have appeared. Unfortunately, $\dot{v}$, v and θ cannot be assumed to be uncorrelated. To specify the covariances would require formulating the joint probability density, which is not possible without adding to the (fairly general) assumptions of (8) and (15).

The variance is a lenghty expression:

$$\begin{aligned}\mathrm{Var}(P_{in}) = \frac{1}{\eta_e^2}\Big(& (mg\bar{v}\sigma_\theta)^2 + \sigma_v^2 \left[(mg)^2\left(\sigma_\theta^2 + \bar{\theta}^2\right)\right. \\ & \left. + (mgf_r + rT_{loss})\left(mg\left(f_r + 2\bar{\theta}\right) + rT_{loss}\right)\right] + C\Big),\end{aligned} \tag{17}$$

with C an assortment of covariances

$$\begin{aligned} C = & -m^* \,\mathrm{Cov}(\dot{v}, v)\left[m^* \,\mathrm{Cov}(\dot{v}, v) + 2mg \,\mathrm{Cov}(v, \theta)\right. \\ & \left.+ 2\bar{v}\left(rT_{loss} + mg(1 + \bar{\theta})\right)\right] + mg \,\mathrm{Cov}(v, \theta)\left[-mg \,\mathrm{Cov}(v, \theta)\right. \\ & \left.+ 2\bar{v}\left(rT_{loss} + mg(\bar{\theta} + f_r)\right)\right] + 2m^* mg \,\mathrm{Cov}\left(\dot{v}v^2, \theta\right) \\ & + m^{*2} \,\mathrm{Cov}\left(\dot{v}^2, v^2\right) + 2m^* \,\mathrm{Cov}\left(\dot{v}, v^2\right)\left[rT_{loss} + mg(f_r + \bar{\theta})\right] \\ & + 2mg\left(rT_{loss} + f_r mg\right)\mathrm{Cov}\left(v^2, \theta\right) + (mg)^2 \,\mathrm{Cov}\left(v^2, \theta^2\right) \\ & + m^{*2}\left[\bar{v}^2 + \sigma_v^2\right]\mathrm{Var}(\dot{v}). \end{aligned} \tag{18}$$

The first term in (17) is the same as in the backward approach, while the others show up because the vehicle speed is no longer constant. Unfortunately, this is as far as we get without more information. However, with the assumption that the variation in road grade is small, it should hold that the variation in speed is too. Then the new terms would be lesser in magnitude than (11) when the target speed is moderate or higher, because the term is proportional to mass, gravitational acceleration and mean speed. In addition, it is not certain that all covariances are positive. Therefore, the forward variation is expected to be greater than the backward variation, but not necessarily much so.

It must be noted that the expressions in (10), (11), (16) and (17) apply to a very simple driving scenario and do not hold without the initial assumptions. In realistic scenarios there are gearshifts, the losses and efficiencies are not constant, and the target speed changes. Furthermore, the air resistance was neglected out of convenience, as it was not necessary to include to show that there is a difference in the predicted statistical moments. Its inclusion would add a considerable number of terms to (16) and (17) but only one to (10) and none to (11).

3. NUMERICAL RESULT

Next, we show a numerical example of typical results from the backward and forward simulation schemes. The mathematical vehicle model in the backward framework of VECTO is implemented in the forward framework of VehProp (Pettersson, 2017). The source code of VECTO is not public and the implementation was done based on its documentation (Joint Research Centre, 2019) only. Though the implemented forward model follows this, it cannot be verified that the original backward model does. Therefore, no guarantee can be given that the models are completely identical mathematically. Also, do note that VECTO itself is used for all numerical experiments and not the simplified model in Section 2.

The vehicle is a 4x2 tractor semi-trailer combination with a stepped gearbox of twelve gears and a 12.7 l diesel engine. The combined mass is 15.7 tonnes. This specification is taken from VECTO's example vehicles, including the model parameters.

