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# Optimal route design of electric transit networks considering travel reliability

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## Abstract

Travel reliability is the most essential determinant for operating the transit system and improving its service level. In this study, an optimization model for the electric transit route network design problem is proposed, under the precondition that the locations of charging depots are predetermined. Objectives are to pursue maximum travel reliability and meanwhile control the total cost within a certain range. Constraints about the bus route and operation are also considered. A Reinforcement Learning Genetic Algorithm is developed to solve the proposed model. Two case studies including the classic Mandl's road network and a large road network in the context of Zhengzhou city are conducted to demonstrate the effectiveness of the proposed model and the solution algorithm. Results suggest that the proposed methodology is helpful for improving the travel reliability of the transit network with minimal cost increase.

## 1 | INTRODUCTION

Reliability is widely regarded as one of the fundamental factors influencing residents' choices of travel mode. Traffic congestion, energy consumption, and air pollution have been significant issues for large cities. Sustainable transportation modes, especially public transport, are expected to offer attractive alternatives to private cars (Gao et al., 2020; Gao et al., 2021; Iliopoulou, Tassopoulos et al., 2019). Transit network is the "infrastructure" of the bus transit system that determines the efficiency and service level of public transport (Yao et al., 2014). The transit route network design problem (TRNDP) is a complex optimization problem, which has been a topic of research interest for more than 40 years (see relevant review studies of Guhaire & Hao, 2008; Ibarra-Rojas et al., 2015; Iliopoulou, Kepaptsoglou et al., 2019; Kepaptsoglou & Karlaftis, 2009). In existing studies, user and operator costs were usually

minimized to reflect the transit network design models (Iliopoulou, Kepaptsoglou et al., 2019). User cost mainly includes travel time, waiting time, and the number of transfers. Operator cost is usually related with the number and length of bus routes, fleet size, and operation hours. Travel reliability is significant for passengers as an essential performance measure for transit systems (C. Liu & Murphey, 2020; Yao et al., 2014). For the unreliable bus transit systems, passengers need to accommodate more time or poor service levels to ensure punctuality. Although there is a significant body of literature on TRNDP, a few studies have addressed the travel reliability involved in this problem. Yan et al. (2013) and Yao et al. (2014) presented robust optimization models considering travel time reliability. Considering travel time and passenger demand uncertainties, Liang et al. (2019) constructed a two-step model framework to determine a set of bus routes in the transit network and corresponding

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departure frequencies. However, in the above research, the random part is expressed by the variance of travel time or delay. Essentially, the travel time of each road link is deterministic when the objective function is calculated. The objective function is still to minimize the total travel time.

Electric vehicles are deemed to be a solution for reducing air pollution and greenhouse gas emissions (Y. Xu et al., 2021). Electric buses based on the battery technologies have been presently introduced into the public transport systems around the world (Häll et al., 2019). They are regarded as the future of public transport in view of their significant environmental and energy advantages (Fusco et al., 2013; Iliopoulou & Kepaptsoglou, 2019; Pternea et al., 2015). The introduction of electric buses brings new complexities to the planning process of public transport as compared to the regular buses (Häll et al., 2019). Optimization models need to consider special attributes, range limitations, and charging facilities of electric buses to successfully integrate them into the transit network (Häll et al., 2019). Iliopoulou, Tassopoulos et al. (2019), Iliopoulou and Kepaptsoglou (2019), and Pternea et al. (2015) optimized both the electric transit network layout and the charging facilities deployment. Y. Liu et al. (2020) put forward a model for the electric TRNDP (E-TRNDP) to minimize the total cost of users and operators, which determined the bus routes, frequencies, and the location of charging depots simultaneously.

In this context, we extend the state-of-the-art by investigating the E-TRNDP with foci on optimizing not only the total cost (including the user cost and the operator cost) but also the travel reliability. The notion of travel reliability is introduced as the ability for bus passengers to reach desired destinations under given travel time budget constraints and service level constraints. An optimization model for the E-TRNDP considering travel reliability is then formulated. A Reinforcement Learning Genetic Algorithm (RLGA) is furthermore developed to solve the proposed model.

The remainder of this paper proceeds as follows. Subsection 1.1 offers a literature review on transit network design and Subsection 1.2 highlights our contributions. Section 2 describes the problem. Section 3 presents the mathematical formulation. The solution algorithm is presented in Section 4. Section 5 conducts two case studies and analyzes the calculation results. Finally, Section 6 provides the conclusion.

## 1.1 | Literature review

The TRNDP usually concentrates on optimizing several objectives denoting the efficiency of transit networks

under the constraints from the perspectives of operation and available resources, including the number and the length of bus routes, associated frequencies, and the number of buses (Chakroborty, 2003; Fan & Machemehl, 2006a, 2006b; Kepaptsoglou & Karlaftis, 2009). As a traditional problem in the transportation field with a large number of existing studies, there have been some review papers on the TRNDP (Desaulniers & Hickman, 2007; Guihaire & Hao, 2008; Ibarra-Rojas et al., 2015; Iliopoulou, Kepaptsoglou et al., 2019).

The TRNDP is generally formulated as a multiobjective model (Fan & Machemehl, 2006a). Most studies try to optimize the overall welfare, which combines the interests of the user and the system (Kepaptsoglou & Karlaftis, 2009). The interests of users mainly include minimizing travel costs or transfers and maximizing the coverage. Benefits for the system are to maximize the level of service or operational profits and minimize operating costs. The overall welfare is reflected in the minimization of user and system costs. There are also studies addressing specific goals from the perspective of the environment. Parameters of TRNDP are also explored in Kepaptsoglou and Karlaftis (2009). Some are decision variables determining the layout and operating features (e.g., frequency and bus size). Other parameters express operating conditions (e.g., demand characteristics, network structure, and modes), operating strategies and rules, and the available resources. These constraints are required to formulate a TRNDP model. They are always fixed, determined, or assumed in advance. Besides, existing studies showed a variety of methods in solving TRNDP, which could be divided into four groups, that is, heuristics, analytical methods, mathematical methods, and meta-heuristics (Baaj & Mahmassani, 1991; Chakroborty & Dwivedi, 2002; Iliopoulou, Kepaptsoglou et al., 2019; Kepaptsoglou & Karlaftis, 2009). Iliopoulou, Kepaptsoglou et al. (2019) reviewed applications of meta-heuristics for solving the TRNDP. Meta-heuristics were further divided into single solution-based and population-based methods. A single solution-based method improves a single candidate solution. A population-based method uses a set of candidate solutions (i.e., the population) (Gendreau & Potvin, 2005; Iliopoulou, Kepaptsoglou et al., 2019). Particle Swarm Optimization (PSO), Genetic Algorithms (GAs), and Bee Colony Optimization are commonly used population-based methods (Iliopoulou, Kepaptsoglou et al., 2019).

Most models for TRNDP were based on the lengths or average travel times of road links. However, bus operations are random because traffic conditions are complex and uncertain (Yao et al., 2014). Yan et al. (2013) proposed a TRNDP that considered the stochasticity of travel time and developed a robust optimization model, which aimed to minimize the sum of operators' expected



cost and its variability multiplied by a weighting value. Yao et al. (2014) introduced the concept of travel time reliability and used the minimum average travel time of passengers in the transit network as the optimization objective. Liang et al. (2019) proposed a stochastic linear programming model to optimize frequencies and flows of passenger paths under the condition of travel time and demand uncertainty. The random or reliability parts in their models were expressed by the expected and variance of travel time or delay. Essentially, the link travel time is still fixed when the objective function is calculated. Besides, Szeto et al. (2011) proposed a nonlinear complementarity problem formulation for the risk-averse stochastic transit assignment problem, considering the in-vehicle travel time, waiting time, capacity, and the effect of congestion as stochastic variables simultaneously. Jiang and Szeto (2016) developed a reliability-based stochastic transit assignment method, considering the supply uncertainty. Shen et al. (2018) studied a reliability-based transit assignment model with capacity constraints that adopted a stochastic overload delay formulation.

In recent years, with electric buses being widely used in the transit system, some researchers have begun to study the E-TRNDP. So far, there have been four published studies. Pternea et al. (2015) developed a model to consider electric buses to minimize the weighted sum of user, operator, and external costs, and introduced a direct bus route design approach with route structure and directness control. Iliopoulou and Kepaptsoglou (2019) studied the combined design of transit network and the location of charging infrastructure and proposed a bi-level formulation. At the upper level, the candidate bus route set was generated and evaluated, while at the lower level, the wireless charging infrastructure was deployed. Iliopoulou, Tassopoulos et al. (2019) formulated a bi-level optimization model to jointly design efficient bus routes and identify the required charging infrastructure. The multiobjective PSO algorithm and the mixed linear integer programming model were combined to solve the model. Y. Liu et al. (2020) presented an optimization model for E-TRNDP, whose objectives were to minimize the total passengers' costs and the total daily operation costs. The decision variables were bus routes, frequencies, and the charging depots location. The solution method was based on the Pareto Artificial Fish Swarm Algorithm (PAFSA), and combined crossover and mutation operators.

## 1.2 | Our contributions

As discussed in the literature review, the E-TRNDP is an important research problem from both the theoretical and practical perspectives. This paper is different from previous

studies that allowed buses to charge during dwelling at bus stops (Iliopoulou & Kepaptsoglou, 2019; Iliopoulou, Tassopoulos et al., 2019). It concentrates on E-TRNDP under the precondition that the layout of charging depots is available. Buses charge at the charging depots, which is in line with the current practice. Specific contributions are as follows:

1. An optimization model for the E-TRNDP is proposed, whose objective is to pursue maximum travel reliability and meanwhile keep the total cost within a certain range. Factors such as charging strategies, the calculation model of the number of electric buses, and the total cost calculation model considering electric buses are all taken into account. Reliable transit network could thus be provided and the service level could be improved.
2. An RLGA is developed to solve the proposed optimization model, which enriches the solution methods of TRNDP. The performance of RLGA is investigated through comparing it with the Multi-Population GA (MGA).
3. Two case studies including the classic Mandl's road network and a large road network of Zhengzhou city are used to illustrate the effectiveness of the proposed methodology.

Besides, this study is different from Yan et al. (2013) in the following four aspects:

1. Objective function (design philosophy): The earlier study by Yan et al. (2013) aims to minimize the total travel time cost using the mean-variance model, while treating travel time reliability in the constraint. In this study, the objective is to find a transit network design scheme with the objective to maximize the travel reliability and meanwhile keep the total cost within a certain range.
2. Definition of travel reliability: Unlike the travel time reliability introduced by Yan et al. (2013), this study captures the travel reliability, considering the travel conditions during the transit journey (Ceder, 2016; Jenelius, 2018).
3. Research target: The research target of this study is an electric transit network, and there are differences in charging strategies, the calculation model of the required number of buses to operate the transit network, as well as the total cost calculation model.
4. Solution algorithm: The simulated annealing solution method was used in Yan et al. (2013). In the solution process, the travel time reliability constraints cannot be dealt with separately. The solution results can only be continuously tested to verify whether the reliability

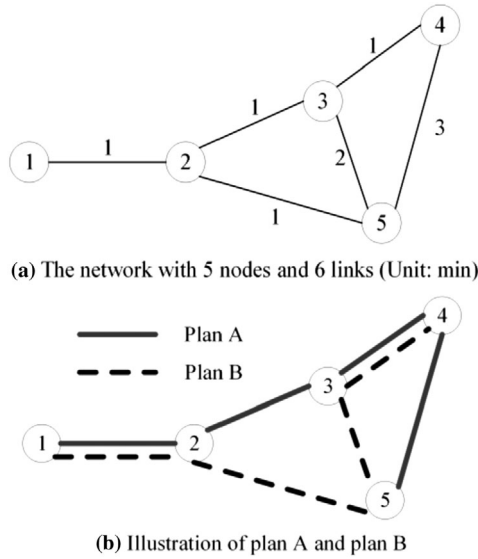


FIGURE 1 The small numerical example

constraints are met. To make the travel between any nodes meet the reliability requirements, this is feasible for a small-size network. However, when the size of the road network is large, it may be difficult to obtain a satisfactory solution. Hence, RLGA is used to solve the optimization model in this study.

