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Towards rigorous boundary value level sensitivity analyses using FEM

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Abstract. The increased complexity of contemporary constitutive models for soils requires a rigorous method to evaluate the effect of the large number of model parameters on the results. Ideally, the interaction effects between the individual parameters should also be quantified. This is achieved by combining a state-of-the-art global sensitivity method with a general purpose Finite Element Method (FEM) for geotechnics. The method is tested for the non-trivial example of coupled hydro-mechanical response of clay in oedometric compression. The results indicate that proposed method for rigorous sensitivity analysis provides a feasible, yet more powerful, alternative to the method commonly used by engineers, i.e. the sequences of one-factor-at-a-time (OFAT) trails.

1. Introduction

Advanced modelling of the soil behaviour is essential to address geotechnical design challenges. The constitutive models, utilised in most Finite Element Methods (FEM), that accurately predict the in-situ soil behaviour near geo-structures are typically complex non-linear relations. Necessary features (e.g. hardening laws) are added to the models to improve the prediction accuracy and describe mechanisms of relevance. Each new feature, on average, requires two additional model parameters. Among the constitutive models for natural sensitive clay, Creep-SCLAY1S offers the best trade-off between prediction accuracy and number of model parameters. In addition to the isotropic Modified Cam clay[1], Creep-SCLAY1S was extended with anisotropy, bonding, and creep properties of normally consolidated soft clay[2, 3]. This resulted in a total of 14 parameters that require careful model calibration. In addition, some of these model parameters might be interdependent. In order to appropriately address these difficulties, a more rigorous method should be applied for sensitivity analysis of model parameter interactions and their impact on the results.

A common sensitivity analysis method among geotechnical engineers and scientists is one-factor-at-a-time (OFAT). Despite its popularity, the OFAT can be inefficient and slow. In contrast to the OFAT approach in which only one parameter is varied whilst maintaining the

other parameters constant, a designed experiment is more economical. [4] argues that a designed experiment is more effective when two or more factors are part of the the analysis.

In this paper the designed experiment approach is combined with FEM to study the sensitivity of Creep-SCLAY1S using a variance-based method. As opposed to prior work of [e.g. 5] the proposed work is developed for boundary value level analyses.

2. Theory
2.1. Creep-SCLAY1S constitutive model
Creep-SCLAY1S is a constitutive model for natural sensitive clay which incorporates creep behaviour using a viscoplastic formulation [3]. The model inherits the anisotropic, and degradation bonding features from preceding variants [2].

The model has three surfaces with the same shape and orientation. A Normal Consolidation Surface (NCS) is defined with the size of p'_m and inclination α, i.e. a representation of anisotropy. Elastic and viscoplastic strains participates at all time within this surface. Another surface, representing the Current (effective) Stress State (CSS) with the size of p'_{eq} is introduced. In this surface the soil behaves mainly elastic. The yield surface of the same soil, with the same stress history, void ratio excluding the bonds is introduced by the name of Intrinsic Compression Surface (ICS). While loading, with a stress path moving the CSS towards NCS, the viscoplastic strains start to become significant and the ICS will expand or shrink depending on the flow rule. The moment the stress path crosses through the NCS surface, large creep strains will develop, see Figure 1.
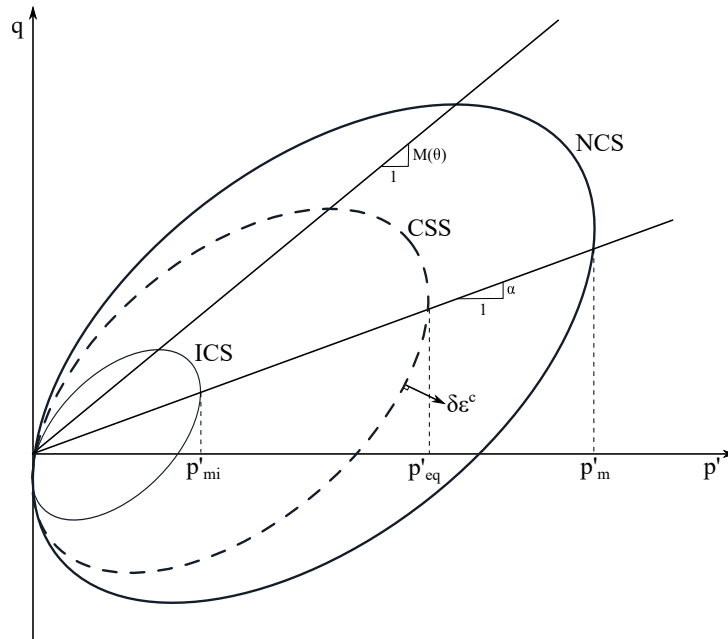
Figure 1. Yield surfaces of Creep-SCLAY1S .