3.1 Comparison on reference data

The reference transport mission was taken from the same dataset as the vehicle specification; it is a 108 km long haul mission that includes a target speed, road gradient, stop locations and auxiliary power requests, all as functions of position. The payload is 19.3 tonnes. For the forward model, the data was reinterpreted on the operating cycle format [3] . The driver model was identical to that presented in Pettersson et al. (2018). Thus it is a PID controller with an anti windup mechanism and a degree of look ahead through a maxium desired acceleration limit.

Figure 2 shows the resulting speed traces when simulating with both models (top and middle), together with the road altitude profile (bottom). With the backward method (top), the vehicle generally follows the target speed exactly but there are some drops, the most notable one between 10-17 km. These appear because the road grade is severe enough that there is equality in the constraint in (5), cf. the altitude profile in the bottom plot, and the backward method in VECTO adjusts itself by reducing the vehicle speed. The result is 33.4 kg of used fuel, corresponding to

[3] The operating cycle format was designed as a comprehensive mission description, independent of both the vehicle and the driver. Therefore, it does not have a set speed but instead describes features of the road, the weather, the traffic and the mission operations. A target speed is left to be decided by an interpretive driver model based on the current and upcoming driving conditions. In this case, there was too little information to separate individual road effects from the target speed and so it was directly interpreted as the speed limit on the road signs, super positioned with stop signs at the standstill locations. The details of the operating cycle format, its implementation and the driver model topology can be found in Pettersson et al. (2018); Pettersson (2017).

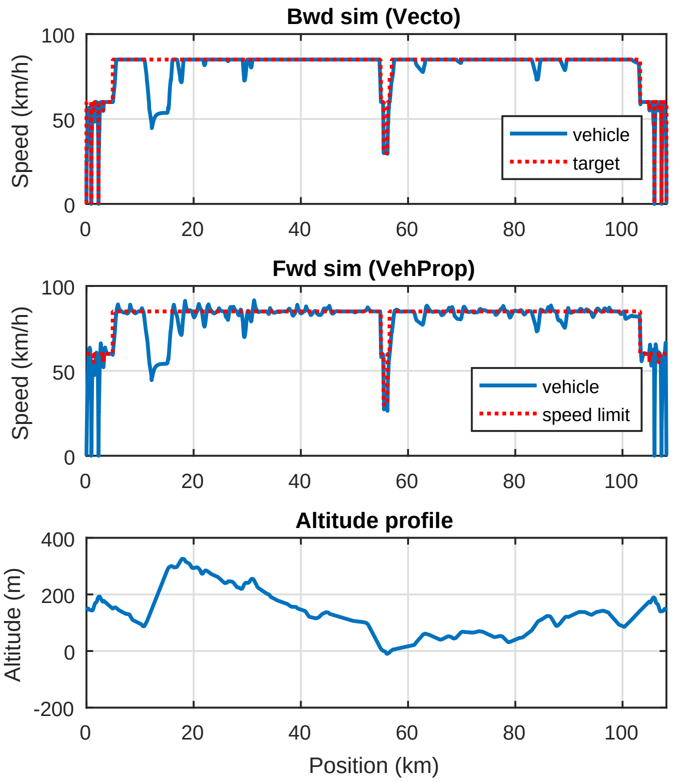

Figure 2. Resulting speed traces from the backward simulation (top) and the forward simulation (middle), with the vehicle speed in solid blue and the target in dotted red; the bottom diagram shows the altitude profile

a (mean) CO_2 emission of 979 g/km or a fuel consumption of 37.1 l/100 km. The mission time was 1.49 h.

The result from the forward method is shown in the middle graph of Fig. 2. The most notable differences are the fluctuations. These are in line with what was described in the theory section: the vehicle speed is controlled by a driver and cannot follow the target exactly because of the varying topography. The variation in speed disappears when the road grade is roughly constant (altitude is linear) and the engine does not work at its maximum power (inequality in (5)), as between 35-37 km and 45-50 km.

