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Article

Passive Front Propagation in Intense Turbulence: Early Transient and Late Statistically Stationary Stages of the Front Area Evolution

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Abstract: Influence of statistically stationary, homogeneous isotropic turbulence (i) on the mean area of a passive front propagating in a constant-density fluid and, hence, (ii) on the mean fluid consumption velocity $\bar{u}_T$ is explored in the case of asymptotically high turbulent Reynolds number and asymptotically high ratio of the Kolmogorov velocity to a constant speed u_0 of the front. First, a short early transient stage is analyzed by assuming that the front remains close to a material surface that coincides with the front at the initial instant. Therefore, similarly to a material surface, the front area grows exponentially with time. This stage, whose duration is much less than an integral time scale of the turbulent flow, is argued to come to the end once the volume of fluid consumed by the front is equal to the volume embraced due to the turbulent dispersion of the front. The mean fluid consumption velocity averaged over this stage is shown to be proportional to the rms turbulent velocity u'. Second, a late statistically stationary regime of the front evolution is studied. A new length scale characterizing the smallest wrinkles of the front surface is introduced. Since this length scale is smaller than the Kolmogorov length scale η_K under conditions of the present study, the front is hypothesized to be a bifractal with two different fractal dimensions for wrinkles larger and smaller than η_K. Finally, a simple scaling of $\bar{u}_T \propto u'$ is obtained for this late stage also.

Keywords: self-propagating front; turbulent consumption velocity; front area; bifractal

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1. Introduction

While turbulent combustion involves various multi-scale and highly non-linear phenomena [1-3] such as turbulence [4-6], complex chemistry [7,8], thermal expansion [9-12] and differential diffusion [13] effects, fundamentals of the influence of turbulence on a flame are often explored by a considering the classical problem of a passive front propagating locally normal to itself at a constant speed u_0 in randomly/turbulent advected media [14-16]. Historically, this problem attracted much attention since 1940s when significant acceleration of flame propagation by turbulence was found. The phenomenon was explained by Damköhler [17] and Shelkin [18] who highlighted random advection of a flame by turbulent flow and reduced the influence of the turbulence on the flame to an increase in the area of the flame surface wrinkled due to large-scale velocity fluctuations. Following those pioneering ideas, various models of flame propagation in a turbulent flow express the mean turbulent consumption velocity $\bar{u}_T$ (i.e., the mean mass rate of reactant consumption per unit area of the mean flame surface, normalized using the fluid density upstream of the flame) to be a function of the front speed u_0 and the rms turbulent velocity u', with a ratio of $\bar{u}_T/u_0$ being controlled by the mean increase in the front surface area. Moreover, recent Direct Numerical Simulation (DNS) study [19] of self-propagation of a passive interface in constant-density turbulence showed a linear relation between $\bar{u}_T - u_0$ and u' at least at $0.5 \le u'/u_0 \le 10$.

However, in spite of long-term investigations of propagation of a front (e.g., a flame) in randomly advected media (e.g., turbulence), physical mechanisms that result in the aforementioned linear relation do not seem to be fully clarified. To resolve the problem, turbulent entrainment, which is controlled by large-scale eddies is commonly highlighted, with small-scale characteristics of any surface (material or self-propagating) being assumed to be adjusted to the influence of large-scale turbulent eddies on the surface. Accordingly, the fractal concept [20] is invoked to describe the surface characteristics at various scales. In particular, for a moderately slow ($u_K < u_0 \ll u'$) front whose fractal dimension $D = 7/3$ and the Gibson length scale $L_G = L(u_0/u')^3 \ll L$ is inside the inertial interval of turbulence spectrum [4,5], i.e. $\eta_K < L_G$, the fractal concept yields $\bar{u}_T \propto u'$ [21,22]. Here, L is an integral length scale of the turbulence, $u_K = (\nu\bar{\varepsilon})^{1/4}$ and $\eta_K = (\nu^3/\bar{\varepsilon})^{1/4}$ designate Kolmogorov velocity and length scales [4], respectively, ν is the kinematic viscosity of the fluid, and $\bar{\varepsilon}$ is the mean rate of viscous dissipation of turbulent kinetic energy. What happens when $u_0/u_K \to 0$ and, consequently, the Gibson length scale is inside the viscous (dissipation) interval, i.e. $\eta_K > L_G$, is an open question. Accordingly, the primary goal of the present communication is to hypothesize a specific physical mechanism that reconciles (i) a scaling of $\bar{u}_T \propto u'$ at $u_0 \ll u_K$, (ii) the concept of turbulent entrainment, and (iii) a well-recognized paradigm that reduces the influence of turbulence on a front to an increase in the front area by turbulent eddies characterized within the framework of the Kolmogorov theory [4,5,23].

