# Peak Sidelobe Level Based Waveform Optimization for OFDM Joint Radar-Communications



Citation for the original published paper (version of record):
Keskin, F., Tigrek, R., Aydogdu, C. et al (2021). Peak Sidelobe Level Based Waveform Optimization for OFDM Joint Radar-Communications. EuRAD 2020 - 2020 17th European Radar Conference: 1-4.
http://dx.doi.org/10.1109/EuRAD48048.2021.00012

# Peak Sidelobe Level Based Waveform Optimization for OFDM Joint Radar-Communications

Musa Furkan Keskin*, R. Firat Tigrek#, Canan Aydogdu*, Franz Lampel#,
Henk Wymeersch*,#, Alex Alvarado#, Frans M.J. Willems#

#Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands
*Department of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden
e-mail: r.f.tigrek@tue.nl

***Abstract*— We propose a novel joint radar communication waveform design method based on OFDM spectrum allocation to trade off radar and communication performance. The proposed approach combines the water filling optimization to maximize the communication rate for slow fading channels with windowing to reduce pulse compression peak sidelobe level (PSL) for the radar. A trade-off between communication and radar performance is demonstrated by modifying the constraints of the optimization problem. The results show that, depending on the channel response, the proposed optimization method can significantly reduce PSL with limited impact on the data rate.**

***Keywords*— Joint radar and communications, OFDM radar, peak sidelobe level**

## I. INTRODUCTION

Autonomous driving requires situational awareness provided by sensors on each vehicle as well as information shared through vehicle to vehicle communication. Radar has emerged as an indispensable element of the sensor suite on autonomous vehicles [1], [2]. Utilization of radars also for purposes of communication is an appealing option and is attracting increasing attention. The research on radar communication (RadCom) appears to follow two directions: *waveform-centered* and *information-centered* approaches.

In *waveform-centered* approaches, the focus is on embedding communication payload on a radar waveform and assessing the performance of radar using conventional performance parameters such as the resolution and sidelobe behavior. The majority of the research under this category focus on preserving radar performance while the waveform is modified by the communication payload. The communication capacity is analyzed for the proposed techniques, whereas other aspects of communication such as channel properties, capacity optimization and synchronization are in general left out of scope [3], [4].

In *information-centered* approaches, the focus is on assessing the information content of the radar signal after it interacts with a target [5]. Optimization methods towards maximizing information on the target while attaining the best possible channel capacity is the focus of [6]–[8]. The results obtained through information theoretic treatment of radar signals usually do not include the ambiguity function for the radar signal. Probability of detection, which corresponds to the main lobe of the signal response on the ambiguity function, is analyzed as performance metric for radar in [9], [10]. Only recently, the peak side-lobe level (PSL) of the autocorrelation function is considered as performance criterion in information-centered research on cooperative radar-communication [11]. However, the problem description in [11] focuses on the performance of rad-com receiver, where the radar and communication signals originate from different transmitters. The majority of the waveform-centered radar-communication research focuses on combining both functions in one transmitted signal.

The reason behind the differences between the waveform-centered and information-centered approaches stems from a fundamental difference between the problem formulations. The waveform-centered approach can be said to concentrate on detection of targets in a complex, multi-target setting. The rule-of-thumb in this setting is to prevent the sidelobes of a strong scatterer from masking the weak scatterers, which prompts the PSL as an important performance measure. The information-centered approach in general formulates the problem for a single target, which has a specific impulse response. The goal in this setting is to estimate the target response with greater accuracy.

In this paper, a middle ground is sought between waveform-centered and information-centered radar communication. To the best of our knowledge, this paper is the first work that makes a joint assessment of pulse compression sidelobes and communication information-theoretic capacity in a point-to-point communication setting, where the transmitted signal serves both the radar and communication functions. The novel joint radar communication waveform design method is based on optimization of power allocation over the OFDM carriers with constraints on both the peak sidelobe level (PSL) and the communication rate.

