THESIS FOR THE DEGREE OF LICENTIATE OF PHILOSOPHY

Mathematical Modeling, Optimization and Scheduling of Aircraft's Components Maintenance and of the Maintenance Workshop

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Abstract

This thesis concerns the simultaneous scheduling of preventive maintenance for a fleet of aircraft and their common components along with the maintenance workshop, to which the components are sent for repair. The problem arises from an industrial project with the Swedish aerospace and defence company Saab.

While an aircraft operates, its components deteriorate and in order for it to remain operational, maintenance of its components is required. Components that are to be maintained are sent to the maintenance workshop, which schedules and performs all maintenance activities. Our modelling is based on a mixed-binary linear optimization model of a preventive maintenance scheduling problem with so-called interval costs over a finite and discretized time horizon. We extend this scheduling model with the flow of components through the repair workshop, including stocks of spare components as well as of damaged components to be repaired. Along with the scheduling problem, we address and analyze two different contracting forms between the two stakeholders: aircraft operator and maintenance workshop. Namely, an availability of repaired components contract and a repair turn–around–time contract of components sent to the maintenance workshop. We present both an individual and a type-based component flow modeling. Our model is able to capture important properties of the results from the contracting forms and it can be utilized for obtaining a lower limit on the optimal performance of a contracted collaboration between the stakeholders.

Keywords: Maintenance Optimization, Workshop Scheduling, Mixed-Binary Linear Optimization Model, Contracting Forms, Simultaneous Scheduling, Multi-Objective Optimization, Mathematical modelling

List of publications

This thesis is based on the work represented by the following papers:

I. **Obradović, G.**, Strömberg, A.-B, Lundberg, K. Simultaneous scheduling of replacement and repair of individual components in systems subject to operations.
Submitted to *Annals of Operations Research* (2021)

II. **Obradović, G.**, Strömberg, A.-B, Lundberg, K. Replacement and repair of common components in systems subject to operations.
Manuscript (2021)

Additional papers not included in this thesis:

III. **Obradović, G.**, Strömberg, A.-B, Lundberg, K. Simultaneous scheduling of preventive system maintenance and of the maintenance workshop.
Published in PLANs forsknings- och tillämpningskonferens (2020)

Author contributions

I., II., III. I worked on modeling, implementation, creation of data sets, running simulations and creating experiments, writing the manuscript.

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Gothenburg, 2021

Contents

1 Introduction

When planning the maintenance for any system (Wang and Pham, 2006, Ch. 3), the decisions to be made concern when each of its components should be maintained (i.e., replaced, repaired, or serviced) and what kind of maintenance should then be performed, with respect to the operational schedule of the system. *Preventive maintenance* (PM) (Tzvetkova and Klaassens, 2001) can often be planned well in advance, while *corrective maintenance* (CM) is done after a failure has occurred, which may come on very short notice. Typically, CM is costly, partly due to the short notice, partly because it may also cause damages to the system. On the other hand, an unexpected but necessary CM action may provide an opportunity for the PM at which the maintenance actions can be rescheduled, starting from the system's current state. While both PM and CM are aimed at restoring the components in order to put the system back in an operational state, CM is often much more costly than PM, due to a longer system down-time and also due to possible damages to other components caused by the failure.

Maintenance optimization means deciding which maintenance activities to perform, and when, such that one or several objectives are optimized. Maintenance optimization models are extensively studied in the literature (see the surveys Dekker et al. (1997), Nicolai and Dekker (2008)) and have impact on both costs and efficiency of the maintenance actions.

1.1 Problem definition

We present an application from the aerospace industry, in collaboration with a Swedish aerospace and defence company Saab. On one side, we consider a system of aircraft that has an operational demand to fulfill, and on the other, the maintenance workshop (Saab) that repairs the components coming from the aircraft and makes them available for usage again, as well as its supply chain. Hence, there are two stakeholders, an aircraft operator and a maintenance workshop (i.e. maintenance supplier), whose collaboration is normally predefined by a contract. We define and discuss a number of optimization objectives corresponding to two different contract types, so-called *availability* and *turn–around–time* contracts.

In Figure 1.1 we illustrate the system–of–systems governing repair and replacement of components from an aircraft. The Swedish Air Force is assigned a flight hour requirement to be distributed among the fleet of aircraft, which defines a flight assignment/scheduling problem (e.g. Gavranis and Kozanidis (2015)). After an operational/flying schedule is made, each aircraft is assigned to a timetable. Since maintenance can be done only when the aircraft is grounded, time windows of opportunities for doing maintenance are generated based on the operational timetables. Maintenance scheduling is done on the *operational level* (Level O in Fig. 1.1) where each component to be repaired is replaced with a (as good as) new component of the same component type. The component to be repaired goes to the maintenance workshop (MRO — Maintenance, Repair and Overhaul in Fig. 1.1), where it is to be scheduled for repair. The maintenance workshop is governed by Saab but there are also Original Equipment Manufacturers (OEM_i in Fig. 1.1), to which components can be sent for repair, as well as external subcontractors. Components can be repaired in the MRO but they can also be sent further, to one of the OEMs or external subcontractors outside of Saab's supply chain and maintenance operations. Joint activities between any two stakeholders are governed by a contract.

1.2 Motivation

The motivation behind this research lies in real-world applications. Any system that performs some sort of operations and undergoes maintenance can be considered in our modelling; some of numerous examples are railway and air traffic, commercial heavy vehicles, and manufacturing machines in industry (see, e.g., Robert et al. (2018), Verhoeff et al. (2015), Boliang et al. (2019), Papakostas et al. (2010) and Cassady and Kutanoglu (2005)). Performing maintenance operations in a good fashion has a high importance. Maintenance budgets represent, on average, a significant portion of the total plant operating budget, varying from a few percent in lighter manufacturing to a high percentage in equipment-intensive industries. Moreover, hidden costs are usually not accounted for. Ineffective maintenance management policies often lead to big increases in costs and most importantly, decreases in efficiency.