The present study addresses two limiting stages of front area evolution: (i) an early transient stage, whose duration is much less than an eddy-turn-over time scale $\tau_T = L/u'$, and (ii) a late fully-developed stage when the front area reaches a statistically stationary state. During the late stage, growth of the front surface area due to turbulent straining is counterbalanced by reduction of the front surface area due to joint actions of (i) folding of finite-length front elements, caused by strong advection, and (ii) subsequent collisions of self-propagating fronts.

2. Analysis and Results

2.1. Earlier transit stage

Let us consider an infinitely thin front that propagates locally normal to itself at a constant speed u_0 in statistically stationary, homogeneous, isotropic turbulence, which (i) is not affected by the front, (ii) is characterized by a high turbulent Reynolds number $Re_L = u'L/\nu \gg 1$ and, therefore, (iii) is described by the Kolmogorov theory [4,5,23]. Moreover, in order to obtain analytical results, let us assume that the Kolmogorov velocity u_K is much larger than u_0. In this section we consider the early transient stage, i.e. $t \ll \tau_T$, of the growth of the surface of an initially planar front embedded into the turbulence at $t = 0$.

The following analysis is based on (i) the theory of surface area growth, developed by Batchelor [24] for an infinitesimal element of a material surface in the Kolmogorov turbulence, (ii) results of DNS studies [25,26] of the same phenomenon, (iii) theoretical and DNS results [27] on the growth of the area of a finite-length element of a material surface, i.e., an element whose area is much larger than L^2, and (iv) the theory of turbulent diffusion, developed by Taylor [28]. As we will see later, the earlier transit stage takes a time interval much shorter than the eddy turnover time τ_T. During such a short time interval, the area growth rates are almost the same for the infinitesimal and finite-length elements of a material surface [27], because folding of the finite-length elements, caused by large-scale eddies, is relatively slow process.

Therefore, on the one hand, if a planar material surface is embedded into the Kolmogorov turbulence normally to the x-axis (streamwise direction in the following), then, after a short transient time interval of $t \geq t_i \approx (2.5-3)\tau_K$ during that the surface adapts itself to the turbulent field, the mean (ensemble-averaged) surface area $A_M(t)$ is well known to grow exponentially with time [24-27], i.e.,

$$A_M(t) = A_0 \exp\left(\frac{\xi t}{\tau_K}\right), \tag{1}$$

where $\tau_K = (\nu/\bar{\varepsilon})^{1/2}$ is the Kolmogorov time scale, ξ is a constant close to 0.28 [25,26], and $A_0 \gg L^2$ is the area of the element of the initial planar material surface at $t = 0$. It is worth noting that a subsequent DNS study by Goto and Kida [27] indicates that, due to folding of finite-length material surface elements during a later stage, $\xi = 0.345 + 0.00525 Re_L^{1/2}$ in a range of $10 < Re_L^{1/2} < 25$ when $t \approx \tau_T$.

On the other hand, the streamwise dispersion $\Delta_M(t)$ of a material surface grows linearly with time [28]

$$\Delta_M(t) = u' t \tag{2}$$

at $0 < t \ll \tau_T$, with a similar linear dependence of a mean turbulent flame brush thickness on flame-development time being documented in various experiments reviewed elsewhere [29]. It is worth noting that constraints of $t \geq t_i \approx (2.5-3)\tau_K$ and $t \ll \tau_T$ are consistent with one another in the considered case of $Re_L \gg 1$.

