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Passive Front Propagation in Intense Turbulence: Early Transient and Late Statistically Stationary Stages of the Front Area Evolution

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Abstract: Influence of statistically stationary, homogeneous isotropic turbulence (i) on the mean 11 area of a passive front propagating in a constant-density fluid and, hence, (ii) on the mean fluid 12 consumption velocity \bar{u}_T is explored in the case of asymptotically high turbulent Reynolds number 13 and asymptotically high ratio of the Kolmogorov velocity to a constant speed u_0 of the front. First, 14 a short early transient stage is analyzed by assuming that the front remains close to a material sur-15 face that coincides with the front at the initial instant. Therefore, similarly to a material surface, the 16 front area grows exponentially with time. This stage, whose duration is much less than an integral 17 time scale of the turbulent flow, is argued to come to the end once the volume of fluid consumed by 18 the front is equal to the volume embraced due to the turbulent dispersion of the front. The mean 19 fluid consumption velocity averaged over this stage is shown to be proportional to the rms turbulent 20 velocity u'. Second, a late statistically stationary regime of the front evolution is studied. A new 21 length scale characterizing the smallest wrinkles of the front surface is introduced. Since this length 22 scale is smaller than the Kolmogorov length scale η_K under conditions of the present study, the 23 front is hypothesized to be a bifractal with two different fractal dimensions for wrinkles larger and 24 smaller than η_K . Finally, a simple scaling of $\bar{u}_T \propto u'$ is obtained for this late stage also. 25

Keywords: self-propagating front; turbulent consumption velocity; front area; bifractal

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1. Introduction

While turbulent combustion involves various multi-scale and highly non-linear phe-29 nomena [1-3] such as turbulence [4-6], complex chemistry [7,8], thermal expansion [9-12] 30 and differential diffusion [13] effects, fundamentals of the influence of turbulence on a 31 flame are often explored by a considering the classical problem of a passive front propa-32 gating locally normal to itself at a constant speed u_0 in randomly/turbulent advected me-33 dia [14-16]. Historically, this problem attracted much attention since 1940s when signifi-34 cant acceleration of flame propagation by turbulence was found. The phenomenon was 35 explained by Damköhler [17] and Shelkin [18] who highlighted random advection of a 36 flame by turbulent flow and reduced the influence of the turbulence on the flame to an 37 increase in the area of the flame surface wrinkled due to large-scale velocity fluctuations. 38 Following those pioneering ideas, various models of flame propagation in a turbulent 39 flow express the mean turbulent consumption velocity \bar{u}_T (i.e., the mean mass rate of 40 reactant consumption per unit area of the mean flame surface, normalized using the fluid 41 density upstream of the flame) to be a function of the front speed u_0 and the rms turbu-42 lent velocity u', with a ratio of \bar{u}_T/u_0 being controlled by the mean increase in the front 43

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Copyright: © 2021 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/license s/by/4.0/). surface area. Moreover, recent Direct Numerical Simulation (DNS) study [19] of self-propagation of a passive interface in constant-density turbulence showed a linear relation between $\bar{u}_T - u_0$ and u' at least at $0.5 \le u'/u_0 \le 10$.

However, in spite of long-term investigations of propagation of a front (e.g., a flame) 47 in randomly advected media (e.g., turbulence), physical mechanisms that result in the 48aforementioned linear relation do not seem to be fully clarified. To resolve the problem, 49 turbulent entrainment, which is controlled by large-scale eddies is commonly highlighted, 50 with small-scale characteristics of any surface (material or self-propagating) being as-51 sumed to be adjusted to the influence of large-scale turbulent eddies on the surface. Ac-52 cordingly, the fractal concept [20] is invoked to describe the surface characteristics at var-53 ious scales. In particular, for a moderately slow ($u_K < u_0 \ll u'$) front whose fractal dimen-54 sion D = 7/3 and the Gibson length scale $L_G = L(u_0/u')^3 \ll L$ is inside the inertial inter-55 val of turbulence spectrum [4,5], i.e. $\eta_K < L_G$, the fractal concept yields $\bar{u}_T \propto u'$ [21,22]. 56 Here, *L* is an integral length scale of the turbulence, $u_K = (v\bar{z})^{1/4}$ and $\eta_K = (v^3/\bar{z})^{1/4}$ 57 designate Kolmogorov velocity and length scales [4], respectively, ν is the kinematic vis-58 cosity of the fluid, and $\bar{\varepsilon}$ is the mean rate of viscous dissipation of turbulent kinetic en-59 ergy. What happens when $u_0/u_K \rightarrow 0$ and, consequently, the Gibson length scale is inside 60 the viscous (dissipation) interval, i.e. $\eta_K > L_G$, is an open question. Accordingly, the pri-61 mary goal of the present communication is to hypothesize a specific physical mechanism 62 that reconciles (i) a scaling of $\bar{u}_T \propto u'$ at $u_0 \ll u_K$, (ii) the concept of turbulent entrain-63 ment, and (iii) a well-recognized paradigm that reduces the influence of turbulence on a 64 front to an increase in the front area by turbulent eddies characterized within the frame-65 work of the Kolmogorov theory [4,5,23]. 66