## 2 | PROBLEM DESCRIPTION

Most existing TRNDP studies seek to minimize the user or operator cost, without considering the travel reliability. Travel time is an implicit objective for minimization (Kepaptsoglou & Karlaftis, 2009). Such an objective can obtain a good solution of the total travel time. However, excessive travel delays may happen for some travelers. A road network with five nodes and six links in Figure 1 is used as a simple numerical example to illustrate the problem.  $\mathbf{Q}$  in Equation (1) is the bus travel demand matrix of origin-destination (OD) pairs.  $q_{ij}$  ( $i = 1, 2, \dots, 5; j = 1, 2, \dots, 5$ ) is the bus travel demand between node  $i$  and node  $j$ . The number on link  $l_{ij}$  is the travel time between node  $i$  and node  $j$ . Assume the TRNDP is to design a transit network consisting of one bus route, with the objective to minimize the total travel time of passengers. Using the enumeration method, the optimal solution (denoted as plan A) can be obtained, that is, a bus route with nodes sequence 1-2-3-4-5. For plan A, the total travel time of bus passengers is  $f_A = 266$  min, whereas for plan B (i.e., a bus route with nodes sequence 1-2-5-3-4), the total travel time of bus passengers is  $f_B = 274$  min. Apparently, the total travel time of plan A is smaller than that of plan B. However, travelers' acceptability of delays is limited. With the increase in bus

travel delays, the service level and travelers' willingness to choose public transport will be greatly reduced. The delay value acceptable to travelers is not a fixed value, but a variable related to the minimum travel time (X. Xu et al., 2018). Travel delay here is used to reflect the travel time difference between the shortest bus path in the designed transit network and the shortest path in the road network. As shown in matrix  $\mathbf{D}^{(A)}$  and  $\mathbf{D}^{(B)}$ , for plan A, there are more OD pairs with travel delay per passenger equal to 4 min; for plan B, the largest travel delay is only 2 min. Moreover, we define the tolerance threshold as the ratio of the shortest bus travel time in the designed bus network against the shortest travel time in the road network to compare the two plans. For plan A, the proportion of travelers whose travel time of the shortest bus path is more than twice the shortest path in the road network is up to 29.2%. When the ratio is more than three times, the proportion is 8.3%. For plan B, these two values are only 20.8% and 0%, respectively. The method pursuing the minimum total travel time of the transit system may cause delays of some passengers to be high and unacceptable. Besides, the operation and construction of public transportation are mostly based on government grants in China; local transit agencies take more priorities on the benefit of passengers than the benefit of themselves (Yao et al., 2014). Therefore, a method that aims at maximizing the travel reliability of the transit system is expected to be proposed to reduce the risk of such situations:

$$\mathbf{Q} = [q_{ij}]_{5 \times 5} = \begin{bmatrix} 0 & 2 & 5 & 5 & 4 \\ 2 & 0 & 10 & 10 & 10 \\ 5 & 10 & 0 & 2 & 0 \\ 5 & 10 & 2 & 0 & 0 \\ 4 & 10 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

$$\mathbf{D}^{(A)} = [d_{ij}^{(A)}]_{5 \times 5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 2 & 0 & 0 \end{bmatrix} \quad (2)$$

$$\mathbf{D}^{(B)} = [d_{ij}^{(B)}]_{5 \times 5} = \begin{bmatrix} 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

where  $d_{ij}^{(A)}$  is the travel delay per passenger from node  $i$  to node  $j$  in plan A;  $d_{ij}^{(B)}$  is the travel delay per passenger from node  $i$  to node  $j$  in plan B.





Many studies have shown that travel time reliability is an important attribute for the satisfaction of public transport users and most public transport reliability have focused quite narrowly on travel times (Jenelius, 2018). However, there are good reasons to expect that travelers value a reliable public transport service also in terms of travel conditions during the journey (Ceder, 2016; Jenelius, 2018). It is well established that travelers experience in-vehicle travel time more negatively if the vehicle is crowded, especially when it is impossible to get a seat (Cantwell et al., 2009; dell'Olio et al., 2011; Jenelius, 2018). Hence, the probabilistic approach is used to model the transit travel reliability. It is defined as the probability for bus passengers to reach desired destinations within an acceptable travel time and under an acceptable crowdedness condition. However, the travel reliability and the total cost are not negatively correlated. Targeting travel reliability may or may not cause an increase in the total cost; similarly, an increase in the overall cost may not necessarily improve travel reliability. A bi-objective optimization model is therefore established to integrate the travel reliability and the total cost.

Moreover, the E-TRNDP is to determine a set of bus routes that are operated by electric buses. Electric buses recharge at charging depots. Operational characteristics such as range limitations, battery charging duration, and state of charge (SOC) constraints should also be considered. The following assumptions are used. First, buses can only be charged in predetermined charging depots. A bus traveling along a bus route from the start bus stop to the end bus stop in the upstream direction of the route and then returning along the route (i.e., the downstream direction of the route) is known as a service trip (C. Liu & Murphey, 2020). Idle trips denote nonservice trips where buses run between the charging depots and the start/end bus stops of service trips. At the beginning of operation, a bus will leave the nearest charging depot with full battery power for a service trip, and then return to the same charging depot for charging at the end of a service trip. This charging strategy is more favorable for maintaining the battery life (Zhang et al., 2020). Therefore, the cost of the battery can be greatly reduced. Second, the bus fleet is assumed to be homogeneous. Buses have the same battery characteristics and performance (Iliopoulou, Tassopoulos et al., 2019).

### 3 | MATHEMATICAL FORMULATION

#### 3.1 | Objective function

The road network used by the transit system is denoted by an undirected network  $G = (V, E)$ , where  $V$  is the set of

nodes and  $E$  is the set of links.  $Q$  is the set of travel demands of OD pairs.  $q_{ij}$  is the travel demand from node  $i$  to node  $j$ . A bus route with two reversed passenger transport directions is defined by a sequence of nodes in the road network passed by this bus route. A candidate solution to the E-TRNDP is specified by a set of bus routes  $X_y$ . For a designed transit network, a bus path from node  $i$  to node  $j$  consists of a set of bus route segments that are linked in sequence. Dijkstra algorithm, for example, can be used to obtain the bus path for each OD pair (Yao et al., 2014). Assuming the travel times of links are mutually independent, travel time of the bus path for OD pair between node  $i$  and node  $j$  can be calculated as the sum of travel times of links covered by the used bus route segments, the delays at the intersections, the dwelling times at bus stops, and the transfer times at the transfer bus stops:

$$T_{ij} = \sum_{l \in L_{ij}} T_l + \sum_{wt \in WT_{ij}} T_{wt} + \sum_{dw \in DW_{ij}} T_{dw} + \sum_{tr \in TR_{ij}} T_{tr} \quad (4)$$

where  $T_{ij}$  is the stochastic travel time of the shortest bus path between node  $i$  and node  $j$ ;  $L_{ij}$  is the set of links covered by the used bus route segments in the shortest bus path between node  $i$  and node  $j$ ;  $WT_{ij}$ ,  $DW_{ij}$ , and  $TR_{ij}$  are the set of intersections, the set of bus stops, and the set of transfer bus stops along the bus path, respectively;  $T_l$  is the travel time of link  $l$ ;  $T_{wt}$  is the delay at intersection  $wt$ ;  $T_{dw}$  is the dwelling time at bus stop  $dw$ ;  $T_{tr}$  is the transfer time at transfer bus stop  $tr$ .

The objective function in Equation (5) maximizes the travel reliability and the objective function in Equation (6) minimizes the total cost. As summarized in existing studies, the total cost of a transit system usually consists of two parts: the user cost and the operator cost. As there is no emission in the operation of electric buses, the environmental cost is neglected:

$$\max Z_1 = \sum_{i=1}^n \sum_{j=1}^n R_{ij} \cdot \beta_{ij} \quad (5)$$

$$\min Z_2 = C^{(1)} + C^{(2)} \quad (6)$$

where  $n$  is the number of nodes in set  $V$ ;  $R_{ij}$  is the travel reliability for OD pair from node  $i$  to node  $j$ ;  $\beta_{ij}$  is the weight factor of travel demand  $q_{ij}$ ;  $C^{(1)}$  is the user cost;  $C^{(2)}$  is the operator cost:

$$\beta_{ij} = \frac{q_{ij}}{\sum_{i=1}^n \sum_{j=1}^n q_{ij}} \quad (7)$$

$$R_{ij} = P \left\{ T_{ij} \leq T_{ij}^{\max}, \delta_{ij} \leq \delta_{\max} \right\} \quad (8)$$

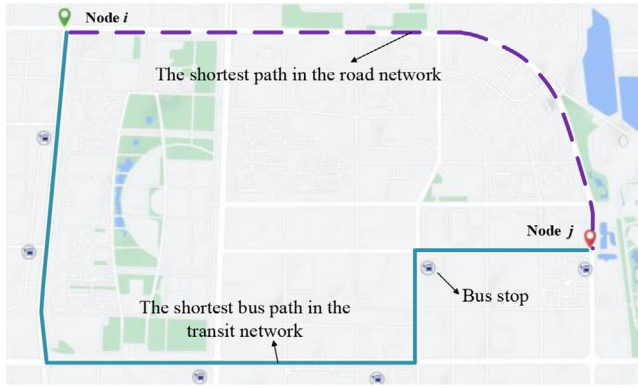


FIGURE 2 Shortest bus path in the designed transit network and the shortest path in the road network

where  $T_{ij}$  is the stochastic travel time of the shortest bus path between node  $i$  and node  $j$ ;  $T_{ij}^{\max}$  is the threshold of  $T_{ij}$ ;  $\delta_{ij}$  is the ratio of the actual number of passengers in the bus to its capacity on the shortest bus path between node  $i$  and node  $j$ ;  $\delta_{\max}$  is the threshold of  $\delta_{ij}$ .

The setting of  $T_{ij}^{\max}$  depends on the travelers' tolerance for travel delays. Travelers have a certain degree of subjectivity regarding the acceptable delay of the selected bus path, and their main psychological reference is the travel time of the shortest path in the road network. Figure 2 shows the difference between the shortest bus path in the designed transit network and the shortest path in the road network. A bus path may involve transfer and use more than one bus routes. Hence,  $T_{ij}^{\max}$  is set as follows:

$$T_{ij}^{\max} = \zeta \cdot \sum_{l \in PS_{ij}} \frac{L_l}{v_l} \quad (9)$$

where  $T_{ij}^{\max}$  is the threshold of  $T_{ij}$ ;  $T_{ij}$  is the stochastic travel time of the shortest bus path in the designed transit network;  $\zeta$  is the tolerance coefficient,  $\zeta \geq 1$ ;  $PS_{ij}$  is the set of links covered by the shortest path between node  $i$  and node  $j$  in the road network;  $L_l$  is the length of link  $l$ ;  $v_l$  is the free-flow speed on link  $l$ .