The description of each model parameter is presented in Table 1. [3] elaborates the definitions and assumptions of Creep-SCLAY1S model parameters in more detail. The parameter set is similar to [5]. However, in this work the sensitivity of the model parameters are evaluated for a virtual oedometer test.

Table 1. Description of Creep-SCLAY1S model parameters.

Parameter	unit	Description
κ^*	$-$	Modified swelling index
ν'	$-$	Drained Poisson's ratio
λ_i^*	$-$	Modified Intrinsic parameter related to irrecoverable compression
M_c	$-$	The slope of critical state line in compression
M_e	$-$	The slope of the critical state line in extension
ω	$-$	Absolute effectiveness of rotational hardening
ω_d	$-$	Relative effectiveness of rotational hardening
a	$-$	The absolute rate of destructuration
b	$-$	The relative rate of the destructuration
OCR	$-$	Over-consolidation ratio
σ_{p_0}	$-$	The initial pre-consolidation pressure
e_0	$-$	Initial void ratio
α_0	$-$	The initial inclination of the reference surface
χ_0	$-$	The initial amount of bonding
τ	d	The reference time in days
μ_i^*	$-$	The intrinsic modified creep index

2.2. Sensitivity analysis
Sensitivity analysis is used to capture the importance of each model parameter (input factor) on the response of the model that is studied. These techniques may be divided into two classes, i.e. Local and Global analysis. In the Local analysis the changes of the output with respect to each input parameter is studied. The Local analysis is efficient for simple functions, however, they are potentially inaccurate for non-linear models. Global sensitivity approaches capture the uncertainty in the output of mathematical or numerical models through the space of input factors. All parameters are altered at the same time to cover the parametric space. Moreover, they have the advantage of capturing the interaction effects among the input factors.

Sobol method is a Global sensitivity analysis approach[6]. The method attributes how the variance of the output depends on the uncertain model parameters. Therefore, the decomposition of the variance is written as eq. (1):

$$V(f) = \sum_{p=1}^{p} V_i + \sum_{1 \leq i \leq j \leq p} V_{ij} + \ldots + V_{1,\ldots,p} \tag{1}$$

Where $V(f)$ is the total variance of the function f; p is the number of parameters within function f; V_i is the first-order variance from i-th parameter, V_{ij} is the second-order contribution from interaction between parameters i and j; and $V_{1,\ldots,p}$ represents the higher-order interactions [7]. The first-order Sobol index is defined as:

$$S_i = \frac{V_i}{V} \tag{2}$$

The total sensitivity index shows the total contribution to the output variation, i.e. the sum of first-order plus all higher-order effects due to interactions [7].

$$S_{T_i} = S_i + S_{ij} + S_{ij\ldots p} \tag{3}$$

The difference between the first index and total index of a parameter, represents its interaction with the other parameter within the function.