As argued by Yeung et al. [26], Eq. (1), which holds for an infinitesimal element of a material surface, describes also the growth of the area $A_F(t)$ of an infinitesimal element of a dynamically passive self-propagating front provided that $u_0 \ll u_K$ and $t \geq t_i \approx (2.5-3)\tau_K$. Moreover, during the studied short earlier stage ($t \ll \tau_T$), the same equation holds for finite-length surface elements [27], as already noted earlier. Thus, at $u_0 \ll u_K$ and $t_i < t \ll \tau_T$, the following two equations

$$A_F(t) = A_0 \exp\left(\frac{\xi t}{\tau_K}\right), \tag{3}$$

$$\Delta_F(t) = u' t \tag{4}$$

are assumed to hold simultaneously. Comparison of Eqs. (1)-(2) with Eqs. (3)-(4) shows that material and self-propagating surfaces that coincide at $t = 0$ remain to be close to one another at $0 < t \ll \tau_T$, with the distance between them being smaller than the Kolmogorov length scale η_K with a high probability [26].

This feature could be attributed to the well-known statistical dominance of positive rates of strain of a material surface in the Kolmogorov turbulence. Because (i) the magnitude of the local velocity normal to a material surface is increased with distance from the surface in the case of a positive local strain rate and (ii) the normal velocity vector $\mathbf{u}_n$ points to the surface, the velocity $|\mathbf{u}_n|$ can be much larger than $u_0 \ll u_K$ at a small distance from the surface, thus, impeding further divergence of the material and self-propagating surfaces.

However, there are fundamental differences between the two surfaces. Indeed, first, there is no cusp formation at the material surface and, second, the neighboring/adjoining elements of folded (folds are produced by strong advection) material surface never collide. Therefore, the area of the material surface grows exponentially and the distance d between different elements of the surface can be very small, as small as we wish. For instance, DNS data by Yeung et al. [26] show that the distance d is randomly distributed in a wide interval of length scales, see Fig. 6 in the cited paper. On the contrary, the cusp formation and collisions of elements of a self-propagating surface result in the local surface annihilation if the local distance between the elements is sufficiently small. However, during the studied short earlier stage, both effects may be neglected, as discussed earlier.

Let us compare the fluid volume consumed by the front at instant t with the volume of the streamwise turbulent dispersion of the front, i.e., a volume bounded by the leading and trailing edges of the front. The former volume can be estimated as follows

$$V_F(t) = u_0 \int_{t_i}^{t} A_F(\theta) d\,\theta = u_0 A_0 \int_{t_i}^{t} \exp\left(\xi \frac{\theta}{\tau_K}\right) d\,\theta = u_0 \tau_K \xi^{-1} A_0 \left[\exp\left(\xi \frac{t}{\tau_K}\right) - \exp\left(\xi \frac{t_i}{\tau_K}\right)\right], \quad (5)$$

where $u_0 A_F(\theta)$ is the volume rate of the fluid consumption at instant θ.

If $t_i \ll t$, the second term in square brackets may be neglected and we arrive at

$$V_F(t) = u_0\, \tau_K \xi^{-1} A_0 \exp\left(\frac{\xi t}{\tau_K}\right), \quad (6)$$

i.e., the volume of the consumed fluid is controlled by the small-scale turbulence and grows exponentially with time.

By virtue of Eq. (4), the volume of the streamwise dispersion of the front is equal to

$$V_T(t) = A_0 \Delta_F(t) \propto A_0 u' t \quad (7)$$

and, consequently, is controlled by large-scale turbulent eddies. This volume grows linearly with time at $0 < t \ll \tau_T$, contrary to the exponential growth of $V_F(t)$. Therefore, in spite of $V_F(t) \ll V_T(t)$ at $\xi t/\tau_K = 0(1)$, because $u_0 \ll u_K \ll u'$, the exponentially growing volume $V_F(t)$ and the linearly growing volume $V_T(t)$ should become equal to one another at certain instant t^*. In other words, at instant t^*, the fluid consumed by the front fills the volume formed by the streamwise dispersion of the front.