It can be observed that in all places where the speed drops in the backward simulation, it also drops in the forward simulation. This is expected: the speed error grows fast and the driver model therefore reacts quickly. The engine power is again limited by (5) and the driver model continues to request full pedal actuation until the speed error disappears, which happens first when the road gradient decreases in magnitude. Hence, for full (prime mover) load cases, there is little difference between the two simulation schemes.

The result in the forward scheme is 34.0 kg of used fuel, corresponding to a CO_2 emission of 997 g/km or a fuel consumption of 37.7 l/100 km. This is 1.8% higher than the backward consumption. The mission time was 1.48 h, which is 0.6% lower.

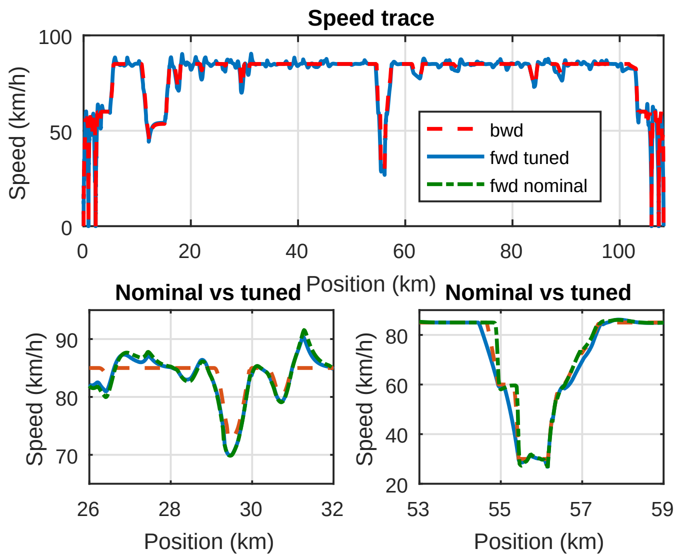

Figure 3. Speed trace from the tuned model; top graph shows the tuned forward simulation in solid blue and the backward trace (same as in Fig 2) in dashed red; the two bottom graphs show the difference between the nominal and tuned forward simulations in two instances, between 26-32 km (left) and around the reduced speed limit on 53-59 km (right)

3.2 Driver model tuning

The choice of numerical values for the driver parameters influence both the energy usage and the transport time. The forward scheme is sometimes criticised for this fact, as the parameter values may seem arbitrary. We can use that to our advantage here and tune the numerical values such that the CO_2 emissions coincide with those of the backward scheme. Therefore, the driver control P, I, D gains and the maximum desired deceleration are adjusted with a particle swarm optimisation routine. The fitness function is the absolute value of the difference between the CO_2 emissions and the backward reference. Figure 3 shows the tuned speed trace, in relation to the backward scheme (top) and two instances where it is compared to the nominal (untuned) speed trace. The CO_2 emissions from the tuned forward simulation are the same as the backward model's to within 0.02%. The transport time has instead increased to 1.52 h, which is 1.7% longer than the backward reference.

The speed traces of the nominal and tuned forward simulations are very similar overall, as can be seen from the bottom diagram of Fig. 2 when compared to the top diagram of Fig. 3. The bottom plots in Fig. 3 show two instances in higher resolution. In the left situation, the target speed is constant, and the two forward simulations are next to identical. In the right, the effect of the increased longitudinal deceleration is apparent, as the tuned model driver decreases the speed earlier but more slowly. This is the main reason for the reduction in energy usage, as the friction brakes are used less and therefore dissipate less energy (about 8.5% less, or 3.1 kWh of the total output energy of 169 kWh).