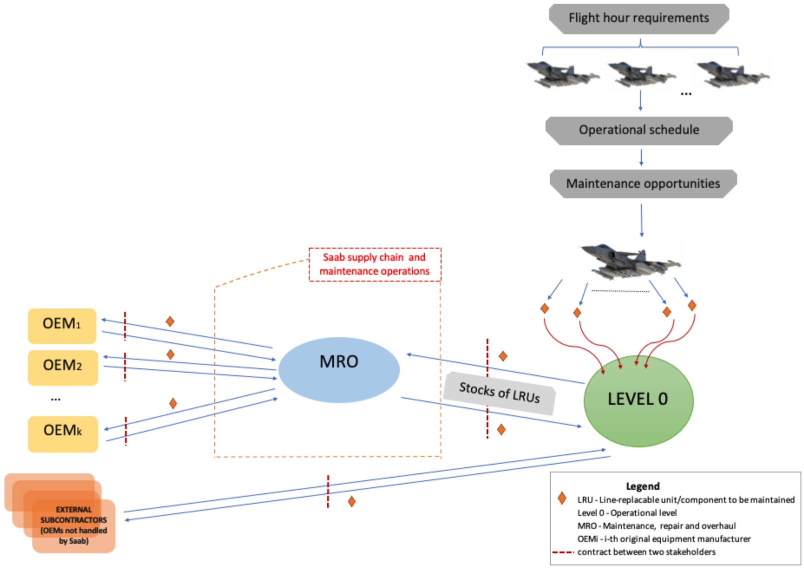

Figure 1.1: Illustration of the problem studied.

Advanced optimization models have been developed for each part of the supply chain of aircraft maintenance—from tactical scheduling of aircraft to missions or maintenance, to depot level maintenance planning and scheduling (see, e.g. Gavranis and Kozanidis (2015), Erkoc and Ertogral (2016), Brucker and Knust (2012), and Kurz (2016)). Even though there exists an interdependent relationship between production scheduling and maintenance planning, the two are mostly planned and executed separately in literature and industry. Most of the time, there is a lack in communication between the maintenance planning and the production scheduling side (Weinstein and Chung, 1999), which usually results in unmet demand and/or supply on either side, implying lower efficiency and higher costs.

The motivation for considering a tight integration of the maintenance planning for the systems and the production scheduling of the maintenance workshop, meaning that the organizations and the information they work with are fully transparent and that decisions are taken simultaneously for both stakeholders, is threefold. First, a tight integration provides a planning tool for systems in which the maintenance workshop is in reality integrated with the operating system. That would mean that the stakeholder operating the aircraft is also responsible and performs maintenance of its components. Secondly, when there is more than one stakeholder, a tightly integrated model will provide an optimistic estimate of the results—in terms of costs for maintenance, of costs for lateness (under a turn-around contract), or of the lower limit of items on the stock and/or the average availability (under an availability contract)—that could be obtained in reality and which can be used as a benchmark. Lastly, the integration enables an investigation and comparison of different types of contracts that can be set-up between the stakeholders.

1.3 Research objectives

The main goal of the project leading to this thesis is to model the integrated scheduling of the preventive maintenance of the systems and of the maintenance workshop. We approach this by formulating two models, one that takes into account individual component flows and one that doesn't but instead uses component types only. As a result, we arrive at a formal modeling of a lower bound on optimal performance for evaluating different contract types between stakeholders and a planning tool that can be further utilized as a decision support. For our modeling to be utilized further as a decision making tool, we address its complexity and aim at reducing the computing times.

We consider two stakeholders and two types of contracts between them, which are formulated as two bi-objective optimization problems. One question to answer is whether different types of contracts will—through the optimization of the corresponding objectives—advocate different planning patterns. A related and important aspect is the resulting implication on the collaboration between different stakeholders, which is highly applicable to general maintenance and supply chain problems and well suited for our modelling framework.

We analyze how changes in different parts of the system affect the solution. Examples are variations of the maintenance workshop capacity, of the requirements on the stock of components and of maintenance costs etc.

1.4 Limitations

Focusing on preventive maintenance modelling enables us to minimize risks of unexpected failures but it does not eliminate them. It may still happen that a component breaks or stops functioning unexpectedly, when there is no scheduled preventive maintenance event. Our modeling can address this situation in two ways. First, once an unexpected failure occurs, one can re-plan from that point in time while leaving the part of the schedule in the near future unchanged (to avoid big disturbances on the system operator's side). Since short-term changes in the operational schedules for the systems, as well as in the schedules for the maintenance workshop, are often inconvenient and sometimes not even feasible, the rescheduling should (if possible) be such that the solution remains fixed for a certain number of time steps. Secondly, by keeping up the level of available components, we ensure that once a component needs to be replaced, there will be no long waiting time. Furthermore, by ensuring that aircraft are available for replacing an aircraft that needs to be maintained when/if required, we would minimize the disturbance even more. However, this is not sufficient to account for the possible unexpected events and to replace the corrective maintenance planning.

Since we address an industrial problem, there are lots of parameters to be included and parts of the systems to be modelled. This makes both modeling and implementation more complicated. Therefore, we make certain simplifications. For example, instead of modeling all component types, we focus on the most important, the safety critical ones[1] (A similar situation is present e.g. in nuclear power plants; see Day and George (1981)).

Another obstacle for obtaining more interesting results is the lack of real data. We tried to get access to real data form Saab but due to data classification and confidentiality, we did not succeed yet. Instead, we create and randomize all the data used, which makes it harder to make conclusions about (some of) the results. In addition, we do not model the parameters that are judged not to be in the scope of our research (e.g., the cost of maintaining a component is fixed and its effect on the solution in the tight integration can thus be neglected). If needed, these parameters could easily be included.

As we introduce more features to our model, its complexity grows. In order for it to be utilised as a decision support tool, computing times should be reasonably small. If, on the other hand, our model would be used as a planning tool (which means it would be used for example few times per year), longer running times would not be a big disadvantage.

1.5 Outline

The outline of this thesis is as follows. In Chapter 1 the problem stated by Saab, which provided the starting point for determining the framework, is described. The mathematical optimization background needed to understand

[1] A component is called *safety critical* if its failure could lead to an engine breakdown, possibly with catastrophic consequences.

the work presented in the thesis is given in Chapter 2. Then, we present the mathematical modeling of the problem in Chapter 3, including decision variables, optimization constraints and objective functions. The appended papers are summarized in Chapter 4 while the main conclusions and future research questions are presented in Chapter 5.