The present study addresses two limiting stages of front area evolution: (i) an early 67 transient stage, whose duration is much less than an eddy-turn-over time scale $\tau_T = L/u'$, 68 and (ii) a late fully-developed stage when the front area reaches a statistically stationary 69 state. During the late stage, growth of the front surface area due to turbulent straining is 70 counterbalanced by reduction of the front surface area due to joint actions of (i) folding of 71 finite-length front elements, caused by strong advection, and (ii) subsequent collisions of 72 self-propagating fronts. 73

2. Analysis and Results

2.1. Earlier transit stage

Let us consider an infinitely thin front that propagates locally normal to itself at a 76 constant speed u_0 in statistically stationary, homogeneous, isotropic turbulence, which 77 (i) is not affected by the front, (ii) is characterized by a high turbulent Reynolds number 78 $Re_L = u'L/v \gg 1$ and, therefore, (iii) is described by the Kolmogorov theory [4,5,23]. 79 Moreover, in order to obtain analytical results, let us assume that the Kolmogorov velocity 80 u_{K} is much larger than u_{0} . In this section we consider the early transient stage, i.e. $t \ll t$ 81 τ_{T} , of the growth of the surface of an initially planar front embedded into the turbulence 82 at t = 0. 83

The following analysis is based on (i) the theory of surface area growth, developed 84 by Batchelor [24] for an infinitesimal element of a material surface in the Kolmogorov tur-85 bulence, (ii) results of DNS studies [25,26] of the same phenomenon, (iii) theoretical and 86 DNS results [27] on the growth of the area of a finite-length element of a material surface, 87 i.e., an element whose area is much larger than L^2 , and (iv) the theory of turbulent diffu-88 sion, developed by Taylor [28]. As we will see later, the earlier transit stage takes a time 89 interval much shorter than the eddy turnover time τ_T . During such a short time interval, 90 the area growth rates are almost the same for the infinitesimal and finite-length elements 91 of a material surface [27], because folding of the finite-length elements, caused by large-92 scale eddies, is relatively slow process. 93

Therefore, on the one hand, if a planar material surface is embedded into the Kolmo-94 gorov turbulence normally to the *x*-axis (streamwise direction in the following), then, 95

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after a short transient time interval of $t \ge t_i \approx (2.5 - 3)\tau_K$ during that the surface adapts 96 itself to the turbulent field, the mean (ensemble-averaged) surface area $A_M(t)$ is well 97 known to grow exponentially with time [24-27], i.e., 98

$$A_M(t) = A_0 \exp\left(\frac{\xi t}{\tau_K}\right),\tag{1}$$

where $\tau_K = (\nu/\bar{\varepsilon})^{1/2}$ is the Kolmogorov time scale, ξ is a constant close to 0.28 [25,26], 99 and $A_0 \gg L^2$ is the area of the element of the initial planar material surface at t = 0. It is 100 worth noting that a subsequent DNS study by Goto and Kida [27] indicates that, due to 101 folding of finite-length material surface elements during a later stage, $\xi = 0.345 +$ 102 $0.00525Re_L^{1/2}$ in a range of $10 < Re_L^{1/2} < 25$ when $t \approx \tau_T$.