3. Geotechnical example

3.1. Finite Element model

The 2D axisymmetric model with 50 mm diameter and 20 mm height is simulated at boundary value level using [8]. A coupled analysis is performed where for the clay, Creep-SCLAY1S is used and for the groundwater flow and consolidation, the storage equation is solved simultaneously. The model presented in Figure 2 is meshed with 56 *tria6* elements which have 2nd order shape functions. The sample is horizontally fixed on the sides, and vertically fixed on the bottom. The top edge flow boundary is open with zero pore pressures $u=0$ kPa. The remaining flow boundaries are closed. Therefore, the sample is consolidated from the top only, see Figure 2. A distributed load is applied on the top edge of the sample following the time-series (representing an incremental loading oedometer test programme) in Figure 3.
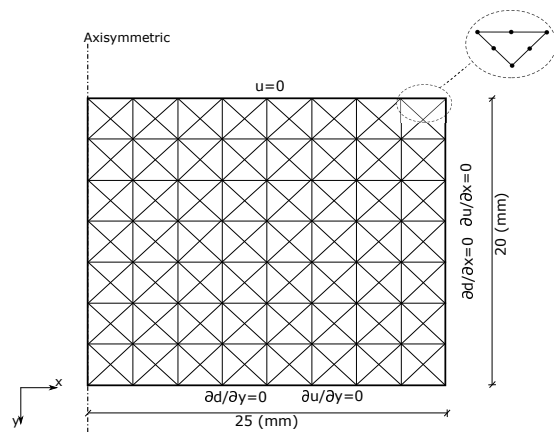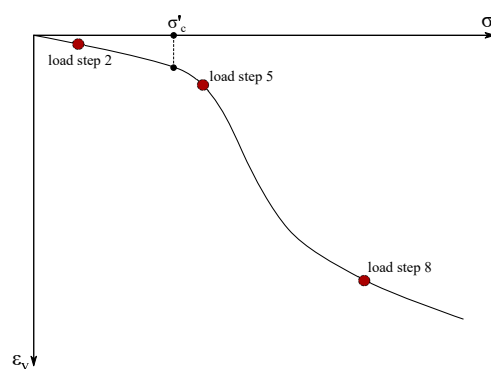

Figure 2. Geometry, mesh, and boundary conditions of the sample.
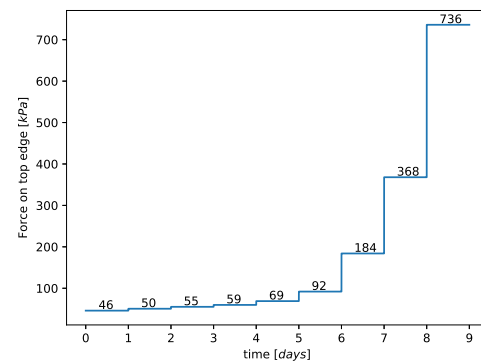
Figure 3. Loading-sequence of the oedometer test.

The consolidation is a transient time dependent phenomenon. Three points on the $\varepsilon_v - \sigma'$ are used for the subsequent sensitivity analysis. These points are sampled at the end of load steps 2, 5 & 8 and are shown in Figure 4.

Figure 4. Choices of load steps.

3.2. Sobol analyses

The preliminary study is performed using $N = 20$ trajectories for the 15 factors presented in Tables 2 and 3. The resulting total number of realisations using Sobol method is $(2k + 2) \times N$, where k is total number of parameters and N is number of trajectories. Therefore, in this study a total number of 640 realisations are executed for the sensitivity analyses. The sample creation and analyses are performed using Sensitivity Analysis Library (SALib)[9].

Table 2. Creep-SCLAY1S ranges

Parameters	unit	mid-value	min	max
κ^*	–	0.025	0.0225	0.0275
ν'	–	0.2	0.18	0.22
λ_i^*	–	0.085	0.077	0.094
M_c	–	1.3	1.17	1.43
M_e	–	0.9	0.81	0.99
ω	–	33	29.7	36.3
ω_d	–	0.872	0.785	0.959
a	–	10	9	11
b	–	0.4	0.36	0.44
OCR	–	1.386	1.247	1.525
α_0	–	0.42	0.378	0.462
χ_0	–	20	18	22
μ_i^*	–	1.5×10^{-3}	1.35×10^{-3}	1.65×10^{-3}

The ranges in Table 2 are taken from *set1-range1* of [5]. In addition the boundary value analysis in FEM allows to study the effect of the hydraulic parameters, such as the horizontal and vertical hydraulic conductivity. The hydraulic conductivity of a comparable sensitive clay is used in the simulations, i.e. $10^{-9} \, \mathrm{m\,s^{-1}}$. Both the horizontal (x) and vertical (y) component are varied following Table 3. The upper and lower limit respectively represent 90% and 110% of the mid-value.