2 Mathematical modeling

A general optimization problem can be formulated ass

$$\text{minimize} \quad f(\mathbf{x}), \tag{2.1a}$$
$$\text{such that} \quad \mathbf{x} \in \mathcal{X}, \tag{2.1b}$$

where $f : \mathbb{R}^n \mapsto \mathbb{R}$ is an *objective function* and the *decision variables* are denoted by $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$. The set $\mathcal{X} \subset \mathbb{R}^n$ defines the *feasible solutions* to the problem. Usually it has the form $\mathcal{X} := \mathbf{x} \in \mathbb{R}^n : \{g_i(\mathbf{x}) \leq b_i, \quad i = 1, \dots, m\}$, where $g_1, \dots, g_m$ are functions and $b_1, \dots, b_m$ are given parameters. Depending on how the functions mentioned are specified and which assumptions are made regarding feasible values on the variables, we obtain different problem classes as, e.g., linear optimization (LP), non-linear optimization (NLP), integer linear optimization (ILP).

This section presents a background for the mathematical modeling and optimization methods used within the thesis.

2.1 Mixed integer linear programming (MILP)

A mixed-integer linear program is an optimization problem with affine/linear objective and constraint functions and integral requirements on some of the variables. Every MILP problem can be expressed as

$$z^* := \text{minimum} \quad \mathbf{c}^{\mathrm{T}}\mathbf{x}, \tag{2.2a}$$
$$\text{such that} \quad A\mathbf{x} \geq \mathbf{b}, \tag{2.2b}$$
$$\mathbf{x} \in \mathbb{Z}^{n_1} \times \mathbb{R}^{n_2}, \tag{2.2c}$$

where $n = n_1 + n_2$ is the dimension of the variable space, m is the number of inequality constraints, A is an $m \times n$ matrix and **b** and **c** are vectors. MILP problems are NP-hard (e.g. Ch. 1.3 Conforti et al. (2014), Section 1.3.3), which means that the time to solve the model (in the worst case) is exponential as a function of the instance size (i.e., number of variables and/or constraints).

A classical example of a problem that can be formulated as a MILP is the travelling salesperson problem (TSP). Let $G = (\mathcal{V}, \mathcal{A})$ be a directed graph, where the nodes $v \in \mathcal{V}$ represent the cities and arcs $a \in \mathcal{A}$ represent the roads. There is a traveling time cost c_a associated with every arc a. If the traveling cost from city i to city j is equal to the cost from j to i, the problem is called symmetric TSP. Otherwise, it is asymmetric. TSP can be modelled as MILP and one of the formulations given by (Miller et al., 1960), is expressed as to

$$\text{minimize} \quad \sum_{a \in \mathcal{A}} c_a x_a, \tag{2.3a}$$

$$\text{such that} \quad \sum_{a \in \delta^+(i)} x_a = 1, \qquad i \in \mathcal{V}, \tag{2.3b}$$

$$\sum_{a \in \delta^-(i)} x_a = 1, \qquad i \in \mathcal{V}, \tag{2.3c}$$

$$u_i - u_j + (n-1)x_{(ij)} \leq n - 2, \qquad (ij) \in \mathcal{A} | i, j \neq s, \tag{2.3d}$$

$$u_i \in [1, n-1], \qquad i \in \mathcal{V} \setminus \{s\}, \tag{2.3e}$$

$$\mathbf{x} \in \{0, 1\}^m. \tag{2.3f}$$

The decision variables u_i denote the order in which the nodes (cities) are being visited. The constraints (2.3b) and (2.3c) ensure that each node has one entering and one leaving arc, where $\delta^-(i)$ and $\delta^+(i)$ denote the set of arcs entering and leaving node i, respectively. To prevent subtours, i.e., to ensure that the solution admits only one connected tour and not multiple disjoint tours, the constraints (2.3d).

2.2 Multi-objective optimization

In multi-objective mathematical programming there are more than one objective functions and most of the time, there is no single optimal solution that optimizes all objective functions at the same time. Then, it is the decision maker who chooses the most preferred solution. Consider the optimization problem to

$$\begin{aligned} \text{minimize} \quad & \{f_1(\mathbf{x}), \ldots, f_K(\mathbf{x})\} && (2.4a) \\ \text{such that} \quad & \mathbf{x} \in \mathcal{X} && (2.4b) \end{aligned}$$

where $K \geq 2$ is the number of possibly conflicting objective functions $f_k : \mathbb{R}^m \to \mathbb{R}, \quad k = 1, \ldots, K$, that are to be optimized simultaneously. The optimization problem (2.4) is a so-called ***multi-objective optimization problem*** (Ehrgott, 2005). If there exists a solution that is optimal with respect to all K objectives, that is a trivial case, since there is no conflict between objectives. We assume that such solutions do not exist in the model (2.4).

Naturally, the question of defining optimality for multi-objective problem arises. For that, we define *Pareto optimality* (see, e.g., Luc (2008)).

A point $\mathbf{x}^* \in \mathcal{X}$ is ***Pareto optimal*** in the multi-objective optimization problem (2.4) if and only if there does *not* exist any point $\mathbf{x} \in \mathcal{X}$ such that $f_k(\mathbf{x}) \leq f_k(\mathbf{x}^*), \quad k \in \{1, \ldots, K\}$, and $f_\ell(\mathbf{x}) < f_\ell(\mathbf{x}^*)$ for at least one $\ell \in \{1, \ldots, K\}$. All Pareto optimal points (possibly an infinite number) constitute ***Pareto optimal set*** or *Pareto front*, which is usually at least one dimension less than the variable space. There are many ways of exploring a Pareto front (see Marler and Arora (2004)). The most common one is to solve single objective problems created from the multi-objective problem through (some sort of) scalarization procedure (e.g. the weighted sum method, or the ϵ - constrained method; see Ehrgott (2006)). Since all solutions on the Pareto front are equally good, it is a *decision maker* who is required to choose one out of the set of all Pareto optimal solutions. In practice, sometimes not all solutions on the Pareto front may be computed, so the decision maker will choose one of the solutions computed.