It should be noted that  $\delta_{ij}$  is changing dynamically for different links on a bus path. After transit assignment, the passenger flow of each link on each bus route can be obtained. Then  $\delta_{l, ij}$  for each link  $l$  on the shortest bus path between node  $i$  and node  $j$  can be calculated. The maximum  $\delta_{l, ij}$  is used as  $\delta_{ij}$  for travel reliability calculation.

Charging is essential for extending the service life of electric buses, because a large discharge range will accelerate the reduction of on-board battery capacity. Therefore, impacts of charging factors should be considered in terms of operator cost. The number of electric buses required for operation will be larger. There will also be idle time during the charging round trip from the start or end bus stop to

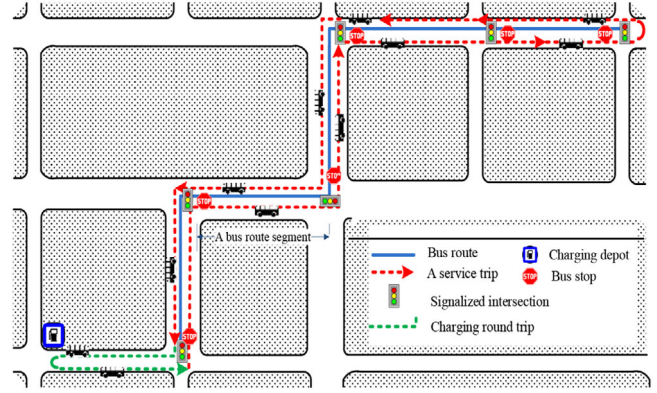


FIGURE 3 Bus operation process

the charging depot, as shown in Figure 3:

$$C^{(1)} = w_1 \cdot \sum_{i=1}^n \sum_{j=1}^n q_{ij} \cdot T_{ij} \quad (10)$$

$$C^{(2)} = C_{\text{bus}}^{(2)} + C_{\text{ope}}^{(2)} + C_{\text{idl}}^{(2)} \quad (11)$$

$$C_{\text{bus}}^{(2)} = w_{\text{bus}}^{(2)} \cdot \sum_{k \in X_y} N_k \quad (12)$$

$$C_{\text{ope}}^{(2)} = 2 \cdot w_{\text{ope}}^{(2)} \cdot \sum_{k \in X_y} LEN_k \cdot \frac{T_k^{\text{ope}}}{f_k} \quad (13)$$

$$C_{\text{idl}}^{(2)} = w_{\text{idl}}^{(2)} \cdot \sum_{k \in X_y} (2 \cdot L_k^{\text{idl}}) \cdot \frac{T_k^{\text{ope}}}{f_k} \quad (14)$$

$$f_k = \frac{60 \cdot N_c}{Q_k^{\max}} \quad (15)$$

$$N_k = 2 \cdot \left[ \frac{LEN_k}{v_k \cdot f_k} + \frac{2 \cdot T_k^{\text{idl}} + T_k^{\text{rec}}}{f_k} + 1 \right] \quad (16)$$

where  $C_{\text{bus}}^{(2)}$ ,  $C_{\text{ope}}^{(2)}$ , and  $C_{\text{idl}}^{(2)}$  are the cost of electric buses, the cost of bus operation, and the cost of idle driving, respectively;  $N_k$  is the number of electric buses required for operation of bus route  $k$ ;  $T_k^{\text{ope}}$  is the daily operation hours of bus route  $k$ ;  $LEN_k$  is the length of bus route  $k$ ;  $v_k$  is the average operation speed of bus route  $k$ ;  $f_k$  is the departure headway



of bus route  $k$ ;  $Q_k^{\max}$  is the peak load point of bus route  $k$ ;  $N_c$  is the capacity of an electric bus;  $L_k^{\text{idl}}$  is the distance of idling running;  $T_k^{\text{idl}}$  is the time of idling running;  $T_k^{\text{rec}}$  is the charging time;  $w_1$ ,  $w_{\text{bus}}^{(2)}$ ,  $w_{\text{ope}}^{(2)}$ , and  $w_{\text{idl}}^{(2)}$  denote the cost rates for  $C^{(1)}$ ,  $C_{\text{bus}}^{(2)}$ ,  $C_{\text{ope}}^{(2)}$ , and  $C_{\text{idl}}^{(2)}$ , respectively.

## 3.2 | Constraints

### 3.2.1 | Length of the bus route

The length of a bus route should be moderate. An over-length bus route may lead to uneven distribution of passenger flows, which further reduces the operation efficiency. A too short bus route will reduce the utilization rate of buses and increase the transfer time:

$$LEN_{\min} \leq LEN_k \leq LEN_{\max} \quad (17)$$

where  $LEN_{\min}$  and  $LEN_{\max}$  are the minimum and maximum length of the bus route, respectively.

### 3.2.2 | Nonlinear coefficient of the bus route

The nonlinear coefficient  $\eta_k$  is the ratio of the actual length of a bus route to the Euclidean distance (i.e., straight-line distance) between the start and end nodes of the bus route. The greater the nonlinear coefficient, the more tortuous is the bus route. The nonlinear coefficient constraint can effectively limit the size of the set of candidate bus routes and improve the calculation efficiency:

$$\eta_k = \frac{LEN_k}{dis_k} \quad (18)$$

$$1.0 \leq \eta_k \leq 1.4 \quad (19)$$

where  $\eta_k$  is the nonlinear coefficient of bus route  $k$ ;  $dis_k$  is the straight-line distance between the start and end nodes of bus route  $k$ .

### 3.2.3 | Percentage of passenger demand satisfied without transfer

$$\frac{q_{\text{dir}}}{\sum_{i=1}^n \sum_{j=1}^n q_{ij}} \geq \lambda_{\min} \quad (20)$$

where  $q_{\text{dir}}$  is the total passenger demand that can be satisfied without transfer;  $\lambda_{\min}$  is the minimum percentage of passenger demand that should be satisfied without transfer.

### 3.2.4 | Departure headway

The departure headway constraint, to a certain extent, helps avoid extreme situations where the passenger flow is too concentrated:

$$f_{k,\min} \leq f_k \leq f_{k,\max} \quad (21)$$

where  $f_{k,\min}$  and  $f_{k,\max}$  are the minimum and maximum values of the departure headway of bus route  $k$ , respectively.

### 3.2.5 | Flow conservation

The total bus travel demand boarding at each node is equal to the sum of passengers boarding at this node on all bus routes:

$$\sum_{j \in E} q_{ij} = \sum_{k \in \mathbf{RK}_i} q_{i,k} \quad (22)$$

where  $\mathbf{RK}_i$  is the set of bus routes passing through node  $i$ ;  $q_{i,k}$  is the passenger flow choosing bus route  $k$  at node  $i$ .

The passenger flow on each link  $l$  is equal to the sum of the passenger flows of all OD pairs that travel through link  $l$ :

$$q_l = \sum_{i \in E} \sum_{j \in E} q_{ij} \cdot \tau \quad (23)$$

$$\tau = \begin{cases} 0; & \text{when } l \notin L_{ij} \\ 1; & \text{when } l \in L_{ij} \end{cases} \quad (24)$$

where  $q_l$  is the passenger flow on link  $l$ .

$$q_{k,l} = \sum_{i \in E} \sum_{j \in E} q_{ij} \cdot \tau \cdot \rho \quad (25)$$

$$\rho = \begin{cases} 0; & \text{when } l \notin L_{ij} \text{ or } k \neq \text{line}_{i,j,l} \\ 1; & \text{when } l \in L_{ij} \text{ and } k = \text{line}_{i,j,l} \end{cases} \quad (26)$$

where  $q_{k,l}$  is the passenger flow of bus route  $k$  on link  $l$ ;  $\text{line}_{i,j,l}$  is the number of the bus route used by the travel demand between nodes  $i$  and  $j$  on link  $l$ .



## 4 | RLGA-BASED SOLUTION ALGORITHM

The relationship between objectives  $Z_1$ ,  $Z_2$ , and the control variable  $\mathbf{X}_y$  cannot be expressed in the form of a specific function. Hence, the solution process will be complex and difficult to control. The single-objective approximation is thus selected to solve the proposed bi-objective problem.  $Z_1$  in Equation (5) is used as the objective function while the objective function  $Z_2$  in Equation (6) is transformed into a constraint. The bi-objective model turns into the following single objective model:

$$\max Z_1 \quad (27)$$

subject to Equations (17)–(26), and

$$Z_2 \leq \varphi \cdot Z_{2,\min} \quad (28)$$

where  $\varphi$  is a coefficient and  $\varphi \geq 1$ ;  $Z_{2,\min}$  is the minimum total cost. The value of  $\varphi$  depends on the decisionmaker's acceptability of the total cost. The minimum total cost  $Z_{2,\min}$  is not given in advance. Its value changes with the change of the transit network size.

TRNDP is a kind of NP-hard combination optimization problem. The heuristic search algorithm, especially GA, is usually used for solution. However, when the number of nodes is large, the traditional GA is prone to premature convergence, which makes it difficult to obtain a good result. In this study, an RLGA is developed to solve the E-TRNDP. In the solution process, after determining the number of bus routes in the transit network,  $Z_2$  in Equation (6) is taken as the objective function. Then the traditional GA is used to calculate  $Z_{2,\min}$ .

### 4.1 | Solution framework

Q-learning is a commonly used reinforcement learning (RL) method. Watkins and Dayan (1992) first proposed Q-learning as a kind of RL method similar to the dynamic programming. It is used to describe and solve the agent problem in the process of interacting with the environment through learning strategies to maximize returns or achieve specific goals. Q-learning is also another expression of Markov Decision Process (C. Liu & Murphey, 2020; Strehl et al., 2009). The principle of the method is shown in Figure 4. Core concepts of the Q-learning approach are that the decision-making system selects one action from the finite set of actions at each step, and the action influences the environment. When the environment accepts the action, the state shifts and a reward is given.