On the other hand, the streamwise dispersion $\Delta_M(t)$ of a material surface grows linearly with time [28] 104

$$\Delta_M(t) = u't \tag{2}$$

at $0 < t \ll \tau_T$, with a similar linear dependence of a mean turbulent flame brush thickness 106 on flame-development time being documented in various experiments reviewed elsewhere [29]. It is worth noting that constraints of $t \ge t_i \approx (2.5 - 3)\tau_K$ and $t \ll \tau_T$ are consistent with one another in the considered case of $Re_L \gg 1$.

As argued by Yeung et al. [26], Eq. (1), which holds for an infinitesimal element of a 110 material surface, describes also the growth of the area $A_F(t)$ of an infinitesimal element 111 of a dynamically passive self-propagating front provided that $u_0 \ll u_K$ and $t \ge t_i \approx$ 112 $(2.5 - 3)\tau_K$. Moreover, during the studied short earlier stage $(t \ll \tau_T)$, the same equation 113 holds for finite-length surface elements [27], as already noted earlier. Thus, at $u_0 \ll u_K$ 114 and $t_i < t \ll \tau_T$, the following two equations 115

$$A_F(t) = A_0 \exp\left(\frac{\xi t}{\tau_K}\right),\tag{3}$$

$$\Delta_F(t) = u't \tag{4}$$

are assumed to hold simultaneously. Comparison of Eqs. (1)-(2) with Eqs. (3)-(4) shows 116 that material and self-propagating surfaces that coincide at t = 0 remain to be close to 117 one another at $0 < t \ll \tau_T$, with the distance between them being smaller than the Kolmogorov length scale η_K with a high probability [26]. 119

This feature could be attributed to the well-known statistical dominance of positive 120 rates of strain of a material surface in the Kolmogorov turbulence. Because (i) the magnitude of the local velocity normal to a material surface is increased with distance from the surface in the case of a positive local strain rate and (ii) the normal velocity vector \mathbf{u}_n 123 points to the surface, the velocity $|\mathbf{u}_n|$ can be much larger than $u_0 \ll u_K$ at a small distance from the surface, thus, impeding further divergence of the material and self-propagating surfaces. 126

However, there are fundamental differences between the two surfaces. Indeed, first, 127 there is no cusp formation at the material surface and, second, the neighboring/adjoining 128 elements of folded (folds are produced by strong advection) material surface never col-129 lide. Therefore, the area of the material surface grows exponentially and the distance d130 between different elements of the surface can be very small, as small as we wish. For in-131 stance, DNS data by Yeung et al. [26] show that the distance d is randomly distributed in 132 a wide interval of length scales, see Fig. 6 in the cited paper. On the contrary, the cusp 133 formation and collisions of elements of a self-propagating surface result in the local sur-134 face annihilation if the local distance between the elements is sufficiently small. However, 135 during the studied short earlier stage, both effects may be neglected, as discussed earlier. 136

Let us compare the fluid volume consumed by the front at instant t with the volume 137 of the streamwise turbulent dispersion of the front, i.e., a volume bounded by the leading 138 and trailing edges of the front. The former volume can be estimated as follows 139

$$W_F(t) = u_0 \int_{t_i}^t A_F(\theta) d\theta = u_0 A_0 \int_{t_i}^t \exp\left(\xi \frac{\theta}{\tau_K}\right) d\theta = u_0 \tau_K \xi^{-1} A_0 \left[\exp\left(\xi \frac{t}{\tau_K}\right) - \exp\left(\xi \frac{t_i}{\tau_K}\right)\right],\tag{5}$$

where $u_0 A_F(\theta)$ is the volume rate of the fluid consumption at instant θ .

If $t_i \ll t$, the second term in square brackets may be neglected and we arrive at

$$V_F(t) = u_0 \tau_K \xi^{-1} A_0 \exp\left(\frac{\xi t}{\tau_K}\right),\tag{6}$$

i.e., the volume of the consumed fluid is controlled by the small-scale turbulence and 142 grows exponentially with time. 143

By virtue of Eq. (4), the volume of the streamwise dispersion of the front is equal to 144

$$V_T(t) = A_0 \Delta_F(t) \propto A_0 u' t \tag{7}$$

and, consequently, is controlled by large-scale turbulent eddies. This volume grows linearly with time at $0 < t \ll \tau_T$, contrary to the exponential growth of $V_F(t)$. Therefore, in spite of $V_F(t) \ll V_T(t)$ at $\xi t/\tau_K = 0(1)$, because $u_0 \ll u_K \ll u'$, the exponentially growing volume $V_F(t)$ and the linearly growing volume $V_T(t)$ should become equal to one another at certain instant t^* . In other words, at instant t^* , the fluid consumed by the front fills the volume formed by the streamwise dispersion of the front.