Table 2. The estimated sOC

Model	Parameter	Value (per road type) *Rural*	*Highway*	Unit
Road type	Mean dist.	3.8	48.4	km
	Row in trans. mat	[0 1]	[1 0]	-
Speed signs	States	$(30, 55, 60)$	85	km/h
	Mean dist.	$(0.7, 0.1, 1.7)$	-	km
	Trans. mat.	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$	-	-
Topography	Hill length	1.6	2.5	km
	Amplitude	2.85	1.77	%
Stop signs	Intensity	0.35	0	1/km
	Time	[54, 66]	-	s

3.3 Comparison on generated data

To numerically attack the question of whether the backward and forward methods yield differences in expected energy usage and variation, we use a method presented in Pettersson et al. (2019). The transport operation is interpreted statistically through stochastic processes. Each physical property of the road and the environment is seen as a random sequence that is modelled by a particular process. The topography, for example, is modelled by a first order autoregressive relation (see also Johannesson et al. (2017)), while the speed signs are modelled by a marked Poisson process with an embedded Markov chain. The parameters of the stochastic models, including those describing the probability distributions, are a compact description of the transport operation, called a stochastic operating cycle (sOC). An extensive explanation of the stochastic models, their physical interpretation and the generation process can be found in Pettersson et al. (2019).

An sOC is estimated from the reference mission, shown in Table 2. With this setup, a generated road contains speed signs, stop signs and topography. The road type is introduced to separate between the long highway-like sections with high speed limits and the shorter rural-like sections. With the statistical parameters known, any number of new transport operations can be created by generating random sequences. The resulting missions are all equivalent in a statistical sense, both to each other and to the reference, because they originate from the same processes and distributions. The representations are still different physically, however. A simulated vehicle will operate differently between individual missions and the CO_2 emissions are not all the same. Instead, these form a distribution that represents the emissions from similar missions in a similar landscape. Both an average CO_2 emission and a spread can be read out from there.

The sOC in Table 2 is used to generate 100 missions. Like the reference, these were all 108 km long, with 19.3 tonne payload and constant 2 kW power take-off demand. They were simulated both with the backward model and the (tuned) forward model. Figure 4 shows the probability density of the CO_2 emissions (meaning the average amount of CO_2 per mission in units of g/km), using a kernel density estimate. The density from the backward model is shown in dashed red and that from the forward model in solid blue.

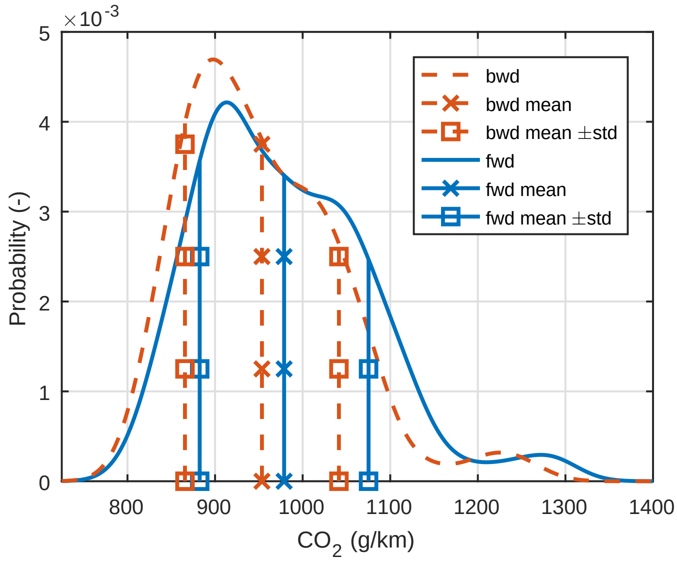

Figure 4. Estimated probability densities of the CO_2 emissions on the 100 generated missions

Table 3. Simulation result summary

Statistic	Measure	Simulation result *Bwd*	*Fwd Nominal*	*Fwd Tuned*	Unit
Ref.	CO_2	979	997	979	g/km
Mean	CO_2	954	1002	979	g/km
SD	CO_2	87.9	102	96.4	g/km
RSD	CO_2	9.2	10.2	9.8	%
Ref.	time	1.49	1.48	1.52	h
Mean	time	1.53	1.53	1.57	h
SD	time	0.19	0.19	0.21	h
RSD	time	12.1	12.1	13.3	%