This instant could be estimated invoking the following criterion

$$V_F(t^*) = V_T(t^*). \quad (8)$$

Henceforth, numerical factors are skipped for simplicity.

It is worth noting that the criterion given by Eq. (8) can be rewritten in the following way

$$\frac{V_F(t^*)}{A_F(t^*)} = \bar{l}(t^*) = \bar{d}(t^*) = \frac{V_T(t^*)}{A_F(t^*)}, \quad (9)$$

where $\bar{l}(t^*)$ is the mean distance between neighboring elements of the material and self-propagating surfaces and $\bar{d}(t^*)$ is the mean distance between opposed elements of either the material or the self-propagating surface. The mean distance $\bar{l}(t^*) = V_F(t^*)/A_F(t^*)$ given by Eqs. (3) and (6) is simply equal to

$$\bar{l}(t^*) = \xi^{-1} \ell_0, \quad (10a)$$

$$\ell_0 = u_0 \tau_K = \frac{u_0}{u_K} \eta_K \ll \eta_K. \quad (10b)$$

Both the distance $\bar{l}(t^*)$ and the microscale ℓ_0 are much less than the Kolmogorov length scale, i.e., they are inside the dissipation subrange of the turbulence spectrum. This estimate agrees with the DNS data by Yeung et al. [26], thus, indicating consistency of the present analysis. Note that the microscale ℓ_0 will also play an important role in an analysis of statistically stationary state of the front evolution, discussed in the next subsection.

Substitution of Eqs. (6) and (7) into Eq. (8) or substitution of Eqs. (1), (3), (6), and (7) into Eq. (9) yields

$$\left(\frac{u'}{u_0}\right)\left(\frac{\xi t^*}{\tau_K}\right) \approx \exp\left(\xi \frac{t^*}{\tau_K}\right). \quad (11)$$

Taking logarithm of Eq. (11), we arrive at

$$\xi \frac{t^*}{\tau_K} \approx \ln\left(\frac{u'}{u_0}\right) + \ln\left(\frac{\xi t^*}{\tau_K}\right). \quad (12)$$

Under the considered conditions of $u_0 \ll u_K \ll u'$, term $\ln(u'/u_0) \gg 1$. Therefore, $\xi t^*/\tau_K \gg 1$, the last term on the right-hand side of Eq. (12) may be neglected when compared to the term on the left-hand side. Consequently, an approximate solution to the nonlinear Eq. (11) reads

$$t^* \approx \xi^{-1}\tau_K \ln\left(\frac{u'}{u_0}\right) \approx \xi^{-1}\tau_K\, Re_L^{-1/2} \ln\left(\frac{u'}{u_0}\right). \quad (13)$$

In order for the time t^* given by Eq. (13) to satisfy the constraint of $t \ll \tau_T$, required for the validity of Eqs. (2) and (4), the following estimate should hold

$$\ln\left(\frac{u'}{u_0}\right) \ll Re_L^{1/2}. \quad (14)$$

At instant t^*, the front area given by Eqs. (3) and (11) is equal to

$$\frac{A_F(t^*)}{A_0} = \xi\left(\frac{u'}{u_0}\right)\left(\frac{t^*}{\tau_K}\right) \approx \left(\frac{u'}{u_0}\right)\ln\left(\frac{u'}{u_0}\right), \quad (15)$$

the turbulent consumption velocity is equal to

$$u_T(t^*) = u_0\frac{A_F(t^*)}{A_0} = \xi\left(\frac{t^*}{\tau_K}\right)u' \approx u'\ln\left(\frac{u'}{u_0}\right), \quad (16)$$

and the volume of the consumed fluid, given by Eqs. (7) and (8), is equal to

$$V_F(t^*) = A_0 u' t^*. \quad (17)$$

Finally, the mean consumption velocity averaged over the time interval of $0 < t < t^*$ is equal to