This instant could be estimated invoking the following criterion

$$V_F(t^*) = V_T(t^*).$$
 (8)

Henceforth, numerical factors are skipped for simplicity.

It is worth noting that the criterion given by Eq. (8) can be rewritten in the following 153 way 154

$$\frac{V_F(t^*)}{A_F(t^*)} = \bar{l}(t^*) = \bar{d}(t^*) = \frac{V_T(t^*)}{A_F(t^*)'}$$
(9)

where $l(t^*)$ is the mean distance between neighboring elements of the material and selfpropagating surfaces and $\bar{d}(t^*)$ is the mean distance between opposed elements of either the material or the self-propagating surface. The mean distance $\bar{l}(t^*) = V_F(t^*)/A_F(t^*)$ 157 given by Eqs. (3) and (6) is simply equal to 158

$$\bar{l}(t^*) = \xi^{-1} \ell_0, \tag{10a}$$

$$\ell_0 = u_0 \tau_K = \frac{u_0}{u_K} \eta_K \ll \eta_K. \tag{10b}$$

Both the distance $\bar{l}(t^*)$ and the microscale ℓ_0 are much less than the Kolmogorov length 159 scale, i.e., they are inside the dissipation subrange of the turbulence spectrum. This estimate agrees with the DNS data by Yeung et al. [26], thus, indicating consistency of the present analysis. Note that the microscale ℓ_0 will also play an important role in an analysis of statistically stationary state of the front evolution, discussed in the next subsection. 163

Substitution of Eqs. (6) and (7) into Eq. (8) or substitution of Eqs. (1), (3), (6), and (7) 164 into Eq. (9) yields 165

$$\left(\frac{\chi u'}{u_0}\right)\left(\frac{\xi t^*}{\tau_K}\right) \approx \exp\left(\xi \frac{t^*}{\tau_K}\right).$$
 (11)

Taking logarithm of Eq. (11), we arrive at

$$\xi \frac{t^*}{\tau_K} \approx \ln\left(\frac{u'}{u_0}\right) + \ln\left(\frac{\xi t^*}{\tau_K}\right). \tag{12}$$

Under the considered conditions of $u_0 \ll u_K \ll u'$, term $\ln(u'/u_0) \gg 1$. Therefore, 167 $\xi t^*/\tau_K \gg 1$, the last term on the right-hand side of Eq. (12) may be neglected when compared to the term on the left-hand side. Consequently, an approximate solution to the nonlinear Eq. (11) reads 170

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$$t^* \approx \xi^{-1} \tau_K \ln\left(\frac{u'}{u_0}\right) \approx \xi^{-1} \tau_K R e_L^{-1/2} \ln\left(\frac{u'}{u_0}\right).$$
(13)

In order for the time t^* given by Eq. (13) to satisfy the constraint of $t \ll \tau_T$, required 171 for the validity of Eqs. (2) and (4), the following estimate should hold 172

$$\ln\left(\frac{u'}{u_0}\right) \ll Re_L^{1/2}.\tag{14}$$

At instant t^* , the front area given by Eqs. (3) and (11) is equal to

$$\frac{A_F(t^*)}{A_0} = \xi \left(\frac{u'}{u_0}\right) \left(\frac{t^*}{\tau_K}\right) \approx \left(\frac{u'}{u_0}\right) \ln \left(\frac{u'}{u_0}\right),\tag{15}$$

the turbulent consumption velocity is equal to

$$u_T(t^*) = u_0 \frac{A_F(t^*)}{A_0} = \xi \left(\frac{t^*}{\tau_K}\right) u' \approx u' \ln\left(\frac{u'}{u_0}\right), \tag{16}$$

and the volume of the consumed fluid, given by Eqs. (7) and (8), is equal to

$$W_F(t^*) = A_0 u' t^*. (17)$$

Finally, the mean consumption velocity averaged over the time interval of $0 < t < t^*$ is 176 equal to 177