The distributions are similar in shape. Both have a (positive) skew, with the forward simulation displaying a somewhat longer tail. The forward distribution is also broader, which was implied by the theoretical result, but the difference is not dramatic. In numbers, the backward method yields an average of 954 g/km, with a standard deviation (SD) of 88 g/km. The forward average is 979 g/km and its SD is 96 g/km, about 10% higher. However, to quantify the difference in spread, it may be more appropriate to use the relative standard deviation (RSD, SD/average), since the averages are different. The backward RSD is 9.2% and the forward RSD is 9.8%; so, the forward method still has a greater dispersion with this metric although the difference is very small.

As already pointed out, the mean values of the two schemes are different, with the backward method having the lower value. The difference is minor at about 25 g/km, or 2.5%, but it does indicate that a forward simulation gives greater average emissions. Another observation is that the forward average coincides with that in the reference mission and, consequently, the backward average ends up below. This is an interesting feature, for which the full reason is not known. Further study is needed to determine whether the difference is systematic or not, and, if so, explain its origin. A summary of the statistics can be found in Table 3.

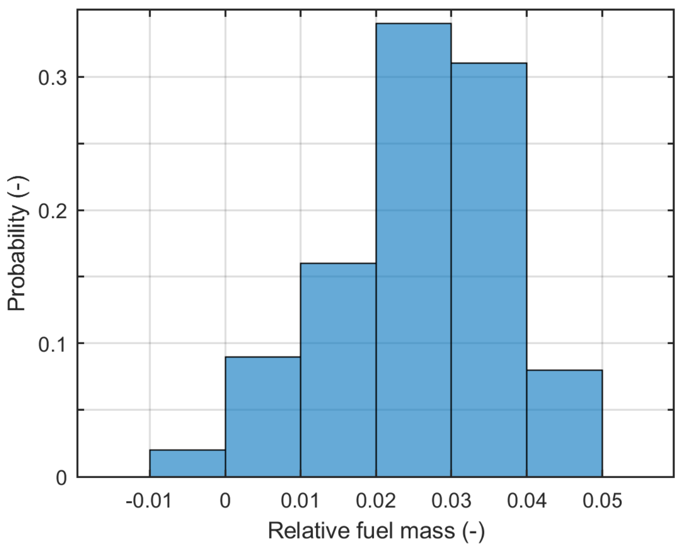

Figure 5. Distribution of the pairwise difference on the generated missions; a positive number means that the forward method had higher emissions

Apart from looking at the distributions of emissions and their statistics, we may look at the relative difference in fuel mass r_f

$$r_f = \frac{m_{f,fwd} - m_{f,bwd}}{m_{f,bwd}}. \tag{19}$$

It measures how much more emissions are predicted by the forward method in units of the backward method. A value of 0.1 means that the forward scheme shows 10% higher emissions, while a negative value means a lower emission. Also, r_f is independent of whether the amount of CO_2 or the fuel is studied, whether volume or mass is used and, if the distance travelled is the same for both methods, whether the total amount or consumption per distance is considered.

Fig. 5 shows a histogram of the pairwise difference over the 100 missions, using r_f. In most of the cases, the forward approach estimates 2% to 4% higher values than the backward approach. For the edge values, it can be observed that the largest difference is still below 5%, while the left edge show that there are a few instances where the backward method yields a higher value than the forward method.

The mean of the pairwise comparison is 0.026 and its SD 0.011. An approximate 95% confidence interval for the mean is thus

$$\mathrm{CI}(\bar{r}_f) = 0.026 \pm 0.002. \tag{20}$$

As zero is not covered by the interval, it may be concluded that the differences between the two simulation schemes are significant at the 95% confidence level and the null hypothesis can be rejected. It must be pointed out that the reference mission displays a low degree of variation, as can been seen from the parameters in Table 2. The difference between the backward and forward schemes should be expected to increase for transport operations with higher variation.