## II. OFDM RADCOM SIGNAL

To investigate the effect of power allocation on radar performance and channel capacity, a continuous transmission scheme is adopted. OFDM symbols with cyclic prefixes are transmitted continuously and in the radar receiver the reception windows are aligned with the transmitted OFDM symbols.

### A. OFDM Pulse Compression by Matched Filter

The conventional radar receiver utilizes matched filter, which gives the maximum signal-to-noise ratio for detecting signals in additive white Gaussian noise. The OFDM pulse compression described in [12] implements matched filter in frequency domain. The formulation of the OFDM pulse compression starts with an OFDM radar signal that is reflected back from a point target [12, Eq. (3)]:

$$s(t) = \sum_{k=0}^{K-1} \alpha \sqrt{P_k}\, x_k\, e^{j2\pi k \Delta f (t-\tau)}\,, \quad 0 \le t < T\,, \tag{1}$$

where $\alpha$ is the complex target coefficient including path loss and radar cross section, $k$ is the subcarrier index, $K$ is the number of subcarriers, $P_k$ and $x_k$ represent, resp., the power and the data symbol on the $k$th subcarrier, $\tau$ is the delay of the radar echo and $\Delta f = 1/T$ is the OFDM carrier spacing. The formulation in (1) assumes that the cyclic prefix duration for the OFDM symbol is greater than the delay $\tau$.

The OFDM pulse compression operates on the sampled echo by applying a discrete Fourier transform (DFT), multiplying each carrier with the complex conjugate of its communication payload and applying an inverse discrete Fourier transform (IDFT) to obtain the pulse compression output, which can be formulated as [12, Eq. (19)]

$$r_n = \sum_{k=0}^{K-1} P_k\, |x_k|^2\, e^{-j2\pi k \Delta f \tau} e^{j2\pi \frac{nk}{N}} + w_n,\, 0 \le n \le N-1, \tag{2}$$

where $N \ge K$ is the number of samples and $\alpha$ is removed as it only scales the entire pulse compression output. The Gaussian noise $w_n$ is disregarded at the radar receiver to provide high dynamic range for the analysis of pulse compression sidelobes.

The Doppler effect is not taken into account in (1)–(2). Range sidelobes for non-zero Doppler are determined not only by power allocation but also by phases of the carriers. The scope of this investigation will be limited to those cases where Doppler effect does not significantly alter the carrier phase. Under this assumption, the delay profile obtained from the OFDM waveform is the inverse Fourier transform of the power spectrum, which is determined by the amplitudes of the OFDM carriers. The complex exponential term in (2) shifts the delay profile by $\tau$, but does not alter the range sidelobes.

### B. *Application of Windowing for Range Sidelobe Suppression*

Spectral leakage in harmonic analysis and application of windows to suppress the leakage arising from the finite duration of the sampled signal is covered extensively in [13]. The same principle is applicable in pulse Doppler radars, where the strong leakage along the Doppler axis associated with stationary reflectors, often regarded as clutter, can easily mask the weaker response from moving targets. Windowing is also applicable to pulse compression of OFDM radar signals. It is explained in the previous paragraph that the pulse compression output is obtained by the IDFT of the power spectral density of the OFDM waveform. Hence, windows designed to suppress spectral leakage in spectral analysis are applicable to pulse compression in order to suppress range sidelobes.

The window functions offer a trade-off between the range resolution and PSL. When the window functions are applied to the received signal without assuming any control on the transmitted signal, processing loss occurs [13]. In radar, it is possible to apply the window at both the transmitter and the receiver, such that the pulse compression filter is still matched to the transmitted signal. Hence, there is no processing loss due to mismatch between the transmitted signal and the pulse compression filter. Besides the window functions listed in [13], a window function can be designed by optimization, as investigated in Section III.