$$\bar{u}_T = \frac{V_F(t^*)}{*} = u'. \tag{18}$$

Independence of the mean consumption velocity on the Kolmogorov scales does not 178 mean that the Kolmogorov eddies are unimportant. On the contrary, it is the Kolmogorov 179 eddies that create front surface within the framework of the above analysis. Nevertheless, 180 the outcome, i.e., the mean $\bar{u}_{T'}$ is independent of the Kolmogorov scales. This apparent 181 paradox is basically similar to well-known independence of the mean dissipation rate on 182 viscosity in the Kolmogorov turbulence at $Re_L \rightarrow \infty$ or independence of the mean rate of 183 entrainment of ambient irrotational fluid into turbulent fluid on viscosity in shear flows 184 [30]. While both the dissipation and entrainment occur due to viscosity, the mean rates of 185 the two processes are controlled by large-scale velocity fluctuations at $Re_L \rightarrow \infty$, whereas 186 small-scale phenomena adjust themselves to these mean rates. As noted by Tsinober [6], 187 "small scales do the 'work', but the amount of work is fixed by the large scales in such a way that 188 the outcome is independent of viscosity". 189

2.2. Statistically stationary state

The method used in section 2.1 to analyze the early ($t \ll \tau_T$) transient stage of front 191 propagation under conditions of $u_0 \ll u_K \ll u'$ is based on the hypothesis that material 192 and self-propagating surfaces that coincide at t = 0 remain to be close to one another 193 during a short ($t \ll \tau_T$) time interval, with the distance between the two surfaces being 194 smaller than the Kolmogorov length scale. This hypothesis allows us to model temporal 195 growth of the front surface area by invoking results well known for material surfaces. 196 However, this hypothesis does not hold at $t \gg \tau_T$ when the front area reaches a statisti-197 cally stationary state. In this limit, the growth of the front surface area due to turbulent 198 straining is counterbalanced by reduction of the front surface area due to joint actions of 199 folding of finite-length front elements, caused by strong advection, and subsequent colli-200 sions of self-propagating fronts. As a result, neighboring front surface elements collide, 201 and the front surface area is reduced. 202

Here, to examine the statistically stationary regime of slow front propagation, we will 203 show that smoothing of small-scale wrinkles occurs in the dissipation range of turbulence 204 spectrum (i.e., at length scales smaller than the Kolmogorov scale). Accordingly, we will 205 consider the front surface to be a bifractal, i.e., two fractals with different dimensions, 206 associated with the dissipation and inertial ranges. A similar scenario was explored by 207 Sreenevasan et al. [20] when discussing turbulent mixing for Schmidt numbers far greater 208

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respectively.

than unity, see Figs. 2a and 6 in the cited paper. Recently, such ideas were developed for 209 flame of a finite thickness [31,32]. In the present communication, the bifractal concept is 210 applied to an infinitely thin front. In particular, to explore the influence of turbulent ed-211 dies on the area of a slowly ($u_0 \ll u_K$) propagating front, the area response to small-scale 212 and large-scale turbulent eddies is modeled by invoking two different fractal submodels. 213 More specifically, both large-scale and small-scale wrinkles of the front are considered to 214 be fractals, but with different dimensions $D_{f,1}$ and $D_{f,2}$ and different cut-off scales. More-215 over, the outer cut-off scale for the small-scale fractal is considered to be equal to the inner 216 cut-off scale for the large-scale fractal, with these two equal cut-off scales being called a 217 crossover length scale in the following. Thus, the focus of the following discussion is 218 placed on the two fractal dimensions, the crossover length scale, as well as the inner ℓ_{in} 219 and outer ℓ_{out} cut-off scales for small-scale and large-scale wrinkles of the front surface, 220

First, following a common supposition [20-22], the large outer cut-off scale ℓ_{out} is 222 assumed to be proportional to a turbulent integral length scale *L*. 223

Second, the crossover length scale is associated with the boundary between inertial 224 and dissipation ranges of the turbulence spectrum. Therefore, the crossover length scale 225 is proportional to the Kolmogorov length scale η_K . Thus, the large-scale fractal covers the 226 following range $\eta_K < r < L$ of wrinkle scales r. It is worth noting that η_K is considered 227 to be the inner cut-off scale not only in single-fractal models of non-reacting turbulent 228 flows [20] or a bifractal model of turbulent mixing at a large Schmidt number [20], but 229 also in certain single-fractal models of highly turbulent flames [33]. Contrary to the latter 230 models, the front is hypothesized to be another fractal even at smaller length scales ℓ_{in} < 231 $r < \eta_K$, rather than a smooth interface. The point if that, under the considered conditions 232 of an infinitely thin and slowly propagating (i.e., $u_0 \ll u_K$) front there is no physical mech-233 anism that can smooth the front surface at scales larger than the Kolmogorov length scale. 234