The most commonly used scalarization method for obtaining points on Pareto fronts is the ϵ - constraint method where one of the objectives is optimized while the other objectives are turned into constraints and expressed as to

$$\begin{aligned} \text{minimize} \quad & f_j(\mathbf{x}), && (2.5a) \\ \text{such that} \quad & f_k(\mathbf{x}) \leq \epsilon_k, \quad k \in \{1, \ldots, K\}, k \neq j && (2.5b) \\ & \mathbf{x} \in \mathcal{X}. && (2.5c) \end{aligned}$$

By parametrical variation in the RHS of the constrained objective functions (ϵ_k) the efficient solutions of the problem are obtained. Results about the method can be found at Chankong and Haimes (1983). Since upper bound constraints on objective values, as expressed in (2.5b), are knapsack constraints[1], the problem (2.5) is usually an NP-hard problems (Conforti et al., 2014, Ch. 1.3), which means that we face computationally expensive problems.

2.3 Complexity

Complexity theory is used to determine how long time it takes to solve certain classes of problems. The *algorithmic complexity* tells us about the dependence of the computational time for an algorithm on the problem size. We are interested in estimating how the computational time changes (usually, increases) as the problem size increases. The *problem complexity*, on the other hand, helps us to classify how easy or difficult various problem classes are to solve.

When we analyze and classify optimization problems with respect to their problem complexity, we study a transformation of the problem called *a decision problem*. A decision problem is formulated such that the answer is always either *yes* or *no*. In general, an optimization problem is not harder to solve than its corresponding decision problem.

An algorithm is said to be of polynomial time if its running time is upper bounded by a polynomial expression in the size of the input for the algorithm. P is a complexity class that includes the set of all optimization problems whose corresponding decision problems can be solved in polynomial time. That is, given an instance of the problem, the answer yes or no can be decided in polynomial time. These problems are usually refereed to as "easy" problems.

A larger class of problems, including the class P, is *non-deterministic polynomial*, denoted by NP. It is a complexity class that represents the set of all decision problems with the property that for each given solution and corresponding yes answer, there exists a polynomial algorithm that can be used to verify that the yes answer is correct. A decision problem is NP-hard if any NP problem can be reduced to it in polynomial time. A decision problem is NP-complete if it is in NP and it is NP-hard (Conforti et al., 2014, Ch. 1.3).

[1]Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

3 Problem description

We present the large scale system-of-systems (see Fig. 1.1, Ch. 1) and a subset of it is subject to the work presented in this thesis. From now on, we denote by MRO as the Maintenance Workshop and Level O as the Operational level. In this chapter, we take a closer look at each part of the problem we are dealing with, namely aircraft maintenance scheduling, maintenance workshop, stocks of components, and operational demand. We present a mathematical modeling of each part of the system, as well as their interconnections, and formulate optimization objectives for the respective stakeholders.

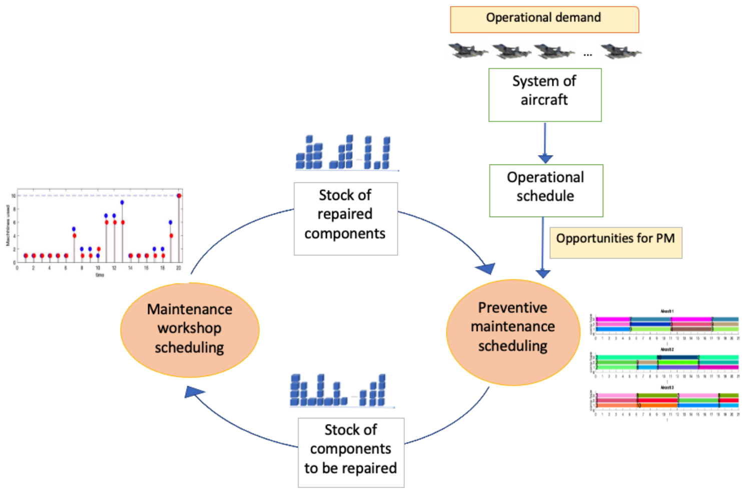

Figure 3.1: Aircraft (preventive) maintenance scheduling and the maintenance workshop scheduling, with the operational demand as input and the scheduling of component replacement and repair as output.

3.1 Aircraft maintenance scheduling

The model of the maintenance scheduling problem presented is partly based on the *preventive maintenance scheduling problem with interval costs* (PMSPIC) model presented in (Gustavsson et al., 2014). The PMSPIC considers a system with multiple component types and for which the costs for replacement of components take into account the interval between any two consecutive replacements/maintenance occasions; we generalize this model such that we allow for more systems and individual component modeling. The PMSPIC is partly an extension of the opportunistic replacement problem (ORP) studied in (Almgren et al., 2012), described as follows: "The system consists of a set of components. The time between two consecutive replacements of a component may not exceed its assigned maximum replacement interval. To each time point in the planning period corresponds a fixed maintenance set-up cost and replacement costs for each component. The problem is to schedule the component replacements over a finite set of time points in order to minimize the total maintenance cost." Unlike the ORP model, the PMSPIC takes into account the intervals between two replacements/maintenance occasions for each component and assigns a cost depending on the length of this maintenance interval.

We consider a fleet of $|\mathcal{K}|$ aircraft with $|\mathcal{I}|$ component types and $|\mathcal{J}_i|$ individual components of each type $i \in \mathcal{I}$. Maintenance can be scheduled at any time step t within the finite and discretized planning horizon $\mathcal{T}$. A maintenance occasion of an aircraft k at time step t generates a maintenance cost. The maintenance interval (i.e., the interval between two maintenance occasions) of a component generates an interval cost, which is non-decreasing with the length of the interval. For each component type, by defining substantially higher costs for scheduling maintenance after —and also close before—the end of its life, unexpected failures are avoided; thereby our approach may stay within the scope of PM scheduling. We model this problem, denoted as GPMSPIC, as a 0-1 mixed-integer linear optimization problem (see Conforti et al. (2014)); the decision variables are described below.