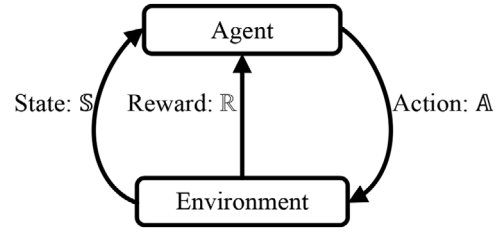


FIGURE 4 The principle of Q-learning

TABLE 1 Procedure of the solution methodology

- |         |  |
|---------|--|
| Step 1: | Generate the set $\mathbf{S}$ including all candidate bus routes satisfying constraints in Equations (17) and (19).  |
| Step 2: | Initialize the Q_Table and define the concept of population diversity $M$ .  |
| Step 3: | $G_t$ is the $t$ th generation population. $M_t$ is the diversity of population $G_t$ . Generate an initial population randomly. Give $t$ an initial value ( $t = 1$ ). Each chromosome $\mathbf{X}_y$ is a subset of $\mathbf{S}$ . Set $\alpha$ and $\beta$ . According to the fitness $Z_1(\mathbf{X}_y)$ and $M_t$ , divide $G_t$ into three subpopulations. |
| Step 4: | Each subpopulation is inherited once by GA and the new ( $t + 1$ )th population is recombined.   |
| Step 5: | Calculate the diversity $M$ of the new population. Update the Q_Table and select an action to determine the new $\alpha$ and $\beta$ . Redistribute three subpopulations.  |
| Step 6: | Judge whether the GA has converged. If yes, output the optimal transit network; otherwise, return to Step 3.   |
| Step 7: | Judge whether constraints in Equations (20) and (21) are satisfied by the optimal solution. If yes, output the optimal solution; otherwise, judge the second-best solution.  |

RLGA is a method of nesting RL outside MGA with GA as the core. Combining RL with GA is to reduce the effects of premature convergence. First, a population diversity function  $M$  (as shown in Equation (29)) is introduced, which represents the degree of difference between individuals in the genetic population (Wang et al., 2011). Then, according to the population diversity  $M$  and two proportion values (i.e., the proportion of breeding population  $\alpha$  and the proportion of retained population  $\beta$ ), the population can be divided into three subpopulations. GA is then applied to each subpopulation. The role of RL in the procedure is to guide the distribution of subpopulations according to the diversity of existing populations, so as to obtain the next generation with higher diversity. The procedure of the solution methodology and principles of RLGA are shown in Table 1 and Figure 5. A candidate solution, that is, a plan of transit network, to the E-TRNDP is determined by a set of bus routes  $\mathbf{X}_y$ . When the initial population is generated, the size of the population (denoted by  $Y$ ), that is, how many individuals (i.e.,  $\mathbf{X}_y$ ) the initial population  $G_1$  contains, is first determined. Then the size of the

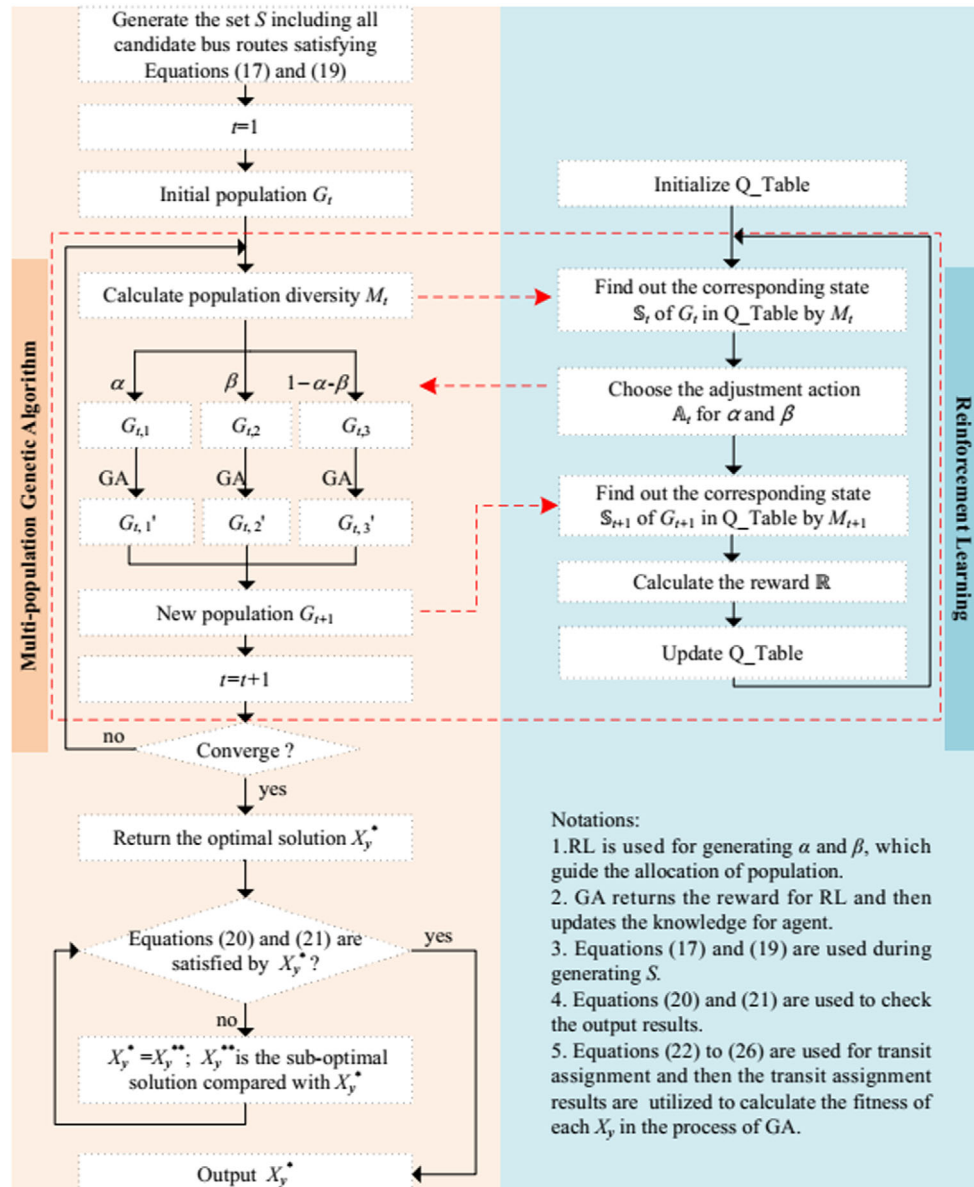


FIGURE 5 RLGA-based solution framework

target transit network  $N_X$  (i.e., the number of candidate bus routes in  $X_y$ ) is determined. After that,  $Y$  individuals with each individual  $X_y$  containing  $N_X$  candidate bus routes are randomly generated from the candidate bus route set  $S$ . In the solution process, the population size  $Y$  is set to be large, which can greatly cover all bus routes in the candidate bus route set. Then in the genetic process, it will continue to mutate, so that the unselected bus routes in the first generation have the possibility to join heredity:

$$M = \frac{1}{Y} \cdot \sum_{y=1}^Y \left( \left| Z_1(X_y) - \frac{1}{Y} \cdot \sum_{u=1}^Y Z_1(X_u) \right| \cdot \text{Num}_y \right) \quad (29)$$

where  $Y$  is the population size, that is, the number of individuals (i.e., chromosomes) in the population;  $X_y$  is the  $y$ th individual that denotes a plan of transit network;  $Z_1(X_y)$  is the fitness of  $X_y$ , which denotes the objective function in Equation (5);  $\text{Num}_y$  is the number of individuals different from  $X_y$  in the population.

In the genetic process, the fitness values of individuals in each generation are calculated and recorded. Therefore, constraints (20) and (21) can be judged sequentially from the optimal individual according to the fitness value, until the best individual that meets these two constraints is found.



TABLE 2 The set of states

State	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$
$\frac{M_t}{M_1}$	[0, 0.2]	(0.2, 0.4]	(0.4, 0.6]	(0.6, 0.8]	(0.8, +∞]

## 4.2 | Q-learning

Compared with the traditional single-population GA, the MGA can reduce the risk of premature convergence to a certain extent. Set the proportional coefficients  $\alpha$  and  $\beta$  ( $\alpha > 0$ ,  $\beta > 0$ , and  $\alpha + \beta = 0.95$ ). Divide the first generation of populations into three subpopulations  $G_{t,1}$ ,  $G_{t,2}$ , and  $G_{t,3}$ . Among them,  $G_{t,1}$  is the genetic population, in which the individuals have higher reproductive ability, that is, they have higher fitness. The newly set difference function  $U(X_y)$  is used as the standard to generate the retained population  $G_{t,2}$ .  $G_{t,3}$  is the optimal population of individuals, and its proportion is set to be a small value. Hence, individuals with the best fitness values could be retained.

The genetic population  $G_{t,1}$  is the main population, which guarantees the evolution direction of the population during the genetic process. The retained population  $G_{t,2}$  retains the more diverse individuals in the population, and to a certain extent, allows the inferior individuals in the population to be retained, so as to ensure the diversity in the evolution of the overall population and delay the premature convergence in the iteration process:

$$U(\mathbf{X}_y) = \sum_{h=1}^Y |Z_1(\mathbf{X}_y) - Z_1(\mathbf{X}_h)| \quad (30)$$

### 4.2.1 | The set of states

According to the ratio of the diversity of each generation population  $M_t$  to the initial population diversity  $M_1$ , the state space  $\mathbb{S}$  is defined as follows:

$$\mathbb{S} = \{S_1, S_2, S_3, S_4, S_5\} \quad (31)$$

The set of states is shown in Table 2.

### 4.2.2 | The set of actions

The action here is to adjust the proportional coefficient, and the action space  $\mathbb{A}$  is defined as:

$$\mathbb{A} = \{A_1, A_2, A_3\} \quad (32)$$

The set of actions is shown in Table 3.

TABLE 3 The set of actions

Action	$A_1$	$A_2$	$A_3$
Implication	$\alpha = \alpha + 0.05$ $\beta = \beta - 0.05$	$\alpha = \alpha + 0$ $\beta = \beta - 0$	$\alpha = \alpha - 0.05$ $\beta = \beta + 0.05$

TABLE 4 Pseudocode of GA

```

Encoding
Input an initial population  $G = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_y, \dots, \mathbf{X}_Y\}$ 
While  $t \leq 10,000$ :  $\forall t$  is a counting variable, which denotes
the generation of GA.
  For  $\mathbf{X}_y$  in the population  $G$ :
    Calculate fitness  $Z_1(\mathbf{X}_y)$  by Equation (5)
  After selection, crossover, and mutation
   $G \leftarrow G'$ 
   $t = t + 1$ 
If  $Z_1(\mathbf{X}_y)$  converge:
  Return  $\mathbf{X}_y \leftarrow \arg \max Z_1(\mathbf{X}_y)$ 

```

### 4.2.3 | Rewards

The solving process of GA is a process of continuously iterating to find better individuals. The fitness value of the best individual in the two generations of populations is used as the standard, and the reward rules are set as follows:

$$\mathbb{R} = \begin{cases} 0.5; \max_{X_y \in G_t} (Z_1(\mathbf{X}_y)) < \max_{X_y \in G_{t+1}} (Z_1(\mathbf{X}_y)) \\ 0; \max_{X_y \in G_t} (Z_1(\mathbf{X}_y)) = \max_{X_y \in G_{t+1}} (Z_1(\mathbf{X}_y)) \\ -1; \max_{X_y \in G_t} (Z_1(\mathbf{X}_y)) > \max_{X_y \in G_{t+1}} (Z_1(\mathbf{X}_y)) \end{cases} \quad (33)$$

## 4.3 | Genetic algorithm

As mentioned earlier, GA is the core of the solution method. The pseudocode of GA is illustrated in Table 4.

### 4.3.1 | Encoding and fitness calculation

The form of integer coding is adopted. Individual  $\mathbf{X}_y$  is represented by an array of candidate bus routes. Each element in the array represents a candidate bus route. Calculating the fitness of individual  $\mathbf{X}_y$  is the most important step of GA, as shown in Table 5. The objective function, that is, the travel reliability of the transit network scheme  $\mathbf{X}_y$ , is used as the fitness value.

**TABLE 5** The basic flow of fitness calculation

- Step 1: According to the transit network scheme  $\mathbf{X}_y$ , the shortest bus path of each OD pair is calculated. Links, intersections, and transfer information for the shortest bus path is meanwhile recorded.
- Step 2: Transit assignment is completed based on the input of the OD demand matrix and constraints (22)–(26). Passenger flows on segments of each bus route in  $\mathbf{X}_y$  is calculated.
- Step 3: On the basis of transit assignment results, the departure headways and the number of electric buses required for operation for each bus route can be calculated.
- Step 4: The travel reliability  $R_{ij}$  between each OD pair is calculated using Equations (8) and (9).
- Step 5: The objective function  $Z_1(\mathbf{X}_y)$  is calculated using Equation (5).