$$\bar{u}_T = \frac{V_F(t^*)}{t^*} = u'. \quad (18)$$

Independence of the mean consumption velocity on the Kolmogorov scales does not mean that the Kolmogorov eddies are unimportant. On the contrary, it is the Kolmogorov eddies that create front surface within the framework of the above analysis. Nevertheless, the outcome, i.e., the mean $\bar{u}_T$, is independent of the Kolmogorov scales. This apparent paradox is basically similar to well-known independence of the mean dissipation rate on viscosity in the Kolmogorov turbulence at $Re_L \to \infty$ or independence of the mean rate of entrainment of ambient irrotational fluid into turbulent fluid on viscosity in shear flows [30]. While both the dissipation and entrainment occur due to viscosity, the mean rates of the two processes are controlled by large-scale velocity fluctuations at $Re_L \to \infty$, whereas small-scale phenomena adjust themselves to these mean rates. As noted by Tsinober [6], "*small scales do the 'work', but the amount of work is fixed by the large scales in such a way that the outcome is independent of viscosity*".

2.2. Statistically stationary state

The method used in section 2.1 to analyze the early $(t \ll \tau_T)$ transient stage of front propagation under conditions of $u_0 \ll u_K \ll u'$ is based on the hypothesis that material and self-propagating surfaces that coincide at $t = 0$ remain to be close to one another during a short $(t \ll \tau_T)$ time interval, with the distance between the two surfaces being smaller than the Kolmogorov length scale. This hypothesis allows us to model temporal growth of the front surface area by invoking results well known for material surfaces. However, this hypothesis does not hold at $t \gg \tau_T$ when the front area reaches a statistically stationary state. In this limit, the growth of the front surface area due to turbulent straining is counterbalanced by reduction of the front surface area due to joint actions of folding of finite-length front elements, caused by strong advection, and subsequent collisions of self-propagating fronts. As a result, neighboring front surface elements collide, and the front surface area is reduced.

Here, to examine the statistically stationary regime of slow front propagation, we will show that smoothing of small-scale wrinkles occurs in the dissipation range of turbulence spectrum (i.e., at length scales smaller than the Kolmogorov scale). Accordingly, we will consider the front surface to be a bifractal, i.e., two fractals with different dimensions, associated with the dissipation and inertial ranges. A similar scenario was explored by Sreenevasan et al. [20] when discussing turbulent mixing for Schmidt numbers far greater than unity, see Figs. 2a and 6 in the cited paper. Recently, such ideas were developed for flame of a finite thickness [31,32]. In the present communication, the bifractal concept is applied to an infinitely thin front. In particular, to explore the influence of turbulent eddies on the area of a slowly ($u_0 \ll u_K$) propagating front, the area response to small-scale and large-scale turbulent eddies is modeled by invoking two different fractal submodels. More specifically, both large-scale and small-scale wrinkles of the front are considered to be fractals, but with different dimensions $D_{f,1}$ and $D_{f,2}$ and different cut-off scales. Moreover, the outer cut-off scale for the small-scale fractal is considered to be equal to the inner cut-off scale for the large-scale fractal, with these two equal cut-off scales being called a crossover length scale in the following. Thus, the focus of the following discussion is placed on the two fractal dimensions, the crossover length scale, as well as the inner ℓ_{in} and outer ℓ_{out} cut-off scales for small-scale and large-scale wrinkles of the front surface, respectively.

First, following a common supposition [20-22], the large outer cut-off scale ℓ_{out} is assumed to be proportional to a turbulent integral length scale L.

Second, the crossover length scale is associated with the boundary between inertial and dissipation ranges of the turbulence spectrum. Therefore, the crossover length scale is proportional to the Kolmogorov length scale η_K. Thus, the large-scale fractal covers the following range $\eta_K < r < L$ of wrinkle scales r. It is worth noting that η_K is considered to be the inner cut-off scale not only in single-fractal models of non-reacting turbulent flows [20] or a bifractal model of turbulent mixing at a large Schmidt number [20], but also in certain single-fractal models of highly turbulent flames [33]. Contrary to the latter models, the front is hypothesized to be another fractal even at smaller length scales $\ell_{in} < r < \eta_K$, rather than a smooth interface. The point if that, under the considered conditions of an infinitely thin and slowly propagating (i.e., $u_0 \ll u_K$) front there is no physical mechanism that can smooth the front surface at scales larger than the Kolmogorov length scale.