Decision variables. To determine the maintenance intervals of the components, we let the decision variable x_{st}^{ijk} take the value 1 if the individual component j of type i from aircraft k receives PM at time steps s and t, but not in-between. Otherwise, $x_{st}^{ijk} = 0$. We further let the decision variable z_t^k take the value 1 if aircraft k is scheduled for maintenance at time t, and 0 otherwise:

$$x_{st}^{ijk} = \begin{cases} 1, & \text{if individual component } j \text{ of type } i \\ & \text{in aircraft } k \text{ receives PM at times } s \\ & \text{and } t \text{, but not in-between,} \\ 0, & \text{otherwise,} \end{cases} \qquad \begin{array}{l} j \in \mathcal{J}_i,\ i \in \mathcal{I},\ k \in \mathcal{K}, \\ 0 \le s < t \le T+1, \end{array}$$

$$z_t^k = \begin{cases} 1, & \text{if maintenance of aircraft } k \text{ occurs} \\ & \text{at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad k \in \mathcal{K},\ t \in \mathcal{T}.$$

Constraints. The feasible set of the maintenance planning is modelled by the following equality and inequality constraints:

$$\sum_{j \in \mathcal{J}_i} \sum_{s=0}^{t-1} x_{st}^{ijk} = \sum_{j \in \mathcal{J}_i} \sum_{r=t+1}^{T+1} x_{tr}^{ijk}, \qquad i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}, \tag{3.1a}$$

$$\sum_{j \in \mathcal{J}_i} \sum_{r=1}^{T+1} x_{0r}^{ijk} = 1, \qquad i \in \mathcal{I}, k \in \mathcal{K}, \tag{3.1b}$$

$$\sum_{j \in \mathcal{J}_i} \sum_{s=0}^{t-1} x_{st}^{ijk} \le z_t^k, \qquad i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}, \tag{3.1c}$$

$$\sum_{k \in \mathcal{K}} \sum_{s=0}^{t-1} x_{st}^{ijk} \le 1, \qquad j \in \mathcal{J}_i, i \in \mathcal{I}, t \in \mathcal{T}, \tag{3.1d}$$

$$x_{st}^{ijk} = 0, \qquad j \in \mathcal{J}_i, k \in \mathcal{K}, \quad \bar{t}_i \le s + \bar{t}_i < t \le T+1, i \in \mathcal{I}. \tag{3.1e}$$

For each system k and component type i, a maintenance interval starts at time 0, which is modeled by (3.1b), while the constraints (3.1a) ensure that the same number (i.e., 0 or 1) of maintenance intervals ends and starts at time t. The constraints (3.1c) model that if a maintenance interval of component type i in system k ends at time t, then maintenance of system k must occur at time t. The constraints (3.1d) ensure that each component (i, j) is in at most one system k at each time t. The constraints (3.1e) prevent any maintenance interval for component type $i \in \mathcal{I}$ from being longer than $\bar{t}_i \le T$, which prevents from having to perform corrective maintenance.

According to (Gustavsson et al., 2014)—see also (Arkin et al., 1989; Boctor et al., 2004)—the PMSPIC is NP-hard, which then implies that our aircraft maintenance scheduling problem is NP-hard. This means that the optimal scheduling of the PM occasions for the components of the aircraft is a computationally demanding problem.

3.2 Maintenance workshop scheduling

Components that should be maintained are sent to the maintenance workshop, which contains a number (L) of (identical) parallel repair lines for component repair, each of which has a repair capacity of one unit while each component repair requires one unit of this capacity per time step during a prespecified (component type-specific) and consecutive (i.e., preemption is not allowed) number of time steps. When a component arrives at the workshop, it is available for repair and (in the case of a turn-around time contract) assigned a due date, at which the repair should be finished, and the component be returned back to the aircraft operator. This problem is identified as an identical parallel machines scheduling problem (IPMSP; Brucker and Knust (2012)). For a survey of parallel machine scheduling problems, see Mokotoff (2001). In the classical deterministic IPMSP, there is a number of independent jobs to be processed on a range of identical machines. Each job has to be carried out on one of the machines during a fixed processing time, without preemption [1]. A component that finishes repair prior to (after) its due date generates a non-positive (non-negative) penalty cost, which applies only in the case of a turn-around time contract (see Section 3.5). A solution to the maintenance workshop scheduling problem specifies at which time each component arriving at the workshop should start maintenance.

Decision variables. For each individual component j of each type i and for each time step t, we define $u_t^{ij} \in \{0, 1\}$ which takes value 1 if component (i, j) starts repair at time t, 0 otherwise. The number of active parallel repair lines at each time step t is defined by the non-negative integer variable l_t:

$$u_t^{ij} = \begin{cases} 1, & \text{if component } (i,j) \text{ starts repair at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad i \in \mathcal{I},\ j \in \mathcal{J}_i,\ t \in \mathcal{T}.$$

[1] If preemption (i.e. job splitting) is allowed, the processing of any operation may be interrupted and resumed at a later time.

The number ℓ_t of active parallel machines at time t should fulfill the constraints

$$0 \leq \ell_t = \ell_{t-1} + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \left(u_t^{ij} - u_{t-p^i}^{ij} \right) \leq L, \qquad t \in \mathcal{T}, \tag{3.2}$$

where p^i is the processing time in the maintenance workshop for a component of type i and ℓ_0 and u_t^{ij}, $t \leq 0$, are initial (fixed) values that constitute input to the model. The number of active parallel machines at time t equals the number of active machines at the previous time step $t-1$ plus the difference between the ones becoming unavailable at time t (i.e., $\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} u_t^{ij}$) minus the ones becoming available at time t (i.e., $\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} u_{t-p^i}^{ij}$). At every time step t, ℓ_t is limited by the workshop capacity L.

The IPMSP with a (weighted) sum objective is polynomially solvable (Lawler et al., 1993, Ch. 8.0), whereas its version with a minimax, i.e., makespan[2], objective is NP-hard (Brucker and Knust, 2012, Ch. 2.1).

To model the interface between the variables defined for the two respective problems, we next introduce the stock dynamics.

3.3 Stock dynamics modeling

When an individual component is taken out of an aircraft it is sent—with no time delay—to the stock of damaged components, where it stays until it is scheduled for repair. The transport time between the stock of damaged components and the maintenance workshop δ_a^i is prespecified. Upon being repaired, the component goes to the stock of repaired, so called as good as new components, again with a prespecified transport time between the workshop and stock of repaired components δ_b^i, where it is kept until its scheduled time for placement into an(other) aircraft. We assume that all transport times are represented by non-negative integers.