### C. *OFDM Communication Capacity and Rate Optimization*

The RadCom systems under consideration are facing each other through a channel that has line-of-sight (LOS) propagation path besides other propagation paths that involve reflection from scattering surfaces in the environment. We assume that the communication channel is slow-fading and remains constant for several OFDM symbols. Given complex channel gains $\mathbf{h} = [h_0, \ldots, h_{K-1}]^T$ across the subcarriers, the channel capacity is calculated through the Shannon-Hartley theorem [14]:

$$C(\mathbf{p};\mathbf{h}) = \sum_{k=0}^{K-1} \log_2 \left\{ 1 + \frac{P_k\, |h_k|^2}{N_0\, \Delta f} \right\} \tag{3}$$

where $N_0$ is the noise power spectral density and $\mathbf{p} = [P_0, \ldots, P_{K-1}]^T$ is the power allocation. If the transmitter knows the channel gains, capacity can be optimized w.r.t. $\mathbf{p}$, leading to the well-known water filling solution [15].

## III. Joint Radar-Communications Waveform Design

### A. *Pulse Compression Output and PSL*

The pulse compression output in (2) can be represented as

$$\mathbf{r} = \mathbf{W}^H (\mathbf{p} \odot \mathbf{x}) \tag{4}$$

where $\odot$ denotes the Hadamard product, $\mathbf{r} \triangleq [r_0, \ldots, r_{N-1}]^T$, $\mathbf{W} \in \mathbb{C}^{K\times N}$ with entries $w_{k,n} \triangleq e^{-j2\pi k (n/N - \Delta_f \tau)}$, and $\mathbf{x} = \left[|x_0|^2, \ldots, |x_{K-1}|^2\right]$. The PSL of a pulse compression output $\mathbf{r}$ can be defined as [16]

$$\mathrm{PSL} = \max_{n\in\mathcal{S}} |r_n|^2 \tag{5}$$

where $\mathcal{S} \subseteq \{0, \ldots, N-1\}$ denotes the side-lobe region of interest. From (4), the PSL of the pulse compression output depends on the power allocation $\mathbf{p}$ and the magnitude of the random data $\mathbf{x}$. Since radar processing occurs over multiple OFDM symbols during which the channel is constant, the PSL of the average pulse compression output becomes a relevant metric:

$$\max_{n\in\mathcal{S}} \left|\mathbb{E}\left(r_n\right)\right|^2 \tag{6}$$

where the expectation is over the distribution of $\mathbf{x}$.

We assume independent and identically distributed, zero-mean unit-variance complex Gaussian data symbols, i.e.,

$x_k \sim \mathcal{CN}(0,1)$. The expected pulse compression output is then given by

$$\mathbb{E}(\mathbf{r}) = \frac{1}{2}\mathbf{W}^H\mathbf{p} \; . \tag{7}$$

The variance of the pulse compression output in the $n$th bin can be computed as

$$\begin{aligned} \sigma^2_{r_n} &= \mathbb{E}\left(|\mathbf{w}_n^H(\mathbf{p}\odot\mathbf{x})|^2\right) - \frac{1}{4}|\mathbf{w}_n^H\mathbf{p}|^2 && (8a)\\ &= \mathbf{w}_n^H\left(\mathbf{p}\mathbf{p}^H \odot \mathbb{E}(\mathbf{x}\mathbf{x}^H)\right)\mathbf{w}_n - \frac{1}{4}\mathbf{w}_n^H\mathbf{p}\mathbf{p}^H\mathbf{w}_n && (8b)\\ &= \frac{1}{4}\mathbf{w}_n^H \mathrm{diag}(P_0^2,\dots,P_{K-1}^2)\mathbf{w}_n && (8c)\\ &= \frac{1}{4}\sum_{k=0}^{K-1} P_k^2\,|w_{k,n}|^2 = \frac{\|\mathbf{p}\|_2^2}{4}, && (8d)\end{aligned}$$

where $\mathbf{w}_n$ is the $n$th column of $\mathbf{W}$ and $\mathbb{E}(\mathbf{x}\mathbf{x}^H)$ in (8b) is a matrix with $1/2$ on the diagonals and $1/4$ on the off-diagonals. Note that the variance of the pulse compression output does not vary over range bins unlike its expectation.