Indeed, third, the sole physical mechanism of smoothing small-scale wrinkles on the 235 surface of an infinitely thin front consists of kinematic restoration due to self-propagation 236 of the front [21,22]. This is the key difference between the present study and a recently 237 developed bifractal model [32] of a highly turbulent reaction wave that has a mixing zone 238 of a finite thickness. For such waves, the inner cut-off scale is controlled by molecular 239 mixing [32]. For an infinitely thin front, the small inner cut-off scale ℓ_{in} is identified as 240 the Gibson scale corresponding to the front velocity u_0 . Therefore, the scale ℓ_{in} is found 241 using the following constraint 242

$$|\Delta u(\ell_{in})| = u_0, \tag{19}$$

where $\Delta u(\ell_{in})$ designates the velocity difference in two points separated by the distance 243 ℓ_{in} .

The same constraint is adopted in the classical single-fractal models of turbulent 245 flames [21,22], which address the case of $u_0 > u_K$ and, accordingly, estimate the velocity 246 difference following the Kolmogorov scaling for the inertial interval [4,5], i.e., $|\Delta u(\ell_{in})| \propto 247$ $u_K(r/\eta_K)^{1/3} > u_K$. However, under conditions of $u_0 \ll u_K$ examined here, the scale ℓ_{in} 248 belongs to the viscous (dissipation) subrange of the turbulence spectrum. Therefore, the 249 difference $|\Delta u(\ell_{in})|$ should be estimated using the Taylor expansion [4]. Consequently, 250 by retaining the linear term in the expansion, we arrive at 251

$$\Delta u(\ell_{in})| \approx |\nabla \mathbf{u}|\ell_{in} \propto \frac{\ell_{in}}{\sqrt{\bar{\varepsilon}/\nu}} = \frac{\ell_{in}}{\tau_K}.$$
(20)

Equations (19) and (20) yield

$$\ell_{in} = u_0 \tau_K = \frac{u_0}{u_K} \eta_K \ll \eta_K.$$
⁽²¹⁾

Comparison of Eqs. (10) and (21) shows that the inner cut-off scale ℓ_{in} is equivalent to the 253 microscale ℓ_0 introduced in section 2.1. Obviously, the scales ℓ_{in} and ℓ_0 differ from the 254

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common Gibson length scale $L_G = L(u_0/u')^3 = \eta_K (u_0/u_K)^3$ [21,22], which characterizes 255 interaction of the front with turbulent eddies from the inertial range. 256

Fourth, the area of a bifractal surface is evaluated as follows [20,32]

$$A_{f,1} = A_0 \left(\frac{L}{\eta_K}\right)^{D_{f,1}-2},$$
(22)

$$A_f = A_{f,1} \left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2},$$
(23)

where subscripts 1 and 2 refer to the large-scale interval of $\eta_K < r < L$ and the small-scale 258 interval of $\ell_0 < r < \eta_K$, respectively. In terminology by Sreenivasan et al. [20], $A_{f,1}$ is the 259 area measured with resolution η_K , and A_f is the true front surface area increased jointly 260 by large-scale and small-scale wrinkles. 261

Substitution of Eq. (22) into Eq. (23) yields

$$A_{f} = A_{f,1} \left(\frac{\eta_{K}}{\ell_{0}}\right)^{D_{f,2}-2} = A_{0} \left(\frac{L}{\eta_{K}}\right)^{D_{f,1}-2} \left(\frac{\eta_{K}}{\ell_{0}}\right)^{D_{f,2}-2} = A_{0} R e_{L}^{3(D_{f,1}-2)/4} \left(\frac{u_{K}}{u_{0}}\right)^{D_{f,2}-2}.$$
(24)