**Table 3.** Hydraulic conductivity ranges

Parameters	unit	mid-value	min	max
k_x	$\mathrm{m\,d^{-1}}$	8.81×10^{-5}	6.82×10^{-5}	1.34×10^{-4}
k_y	$\mathrm{m\,d^{-1}}$	8.81×10^{-5}	6.82×10^{-5}	1.34×10^{-4}

## 4. Results and discussion

4.1. Oedometer FE simulation

The compression curve for all 640 realisations are plotted in grey in Figure 5. Similar to the laboratory data this curve is constructed from the final settlements for each load step. The black line represents the response for the simulation using the the mid-values for the parameters presented in tables 2 and 3. The curve consists of a fairly flat initial stiff section followed by a nearly straight inclined section after the pre-consolidation pressure is reached. The response of each curve differs as function of the combination of parameters used. This shows that the

model is quite sensitive to the changes of its parameters. This will be investigated carefully in Section 4.2.
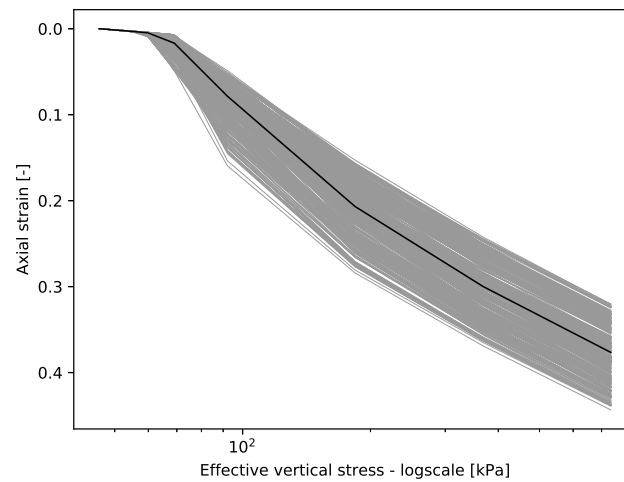
Figure 5. The $\varepsilon_v - \log \sigma'$ compression curve. Realisations from the sensitivity analysis are represented by grey lines. The black line shows the response of the mid-value parameter set.

Figure 6 shows the response of the pore water pressures. The coupled FE formulation correctly captures the generation and dissipation of excess pore water pressures. Larger excess pore water pressures are generated for larger load steps beyond the pre-consolidation pressure.
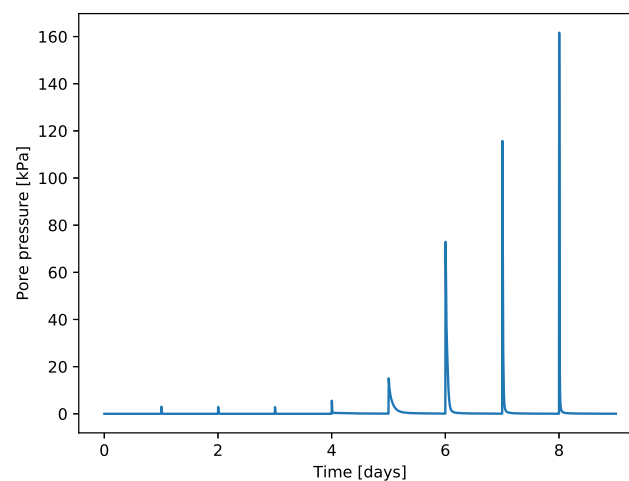

Figure 6. Simulated excess pore water pressures for a typical realisation.