Decision variables. To model the flow of components, we define the following binary variables: a_t^{ij} (b_t^{ij}) takes the value 1 if component (i, j) is on the stock of damaged (as good as new) components at time step t; otherwise, it takes the value 0. Furthermore, α_t^{ij} takes the value 1 if component (i, j) is taken out of some aircraft and placed on the stock of damaged components at time step t, and β_t^{ij} takes the value 1 if component (i, j) leaves the stock of repaired

[2] In manufacturing, makespan is the time difference between the start and finish of a sequence of jobs or tasks.

components and is placed in some aircraft at time step t:

$$a_t^{ij}\ (b_t^{ij}) = \begin{cases} 1, & \text{if individual component } j \text{ of type } i \text{ is on the stock of} \\ & \text{damaged (repaired) components at time } t \in \mathcal{T} \cup \{0\}, \\ 0, & \text{otherwise,} \end{cases}$$

$$\alpha_t^{ij}\ (\beta_t^{ij}) = \begin{cases} 1, & \text{if individual component } j \text{ of type } i \text{ is taken out of} \\ & \text{(placed in) one of the aircraft } k \in \mathcal{K} \text{ at time } t \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases}$$

The stock of damaged components is then modelled by the constraints

$$\alpha_t^{ij} = \sum_{k\in\mathcal{K}} \sum_{s=0}^{t-1} x_{st}^{ijk}, \qquad j \in \mathcal{J}_i,\ i \in \mathcal{I},\ t \in \mathcal{T}, \tag{3.3a}$$

$$a_t^{ij} = a_{t-1}^{ij} + \alpha_t^{ij} - u_{t+\delta_a^i}^{ij} \in \{0,1\}, \quad t \in \{1-\delta_a^i, \ldots, T+1\}, j \in \mathcal{J}_i, i \in \mathcal{I}. \tag{3.3b}$$

The constraints (3.3a) connect the variables from the maintenance scheduling with the stock of damaged components: if a component (i,j) is taken out of any of the aircraft $k \in \mathcal{K}$ at time t, α_t^{ij} will take the value 1; otherwise α_t^{ij} takes the value 0. The constraints (3.3b) provide the state of component (i,j) at time t: whether it is on the stock of damaged components (i.e., $a_t^{ij} = 1$) or not (i.e., $a_t^{ij} = 0$). The state of a component at time t depends on its state in the previous time step $t-1$, whether it is taken out of any system k and placed on the stock at time step t, and whether it is starting maintenance at time step $t + \delta_a^i$.

The stock of repaired components is modelled analogously as

$$\beta_t^{ij} = \sum_{k\in\mathcal{K}} \sum_{r=t+1}^{T+1} x_{tr}^{ijk}, \qquad j \in \mathcal{J}_i,\ i \in \mathcal{I},\ t \in \mathcal{T} \tag{3.4a}$$

$$b_t^{ij} = b_{t-1}^{ij} - \beta_t^{ij} + u_{t-\delta_b^i-p^i}^{ij} \in \{0,1\}, \qquad j \in \mathcal{J}_i,\ i \in \mathcal{I},\ t \in \mathcal{T} \tag{3.4b}$$

$$\sum_{j\in\mathcal{J}_i} b_t^{ij} \geq \underline{b}^i, \qquad i \in \mathcal{I},\ t \in \mathcal{T}. \tag{3.4c}$$

The constraints (3.4a) represent the connection between the stock of repaired components and the maintenance scheduling. If component (i,j) is placed into any aircraft k at time t, β_t^{ij} will take the value 1; otherwise β_t^{ij} takes the value 0. In (3.4b) the individual states of the components at time t are updated: a component is either on the stock (i.e., $b_t^{ij} = 1$) or it is not (i.e., $b_t^{ij} = 0$). A component's state on the stock of repaired components is affected by its state in the previous time step $t-1$, whether it is placed in some system k at time t, and whether it will arrive to the stock at time t after being repaired (i.e., if $u_{t-\delta_b^i-p^i}^{ij} = 1$, which means that component (i,j) started maintenance at time $t-\delta_b^i-p^i$ and will arrive to the stock of repaired components at time t). The variables b_0^{ij}, β_0^{ij}, and u_t^{ij}, $t \in \{1-\delta_b^i-p^i, \ldots, 0\}$, comprise (fixed) input data. Then, in (3.4c) it is expressed that the sum of the variables b_t^{ij} over the individual components, i.e., the stock level of repaired components per component type i at time t, may not be below the lower stock limit $\underline{b}^i$.

Looking at the stock dynamics equations as an isolated system, it can be formulated as a network flow model, where each equality constraint is a node balancing constraint. For example, in (3.3b), the flow from node t to node $t+1$ is defined as a_t^{ij}. Also, in an isolated stock model, α_t^{ij} and u_t^{ij} are parameters. The problem is an LP and thus can be solved in polynomial time.

3.4 Operational demand

The system of aircraft considered possesses an operational demand, represented by a flying/operational schedule that should be fulfilled. The schedules define time intervals during which the aircraft is either operating or grounded, i.e., accessible for maintenance. Therefore, the starting point for our modeling is precisely the operational demand.

For our maintenance planning problem, the schedules (i.e., operational demand) are represented in terms of time intervals when the system is either operating—at which times maintenance cannot be performed—or accessible for maintenance. In other words, PM may not be scheduled while a system is operating. In the case of railway systems (Lidén, 2020), each train is assigned time slots when it should operate (i.e., perform transports of goods or passengers); hence, PM may be scheduled only in-between those time slots. In the case of offshore wind turbine maintenance (Shafiee et al., 2013), the operational demand is fulfilled by wind energy production, while maintenance work can be done only during time periods of not too harsh weather conditions. When planning any PM occasion the (predicted or planned) operational schedules for the systems provide time windows during which maintenance may be performed. As input to the integrated GPMSPIC and IPMSP model, for all

$t \in \mathcal{T}$ and all $k \in \mathcal{K}$ we thus use the parameters

$$\overline{z}_t^k = \begin{cases} 1, & \text{if PM is allowed to be scheduled for system } k \text{ at time } t, \\ 0, & \text{otherwise,} \end{cases}$$

and include the following constraints—such that the time windows for PM are respected—in our model:

$$z_t^k \leq \overline{z}_t^k, \qquad t \in \mathcal{T},\ k \in \mathcal{K}. \tag{3.5}$$

An efficient way of generating the operational schedules (e.g., timetables) for the systems considered is presented in (Gavranis and Kozanidis, 2015), in which the availability of a fleet of aircraft is maximized subject to requirements on the transport missions and maintenance of the aircraft and their components. An alternative way is presented in (Cho, 2011), where the maximal number of aircraft in maintenance at any given time during the planning period is minimized.