Indeed, third, the sole physical mechanism of smoothing small-scale wrinkles on the surface of an infinitely thin front consists of kinematic restoration due to self-propagation of the front [21,22]. This is the key difference between the present study and a recently developed bifractal model [32] of a highly turbulent reaction wave that has a mixing zone of a finite thickness. For such waves, the inner cut-off scale is controlled by molecular mixing [32]. For an infinitely thin front, the small inner cut-off scale ℓ_{in} is identified as the Gibson scale corresponding to the front velocity u_0. Therefore, the scale ℓ_{in} is found using the following constraint

$$|\Delta u(\ell_{in})| = u_0, \tag{19}$$

where $\Delta u(\ell_{in})$ designates the velocity difference in two points separated by the distance ℓ_{in}.

The same constraint is adopted in the classical single-fractal models of turbulent flames [21,22], which address the case of $u_0 > u_K$ and, accordingly, estimate the velocity difference following the Kolmogorov scaling for the inertial interval [4,5], i.e., $|\Delta u(\ell_{in})| \propto u_K(r/\eta_K)^{1/3} > u_K$. However, under conditions of $u_0 \ll u_K$ examined here, the scale ℓ_{in} belongs to the viscous (dissipation) subrange of the turbulence spectrum. Therefore, the difference $|\Delta u(\ell_{in})|$ should be estimated using the Taylor expansion [4]. Consequently, by retaining the linear term in the expansion, we arrive at

$$|\Delta u(\ell_{in})| \approx |\nabla \mathbf{u}|\ell_{in} \propto \frac{\ell_{in}}{\sqrt{\bar{\varepsilon}/\nu}} = \frac{\ell_{in}}{\tau_K}. \tag{20}$$

Equations (19) and (20) yield

$$\ell_{in} = u_0\tau_K = \frac{u_0}{u_K}\eta_K \ll \eta_K. \tag{21}$$

Comparison of Eqs. (10) and (21) shows that the inner cut-off scale ℓ_{in} is equivalent to the microscale ℓ_0 introduced in section 2.1. Obviously, the scales ℓ_{in} and ℓ_0 differ from the common Gibson length scale $L_G = L(u_0/u')^3 = \eta_K(u_0/u_K)^3$ [21,22], which characterizes interaction of the front with turbulent eddies from the inertial range.

Fourth, the area of a bifractal surface is evaluated as follows [20,32]

$$A_{f,1} = A_0\left(\frac{L}{\eta_K}\right)^{D_{f,1}-2}, \tag{22}$$

$$A_f = A_{f,1}\left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2}, \tag{23}$$

where subscripts 1 and 2 refer to the large-scale interval of $\eta_K < r < L$ and the small-scale interval of $\ell_0 < r < \eta_K$, respectively. In terminology by Sreenivasan et al. [20], $A_{f,1}$ is the area measured with resolution η_K, and A_f is the true front surface area increased jointly by large-scale and small-scale wrinkles.

Substitution of Eq. (22) into Eq. (23) yields

$$A_f = A_{f,1}\left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2} = A_0\left(\frac{L}{\eta_K}\right)^{D_{f,1}-2}\left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2} = A_0 Re_L^{3(D_{f,1}-2)/4}\left(\frac{u_K}{u_0}\right)^{D_{f,2}-2}. \tag{24}$$

The value of the fractal dimension $D_{f,2}$ of the small-scale wrinkles can be found by noting that the scales $\ell_0 < r < \eta_K$ are inside the dissipation subrange. Accordingly, the small-scale wrinkles of the front surface fill the space between ℓ_0 and η_K, and, hence, $D_{f,2} = 3$ [34], as proposed by E. Hawkes during discussion with the first author in Dubrovnik in April 2017 . For the fractal dimension $D_{f,1}$ of wrinkles whose scale is larger than η_K, the common value [20-22] of $D_{f,1} = 7/3$ may be adopted.