4. DISCUSSION AND CONCLUSION

In this paper we have discussed differences and similarities between two common computational approaches for full vehicle simulation models when estimating energy usage: the backward (or inverse dynamic) scheme and the forward (or causal) scheme. Two research questions were formulated:

(1) Are there differences in vehicle behaviour between the backward and forward schemes?
(2) Do the backward and forward schemes yield the same expected CO_2 emissions and exhibit the same variance?

Concerning the second question, a simple theoretical example showed that the instantaneous input power P_{in} neither has the same expected value nor variance in the two schemes. By the mathematical relation between the input power and the CO_2 mass, it follows that the expected emissions and the variance cannot be identical either, in general. The variation was greater in the forward method, while the expected value could be either, under the fairly loose assumptions used. In the numerical example, the backward model showed both a lower average emission and less variation. It was found that the differences were statistically significant at the 95% confidence level.

For the first question, it is indeed the case that a simulated vehicle behaves differently in the two methods. The difference can be seen in Fig. 2: the backward method relies on the initial assumption that the vehicle speed is identical to the target, while the forward method treats it as a dynamic state variable. When the target speed can be reached within the limitations of the powertrain capabilities, the backward scheme follows it perfectly. When the target cannot be reached, the backward model constrains the power from the prime mover to the greatest possible. In the forward scheme, fluctuations in vehicle speed appear because of the driver model trying to control the vehicle on a road with varying properties: topography, in this case. No special treatment is needed to deal with powertrain constraints. However, operation at the limit of maximum power results in very similar behaviour in the two schemes.

The backward method is easier to work with, in general. It does not require a driver model, which reduces the overall complexity. The scheme is suitable to use when the problem in question can be treated as steady state or quasi steady state, and the dynamics of the mathematical (vehicle) model are relatively simple. However, when the model complexity increases (i.e. more dynamic states), the mathematical problem can be difficult or impossible to solve in this way. Therefore, the backward approach can be infeasible for complex models, for example when the mechancal components of the powertrain are modelled in detail. From a vehicle behaviour perspective, the backward method can follow an input speed exactly given that the powertrain of the vehicle in question is physically able to. It is a useful option when combined with vehicle log data to reconstruct specific situations or entire missions, to analyse component and function actions in retrospect.

The forward method has a greater complexity, in general, because it strictly requires a driver model to function. The scheme is well suited for modelling complicated dynamics, as the mathematical problem is inherently treated as a system of differential algebraic equations. The direct relation between input speed and vehicle speed does not exist in the forward method, which means that it cannot generally reproduce an exact behaviour. However, the effect is that the influence from the environment is reflected in a natural way, like the fluctuations in Fig. 2 and Fig. 3. This type of variation can be seen in log data from real driving (Pettersson et al., 2018), but whether the real and simulated variations match in magnitude (given some metric, say, the root mean square) is a tuning problem of the driver model parameters. The presence of a driver introduces many such parameters, and these are degrees of freedom that can seem arbitrary as it is rarely obvious how to choose their values. This need not be a disadvantage. Instead, it should be seen as an opportunity to represent different driving styles and their effect on the longitudinal vehicle action. The fact that the forward method requires a driver model to control the vehicle interfaces means that it is conceptually closer to the situation in reality than a corresponding backward method, at the cost of greater complexity. In addition, ill-chosen numerical values may result in an unrealistic vehicle behaviour, though this can be ameded to an extent by interpreting the model parameters physically and evaluating their plausibility.

We have avoided to compare the two approaches with respect to computational time because of two reasons. Firstly, the numerical solver algorithm in the backward model (VECTO) is unknown. Secondly, the models are not implemented in the same language. A fair and controlled comparison was therefore not possible. However, it still deserves to be mentioned that the backward simulations ran about ten times faster than the forward simulations.

The forward models are public and available for download from `www.chalmers.se/vehprop`.

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