## B. Problem Formulation

Our goal herein is to determine $\mathbf{p}$ that maximizes the communication rate while keeping the side-lobe levels of the expected pulse compression output in (7) under a preset threshold. To this end, we propose the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{p}} \;\; C(\mathbf{p};\mathbf{h}) && (9a)\\ &\text{s.t.} \;\; \max_{n\in\mathcal{S}} \; |\mathbf{w}_n^H\mathbf{p}|^2 \leq \gamma && (9b)\\ &\qquad \mathbf{1}^T\mathbf{p} = P_T,\; \mathbf{p}\succeq \mathbf{0} && (9c)\end{aligned}$$

where $\gamma$ denotes the upper bound on the PSL over a given side-lobe region $\mathcal{S}$ and $P_T$ is the total power. The problem in (9) is convex and thus can be solved using standard tools of convex optimization [17].

# IV. Numerical Results

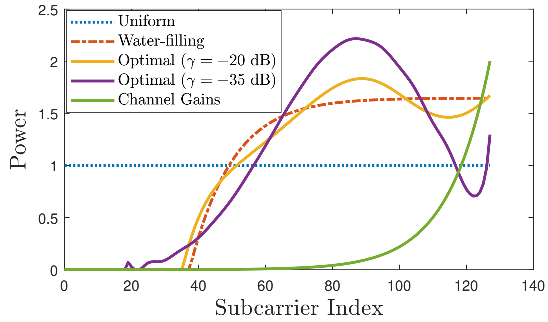
We consider an OFDM system with $K = 128$ and $\Delta f = 1\,\mathrm{MHz}$ for simulations. We investigate two scenarios, each corresponding to a certain channel gain realization, as shown in Fig. 1(a) and Fig. 2(a).

For waveform design, we discuss the results of (9). Fig. 1 shows the subcarrier power allocations and the corresponding expected pulse compression outputs for Scenario 1. It is seen that the optimal power allocation closely follows the water-filling solution for a loose PSL threshold, while it assumes a window-like shape as the PSL threshold gets tighter, which is in compliance with the conventional windowing idea [18]. In addition, we observe from the pulse compression output that the optimal solution converges to the water-filling solution in the side-lobe region while suppressing the PSL around the main-lobe, which reveals the radar-communications trade-off behavior in different portions of the spectrum.

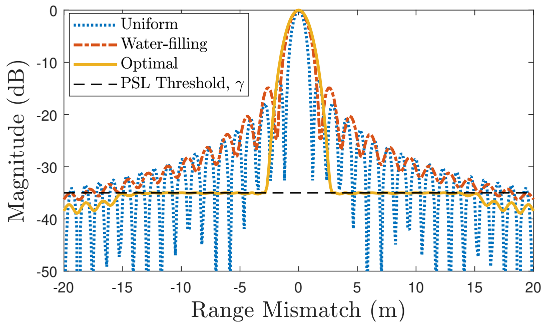
Fig. 2 illustrates the results of Scenario 2. Similar to Scenario 1, the radar-communications optimal power allocation moves from the water-filling solution (i.e., *communication-optimal*) towards the windowing solution (i.e., *radar-optimal*) as the PSL threshold becomes more strict. Compared to Scenario 1, the water-filling creates much higher side-lobes in Scenario 2 since the channel gains become large at both ends of the frequency spectrum. As seen from Fig. 2(b), the proposed waveform design strategy in (9b) can successfully suppress side-lobe levels by imposing a PSL constraint.

Finally, we investigate capacity-PSL trade-offs in Fig. 3 for the two scenarios. Fig. 3 is generated through Monte Carlo simulations. For each window function, the pulse compression outputs are averaged over multiple realizations of random complex communication symbols and the PSL for the averaged pulse compression output is determined.