Transit assignment is an important part in the procedures of fitness calculating. The objective function involves the requirement of crowdedness, and transit assignment is the basis for calculating the number of buses required for operation. It is necessary to calculate the passenger flow in each link on each bus route under different transit network schemes. Besides, because GA is used to solve the model, with each new individual  $\mathbf{X}_y$ , the transit assignment is conducted once. Hence, the number of transit assignment is huge. For each transit assignment, the following rules are first formulated: (1) Passengers prefer to choose the bus path with a shorter distance. (2) When there are several bus paths with the same distance, passengers prefer to choose the bus path that transfers later. (3) When there are several nodes on the bus path that can be used for transfer, passengers prefer to choose the node closer to the destination as the transfer node.

Based on above assumptions, a Python script is written to realize the transit assignment. The pseudocode is shown in Table 6.  $\mathbf{X}_y$  is the transit network scheme with a set of bus routes.  $\mathbf{Q}$  is the bus travel demand matrix.  $k$  represents the bus route number.  $l$  denotes the number of road links.  $d$  represents the travelling direction.  $d = 0$  is the upward direction and  $d = 1$  is the downward direction.  $q_{k,l,d}$  is the passenger flow of link  $l$  on bus route  $k$  travelling in direction  $d$ .  $\{\mathbf{x}_k\}$  is the set of bus routes on the shortest bus path  $L_{ij}$ .  $\{\mathbf{l}_k\}$  is the set of links on bus route  $k$ .  $d_k$  is the direction of  $L_{ij}$  travelling along bus route  $k$ .  $q_{ij}$  is the demand between node  $i$  and node  $j$ .

According to the transit assignment method, the travel demand between all nodes can be traversed. Then all the passenger flows passing through the same bus route and the same link can be superimposed to obtain the final passenger flow for each link of each bus route.

**TABLE 6** Pseudocode of transit assignment

Input  $\mathbf{X}_y$  and  $\mathbf{Q}$   
 $q_{k,l,d} = 0, \forall k \in \mathbf{X}_y, \forall l \in \mathbf{E}, d \in \{0, 1\}$   
 Generate the adjacency matrix  $\mathbf{M}_{\mathbf{X}_y}$  of transit network  $\mathbf{X}_y$   
 For  $i$  in range( $n$ ):  
   For  $j$  in range( $n$ ):  
 Calculate the shortest bus path  $L_{ij}$  from  $i$  to  $j$  using Dijkstra algorithm based on  $\mathbf{M}_{\mathbf{X}_y}$   
 Calculate  $\{\mathbf{x}_k\}$ ,  $\{\mathbf{l}_k\}$ , and  $d_k$   
 For  $k$  in  $\{\mathbf{x}_k\}$ :  
   For  $l$  in  $\{\mathbf{l}_k\}$ :  
      $d = d_k$ :  
      $q_{k,l,d} = q_{k,l,d} + q_{ij}$

**TABLE 7** Probability distributions of random variables

Random variables	Probability distribution	Mathematical description
$T_l$	Normal distribution	$N(\mu_l, \sigma_l^2)$
$T$ When $T_{wt} = 0$	–	$P(T_{wt} = 0) = \frac{g_{wt}}{g_{wt} + r_{wt}}$
When $T_{wt} \neq 0$	Uniform distribution	$U(0, r_{wt})$
$N_{dw}$	Poisson distribution	$NP(\lambda_{dw})$
$T_{tr}$	Uniform distribution	$U(0, f_{tr})$

The calculation of  $R_{ij}$  is achieved utilizing the Monte Carlo simulation. The travel time  $T_l$  of each road link, delay  $T_{wt}$  at each intersection, the number of passengers  $N_{dw}$  on board at each bus stop, and the transfer time  $T_{tr}$  at each transfer bus stop are all set as random variables. The probability distributions of the random variables are shown in Table 7 (Mazloumi et al., 2010; Rahman et al., 2018; Yan et al., 2013).

In Table 7,  $\mu_l$  is the mean travel time on segment  $l$ ;  $\sigma_l^2$  is the variance of travel time on segment  $l$ ;  $g_{wt}$  is the effective green time of the signalized intersection  $wt$ ;  $r_{wt}$  is the red phase time of the signalized intersection  $wt$ ;  $\lambda_{dw}$  is the mean number of passengers arriving within a departure interval at bus stop  $dw$ ;  $f_{tr}$  is the departure headway for the transfer bus route.

The dwelling time  $T_{dw}$  at bus stop  $dw$  is calculated as follows:

$$T_{dw} = N_{dw} \cdot \bar{t} + t_{\text{stop}} + t_{\text{start}} \quad (34)$$

where  $\bar{t}$  is the mean time consumed for a passenger getting on the bus;  $t_{\text{start}}$  is the lost time for bus starting;  $t_{\text{stop}}$  is the lost time for bus stopping.

If there are  $m$  segments,  $h$  signalized intersections,  $r$  bus stops, and  $s$  transfers between an OD pair, the calculation procedure of  $R_{ij}$  is described in Table 8.

**TABLE 8** Pseudocode for the calculation of  $R_{ij}$ 

```

 $R' \leftarrow 0$ 
While  $x \leq Times$ :  $\nabla Times$  is the number of Monte Carlo simulations
    According to Table 7, generate  $m, h, r$ , and  $s$  random numbers
    Calculate  $T_{dw}$  by Equation (34)
    Calculate  $T_{ij}$  by Equation (4)
    Calculate  $\delta_{ij}$  and  $T_{ij}^{\max}$ 
    if  $T_{ij} \leq T_{ij}^{\max}$  and  $\delta_{ij} \leq \delta_{\max}$ :
         $R' \leftarrow R' + 1$ 
     $x \leftarrow x + 1$ 
 $R_{ij} \leftarrow R' / Times$ 
Return  $R_{ij}$ 

```

### 4.3.2 | Selection, crossover, and mutation

Based on the cumulative probability, the population is retained under “roulette rule.” Single-point crossover and single-point mutation patterns are used (Figure 6).

## 5 | CASE STUDIES

### 5.1 | Mandl's network

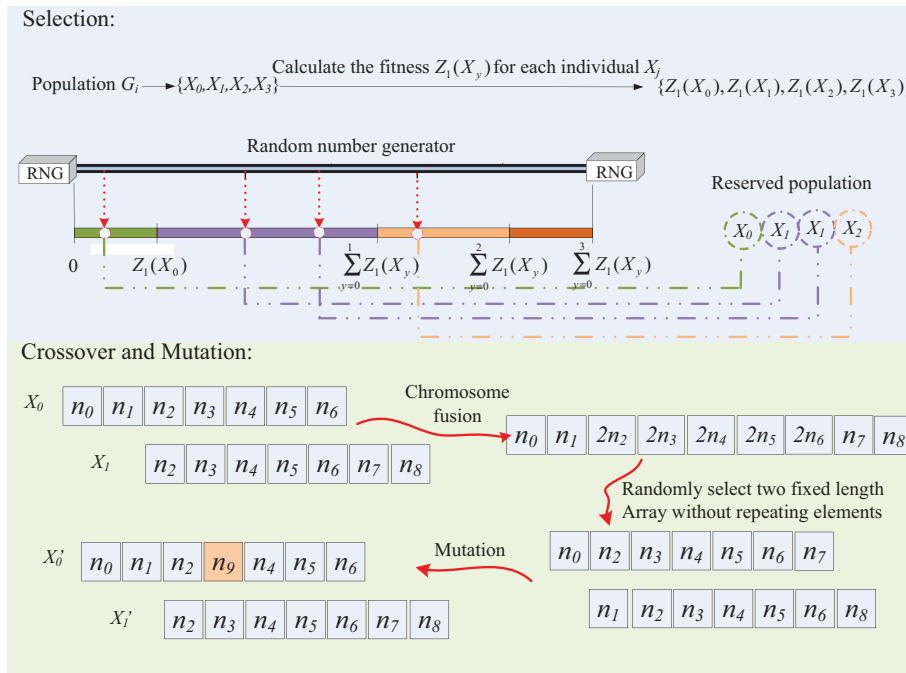
Mandl (1979) used a small network to test his transit network design method, which was adopted subsequently as benchmark network for transit network design problems, as shown in Figure 8a. The proposed method aims at electric transit network design. To conduct a more realistic

**TABLE 9** Parameters setting of Mandl's network

Parameter	Value	Parameter	Value
$\delta_{\max}$	1	$\lambda_{\min}$	0.6
$LEN_{\min}$	6 km	$\varphi$	1.1
$f_{k,\min}$	3 min	$LEN_{\max}$	8 km
$g_{wt}$	40 s	$f_{k,\max}$	30 min
$r_{wt}$	40 s	$\mu_l$	$L_l/v_l$
$\sigma_l$	$0.1L_l/v_l$	$\lambda_{dw}$	$f_k \cdot q_{l,k}/60$

comparison, the following assumptions are made for this benchmark network: (1) Charging facilities in the road network are sufficient, that is, there are enough charging facilities at each node. (2) Intersections are controlled with the same cycles and green splits. (3) The passenger flow at the peak hour accounts for 30% of the total bus travel demand (Yan et al., 2013). Then the same parameters are set (as shown in Table 9) and methods in existing studies (Baaj & Mahmassani, 1991; Chakroborty, 2003; Mahdi et al., 2015; Mandl, 1979; Shih & Mahmassani, 1994; Yan et al., 2013) are used to solve the E-TRNDP of Mandl's road network. OD travel demands can be referred to in Mandl (1979).

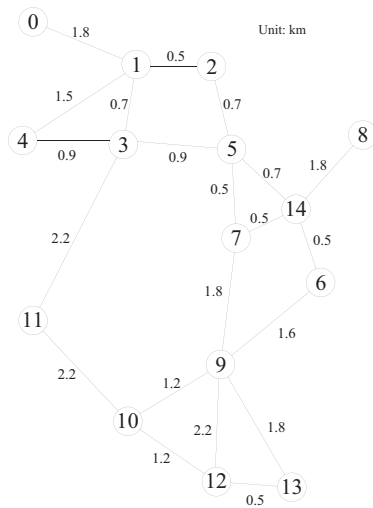
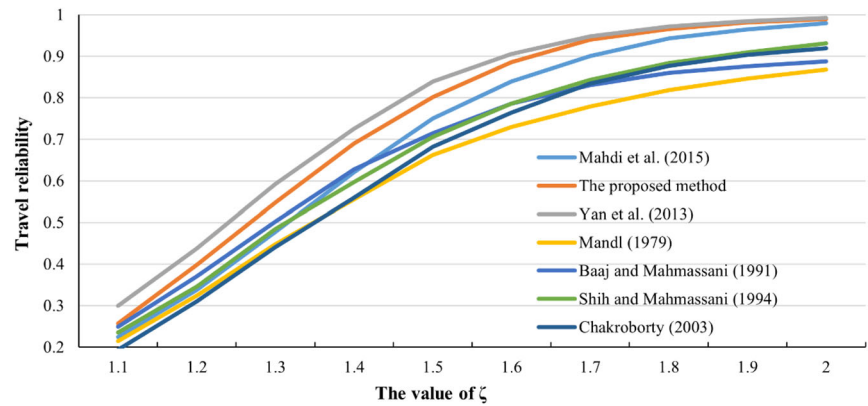
It is suggested that  $\zeta$  may be between 1.3 and 1.6 (Leurent, 1997; X. Xu et al., 2018). Land Transport Masterplan (Land Transport Authority of Singapore, 2008) in Singapore proposed that average public transport journey times would be reduced from 1.7 times of that by car today to 1.5 times by 2020 to make public transport more competitive relative to cars. Hence, the sensitivity