4.2. Sobol results

The Sobol sensitivity indices are presented in Figures 7 to 9. The two most sensitive parameters at the end of load step 2 are shown in the Figure 7. κ^* i.e. the slope of the initial part of the stress-strain curve is the dominating factor following by the Poisson's ratio. It is clear that

the elastic parameters have the main effect since load step 2 is located on the linear branch before the pre-consolidation pressure of compression curve. According to this preliminary study, Poisson's ratio at this load step has some interaction effects with the other parameters involved.
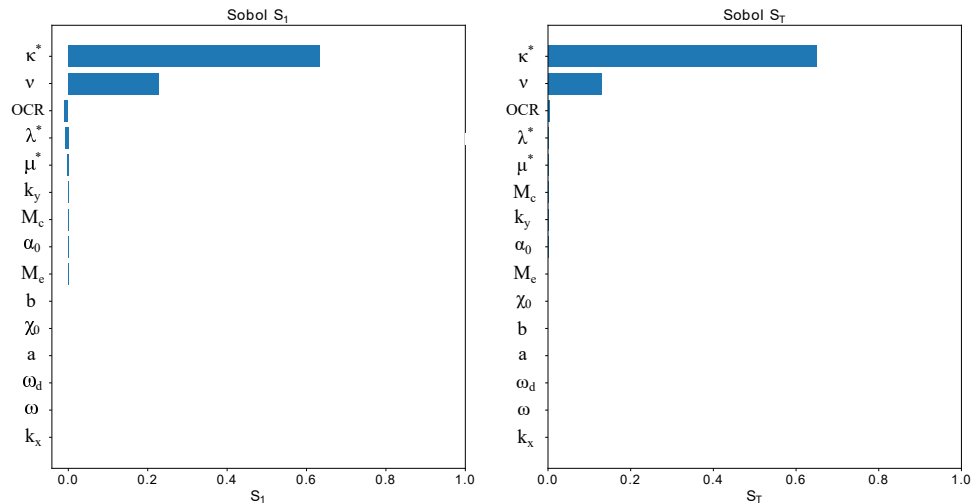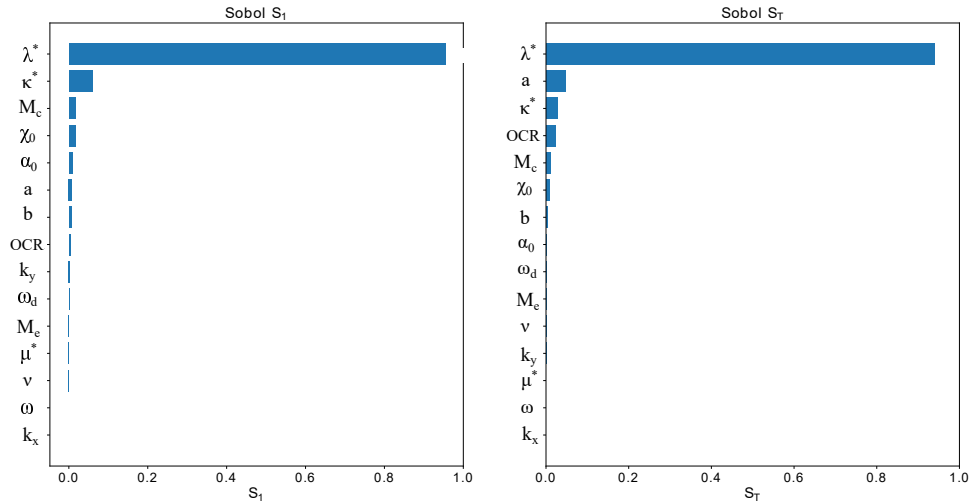

Figure 7. Sobol sensitivity indices on ε_v at the end of load step 2.