Fig. 3 shows that channel realizations associated with less window-like water-filling allocations indicate more severe trade-offs between radar and communication performances. The capacity-PSL trade-off by allocating carrier powers according to Gaussian window function described in [13] also supports this observation. When implementing the Gaussian window, the peak sidelobe level is modified through the window coefficient, which controls the spread of the Gaussian window function. Gaussian window in general realizes lower communication capacity, and preceding the Gaussian window by water filling provides a limited improvement in the

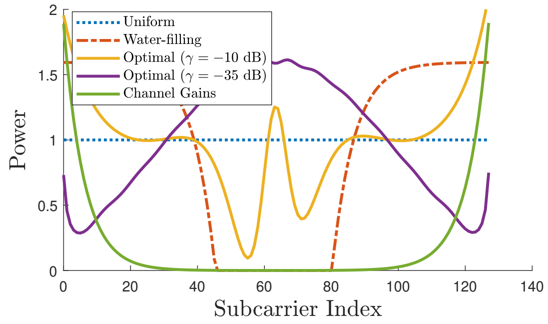



(a)

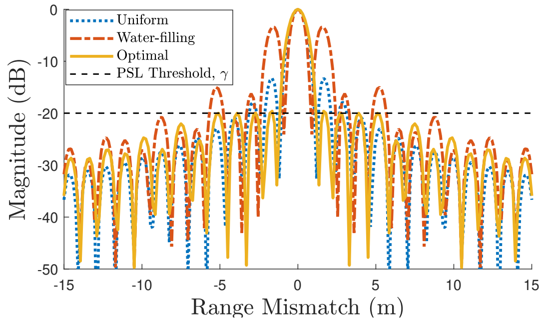



(b)

Fig. 1. Scenario 1: (a) Channel gains $|h_k|^2$ in (9a) and subcarrier powers $\mathbf{p}$, and (b) expected pulse compression output in (7) for the cases of uniform, water-filling and radar-communications optimal design in (9), where the normalized variances in (8) (i.e., $\|\mathbf{p}\|_2/P_T$) are given, respectively, by $-10.53\,\mathrm{dB}$, $-9.63\,\mathrm{dB}$ and $-9.45\,\mathrm{dB}$. The side-lobe region $\mathcal{S}$ is set to be the unambiguous range interval excluding the main-lobe region $[-3,3]\,\mathrm{m}$.




(a)




(b)

Fig. 2. Scenario 2: (a) Channel gains $|h_k|^2$ in (9a) and subcarrier powers $\mathbf{p}$, and (b) expected pulse compression output in (7) for the cases of uniform, water-filling and radar-communications optimal design in (9), where the normalized variances in (8) (i.e., $\|\mathbf{p}\|_2 / P_T$) are given, respectively, by $-10.53\,\mathrm{dB}$, $-9.70\,\mathrm{dB}$ and $-10.45\,\mathrm{dB}$. The side-lobe region $\mathcal{S}$ is set to be the unambiguous range interval excluding the main-lobe region $[-2, 2]\,\mathrm{m}$.

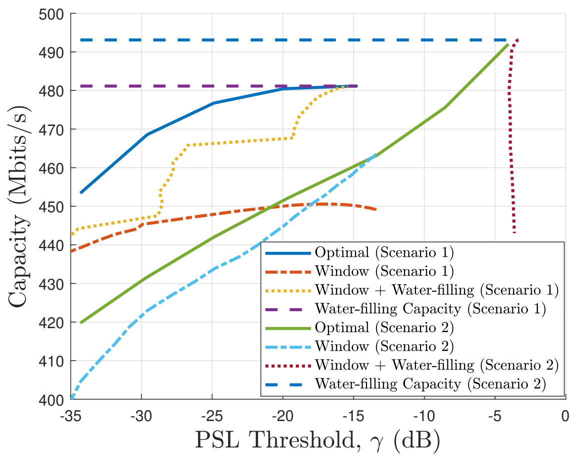



Fig. 3. Capacity versus PSL threshold for the radar-communications optimal design in (9) along with the water-filling lines for Scenario 1 and Scenario 2.

communication capacity. In fading scenarios that appear contrary to window functions, the water-filling completely overcomes the effect of windowing to the point where PSL can no longer be improved. The joint optimization offers the only viable solution in such cases.