**FIGURE 6** Selection, crossover, and mutation

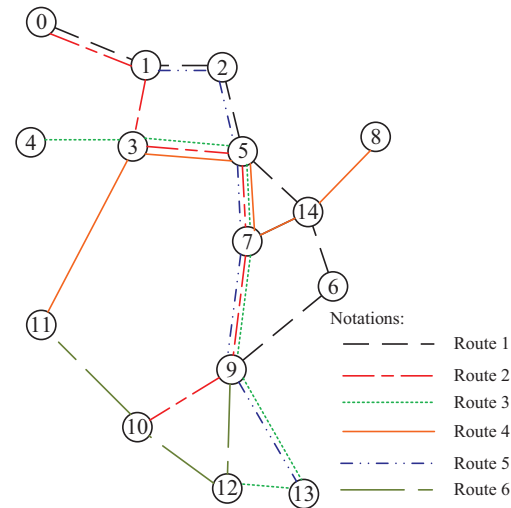




**FIGURE 7** The trend of travel reliability with the change of  $\zeta$



(a) Mandl's road network



(b) Resulted transit network of the proposed method

**FIGURE 8** Mandl's road network and the result for E-TRNDP by the proposed method

analysis of the coefficient  $\zeta$  is conducted. Its value is set to be between 1.1 and 2.0. Calculation results of the travel reliability for different methods in existing studies (Baaj & Mahmassani, 1991; Chakroborty, 2003; Mahdi et al., 2015; Mandl, 1979; Shih & Mahmassani, 1994; Yan et al., 2013) are shown in Figure 7. With the change of  $\zeta$ , the travel reliability for different methods first grows rapidly, and then grows slowly. The travel reliability difference is small in two ends and large in the middle. Hence, it is feasible to use a fixed value of parameter  $\zeta$  as a reference for transit network design.  $\zeta$  is thus set to be 1.6, which is beneficial for distinguishing different individual fitness values in the GA.

The solution of E-TRNDP is realized by RLGA and the resulted transit network using the proposed method with  $\zeta = 1.6$  is illustrated in Figure 8b. Figure 9 shows the convergence process of RLGA. It can be observed that when the curve is close to convergence, the curve still appears a small fluctuation due to the action of RLGA and muta-

tion rules, which can reduce the influence of premature convergence to some extent. The performance of RLGA is investigated through comparing it with MGA, as shown in Figures 10 and 11. It can be seen from Figure 10 that compared with MGA, RLGA has higher population diversity in the convergence process, which is beneficial to avoid premature convergence. Figure 11 shows that MGA converges faster. In RLGA, due to the larger differences among populations in the iterative process, the probability of better individuals continuously emerging is higher, and the convergence process is slower. But the convergence results have higher fitness.

Moreover, based on transit networks resulted from the methods in existing studies (Baaj & Mahmassani, 1991; Chakroborty, 2003; Mandl, 1979; Shih & Mahmassani, 1994; Yan et al., 2013; Mahdi et al., 2015), corresponding travel reliability values and the total costs when  $\zeta = 1.6$  are summarized in Table 10. Through comparative analysis, it can be found that:

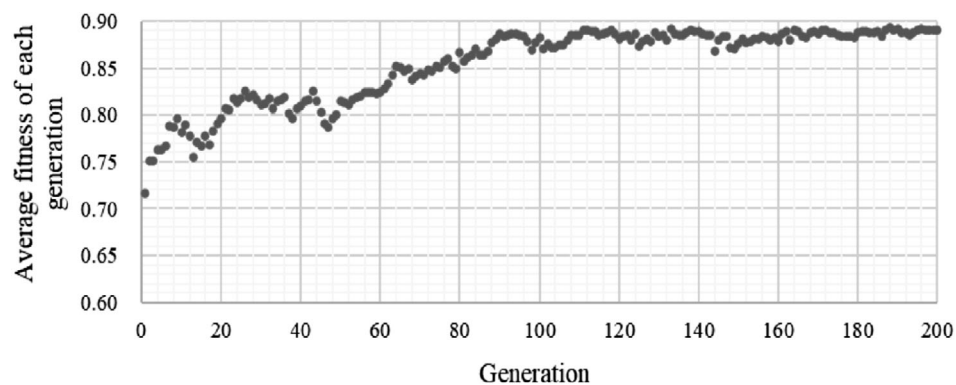


FIGURE 9 Convergence process of RLGA

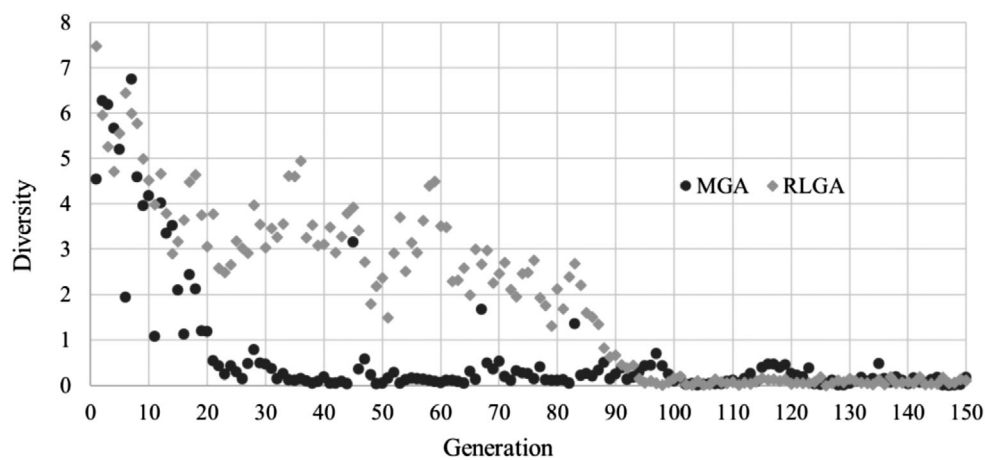


FIGURE 10 Trends of population diversity

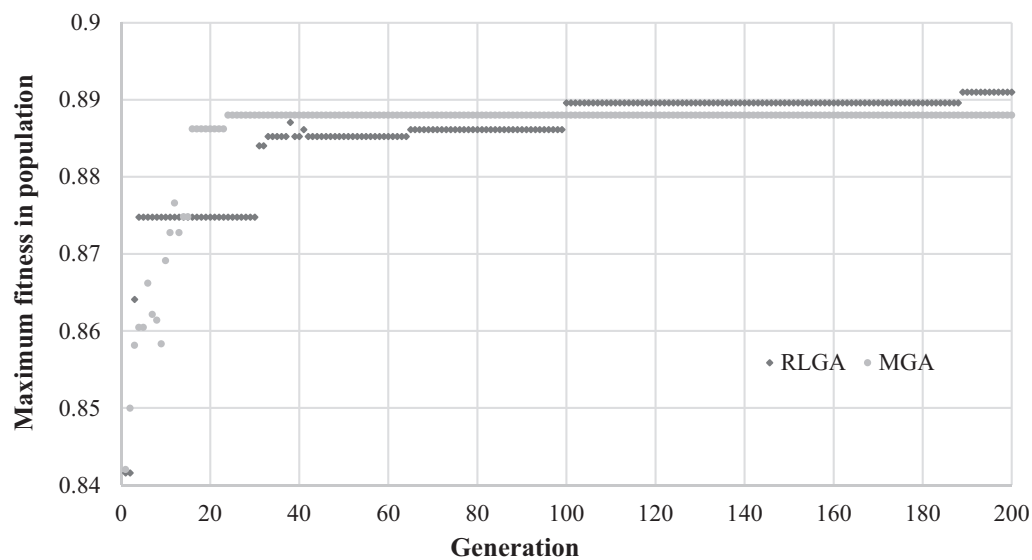


FIGURE 11 The evolution of RLGA and MGA



TABLE 10 Comparison of different methods

Method	Bus routes in the network	The number of bus routes	Total mileage (km)	The number of transfers (%)			The number of electric buses	Average travel time (h/passenger)	Total cost (yuan/day)	Travel reliability
				0	1	$\geq 2$				
Mandl (1979)	R1: 0-1-2-5-7-9-10-12; R2: 4-3-5-7-14-6; R3: 12-13-9; R4: 11-3-5-14-8.	4	18.9	67.7	28.9	3.4	34	0.244	47,701.78	0.730
Baaj and Mahmassani (1991)	R1: 9-12; R2: 9-13; R3: 8-14-6-9; R4: 4-3-5-7-9; R5: 9-10-11; R6: 0-1-2-5-7-9; R7: 0-1-3-4.	7	23.5	81.3	18.7	0	34	0.239	46,932.71	0.785
Shih and Mahmassani (1994)	R1: 5-7-9-10-12-13; R2: 6-14-7-9-10-11; R3: 0-1-2-5-7-9; R4: 6-9-12; R5: 8-14-6-9; R6: 4-3-5-7-9.	6	27.7	82.9	15.6	1.5	36	0.226	46,644.41	0.787
Chakroborty (2003)	R1: 11-3-4-1-2-5-7-9-10-12-13; R2: 0-1-2-5-7-9-12-10-11; R3: 1-3-5-14; R4: 5-7-9-6-14.	4	27.9	87.4	7.0	5.6	34	0.256	48,994.18	0.766
Yan et al. (2013)	R1: 0-1-3-5-14-6-9-13; R2: 2-5-7-14-6-9-12-13; R3: 6-14-7-5-2-1-4-3-11; R4: 4-3-5-7-9-6-14-8; R5: 0-1-2-5-7-9-10-12; R6: 2-1-3-11-10-9-3-11;	6	43.3	94.1	5.9	0	42	0.204	47,946.99	0.901
Mahdi et al. (2015)	R1: 6-14-5-2-1-3-11; R2: 4-3-5-7-9-13-12-10; R3: 0-1-2-5-14-8; R4: 11-10-9-6-14.	4	23.7	80.0	19.0	1.0	36	0.217	45,916.80	0.842
The proposed method	R1: 0-1-2-5-14-6-9; R2: 0-1-3-5-7-9-10; R3: 1-2-5-7-9-13; R4: 4-3-5-7-9-13-12; R5: 8-14-7-5-3-11; R6: 9-12-10-11.	6	35.3	84.6	14.3	1.1	36	0.211	45,885.01	0.890