It is shown in Figure 8 that the overconsolidation ratio (OCR) is the most important parameter among others for the step close to the pre-consolidation pressure. The main effect of λ^*, i.e. the Sobol's first sensitivity index, has increased compared to Figure 7. The OCR, λ^*, and M_c has interaction effect with other parameters according to the total sensitivity index, S_T, shown in Figure 8. Investigating the detailed interaction of these factors can be the subject of future studies.
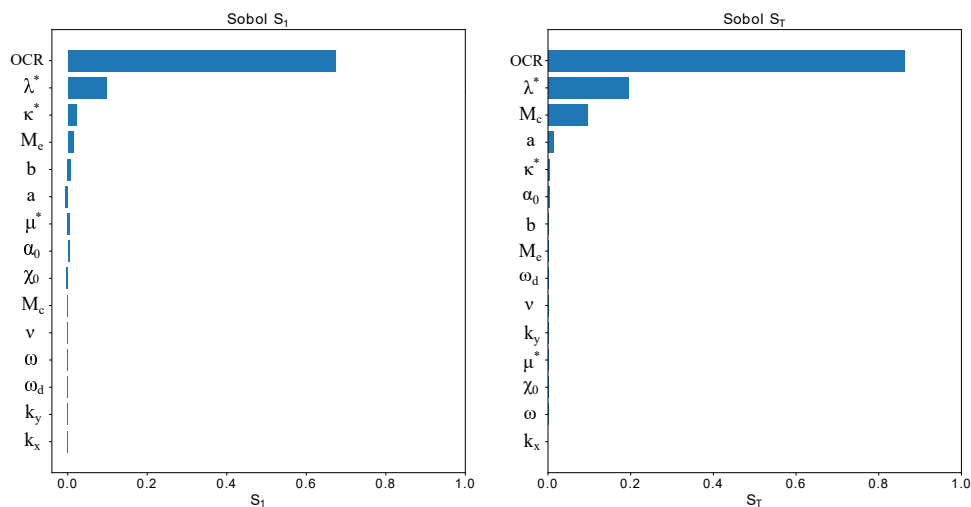
Figure 8. Sobol sensitivity indices on ε_v at the end of load step 5.

As it it presented in Figure 9, λ^* the modified compression index has a dominating effect. Load step 8 is located at the end of the stress-strain plane, therefore, it is reasonable to have the majority of effects from λ^*. Based on the values of the total sensitivity index, factors a, κ^*, and OCR have minor interaction with the other parameters studied.

Figure 9. Sobol sensitivity indices on ε_v at the end of load step 8.

This preliminary investigation demonstrate that the effect of Creep-SCLAY1S parameters evolves during subsequent loading. Depending on the stress state, certain model features become increasingly more important, whilst other parameters reduce in order of importance. For this loading path and for the selected response criterion used in the Sobol analyses (i.e. the vertical strain) only a few number of the studied parameters are important on the chosen response. This is true for all the selected load steps studied, i.e. Figure 4. The parameters for hydraulic conductivity , i.e. k_x and k_y, do not show a remarkable sensitivity among the others. The main reason is the fact that the range studied for the hydraulic conductivity was relatively small. Furthermore, the load step duration was chosen sufficiently long (similar as in the lab) to ensure full dissipation of the excess pore water pressures in all realisations.

The Sobol's total index in this study differs from that in [5]. This discrepancy can be attributed to the loading path used in the simulation. [5] investigated a CRS loading path at a single integration point, hence explicit modelling of the water flow. Moreover, an error function is chosen as the response to carry out the analysis in [5] which is not the case in this study.

5. Conclusions

An advanced method for design of experiment was successfully implemented for boundary value level geotechnical FEM analyses. Compared to prior work, this allows to incorporate both the model parameters of the Creep-SCLAY1S and additional numerical variables from the FE formulation in the sensitivity analysis. The implementation is demonstrated by using a Sobol analysis in conjunction with the non-trivial problem of a coupled analysis of a 1D incremental loading oedometer test.

The results indicate that the sensitivity of certain model parameters are dependent on the load step, i.e. κ^* governs the initial response of the compression curve, whereas the OCR

dominates in the step near the pre-consolidation pressure and λ^* for the load step representing the largest stress level.

In conclusion, the results indicate that proposed method for rigorous sensitivity analysis provides a feasible, yet more powerful, alternative to the method commonly used by engineers, i.e. the sequences of one-factor-at-a-time (OFAT) trails.

6. Acknowledgments

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