The value of the fractal dimension $D_{f,2}$ of the small-scale wrinkles can be found by noting 263 that the scales $\ell_0 < r < \eta_K$ are inside the dissipation subrange. Accordingly, the small-264 scale wrinkles of the front surface fill the space between ℓ_0 and η_K , and, hence, $D_{f,2} = 3$ 265 [34], as proposed by E. Hawkes during discussion with the first author in Dubrovnik in 266 April 2017 . For the fractal dimension $D_{f,1}$ of wrinkles whose scale is larger than η_K , the 267 common value [20-22] of $D_{f,1} = 7/3$ may be adopted. 268

Subsequently, Eqs. (22) and (24) read

$$A_{f,1} = A_0 R e_L^{1/4}, (25)$$

$$A_f = A_0 R e_L^{1/4} \frac{u_K}{u_0} = A_0 \frac{u'}{u_0}.$$
 (26)

Thus, the turbulent consumption velocity is equal to

$$\bar{u}_T = u_0 \frac{A_f}{A_0} = u'.$$
(27)

Finally, it is worth noting the following point. If we consider the entire small-scale 271 $(\ell_0 < r < \eta_K)$ fractal to be a broadened front propagating at an increased speed 272

$$u_{f,2} = u_0 \left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2} = u_0 \frac{\eta_K}{\ell_0} = u_K,$$
(28)

then, the Gibson length scale for this front is equal to η_K , which, in its turn, is equal to the 273 crossover length scale or the inner cut-off scale for the large-scale ($\eta_K < r < L$) fractal. This 274 example shows self-consistency of the present estimates of the two inner cut-off scales ℓ_0 275 and η_K , as they both are associated with Gibson scales obtained by comparing the front 276 speed and velocity difference for the appropriate range of the turbulence spectrum. Moreover, the turbulent consumption velocity is again equal to u'. Indeed, 278

$$\bar{u}_T = u_K \frac{A_{f,1}}{A_0} = u'.$$
⁽²⁹⁾

3. Discussion

When small-scale turbulent eddies stretch a slowly $(u_0 \ll u_K)$ propagating front and280increase its area, such an increase in the area cannot be continuous during a long time.281Due to the exponential growth of the area, the front packing in a finite volume is limited282by annihilation of the front elements in mutual collisions. Accordingly, a stage character-283ized by rapidly growing front area and consumption velocity should be followed by a284

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stage during that the area partly disappears and the velocity drops. Due to this physical 285 mechanism, transient effects (oscillations) could play a substantial role even during fully 286 developed stage of the front propagation. Moreover, due to this physical mechanism and 287 the transient effects caused by it, the mean turbulent consumption velocity \bar{u}_T may adjust 288 itself to the rate of turbulent entrainment, i.e., to the rms turbulent velocity u', which char-289 acterizes large-scale eddies. The smallest eddies of the Kolmogorov scales do not affect 290 the mean area of the front and the turbulent consumption velocity, respectively, in spite 291 of the fact that an increase in the front area and, hence, an increase in a ratio of $u_T(t)/u_0$ 292 are mainly controlled by such eddies. In some sense, the Kolmogorov eddies behave like 293 Cheshire cat from Alice in Wonderland. 294

If the speed u_0 of a self-propagating infinitely thin front is less than the Kolmogorov 295 velocity u_{K} , the front surface should be wrinkled even by eddies smaller than the Kolmo-296 gorov ones, because the sole (for the infinitely thin front) mechanism of smoothing the 297 surface wrinkle, i.e., kinematic restoration, can only be efficient at scales smaller than the 298 Kolmogorov length scale η_K under the considered conditions. Due to this mechanism, 299 wrinkles with a length scale smaller than $\ell_0 = u_0 \tau_K = \eta_K (u_0/u_K) \ll \eta_K$ are smoothed out. 300 In other words, the newly introduced length scale ℓ_0 characterizes the smallest possible 301 wrinkles of the surface of a slowly propagating front. Since eddies from both inertial and 302 dissipation ranges of turbulence spectrum wrinkle the front surface, the surface is ex-303 pected to be a bifractal with two different fractal dimensions for scales smaller (i.e., $\ell_0 <$ 304 $r < \eta_K$) and larger (i.e., $\eta_K < r < L$) than the crossover scale, which is equal to η_K under 305 the considered conditions. In spite of apparent complexity of the above scenario, the mean 306 fluid consumption velocity is simply controlled by the rms turbulent velocity $u' \gg u_0$ 307 during the late statistically stationary phase of the evolution of the front. 308 Author Contributions: Both authors contributed to the analysis, discussion of results, and writing. 309

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