## V. Conclusions

A waveform optimization method for OFDM joint radar-communications is demonstrated for two channel realizations. The novel optimization method performs significantly better than the conventional windowing and water filling techniques under severe fading conditions, resulting in moderate reductions in communication capacity for PSL thresholds as low as $-35$ dB for the considered scenarios. Assessment of the joint radar communications waveform design method under non-negligible Doppler shifts is considered as the next step towards realizing radar communication systems.

**Acknowledgments:** This work was supported, in part, by Vinnova grant 2018-01929. Contribution of Eindhoven University of Technology is supported by Netherlands Organisation for Scientific Research (NWO) under the contract "Integrated Cooperative Automated Vehicles" (i-CAVE).

## References

[1] S. M. Patole, M. Torlak *et al.*, "Automotive radars: A review of signal processing techniques," *IEEE Signal Processing Magazine*, vol. 34, no. 2, pp. 22–35, March 2017.

[2] F. Roos, J. Bechter *et al.*, "Radar sensors for autonomous driving: Modulation schemes and interference mitigation," *IEEE Microwave Magazine*, vol. 20, no. 9, pp. 58–72, Sep. 2019.

[3] C. Sahin, J. G. Metcalf, and S. D. Blunt, "Characterization of range sidelobe modulation arising from radar-embedded communications," in *International Conference on Radar Systems (Radar 2017)*, Oct 2017, pp. 1–6.

[4] S. H. Dokhanchi, M. R. B. Shankar *et al.*, "Joint automotive radar-communications waveform design," in *2017 IEEE 28th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, Oct 2017, pp. 1–7.

[5] M. R. Bell, "Information theory and radar waveform design," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1578–1597, Sep. 1993.

[6] D. W. Bliss, "Cooperative radar and communications signaling: The estimation and information theory odd couple," in *2014 IEEE Radar Conference*, May 2014, pp. 0050–0055.

[7] A. R. Chiriyath, B. Paul, and D. W. Bliss, "Radar-communications convergence: Coexistence, cooperation, and co-design," *IEEE Transactions on Cognitive Communications and Networking*, vol. 3, no. 1, pp. 1–12, March 2017.

[8] M. Kobayashi, G. Caire, and G. Kramer, "Joint state sensing and communication: Optimal tradeoff for a memoryless case," in *2018 IEEE International Symposium on Information Theory (ISIT)*, June 2018, pp. 111–115.

[9] A. D. Harper, J. T. Reed *et al.*, "Performance of a joint radar-communication system in doubly-selective channels," in *2015 49th Asilomar Conference on Signals, Systems and Computers*, Nov 2015, pp. 1369–1373.

[10] Z. Zhu, S. Kay, and R. S. Raghavan, "Information-theoretic optimal radar waveform design," *IEEE Signal Processing Letters*, vol. 24, no. 3, pp. 274–278, March 2017.

[11] A. R. Chiriyath, S. Ragi *et al.*, "Novel radar waveform optimization for a cooperative radar-communications system," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 3, pp. 1160–1173, June 2019.

[12] R. F. Tigrek, W. J. A. De Heij, and P. Van Genderen, "OFDM signals as the radar waveform to solve Doppler ambiguity," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 130–143, Jan 2012.

[13] F. J. Harris, "On the use of windows for harmonic analysis with the discrete fourier transform," *Proceedings of the IEEE*, vol. 66, no. 1, pp. 51–83, Jan 1978.

[14] Proakis, *Digital Communications 5th Edition*. McGraw Hill, 2007.

[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory (Wiley Series in Telecommunications and Signal Processing)*. USA: Wiley-Interscience, 2006.

[16] L. Wu and D. P. Palomar, "Sequence design for spectral shaping via minimization of regularized spectral level ratio," *IEEE Transactions on Signal Processing*, vol. 67, no. 18, pp. 4683–4695, Sep. 2019.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge university press, 2004.

[18] M. A. Richards, *Fundamentals of Radar Signal Processing*. Tata McGraw-Hill Education, 2005.