Subsequently, Eqs. (22) and (24) read

$$A_{f,1} = A_0 Re_L^{1/4}, \tag{25}$$

$$A_f = A_0 Re_L^{1/4}\frac{u_K}{u_0} = A_0\frac{u'}{u_0}. \tag{26}$$

Thus, the turbulent consumption velocity is equal to

$$\bar{u}_T = u_0\frac{A_f}{A_0} = u'. \tag{27}$$

Finally, it is worth noting the following point. If we consider the entire small-scale ($\ell_0 < r < \eta_K$) fractal to be a broadened front propagating at an increased speed

$$u_{f,2} = u_0\left(\frac{\eta_K}{\ell_0}\right)^{D_{f,2}-2} = u_0\frac{\eta_K}{\ell_0} = u_K, \tag{28}$$

then, the Gibson length scale for this front is equal to η_K, which, in its turn, is equal to the crossover length scale or the inner cut-off scale for the large-scale ($\eta_K < r < L$) fractal. This example shows self-consistency of the present estimates of the two inner cut-off scales ℓ_0 and η_K, as they both are associated with Gibson scales obtained by comparing the front speed and velocity difference for the appropriate range of the turbulence spectrum. Moreover, the turbulent consumption velocity is again equal to u'. Indeed,

$$\bar{u}_T = u_K\frac{A_{f,1}}{A_0} = u'. \tag{29}$$

3. Discussion

When small-scale turbulent eddies stretch a slowly ($u_0 \ll u_K$) propagating front and increase its area, such an increase in the area cannot be continuous during a long time. Due to the exponential growth of the area, the front packing in a finite volume is limited by annihilation of the front elements in mutual collisions. Accordingly, a stage characterized by rapidly growing front area and consumption velocity should be followed by a stage during that the area partly disappears and the velocity drops. Due to this physical mechanism, transient effects (oscillations) could play a substantial role even during fully developed stage of the front propagation. Moreover, due to this physical mechanism and the transient effects caused by it, the mean turbulent consumption velocity $\bar{u}_T$ may adjust itself to the rate of turbulent entrainment, i.e., to the rms turbulent velocity u', which characterizes large-scale eddies. The smallest eddies of the Kolmogorov scales do not affect the mean area of the front and the turbulent consumption velocity, respectively, in spite of the fact that an increase in the front area and, hence, an increase in a ratio of $u_T(t)/u_0$ are mainly controlled by such eddies. In some sense, the Kolmogorov eddies behave like Cheshire cat from Alice in Wonderland.

If the speed u_0 of a self-propagating infinitely thin front is less than the Kolmogorov velocity u_K, the front surface should be wrinkled even by eddies smaller than the Kolmogorov ones, because the sole (for the infinitely thin front) mechanism of smoothing the surface wrinkle, i.e., kinematic restoration, can only be efficient at scales smaller than the Kolmogorov length scale η_K under the considered conditions. Due to this mechanism, wrinkles with a length scale smaller than $\ell_0 = u_0\tau_K = \eta_K(u_0/u_K) \ll \eta_K$ are smoothed out. In other words, the newly introduced length scale ℓ_0 characterizes the smallest possible wrinkles of the surface of a slowly propagating front. Since eddies from both inertial and dissipation ranges of turbulence spectrum wrinkle the front surface, the surface is expected to be a bifractal with two different fractal dimensions for scales smaller (i.e., $\ell_0 < r < \eta_K$) and larger (i.e., $\eta_K < r < L$) than the crossover scale, which is equal to η_K under the considered conditions. In spite of apparent complexity of the above scenario, the mean fluid consumption velocity is simply controlled by the rms turbulent velocity $u' \gg u_0$ during the late statistically stationary phase of the evolution of the front.

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