We explore another way of modeling (3.5) and that is to use slightly softer constraints to model the opportunities for performing maintenance, as follows:

$$\sum_{k \in \mathcal{K}} z_t^k \leq M, \qquad t \in \mathcal{T}, \tag{3.6}$$

where (3.6) limits the number of maintenance occasions for each time step t to at most M aircraft at a time. The main benefit of using this approach is that it gives more freedom to the model to choose the optimal maintenance schedules.

3.5 Optimization objectives

We consider two stakeholders, the aircraft operator and the maintenance workshop. In Paper I, we study two contract types governing their activities (availability and turn-around time) by defining two bi-objective optimization problems (see Ehrgott (2005)). The first problem is composed by the minimization of the maintenance cost (i.e., set-up cost and interval cost) and the maximization of the availability of components on the stock of repaired components. The second problem is composed by the minimization of the maintenance costs and the minimization (maximization) of the penalties for lateness (earliness). The minimization of the maintenance cost is of interest for the aircraft operator while the other two objectives are relevant for the maintenance workshop and represent the risk for lack of spare components. In Paper II, we focus on an availability contract, defined such that the lower limit on the number of available components of type i is maximized. We again have a bi-objective formulation, with one objective being minimization of the maintenance costs and the other one maximization of availability.

The modeling of the different objectives is described below.

Minimizing costs for maintenance set-up and intervals. Each maintenance occasion yields a set-up/maintenance cost for the aircraft operator. It can be either the cost of having an aircraft grounded/unavailable for flight operations, or the cost of performing any maintenance activity. Besides this, there is an interval cost for every component which is determined based on the length of the interval between two consecutive maintenance occasions. We assume that the interval cost is non-decreasing with an increasing length of the interval. Furthermore, the longer the length of the maintenance interval is the more expensive it gets to do maintenance. Using this cost structure enable us to prevent (too) long maintenance intervals which could lead to over usage of a component and thereby, to component failure.

From the aircraft operators' point of view, the objective is to minimize the total costs for maintenance, which includes both set-up and interval costs, and it is modelled as to

$$\text{minimize} \quad \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} d_t z_t^k + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \sum_{t=1}^{T+1} \sum_{s=0}^{t-1} c_{st}^i x_{st}^{ijk}. \tag{3.7}$$

Minimizing the risk for lack of spare parts. To ensure that the operational schedule is undisturbed, or that the disturbance is minimal, it is crucial to have enough spare components available. Then, whenever an unexpected failure occurs, the damaged component can be replaced by an as "good as new" component without the planned operations having to be stopped.

One way of defining this objective is to maximize a weighted average of the number of repaired (or new) components available, which is modelled as to

$$\text{maximize} \quad \frac{1}{T} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} w^i \sum_{j \in \mathcal{J}_i} b_t^{ij}, \tag{3.8}$$

where $w^i > 0$ is an objective weight assigned to component type $i \in \mathcal{I}$.

An alternative way of minimizing the risk for lack of spare components would be to maximize a weighted average of the lower limits on the numbers of available components of each type, subject to a lower bound on the availability of each component type, i.e., to

$$\text{maximize} \quad \sum_{i \in \mathcal{I}} w^i e^i, \tag{3.9a}$$

$$\sum_{j \in \mathcal{J}_i} b_t^{ij} \geq e^i \geq \underline{b}^i, \qquad i \in \mathcal{I},\, t \in \mathcal{T}, \tag{3.9b}$$

where, for each component type $i \in \mathcal{I}$, $w^i > 0$ denotes the weight assigned while the lower limit on the number of available components is denoted by e^i.

Minimizing the risk for exceeding the contracted turn-around times for component repair. The 'turn-around time' v_{tat}^{ij} of an individual component (i, j) is defined as the time from when it is taken out of one of the aircraft in $\mathcal{K}$ until it has become repaired and is available for usage again in one of the aircraft. Letting $c_{\text{delay}}^{ij} > 0$ and $c_{\text{early}}^{ij} \in (0, c_{\text{delay}}^{ij}]$ denote the penalty for late and early, respectively, delivery of a repaired component, this objective is then expressed as to

$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} \left(c_{\text{delay}}^{ij} v_{\text{delay}}^{ij} - c_{\text{early}}^{ij} v_{\text{early}}^{ij} \right), \tag{3.10a}$$

where v_{delay}^{ij} (v_{early}^{ij}) denotes the total delay (earliness) for component (i, j) over the planning period. These variables are due to the constraints

$$v_{\text{early}}^{ij} \leq v_{\text{tat}}^{ij} - q_{\text{due}}^{ij} \left(a_0^{ij} + \sum_{t=1}^{T+1} \alpha_t^{ij} \right) \leq v_{\text{delay}}^{ij}, \tag{3.10b}$$

$$v_{\text{early}}^{ij} \leq 0 \leq v_{\text{delay}}^{ij}, \tag{3.10c}$$

where $q_{\text{due}}^{ij} > 0$ denotes the contracted due date for component (i, j), $j \in \mathcal{J}_i$, $i \in \mathcal{I}$. Due to the construction of (3.10) either v_{early}^{ij} or v_{delay}^{ij} (or both) will attain the value 0 when the objective (3.10a) is optimized (a component will be either early, or late, or on time; in the latter case $v_{\text{early}}^{ij} = v_{\text{delay}}^{ij} = 0$ hold). Therefore, for each component (i, j) the objective (3.10a) minimizes the penalty for total lateness or earliness.

There are more objectives that could be discussed and included in the multi-objective setting, if relevant. Two such examples are given below.