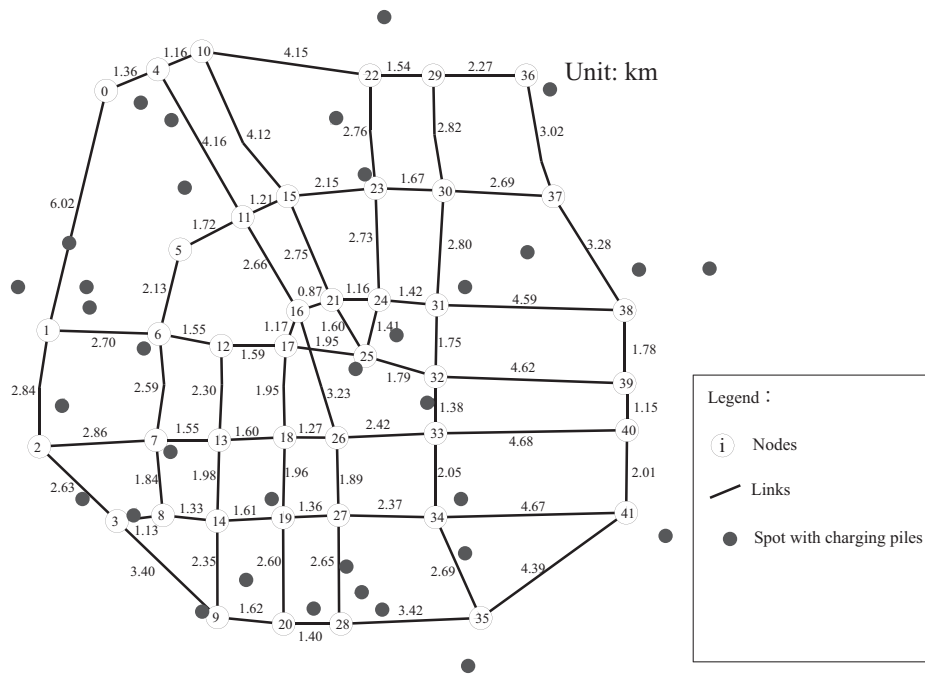


FIGURE 12 Road network and charging facilities

1. For methods of Mandl (1979), Baaj and Mahmassani (1991), Shih and Mahmassani (1994), Chakroborty (2003), and Mahdi et al. (2015), the total mileages of bus routes are shorter. But the average travel time is longer, and thus travel reliability of the resulted transit systems is reduced.
2. Fewer bus routes and shorter bus mileages may not reduce the number of buses required for operation or the total cost. As the travel demand is stable, when the number of bus routes is fewer and the total mileage is shorter, the departure headways of bus routes will increase. The number of buses required for operation does not necessarily decrease significantly. Besides, travel time of passengers may also increase. Hence, though transit networks in Mandl (1979) and Chakroborty (2003) have fewer bus routes, the total costs are relatively higher.
3. Compared with the study by Yan et al. (2013), the proposed method obtains almost the same travel reliability at 6% lower total cost. The proposed method pursues high travel reliability and meanwhile considers cost constraints.

## 5.2 | Network of Zhengzhou city

A large road network based on arterial roads in Zhengzhou city (as shown in Figure 12) is used in this section for case study. The layout of existing charging facilities is also shown in Figure 12. There are total 42 nodes, 73 links, and

TABLE 11 Parameters setting of the network of Zhengzhou city

Parameter	Value	Parameter	Value
$\zeta$	1.6	$\lambda_{\min}$	0.6
$\delta_{\max}$	1	$\varphi$	1.1
$LEN_{\min}$	15 km	$LEN_{\max}$	30 km
$f_{k,\min}$	3 min	$f_{k,\max}$	30 min

38 charging facilities. The cycle lengths for intersections are all set as 80 s, and the green time ratios are all set as 0.5. The travel demand for each OD pair is 10 persons/h. The average operation speed on each link is 40 km/h. The coefficient of variation for travel time of each link is set as 0.3. Other parameters are shown in Table 11.

All calculation scripts are written using Python. Because of the relatively large scale of the road network and the relatively complex structure of the proposed solution algorithm, the calculation complexity is high. The genetic algebra of GA is set to 500 generations. The calculations are conducted using the proposed method when the transit network contains 7–12 bus routes. In the environment of an ordinary small workstation (Intel® Xeon® CPU E5-2637v3@3.5 GHz, RAM: 64 GB), the time cost of script running is 54,756 s when the transit network contains 11 bus routes. The calculation results of the six scenarios are shown in Table 12 and Figure 13. It can be found in Figure 13 that with the increase of the number of bus routes, the travel reliability of the transit system increases. However, after reaching a certain value, the increase in travel



TABLE 12 Calculation results for six scenarios

The number of bus routes	Travel reliability	User cost (yuan/day)	Operator cost (yuan/day)	Total cost (yuan/day)	Total mileage (km)	The number of electric buses
7	0.596	210,251.5301	83,592.05107	293,843.5812	113.5	122
8	0.629	203,596.3221	107,062.7865	310,659.1085	133.4	156
9	0.696	177,965.6816	132,413.6943	310,379.3759	143.3	194
10	0.754	163,227.8483	132,220.0043	295,447.8526	165.5	192
11	0.760	162,924.8435	129,727.093	292,651.9365	190.2	190
12	0.771	161,038.7195	134,008.2107	295,046.9302	191.5	196

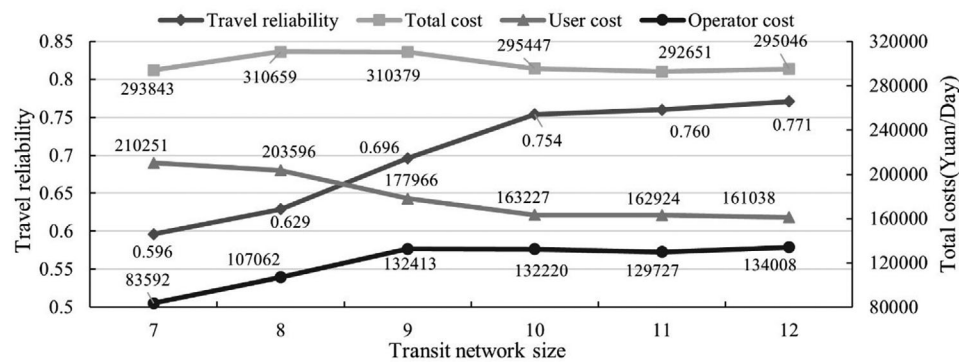


FIGURE 13 Travel reliability values and total costs of six scenarios

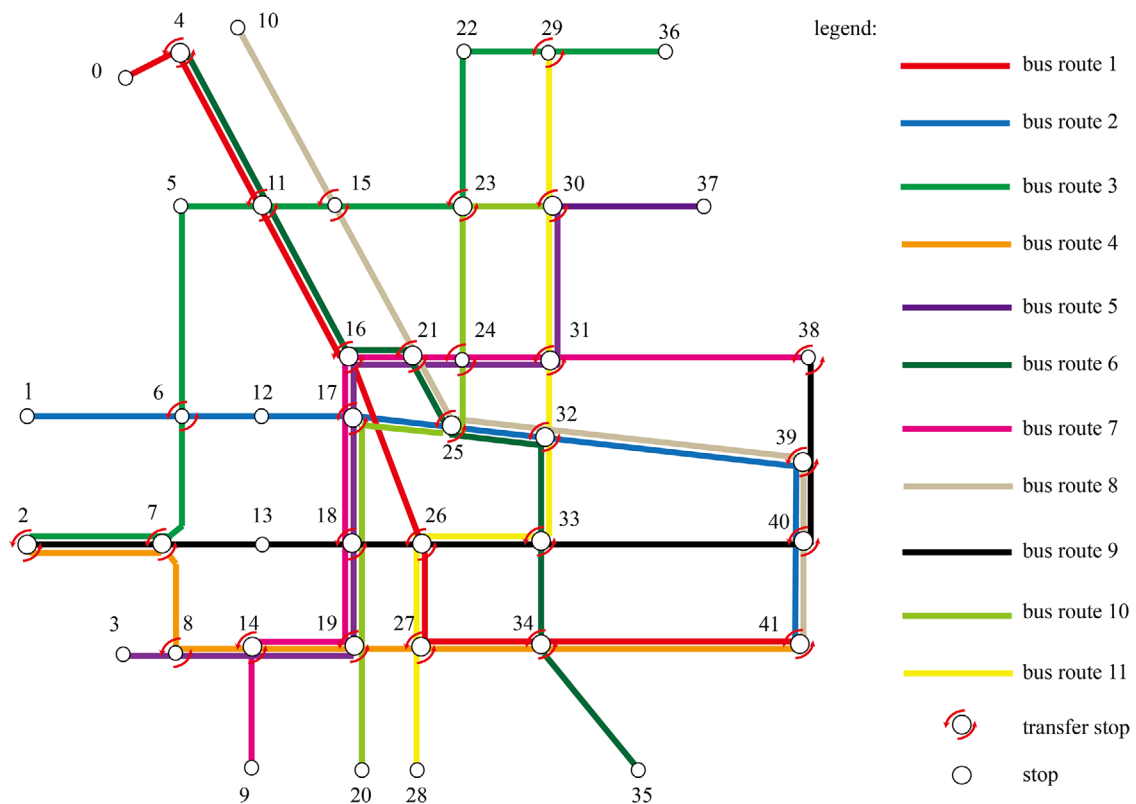


FIGURE 14 Transit network scheme with 11 bus routes



**TABLE 13** Comparison between transit network schemes considering only electric buses or only regular buses

Indexes	Electric buses	Regular buses
The number of bus routes	11	11
Travel reliability	0.760	0.768
The number of buses	190	166
The average distance between the start and end nodes of all bus routes and the nearest charging station (km)	1.484	1.892
Total cost (yuan/day)	292,651.9	271,833.0
Purchase cost of buses (yuan/day)	104,109.6	45,484.0
The cost of operation (yuan/day)	22,682.2	65,111.1
The cost of idle running (yuan/day)	4002.7	0
Energy consumption	100 kWh/100 km	40 L/100 km
Unit price	0.85 yuan/kWh	6.1 yuan/L

reliability starts to slow down. Meanwhile, as the number of bus routes increases, the operator cost increases gradually. But the user cost is reduced accordingly. After going up and then down, the total cost tends to be stable. The scenario with 11 bus routes, which has a travel reliability value of 27.5% higher than the minimum travel reliability and has the minimum total cost, is selected as the optimal transit network scheme.

The transit network scheme with 11 bus routes is shown in Figure 14. Its mileage is 190.2 km. The node coverage is 100%, indicating that passengers at each node can be served by buses. The number of nodes passing by two or more bus routes is 28, accounting for 66.7% of the total nodes. In the case of limited number of bus routes, the layout of the transit network is reasonable.

To obtain an optimal network design outcome when all buses are regular ones, we modify our electric transit model by setting the charging time as zero and using parameters associated with regular buses. The comparison in Table 13 demonstrates that: (1) Travel reliability values are almost the same. (2) As regular buses are not affected by the charging process, the number of buses required for operation is less than that of the transit network considering only regular buses. (3) The average distance between the start and end nodes of bus routes and the nearest charging station for the transit network considering only regular buses is farther, indicating that the charging strategy constraint works in the proposed methodology. (4) Although in terms of the total cost, the transit network considering only electric buses does not show big advantages. The cost of operation for the transit network considering only electric buses is far less than that for the transit network considering only regular buses. Especially with the decrease

of the prices of batteries, electric buses, and charging facilities, and taking the environmental benefits into account, the advantages of electric buses will gradually emerge.

## 6 | CONCLUSION

This study defines travel reliability as the ability for bus passengers to reach desired destinations within given travel time budget constraints and service level constraints. An optimization method for E-TRNDP considering both the travel reliability and the total cost is proposed. To solve the problem, an RLGA-based solution methodology is developed. During the solution process, only the population diversity is used as the standard for state division, and the diversity interval is divided into more intervals. For the reward feedback part, the fitness value of the largest individual in the population is used instead of the average fitness value as the reward feedback standard.

Comparisons with other methods are first made based on Mandl's (1979) road network. Results show that compared with MGA, RLGA has higher population diversity in the convergence process and the convergence result has higher fitness. A large road network in the context of Zhengzhou city is then taken as another case study. Calculation results suggest that the proposed model and solution approach could effectively improve the travel reliability of the transit network and meanwhile consider the total cost constraint.