Minimizing investment costs for repair lines in the workshop. We assume that each repair line in the maintenance workshop comes with an investment cost $c_{\text{inv}} > 0$. One of the objectives on the maintenance workshop side would be to minimize the investment costs for repair lines in the workshop. The workshop capacity costs are then addressed as to

$$\text{minimize} \qquad c_{\text{inv}} L, \tag{3.11}$$

where the parameter L would then be regarded as a decision variable, which takes the role of an upper limit, as expressed in the constraints (3.2). This objective is relevant when investigating the optimal workshop capacity.

Minimizing the costs of performing repairs. Each maintenance activity associated with a component (i, j) normally has a repair cost c^{ij}_{repair}, which could depend on the component's processing time p^{ij}, be non-decreasing with an increasing value of p^{ij} and assigned to the u^{ij}_t variables. The objective is to

$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i} c^{ij}_{\text{repair}}(p^{ij}) u^{ij}_t, \tag{3.12}$$

and it is relevant if we want to optimize the number of maintenance activities. Since in our application, components have to be repaired regardless of the price of repair, and it is not possible to merge two or more repairs together to minimize the costs (opportunistic replacement planning, see e.g. Almgren et al. (2012)), we neglect this objective at the current stage. Moreover, (3.12) would lead to a so-called zero-sum game [3], which would not have an impact on the bi-objective analysis presented in our current work.

[3] In game theory and economic theory, a *zero-sum game* is a mathematical representation of a situation in which an advantage that is won by one of two sides is lost by the other. See e.g. von Neumann and Morgenstern (2007).

4 Summary of appended papers

4.1 Paper I: Simultaneous scheduling of replacement and repair of individual components in systems subject to operations

In this paper we put an emphasis on formulating and analyzing two different contracting forms between the two stakeholders, the system operator and the maintenance workshop. Components in the systems that are to be maintained are sent to the maintenance workshop, which needs to schedule and perform all maintenance activities while satisfying the contract. The workshop's ability to fulfill the contract is dependent on its capacity, which may be distributed on different facilities. Our modelling includes the stocks of damaged and repaired components.

The model we formulated is based on a mixed-binary linear optimization model of a preventive maintenance scheduling problem with so-called interval costs over a finite and discretized time horizon. We extend this scheduling model with the flow of components through the repair workshop, including stocks of spare components, both those components that need repair and the repaired ones. The resulting scheduling model is then utilized in the optimization of two main contracts, namely maximizing the availability of repaired (or new) components, and minimizing the deviation from the contracted turn-around times for the components in the maintenance loop. Each of these objectives are combined—in a bi-objective setting—with the objective to minimize the costs for maintenance of the operating system.

We analyze the two contracting forms between the system operator and the repair workshop by studying and comparing the Pareto fronts resulting from different parameter settings, regarding minimum allowed stock levels and investments in repair capacity of the workshop. Our specific results concern the effect on the levels of the stocks of components. We conclude that our bi-objective mixed-binary linear optimization model is able to capture important properties of the results from the contracting forms. The solutions resulting from our modelling can be used to find a lower limit or an optimal performance of a collaboration between stakeholders who govern a common system-of-system.

This paper is submitted to the Annals of Operations Research and is under review for publication. Initial ideas were presented on The First EUROYoung Workshop, Seville (2019) and some later ideas on the Swedish Operations Research Conference, Nyköping (2019) and PLANs forsknings- och tillämpningskonferens, KTH Södertälje (2020).

4.2 Paper II: Replacement and repair of common components in systems subject to operations

In this paper, we focus on an availability contract governing joint activities between the system operator and the maintenance workshop. Unlike in Paper-I, we do not model individual component flow but instead we only consider component types. Another novelty is the definition of the availability contract. In Paper I, availability was defined as the average number of components on the stock of repaired components over the planning horizon, whereas in this paper, we maximize the lower limit on the number of available components for a component type.

The model we formulated is based on a mixed-binary linear optimization model of a preventive maintenance scheduling problem with so-called interval costs over a finite and discretized time horizon. We generalize and connect this scheduling model with the repair workshop, including stocks of spare components, both those components that need repair and the repaired ones. The resulting scheduling model is then utilized in the optimization of the availability contract, namely maximizing the lower limit on the availability of repaired (or new) components, and minimizing the maintenance (interval and set-up) costs. The two objectives formulate a bi-objective optimization problem. The solutions resulting from our modelling can be used to find a lower limit for an optimal performance of a collaboration between stakeholders who govern a common system-of-systems regulated with an availability contract. We show that our model can be used for analyzing the change in the solution when some parameters (e.g., the maintenance workshop capacity) are varied.

This paper is in the form of a manuscript that is to be submitted to a journal.

5 Conclusions and future research

We start from an NP-hard preventive maintenance scheduling problem, generalize it to aircraft and individual components modeling, incorporate the maintenance workshop, model the stock dynamics to connect these two systems, as well as define several optimization objectives. In Paper I, we define and analyze two different contracting forms governing joint activities between the stakeholders. For larger instances, the model and solution approach become computationally intractable and are subject to further investigation and development, especially in the case of turn-around time contract which introduces non-binary coefficients in the constraint matrix. In Paper II, we remove the individual component modeling and focus on an availability contract type.

There are a few directions in which our research can be extended. An extension, which is important for the intended application of this work, is to introduce corrective maintenance modelling. At the current stage, the means to handle unexpected failures are to reduce the risk for such failures by not allowing too large maintenance intervals and to reschedule the maintenance plan whenever an unexpected event occurs. Since short-term changes in the operational schedules, as well as in the schedules for the maintenance workshop, are often inconvenient and sometimes not even feasible, (if possible) the rescheduling should be such that the solution remains fixed for a certain number of time steps.

The total number of components to be considered can be quite large and it is challenging to model all of them. Moreover, since only a subset of the total number of components is safety critical, they are the ones that are interesting to model and constitute the driving force of the whole system-of-systems. Another extension could be to cluster the non-safety critical components in some way so that they can be incorporated into the modeling.

To model the whole system-of-systems presented in Figure 1.1, we would expand our modeling to more than one maintenance workshop, include external subcontractors and a contract regulating Saab's collaboration with them. Maintenance workshop modeling is simplified in the current work, constituted with repair lines and processing times for components that are repaired, and that could be further expanded. Moreover, instead of having operational schedule as an input to the model, we could incorporate the problem of aircraft operations scheduling.

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