However, there are still some limitations in this work. As the calculation script is written in a dynamic language, that is, Python, the script structure is single-threaded and the calculation time is too long. In the future, methods



such as parallel calculation will be introduced to improve the efficiency of calculation. The transit assignment rules are mainly based on the shortest bus path. Capacity limits are important factors to be considered in the future work. Charging infrastructure is closely related to the transit network design. Considering the charging stations at fixed locations and the charging strategy that the bus will be charged every time it completes a service trip (i.e., charged during layover) are two other limitations of this study. It is expected that the influence of range anxiety is considered in more depth and detail in the future work. Wireless charging technology for an electric bus is helpful for solving the issue of limited driving range (Khan et al., 2019). Besides, the charging schedule and the size of the charging station may cause bus to queue in the station (Mencía et al., 2019). The size of the charging station will be considered in the modeling. Furthermore, this study combines the machine learning with GA, in which machine learning adopts RL. In the follow-up research, the effects of combining different learning methods (Ahmadlou & Adeli, 2010; Pereira et al., 2020; Rafiei & Adeli, 2017; Rokibul Alam et al., 2020; Sørensen et al., 2020) with GA will be investigated. In future extensions, effects of other distribution types for random variables may also be addressed.

## REFERENCES

- Ahmadlou, M., & Adeli, H. (2010). Enhanced probabilistic neural network with local decision circles: A robust classifier. *Integrated Computer-Aided Engineering*, 17(3), 197–210.
- Baaj, M. H., & Mahmassani, H. S. (1991). An AI-based approach for transit route system planning and design. *Journal of Advanced Transportation*, 25(2), 187–209.
- Cantwell, M., Caulfield, B., & O'Mahony, M. (2009). Examining the factors that impact public transport commuting satisfaction. *Journal of Public Transportation*, 12(2), 1–21.
- Ceder, A. (2016). *Public transit planning and operation: Modeling, practice and behavior* (2nd ed.). Boca Raton: CRC Press.
- Chakroborty, P. (2003). Genetic algorithms for optimal urban transit network design. *Computer-Aided Civil and Infrastructure Engineering*, 18(3), 184–200.
- Chakroborty, P., & Dwivedi, T. (2002). Optimal route network design for transit systems using genetic algorithms. *Engineering Optimization*, 34(1), 83–100.
- dell'Olio, L., Ibeas, A., & Cecin, P. (2011). The quality of service desired by public transport users. *Transport Policy*, 18(1), 217–227.
- Desaulniers, G., & Hickman, M. (2007). Public transit. In C. Barnhart & G. Laporte (Eds.), *Handbooks in operation research and management science* (Vol. 14, pp. 69–127). Elsevier.
- Fan, W., & Machemehl, R. B. (2006a). Optimal transit route network design problem with variable transit demand: Genetic algorithm approach. *Journal of Transportation Engineering-ASCE*, 132(1), 40–51.
- Fan, W., & Machemehl, R. B. (2006b). Using a simulated annealing algorithm to solve the transit route network design problem. *Journal of Transportation Engineering-ASCE*, 132(2), 122–132.
- Fusco, G., Alessandrini, A., Colombaroni, C., & Valentini, M. P. (2013). A model for transit design with choice of electric charging system. *Procedia – Social and Behavioral Sciences*, 87, 234–249.
- Gao, K., Yang, Y., Li, A., Li, J., & Yu, B. (2021). Quantifying economic benefits from free-floating bike-sharing systems: A trip-level inference approach and city-scale analysis. *Transportation Research Part A: Policy and Practice*, 144, 89–103.
- Gao, K., Yang, Y., Sun, L., & Qu, X. (2020). Revealing psychological inertia in mode shift behavior and its quantitative influences on commuting trips. *Transportation Research Part F: Traffic Psychology and Behaviour*, 71, 272–287.
- Gendreau, M., & Potvin, J. Y. (2005). Metaheuristics in combinatorial optimization. *Annals of Operations Research*, 140(1), 189–213.
- Guihaire, V., & Hao, J.-K. (2008). Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice*, 42(10), 1251–1273.
- Häll, C. H., Ceder, A., Ekström, J., & Quttineh, N.-H. (2019). Adjustments of public transit operations planning process for the use of electric buses. *Journal of Intelligent Transportation Systems*, 23(3), 216–230.
- Ibarra-Rojas, O. J., Delgado, F., Giesen, R., & Muñoz, J. C. (2015). Planning, operation, and control of bus transport systems: A literature review. *Transportation Research Part B: Methodological*, 77, 38–75.
- Iliopoulou, C., & Kepaptsoglou, K. (2019). Integrated transit route network design and infrastructure planning for on-line electric vehicles. *Transportation Research Part D: Transport and Environment*, 77, 178–197.
- Iliopoulou, C., Tassopoulos, I., Kepaptsoglou, K., & Beligianis, G. (2019). Electric transit route network design problem: Model and application. *Transportation Research Record*, 2673(8), 264–274.
- Iliopoulou, C. A., Kepaptsoglou, K. L., & Vlahogianni, E. I. (2019). Metaheuristics for the transit route network design problem: A review and comparative analysis. *Public Transport*, 11(3), 487–521.
- Jenelius, E. (2018). Public transport experienced service reliability: Integrating travel time and travel conditions. *Transportation Research Part A: Policy and Practice*, 117, 275–291.
- Jiang, Y., & Szeto, W. Y. (2016). Reliability-based stochastic transit assignment: Formulations and capacity paradox. *Transportation Research Part B: Methodological*, 93, 181–206.
- Kepaptsoglou, K., & Karlaftis, M. G. (2009). Transit route network design problem: review. *Journal of Transportation Engineering-ASCE*, 135(8), 491–505.
- Khan, Z., Khan, S. M., Chowdhury, M., Safro, I., & Ushijima-Mwesigwa, H. (2019). Wireless charging utility maximization and intersection control delay minimization framework for electric vehicles. *Computer-Aided Civil and Infrastructure Engineering*, 34(7), 547–568.
- Land Transport Authority of Singapore. (2008). *Land transport masterplan* (Report). [https://www.lta.gov.sg/content/dam/ltagov/who\\_we\\_are/statistics\\_and\\_publications/master-plans/pdf/LTMP-Report.pdf](https://www.lta.gov.sg/content/dam/ltagov/who_we_are/statistics_and_publications/master-plans/pdf/LTMP-Report.pdf)
- Leurent, F. M. (1997). Curbing the computational difficulty of the logit equilibrium assignment model. *Transportation Research Part B: Methodological*, 31(4), 315–326.
- Liang, J., Wu, J., Gao, Z., Sun, H., Yang, X., & Lo, H. K. (2019). Bus transit network design with uncertainties on the basis of a metro network: A two-step model framework. *Transportation Research Part B: Methodological*, 126, 115–138.



- Liu, C., & Murphey, Y. L. (2020). Optimal power management based on Q-learning and neuro-dynamic programming for plug-in hybrid electric vehicles. *IEEE Transactions on Neural Networks*, 31(6), 1942–1954.
- Liu, Y., Feng, X., Zhang, L., Hua, W., & Li, K. (2020). A pareto artificial fish swarm algorithm for solving a multi-objective electric transit network design problem. *Transportmetrica A: Transport Science*, 16(3), 1648–1670.
- Mahdi, A. S. M., Mohaymany, A. S., & Ceder, A. (2015). Optimal modification of urban bus network routes using a genetic algorithm. *Journal of Transportation Engineering-ASCE*, 141(3), 4014081.
- Mandl, C. E. (1979). *Applied network optimization*. London: Academic Press.
- Mazloumi, E., Currie, G., & Rose, G. (2010). Using GPS data to gain insight into public transport travel time variability. *Journal of Transportation Engineering-ASCE*, 136(7), 623–631.
- Mencía, C., Sierra, M. R., Mencía, R., & Varela, R. (2019). Evolutionary one-machine scheduling in the context of electric vehicles charging. *Integrated Computer-Aided Engineering*, 26(1), 49–63.
- Pereira, D. R., Piteri, M. A., Souza, A. N., Papa, J. P., & Adeli, H. (2020). FEMa: A finite element machine for fast learning. *Neural Computing and Applications*, 32(10), 6393–6404.
- Pternea, M., Kepaptsoglou, K., & Karlaftis, M. G. (2015). Sustainable urban transit network design. *Transportation Research Part A: Policy and Practice*, 77, 276–291.
- Rafiei, M. H., & Adeli, H. (2017). A new neural dynamic classification algorithm. *IEEE Transactions on Neural Networks & Learning Systems*, 28(12), 3074–3083.
- Rahman, M. M., Wirasinghe, S. C., & Kattan, L. (2018). Analysis of bus travel time distributions for varying horizons and real-time applications. *Transportation Research Part C: Emerging Technologies*, 86, 453–466.
- Rokibul Alam, K. M., Siddique, N., & Adeli, H. (2020). A dynamic ensemble learning algorithm for neural networks. *Neural Computing and Applications*, 32(12), 8675–8690.
- Shen, L., Shao, H., Li, C., Sun, W., & Shao, F. (2018). Modeling stochastic overload delay in a reliability-based transit assignment model. *IEEE Access*, 7, 3525–3533.
- Shih, M. C., & Mahmassani, H. S. (1994). *Design methodology for bus transit networks with coordinated operations*. <https://www.osti.gov/biblio/20>
- Sørensen, R. A., Nielsen, M., & Karstoft, H. (2020). Routing in congested baggage handling systems using deep reinforcement learning. *Integrated Computer-Aided Engineering*, 27(2), 139–152.
- Strehl, A. L., Li, L., & Littman, M. L. (2009). Reinforcement learning in finite MDPs: PAC analysis. *Journal of Machine Learning Research*, 10, 2414–2444.
- Szeto, W. Y., Solayappan, M., & Jiang, Y. (2011). Reliability-based transit assignment for congested stochastic transit networks. *Computer-Aided Civil and Infrastructure Engineering*, 26(4), 311–326.
- Wang, X. Y., Liu, Q., Fu, Q. M., & Zhang, L. (2011). Multiple policy selection genetic algorithm based on reinforcement learning. *Computer Engineering*, 37(8), 149–152.
- Watkins, C. J. C. H., & Dayan, P. (1992). Technical note Q-learning. *Machine Learning*, 8(3), 279–292.
- Xu, X., Chen, A., Jansuwan, S., Yang, C., & Ryu, S. (2018). Transportation network redundancy: Complementary measures and computational methods. *Transportation Research Part C: Methodological*, 114, 68–85.
- Xu, Y., Zheng, Y., & Yang, Y. (2021). On the movement simulations of electric vehicles: A behavioral model-based approach. *Applied Energy*, 283, 116356.
- Yan, Y., Liu, Z., Meng, Q., & Jiang, Y. (2013). Robust optimization model of bus transit network design with stochastic travel time. *Journal of Transportation Engineering-ASCE*, 139(6), 625–634.
- Yao, B., Hu, P., Lu, X., Gao, J., & Zhang, M. (2014). Transit network design based on travel time reliability. *Transportation Research Part C: Emerging Technologies*, 43, 233–248.
- Zhang, L., Zeng, Z., & Qu, X. (2020). On the role of battery capacity fading mechanism in the lifecycle cost of electric bus fleet. *IEEE Transactions on Intelligent Transportation Systems*. <https://doi.org/10.1109/TITS.2020.3014097>

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